Duration of inflation and possible remnants of the preinflationary Universe

L. P. Grishchuk
McDonnell Center for the Space Sciences, and Department of Physics, Washington University, St. Louis, Missouri 63130
and Sternberg Astronomical Institute, Moscow University, 119899 Moscow V234, Russia
(Received 31 January 1992)

It is shown that the minimally sufficient duration of inflation may happen to be a very probable prediction of certain quantum cosmological models. This can make reasonable and interesting the observational search for the possible remnants of the preinflationary Universe.

PACS number(s): 98.80.Bp, 98.80.Dr

In a recent interesting paper [1], Turner considers cosmological models whose period of inflation lasted a little longer than the minimal duration necessary to increase a preinflationary scale to the size of the present-day Hubble radius. He discusses the possible "remnants" of the preinflationary Universe and their observational effects. Turner warns the reader that in many, if not most, models of inflation the number N of e-foldings of the scale factor during inflation is much, much larger than the minimal $N_{\rm min}$, in which case the volume covered by inflation is much, much larger than the present-day Hubble volume and, hence, the discussed issues are moot.

One can agree with the author that in the framework of classical cosmologies there is no natural reason why the duration of inflation, if it occurred at all, should be so fine-tuned that $N \approx N_{\rm min}$. It seems to be unlikely that inflation has expanded a preinflationary patch precisely to the size of the present-day Hubble radius, or so. The purpose of this Brief Report, however, is to take into account quantum cosmological considerations and to demonstrate that the duration of inflation close to the minimally sufficient amount may happen to be the most probable prediction of respective quantum cosmological models.

Before going to issues of quantum cosmology and inflation, let us set the scene from the point of view of the current observations. The available astronomical data are confined, of course, to the scales smaller than the present-day Hubble radius $l_H \approx 10^{28}$ cm. From these data we deduce that the visible Universe is pretty homogeneous and isotropic. Scales larger than l_H are currently inaccessible to direct astronomical observations. At first sight, this makes it possible to suggest that the Universe can be much more inhomogeneous and anisotropic on scales l which are, say, 10 or 100 times larger than l_H. In other words, this seemingly allows one to have a perturbation with an arbitrarily large amplitude, as soon as the wavelength of the perturbation is longer than l_H.

A remarkable fact is, however, that although such scales are not accessible to direct observations, we know that the Universe is still homogeneous and isotropic in a volume which is about six orders of magnitude larger than the present-day Hubble volume l_H^3. In other words, we know that the relative amplitudes of the possible perturbations are smaller than 1 for all wavelengths that are longer than the present-day Hubble radius and range from $l=l_H$ up to about $l=100l_H$ [2].

The argument is based on the relationship between the expected quadrupole anisotropy of the cosmic microwave background radiation (CMBR) $\Delta T/T$ generated by the gravitational field of the perturbation and the wavelength of the perturbation l:

$$\frac{\Delta T}{T}=h\left[\frac{l_H}{l}\right]^2 . \tag{1}$$

In this formula, h is the amplitude of the gravitational field perturbation associated with the growing mode of the density perturbation, or the amplitude of the growing (nonsingular) mode of the gravitational-wave perturbation. Other modes are less significant. For simplicity we neglect numerical factors of order 1 and assume that the experimental upper limit on $\Delta T/T$ is 10^{-4}. Then, consistency with the observations requires that the number h should remain smaller than 1 for all wavelengths up to $l=100l_H$. In fact, the experimental upper limit $\Delta T/T \lesssim 3\times10^{-5}$ is already more stringent than 10^{-4}, which increases the size of the region of homogeneity and isotropy up to about $l=200l_H$. In general, the lower the upper limits on $\Delta T/T$, the larger the volume of homogeneity and isotropy of our Universe. (See Note added in proof.) This conclusion assumes, of course, that the possible long-wavelength perturbations are not so neatly correlated that we have happened to live inside of an inhomogeneity which is big in its amplitude but strictly spherically symmetric in its form. Such an inhomogeneity, even if it had a radius of order of l_H, would not manifest itself in the quadrupole anisotropy of CMBR. We should also mention that these conclusions, being derived for models with the cosmological parameter $\Omega=1$, are not very sensitive to the variations of Ω, but we exclude extreme situations, such as a logically possible case that the Universe is closed and has the volume of the order of l_H^3 in which case the longer waves just do not exist.

One can see from Eq. (1) that the allowed amplitudes, as a function of the increasing wavelength, can grow according to the relation

$$h=10^{-4}\frac{l}{l_H} . \tag{2}$$

On scales larger than $l=(1-2)\times10^2 l_H$ the Universe can be significantly inhomogeneous and anisotropic without being in conflict with the available observational constraints.

Now we will mention modifications in this picture which arise under the assumption that the inflationary hypothesis is correct. If inflation has encompassed only the present-day Hubble radius then the perturbations associated with the preinflationary "remnants" can exist as long as they agree with the limit imposed by Eq. (2). If inflation has encompased a scale larger than $10^2 l_H$ then only perturbations of quantum-mechanical origin survive and their h's should stay constant at a level not higher than

$$h=10^{-4} \tag{3}$$

for all l in the interval $10^2 l_H > l > l_H$ [3]. The dominant contribution to $\Delta T/T$ comes from the perturbations with $l=l_H$ and gets increasingly smaller for longer wavelengths. Thus, the difference between the minimally sufficient inflation and long-lasting inflation, in terms of the allowed amplitudes of the long-wavelength perturbations, is represented by the difference between Eq. (2) and Eq. (3).

Finally, we turn our attention to the quantum cosmological considerations. Quantum cosmology is supposed to provide initial data for classical cosmological models and resolve such issues as the likelihood of inflation and its probable duration. Obviously, we are still far away from a satisfactory answer. A part of the problem is that there are too many possible wave functions: the trouble of selecting an appropriate classical solution from the space of all possible classical solutions is replaced by an even bigger problem of selecting an appropriate wave function from the space of all possible wave functions. In the absence of a guiding principle allowing one to prefer one cosmological wave function over others, we will probably face a painful job of analyzing all of them trying to introduce a probability measure in the space of the wave functions. However, if a cosmological wave function is chosen, the derivation of the probability distribution of the permitted classical solutions seems to be more straightforward.

A wave function which has received much attention in the literature is the so-called Hartle-Hawking wave function $\psi_{\rm HH}$ [4]. The recipe of constructing this wave function is formulated in an elegant mathematical manner. One cannot say that the $\psi_{\rm HH}$ is in any sense more probable than others. On the contrary, it looks, rather, as an exception. For simple quantum cosmological models allowing inflation, the Hartle-Hawking wave function corresponds to a single point—a pole on the two-sphere representing the space of all physically different wave functions [5]. However, the $\psi_{\rm HH}$ is a real wave function while all others (except the one corresponding to the opposite pole which is also real and which we call the "anti-Hartle-Hawking" wave function) are complex. This exceptional property of the $\psi_{\rm HH}$ alone, if for no other reasons, justifies special attention to this wave function and makes it interesting to see what kind of predictions with regard to inflation follow from it.

For the case of homogeneous isotropic models with the scale factor $a(t)$ and a scalar field $\phi(t)$, the $\psi_{\rm HH}$ predicts a set of classical inflationary solutions which can be described as trajectories in the two-dimensional space $[a(t),\phi(t)]$ [6]. These trajectories begin in the vicinity of a line which is the caustic line for the so-called Euclidean trajectories. The probability distribution $P_{\rm HH}$ for the classical (Lorentzian) inflationary solutions follows from the $\psi_{\rm HH}$ and has the form

$$P_{\rm HH}=N\exp\left[\frac{2}{3H^2(\phi)}\right], \tag{4}$$

where N is the normalization constant and $H(\phi)$ is the Hubble factor at the beginning of inflation. The function $P_{\rm HH}$ varies along the caustic line and increases rapidly toward the smaller values of ϕ. This means that the probability to find a given inflationary solution is higher the lower the initial value of the scalar field $\phi(t)$ (if, of course, this interpretation of $P_{\rm HH}$ is correct). But smaller initial values of $\phi(t)$ correspond to the shorter periods of inflation which makes solutions with a shorter period of inflation much more probable than solutions with a longer period of inflation.

An important fact is, however, that the inflationary period cannot be too short. The reason is that the caustic line does not extend down to the very low values of ϕ; instead, it has a sharp cusp (singularity) at the point of return from which the second branch of the caustic line develops (see Fig. 1) [7]. The point of return on the caustic line divides the Euclidean trajectories into two families which touch the first or the second branch of the caustic, respectively. The Lorentzian inflationary solutions cannot begin with the initial value of the scalar field and the Hubble factor lower than the value corresponding to the point of return ϕ^* and, therefore, their periods of inflation cannot be arbitrarily short. Thus, the $\psi_{\rm HH}$ gives

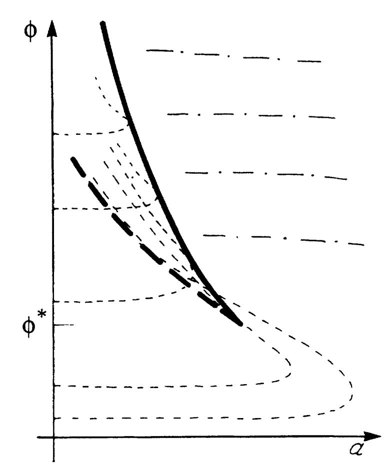

FIG. 1. Two branches of the caustic and its singularity. The first branch of the caustic is shown by a solid line, the second branch by a dashed line with long dashes. The Euclidean trajectories are shown by dashed lines with short dashes. The Lorentzian (inflationary) solutions are shown by dash-dot lines.

more weight to inflationary solutions with lower initial values of $\phi(t)$ but does not accommodate solutions which begin with $\phi(t)$ smaller than ϕ^*. The numerical estimates for the case of the scalar field potentials $V(\phi)=m^2\phi^2/2$ and $V(\phi)=\lambda\phi^4/2$ show [7] that the number ϕ^* falls short a factor 4 or 3, respectively, to ensure the minimally sufficient inflation. The inflated scale turns out to be of order 10^{21} cm instead of the required 10^{28} cm. At the same time, the probability distribution function P_{HH} reaches its maximum value at $\phi=\phi^*$. Thus, it seems that the most probable prediction of the Hartle-Hawking wave function is a "small, underinflated universe." However, it is possible that the discrepancy between l_H and the predicted inflated scale may be weakened or even removed for other scalar field potentials. Apart from that, the deficiency of ϕ^* in being just a numerical factor 4 or 3 smaller than necessary, in the situation where the initial values of the scalar field can vary within a huge interval from ϕ^* up to about $10^5\ \phi^*$, can serve as an indication that the duration of inflation close to the minimally sufficient amount should, probably, be taken seriously, at least, as a prediction of the Hartle-Hawking wave function.

The meaning of the above discussion is that the search for the "remnants" of the preinflationary Universe, in the framework of the inflationary hypothesis, may not necessarily be of a purely academic interest.

Note added in proof. The recent COBE observations indicate that the actually measured quadrupole anisotropy is $\Delta T/T \approx 5\times 10^{-6}$. This anisotropy can, in principle, be predominantly generated by perturbations with wavelengths longer than the Hubble radius. If so, the above argument cannot guarantee that the Universe is still homogeneous and isotropic on scales exceeding $l \approx 500 l_H$.

I am grateful to L. Rozhansky for a useful conversation.

[1] M. S. Turner, Phys. Rev. D **44**, 3737 (1991).
[2] L. P. Grishchuk and Ya. B. Zel'dovich, Astron. Zh. **55**, 209 (1978) [Sov. Astron. **22**, 125 (1978)].
[3] L. P. Grischuk and Yu. V. Sidorov, Class. Quantum Grav. **6**, L155 (1989).
[4] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983).
[5] G. W. Gibbons and L. P. Grishchuk, Nucl. Phys. **B313**, 736 (1989).
[6] S. W. Hawking, Nucl. Phys. **B239**, 257 (1984).
[7] L. P. Grishchuk and L. V. Rozhansky, Phys. Lett. B **234**, 9 (1990).