BRIEF REPORTS

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Bulk viscosity of hot-neutron-star matter from direct URCA processes

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Direct URCA processes, occurring in neutron-star matter with a proton fraction exceeding the critical value of (11–15)%, can strongly enhance the bulk viscosity of the matter.

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The viscosity of neutron-star matter determines damping time scales of radial vibrations of neutron stars [1]. Such vibrations can be excited in the process of the formation of a neutron star, or can result from neutron-star quakes. The viscosity enters in the criteria for the gravitational-wave instabilities in rapidly rotating neutron stars [2], which are essential for the determination of the maximum rate of rotation of neutron stars.

Recently, the problem of the actual viscosity of hot-neutron-star matter has been reexamined by Sawyer [3], who showed that at temperatures higher than 10^9 K the bulk viscosity exceeds significantly the shear viscosity, and discussed the consequences of his findings for hot, rapidly rotating neutron stars. The source of the bulk viscosity of neutron-star matter is the deviation from β equilibrium, and the ensuing nonequilibrium reactions, implied by the compression and rarefaction of the matter in the pulsating neutron star.

An essential assumption in Sawyer's calculation was that at baryon densities n prevailing in the interiors of neutron stars, the equilibrium proton fraction $x=n_p/n$ is so low that the β equilibrium can be reestablished only through the *modified* URCA processes $e^-+p+n\rightarrow n+n+\nu_e$, $n+n\rightarrow p+n+e^-+\bar{\nu}_e$. The participation of an additional "spectator" nucleon in the neutron-decay or electron-capture reactions is necessary to allow for the conservation of energy and momentum in the degenerate neutron-star matter, in which neutron, proton, and electron Fermi momenta are assumed to violate the inequality $p_{Fn}<p_{Fp}+p_{Fe}$ [4]. This standard assumption is corroborated by the simplest free Fermi-gas model of the *npe* plasma, used in the calculation of Sawyer [3], for which $x=5\times10^{-3}n/n_0$ ($n_0=0.16$ fm^{-3} is normal nuclear density).

Very recently it has been shown that for numerous models of dense nucleon matter the momentum condition $p_{Fn}<p_{Fp}+p_{Fe}$ *is* actually satisfied at a sufficiently high n, allowing thus for *direct* URCA processes in neutron-star matter: $n\rightarrow p+e^-+\bar{\nu}_e$, $p+e^-\rightarrow n+\nu_e$ [5]. In this Brief Report we discuss the consequences of the large proton fraction for the bulk viscosity of neutron-star matter.

The simplest model of neutron-star matter at a supranuclear density is the *npe* plasma. For such a model, the energy per baryon is the sum of the nucleon E_N and electron E_e contributions: $E=E_N+E_e$. Many-body calculations of the properties of asymmetric nuclear matter indicate that to a very good approximation E_N is quadratic in the neutron excess parameter, $\alpha=(n_n-n_p)/n=1-2x$ (see, e.g., Refs. [6–9]), and can be thus written as

$$E_N(n,\alpha)=E_0(n)+S(n)\alpha^2 . \tag{1}$$

The β equilibrium condition at a given n determines then the equilibrium proton fraction $x_{\rm eq}$ as a solution of equation

$$x^{1/3}/(1-2x)=4S(n)/\hbar c(3\pi^2)^{1/3}n^{1/3} . \tag{2}$$

For the *npe* matter, the value of $x_{\rm eq}$ is thus determined solely by the nuclear symmetry energy S. A sufficiently rapid increase of S with density implies a sizable proton fraction at $n=2$–$4n_0$, and with $x_{\rm eq}>x_{\rm crit}=\frac{1}{9}$ (corresponding to $p_{Fp}+p_{Fe}=2p_{Fp}>p_{Fn}$) direct URCA reactions can proceed [5].

In what follows we assume that $x_{\rm eq}>\frac{1}{9}$. Our calculation of the bulk viscosity of the *npe* matter follows the formalism of Sawyer [3], with minor modifications. As in Ref. [3] we treat neutrons and protons as normal Fermi liquids (possible effects of neutron superfluidity and proton superconductivity will be discussed in the final part of this Brief Report). We first assume that matter is transparent to neutrinos, so that neutrino absorption can be neglected. The nonequilibrium reactions in compressed and decompressed *npe* matter are driven by the nonzero value of $\Delta=\mu_n-\mu_p-\mu_e$, where μ_n, μ_p, and μ_e are instantaneous neutron, proton, and electron chemical potentials, respectively. Assuming $\Delta \ll kT$ we calculate the linear response of the rate (in 1 cm^3, per 1 s) of the reaction $p+e^- \rightarrow n+\nu_e$ (denoted by Γ_ν), and the inverse $n \rightarrow p+e^- +\bar{\nu}_e$ (denoted by $\Gamma_{\bar{\nu}}$), to the instantaneous nonzero value of Δ. This response is determined by the coefficient λ of Sawyer [3], which we rewrite as

$$\lambda = 2(\partial\Gamma_\nu/\partial\Delta)_{\Delta=0} , \tag{3}$$

so that in the linear approximation $\Gamma_\nu(\Delta)-\Gamma_{\bar{\nu}}(\Delta)=\lambda\Delta$. The value of λ determines the rate of the nonequilibrium URCA reactions in the *npe* matter. We have calculated λ using standard methods and approximations, suitable for the strongly degenerate *npe* plasma. Our result can be summarized in the formula

$$\lambda(\text{URCA}) = -3.5\times 10^{40}\frac{m_n^*}{m_n}\frac{m_p^*}{m_p}\left[x\frac{n}{n_0}\right]^{1/3} T_9^4 \ \text{cm}^{-5}\,\text{g}^{-1}\,\text{s} , \tag{4}$$

where $T_9 = T/10^9$ K. Comparing our results with that of Sawyer [3], we see that $\lambda(\text{URCA}) \sim 10^8 \ T_9^{-2} \ \lambda(\text{mod URCA})$, which reflects the difference in the linear response of the rates of the URCA and modified URCA reactions in the strongly degenerate neutron-star matter to the perturbation of matter density.

The instantaneous value of Δ during neutron-star vibrations depends on the value of the neutron excess parameter α and on the baryon density n, which oscillate around their equilibrium values α_{eq} and n_{eq}. The linear response of Δ to the deviation of α and n from the equilibrium values is determined by Sawyer's parameters B and C, calculated here as

$$B=(\partial\Delta/\partial\alpha)_n=\frac{8}{3}S(1+1/4x) , \tag{5}$$

$$C=n(\partial\Delta/\partial n)_\alpha = 4(1-2x)(nS'-\tfrac{1}{3}S) , \tag{6}$$

where all the quantities are evaluated at the equilibrium (i.e., at $\alpha=\alpha_{\text{eq}}$, $n=n_{\text{eq}}$) and $S'=dS/dn$. The fourth coefficient, relevant for the bulk viscosity of neutron-star matter, measures the linear response of pressure, at fixed n, to the changes in α,

$$D=(\partial P/\partial\alpha)_n , \tag{7}$$

and has to be evaluated at $n=n_{\text{eq}}$, $\alpha=\alpha_{\text{eq}}$. In order to calculate this coefficient for the strongly interacting nucleon matter with an admixture of electrons, we use a general thermodynamic relation, valid also off chemical equilibrium:

$$P=\sum_{i=n,p,e} n_i\mu_i - \varepsilon , \tag{8}$$

where $\varepsilon(n,\alpha)=n[E_N(n,\alpha)+E_e(n,\alpha)]$. Equation (8), used to calculate coefficient D, Eq. (7), leads to

$$D=(\tfrac{1}{2}nc) . \tag{9}$$

The thermodynamical relation between the coefficients C and D, Eq. (9), just relies on the condition of β equilibrium, which implies $(\partial E/\partial\alpha)_{\text{eq}}=0$, $\Delta=0$, and on charge neutrality. It obviously coincides with that found by Sawyer [3] for the free Fermi-gas model of the *npe* matter.

Neutron-star vibrations of angular frequency ω induce the perturbation of n and α around their equilibrium values n_{eq} and α_{eq}. Following the considerations of Sawyer [3] we calculate the time lag of $\alpha-\alpha_{\text{eq}}$ as compared to $n-n_{\text{eq}}$, and the resulting time dependence of the perturbation of the local pressure $P-P_{\text{eq}}$. This enables us to derive an expression for the time average of the energy dissipation rate (in cm^3, per 1 s) due to nonequilibrium URCA reactions:

$$\left\langle\frac{dE}{dt}\right\rangle_{\text{diss}} = -\frac{\omega^2}{2}\left[\frac{\delta n}{n_{\text{eq}}}\right]^2 \frac{\lambda C^2}{\omega^2+4\lambda^2B^2/n_{\text{eq}}^2} . \tag{10}$$

Identifying this dissipation rate with that due to the macroscopic bulk-viscosity coefficient ζ, given in our case by the expression $\frac{1}{2}\zeta\omega^2(\delta n/n_{\text{eq}})^2$, we get the final formula for the bulk-viscosity coefficient of the *npe* matter:

$$\zeta=-\lambda C^2/(\omega^2+4\lambda^2B^2/n^2) , \tag{11}$$

where all the quantities are to be calculated at $x=x_{\text{eq}}(n)$.

Within our approximations, ζ(URCA) is determined by the nuclear symmetry energy $S(n)$ and its derivative $S'(n)$. A detailed discussion of the relevance of the density dependence of S for neutron-star structure can be found in Refs. [10 and 11]. Experimental determinations of the bulk symmetry energy at the saturation density $S_0=S(n_0)$ yield $S_0=27$–36 MeV [12], but the density dependence of S, and its value at, say, $n>2n_0$, are unknown. Therefore, we have to rely on the theoretical determination of $S(n)$. In our calculation of ζ(URCA) we used simple models of the density dependence of S, proposed by Prakash *et al.* [10] in order to simulate the results of various many-body calculations of dense nucleon matter, and parametrized as

$$S(n)=13\,\text{MeV}[u^{2/3}-F(u)]+S_0F(u) , \tag{12}$$

where $u=n/n_0$ and we assume $n_0=0.16$ fm^{-3}. The function F satisfies the condition $F(1)=1$. We considered two models, proposed in Ref. [10]: model I, with $F(u)=u$, and model II, with a stronger density dependence for moderate u, $F(u)=2u^2/(1+u)$. Both models have an asymptotic behavior $S \propto u$ for $u \gg 1$, characteristic of relativistic mean-field-theory models of dense baryonic matter [8–10]. Our results for $x_{\text{eq}}(n)$, B, and C, obtained for a rather conservative choice of $S_0=30$ MeV, are shown in Figs. 1 and 2. Direct URCA processes are

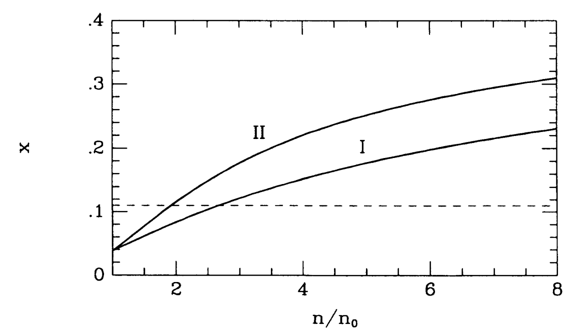

FIG. 1. The equilibrium proton fraction as a function of nucleon density of the *npe* matter, for models I and II of symmetry energy. The dashed line corresponds to $x_{\text{crit}}=\frac{1}{9}$.

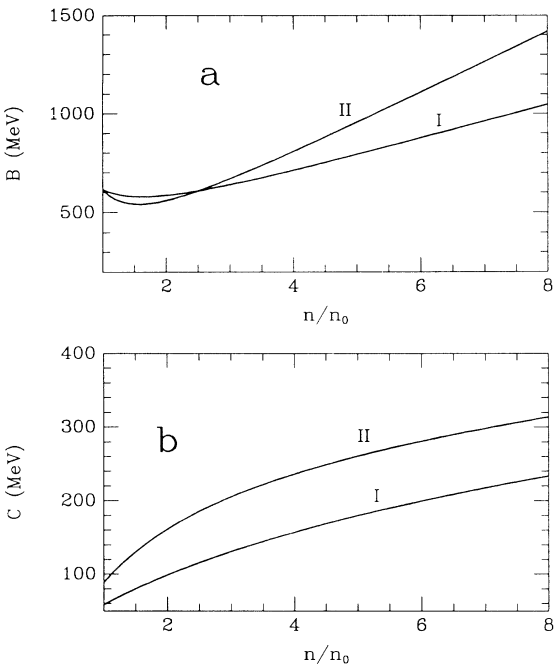

FIG. 2. Linear response parameters B and C, for models I and II of symmetry energy vs nucleon density of the *npe* matter.

operative in the *npe* matter at $n>2n_0$ (model II) or $n>2.7n_0$ (model I)—and therefore can be expected to be relevant for massive neutron stars.

Let us consider the denominator of expression (11) for ζ(URCA). The second term in the denominator is strongly temperature dependent ($\propto T^8$). Using Fig. 2(b) we estimate it, at $n=3n_0$, as $\sim 10^{-3}T_9^8\ \mathrm{s}^{-2}$. In view of the fact that the angular frequency of the fundamental mode of radial pulsations $\omega \sim 10^4\ \mathrm{s}^{-1}$ (and those of the higher modes are even higher), we see that at, say, $T_9<10$, the second term in the denominator is negligibly small compared to ω^2. Therefore, our expression for ζ(URCA) can be cast in a simple "high-frequency" formula:

$$\zeta(\mathrm{URCA})=8.9\times 10^{24}\frac{m_n^*}{m_n}\frac{m_p^*}{m_p}\left[x\frac{n}{n_0}\right]^{1/3}\left[\frac{C}{100\ \mathrm{Mev}}\right]^2\omega_4^{-2}\times T_9^4\ \mathrm{g\,cm^{-1}\,s^{-1}}\ , \tag{13}$$

where $\omega_4=\omega/10^4\ \mathrm{s}^{-1}$.

The calculation of Sawyer [3] leads, at $n=4n_0$, to an estimate

$$\zeta(\mathrm{mod\,URCA})\simeq 7\times 10^{15}\omega_4^{-2}T_9^6\ \mathrm{g\,cm^{-1}\,s^{-1}}\ , \tag{14}$$

so that at such a density one gets an order-of-magnitude estimate $\zeta(\mathrm{URCA})/\zeta(\mathrm{mod\,URCA})\sim 10^9T_9^{-2}$. In the central region of a massive neutron star, where the condition $p_{Fe}+p_{Fp}>p_{Fn}$ is satisfied, modified URCA processes yield a negligibly small correction to the dissipation via the direct URCA reactions.

Up to now our calculation has been made under the assumption that matter is transparent to neutrinos produced in the nonequilibrium reactions. The absorption mean-free path of ν_e of energy $E=pc=ykT$, in the *npe* matter with $x>x_{\mathrm{crit}}$, is (cf., Ref. [13], where, however, the case of *degenerate* neutrinos was considered)

$$\lambda_a(y)\simeq 1.4\times 10^6T_9^{-2}\frac{1+e^{-y}}{\pi^2+y^2}\left[x\frac{n}{n_0}\right]^{-1/3}\mathrm{cm}\ . \tag{15}$$

We see that at $T_9\gtrsim 2$ the absorption of neutrinos becomes important, and we should include it in our calculation. The formalism for the calculation of the bulk viscosity of the *npe* matter which is *opaque* to neutrinos was worked out by Sawyer [14]. After a short stage of deleptonization, which lasts at most some tens of seconds [15], the interior of a newly born neutron star has practically no trapped lepton-number excess, as compared to catalyzed matter. The diffusion of $\nu_e,\bar{\nu}_e$ is then driven by the temperature gradient. Locally, the equilibrium distribution function of $\nu_e,\bar{\nu}_e$ can be approximated by the Fermi-Dirac one, with a zero chemical potential. As shown by Sawyer [14], crucial for the determination of the bulk viscosity of *npe* matter under such conditions is the rate $I(y)$, related to the absorption rate of ν_e in the absence of other neutrinos, $I_a(y)=c/\lambda_a(y)$, by $I(y)=I_a(y)(1+e^{-y})$. Using Eq. (15) we get

$$I(y)=2.1\times 10^4(\pi^2+y^2)(xn/n_0)^{1/3}T_9^2\ \mathrm{s}^{-1}\ . \tag{16}$$

As in Ref. [14] we shall neglect, for the sake of simplicity, the energy dependence of I, putting $I\simeq 2\times 10^5(xn/n_0)^{1/3}T_9^2\ \mathrm{s}^{-1}$. For $T_9\gtrsim 5$ the absorption rate I_a is much higher than the frequency of neutron-star oscillations ($\omega\sim 10^4\ \mathrm{s}^{-1}$), so that the dissipation rate will then become frequency independent. Using the formulas of Ref. [14] we obtain then the following expression for the bulk viscosity of the *npe* matter with trapped neutrinos (but with no trapped lepton excess),

$$\zeta(\mathrm{URCA})=(kT)^2C^2/6(\hbar c)^3I\ , \tag{17}$$

which yields the numerical estimate

$$\zeta(\mathrm{URCA})\simeq 1.3\times 10^{22}\left[\frac{C}{100\ \mathrm{MeV}}\right]^2\left[x\frac{n}{n_0}\right]^{1/3}\mathrm{g\,cm^{-1}\,s^{-1}}\ . \tag{18}$$

We notice a striking difference with respect to ζ(mod URCA), which in the case of a neutrino opaque core with no trapped lepton excess decreases with temperature as T^{-2} [14].

Let us compare the bulk viscosity of the *npe* matter in which the direct URCA process is operative ($x>x_{\mathrm{crit}}$), with that in which only modified URCA processes can proceed ($x<x_{\mathrm{crit}}$), Refs. [3 and 14], assuming zero trapped lepton-number excess. For temperatures up to some 2×10^9 K, we have $\zeta(\mathrm{URCA})/\zeta(\mathrm{mod\,URCA})\sim 10^9T_9^{-2}$, so that the bulk viscosity of the $x>x_{\mathrm{crit}}$ core exceeds that of the $x<x_{\mathrm{crit}}$ one by many orders of magnitude. However, for $T\gtrsim 5\times 10^9$ K the $x>x_{\mathrm{crit}}$ core is opaque to $\nu_e,\bar{\nu}_e$ and we get $\zeta(\mathrm{URCA})/\zeta(\mathrm{mod\,URCA})\sim\omega_4^2T_{10}^{-6}$, so that both ζ become equal at $T\approx 10^{10}$ K. At temperatures exceeding a few times 10^{10} K, also the $x<x_{\mathrm{crit}}$ core is opaque to $\nu_e,\bar{\nu}_e$, and then ζ(mod URCA) starts to decrease with increasing temperature as T^{-2} (see Ref. [14]), while ζ(URCA) stays con-

stant, Eq. (18).

Hyperons are likely to be present in neutron-star matter at a density exceeding a few times normal nuclear density. The hyperon-URCA processes $\Sigma^- \rightarrow \Lambda^0 + e^- + \bar{\nu}_e$, $e^- + \Lambda^0 \rightarrow \Sigma^- + \nu_e$, are then expected to be allowed, and will contribute to the bulk viscosity of matter. The method, which led to the estimate of $\lambda(\text{URCA}) \equiv \lambda(\text{nuc URCA})$, Eq. (4), can be applied in a straightforward manner to estimate $\lambda(\text{hyp URCA})$. In the limit of strong degeneracy, the integrated phase space is the same for the nucleon-URCA and hyperon-URCA processes, but the transition matrix element for the hyperon-URCA process is smaller. The nonequilibrium linear response parameters B, C for the hyperon-URCA processes are different from those for the nucleon-URCA ones, because the $n-p$ symmetry energy is to be replaced by the corresponding $\Sigma - \Lambda$ quantity, which is expected to be very uncertain because of the lack of knowledge of the hyperon-hyperon and hyperon-nucleon interactions in high density matter. It should be stressed that the presence of hyperons will make the condition for the occurrence of the direct nucleon-URCA process easier to satisfy, because of the relative *decrease* of the neutron fraction (see, e.g., Ref. [17]). As soon as it is allowed, the nucleon-URCA contribution to the bulk viscosity may be expected to dominate over the hyperon-URCA one.

The presence of muons in neutron-star matter introduces minor modifications. Muons will be present when $\mu_e > m_\mu c^2 = 105.7$ MeV. In the presence of muons $p_{Fe} < p_{Fp}$. Their presence will therefore slightly increase the value of the critical proton fraction, above which $p_{Fe} + p_{Fp} > p_{Fn}$, but by a small amount, to not more than 15% [5]. On the other hand, at fixed n, the proton fraction $x = x_e + x_\mu$ is larger then for the *npe* matter. This actually *decreases,* for a given $S(n)$, the critical value of n, above which the direct URCA process is operative. One notices that for $p_{Fp} + p_{F\mu} > p_{Fn}$, which occurs somewhat above the threshold density for the appearance of muons [5], direct URCA processes with muons will proceed, $n \rightarrow p + \mu^- + \bar{\nu}_\mu$, $p + \mu^- \rightarrow n + \nu_\mu$, increasing further the bulk viscosity of neutron-star matter.

The shear viscosity of normal neutron-star matter, resulting from the two-body scattering of its constituents, has been calculated by Flowers and Itoh [16], and their result (obtained assuming normal neutrons) can be represented by an analytic fit

$$\eta = 2 \times 10^{18} (\rho_{15})^{9/4} T_9^{-2} \, \text{g cm}^{-1} \text{s}^{-1} , \tag{19}$$

where $\rho_{15} = \rho / 10^{15}$ g cm^{-3}. In the regions where $p_{Fe} + p_{Fp} > p_{Fn}$, and neutrino absorption can be neglected, the ratio of bulk to shear viscosity goes as $\zeta(\text{URCA})/\eta \propto \omega_4^{-2} T_9^6$. At $\omega_4 = 1$ and $n = 4n_0 (\rho_{15} = 1)$, the crossover temperature, above which bulk URCA viscosity dominates over the shear one, is, according to our estimates, about 8×10^7 K.

As far as the evolution of a newly formed neutron star is concerned, we should stress the dramatic difference in the time scales of cooling of the neutron-star core in the $x > x_{\text{crit}}$ and $x < x_{\text{crit}}$ cases. Despite very high neutrino opacity we expect that the $x > x_{\text{crit}}$ core cools to $T_9 \sim 1$ (by thermal diffusion of ν_e, $\bar{\nu}_e$) in a few hours, while the $x < x_{\text{crit}}$ core needs about one year to reach such a temperature. Using neutrino emissivity of Ref. [5], $\epsilon(\text{URCA}) \simeq 4 \times 10^{27} (xn/n_0)^{1/3} T_9^6$ erg cm^{-3} s^{-1}, we see that the $x > x_{\text{crit}}$ core cools to the crossover temperature in about one week. Therefore, enhanced bulk viscosity, resulting from direct URCA processes, influences the properties of the neutron star during a very short period after its formation, increasing then the stability of the rapid rotation of the newly born neutron star.

Our results have been obtained assuming that both neutrons and protons form normal Fermi liquids. We expect that at least at some densities neutrons may be superfluid and/or protons superconducting. The calculated gaps in the single-particle spectra are very uncertain but are typically of the order of a few hundred keV, which corresponds to a critical temperature T_c of the order of a few 10^9 K. For $T \ll T_c$, the URCA reaction rates and consequently also the value of $\zeta(\text{URCA})$ will be significantly reduced as compared to those obtained for normal neutrons and protons. However, the corresponding reduction of $\zeta(\text{mod URCA})$ will then be even stronger.

Finally, let us notice that in the case of superfluid neutrons and $T \ll T_c$ shear viscosity would result from the two-body scattering in (normal) electron gas. The numerical coefficient in Eq. (19) would then be somewhat different, but the general effect of superfluidity of neutrons and/or superconductivity of protons would be to shift the crossover temperature, above which bulk viscosity dominates over the shear one, to a higher value.

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[1] C. Cutler, L. Lindblom, and R. J. Splinter, Astrophys. J. **363**, 603 (1990).
[2] C. Cutler and L. Lindblom, Astrophys. J. **314**, 234 (1987).
[3] R. F. Sawyer, Phys. Rev. D **39**, 3804 (1989).
[4] H. Chiu and E. E. Salpeter, Phys. Rev. Lett. **12**, 413 (1964).
[5] J. M. Lattimer, C. J. Pethick, M. Prakash, and P. Haensel, Phys. Rev. Lett. **66**, 2701 (1991).
[6] I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. **A369**, 470 (1981).
[7] R. B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C **38**, 1010 (1988).
[8] M. Prakash and T. L. Ainsworth, Phys. Rev. C **36**, 346 (1987).
[9] H. Muether, M. Prakash, and T. L. Ainsworth, Phys. Lett. B **199**, 469 (1988).
[10] M. Prakash, T. L. Ainsworth, and J. M. Lattimer, Phys. Rev. Lett. **61**, 2518 (1988).
[11] J. Cooperstein, Phys. Rev. C **37**, 786 (1988).
[12] P. Moeller, W. D. Myers, W. J. Swiatecki, and J. Treiner,

At. Data Nucl. Data Tables **39**, 225 (1988).
[13] R. F. Sawyer and A. Soni, Astrophys. J. **230**, 859 (1979).
[14] R. F. Sawyer, Astrophys. J. **237**, 187 (1980).
[15] A. Burrows and J. M. Lattimer, Astrophys. J. **307**, 178 (1986).
[16] E. Flowers and N. Itoh, Astrophys. J. **153**, 835 (1976).
[17] N. K. Glendenning, F. Weber, and S. A. Moszkowski, Phys. Rev. C **45**, 844 (1992).