Fermion damping in hot gauge theories

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(Received 23 December 1991; revised manuscript received 9 April 1992)

The damping rate to order g^2T for fermions in the long-wavelength limit of hot gauge theories is calculated using the recently developed resummation methods in terms of hard thermal loops. Both a heavy and a massless fermion are considered. Ward identities between the effective propagators and vertices are used to formally prove the gauge independence of the damping rate to this order in a wide class of gauges.

PACS number(s): 11.15.Bt;12.38.Mh

I. INTRODUCTION

The infrared behavior of gauge theories at high temperatures generally leads in perturbation theory to a number of paradoxes [1]. One particular problem that has attracted much attention recently is the calculation of the damping constant, as in the one-loop approximation both the magnitude and the sign of this rate depend on the choice of gauge. Pisarski in particular has argued that such a dependence indicates the breakdown of the loop expansion [2, 3], and would disappear in a consistent expansion incorporating all relevant higher-loop effects. General formal arguments also exist to support the conclusion that the poles in the propagator will be gauge invariant when calculated accurately [4]. For high-temperature QED and QCD such an expansion in terms of the "hard thermal loops" studied by Klimov [5], Weldon [6], Braaten and Pisarski [7], Taylor and Wong [8], and Frenkel and Taylor [9] was developed, and subsequently used to calculate the lowest-order damping rate for gluons [10]. The purpose of this paper is to use these resummation techniques to calculate the analogous rate for fermions.

The reasons for performing this calculation are threefold. Firstly, as in the calculation for gluons, the sign and the order of magnitude of the damping rate should provide an indication that such techniques give tractable and reasonable results. Secondly, an approach is employed that involves intermediate steps different from those that arise using dispersion relation methods [3, 10], and we verify that both approaches lead to the same final answer. In light of the difficulty of such calculations relative to those of the usual loop expansion, this provides a useful technical check on the methods. Finally, as is also found for gluons [7, 10], we show that Ward identities between the effective propagators and vertices simplify significantly the fermion calculation, and at an algebraic level can be used to formally prove the gauge independence of the lowest-order damping rate in a wide class of gauges.

The paper is organized as follows. In Sec. II we list some technical details of the evaluation of the Matsubara frequency sums and the extraction of the imaginary part of the subsequent analytically continued expressions. In Sec. III we give the forms of the effective propagators and vertices that arise in the resummed expressions for the quark self-energy, as well as the corresponding Ward identities. In Sec. IV we use these results to calculate the lowest-order damping rate in the long-wavelength limit for a heavy fermion ($m \gg T$), and in Sec. V we consider a massless fermion. Both results are gauge independent in a wide class of gauges, although recently questions have been raised in this regard concerning the neglect of some terms in certain gauges, both for the fermion and the gluon damping rate calculations to lowest order [11]. Section VI contains a brief summary.

II. TECHNICAL DETAILS

In this section we give some technical details of the evaluation of the Matsubara frequency sums and of the calculation of the imaginary part of the subsequent analytically continued expressions [12]. Consider first a complex function $f(x)$ defined on the real axis. The inverse of this function will have an imaginary part related to its discontinuity across the real axis as

$$\mathrm{Im}_x \frac{1}{f(x)} \equiv \pi \Im_x \frac{1}{f(x)} = \frac{1}{2i}\left(\frac{1}{f(x+i\epsilon)} - \frac{1}{f(x-i\epsilon)}\right). \tag{2.1}$$

Contributions to this imaginary part come from zeros of the function, giving rise to residue terms, and from cut terms due to discontinuities of f across the real axis. A residue contribution for $1/f(x)$ with a simple pole at $x = x_0$ will have the form

$$-\pi \frac{1}{f'(x_0)}\delta(x-x_0) \equiv \pi R_f(x_0)\delta(x-x_0). \tag{2.2}$$

For a cut term, suppose $f(x)$ has a cut for $0 < x < a$, and define in this interval $f(x+i\epsilon) = \bar{f}(x) + i f_I(x)$ and $f(x - i\epsilon) = \bar{f}(x) - i f_I(x)$. The contribution of this cut to Eq. (2.1) will have the form

$$-\frac{f_I(x)}{\bar{f}^2(x) + {f_I}^2(x)}\,\theta(x)\,\theta(a-x) \equiv \pi\beta_f(x)\,\theta(x)\,\theta(a-x). \tag{2.3}$$

Including both the pole and cut terms, Eq. (2.1) then becomes

$$\Im_x \frac{1}{f(x)} = R_f(x_0)\delta(x - x_0) + \beta_f(x)\,\theta(x)\,\theta(a-x). \tag{2.4}$$

This formula can readily be generalized to cases with multiple poles and cuts.

Consider now a frequency sum at finite temperature in the imaginary-time formalism involving a bosonic function $f(k_0 = i2\pi nT)$—the extension of the following to include fermions is straightforward. We assume $f(z)$ is analytic everywhere except on the real axis, and evaluate the frequency sum by [12]

$$T\sum_{n=-\infty}^{+\infty} f(k_0 = i2\pi nT) = \frac{1}{4\pi i}\oint_C dk_0 \coth\left[\frac{k_0}{2T}\right] f(k_0), \tag{2.5}$$

where the continuation $i2\pi nT \to k_0 + i\epsilon$ to real energies is made and the integration contour $\mathcal{C}$ encircles the imaginary axis counterclockwise. The contour can be deformed and split into two pieces: one encircling the real axis and the other forming a great circle at infinity. Since $1/(e^{|k_0|/T} - 1)$ damps at large $|k_0|$, only poles and cuts of f which appear along the real axis will contribute. We then have, using Eq. (2.4),

$$T\sum_{n=-\infty}^{+\infty} f(k_0 = i2\pi nT) = \frac{1}{2}\int_{-\infty}^{+\infty} dk_0 \coth\left[\frac{k_0}{2T}\right] \Im_k f(k_0). \tag{2.6}$$

We apply these considerations to a self-energy contribution involving a single loop integration :

$$F(p_0 = i2\pi nT, \mathbf{p}) = \int dK\, f(P,K) \equiv T\sum_m \int \frac{d^3k}{(2\pi)^3} f(p_0 = i2\pi nT, \mathbf{p}, k_0 = i2\pi mT, \mathbf{k}), \tag{2.7}$$

where the notation $P_\mu = (p_0, \mathbf{p})$ will be used to denote the 4-momentum. The extension of Eq. (2.6) to include functions which involve an external energy $p_0 = i2\pi nT$ is straightforward, and after such an evaluation of the frequency sum the analytic continuation $i2\pi nT \to p_0 + i\epsilon$ is made. This continuation results in the function acquiring an imaginary part, which can subsequently be extracted by use of Eqs. (2.1) and (2.4). Such a procedure, with modifications as necessary to include fermions, shall be used in the following, and will be seen to lead to the same final answer as that found by the spectral representation methods of Refs. [3, 10].

III. EFFECTIVE PROPAGATORS AND VERTICES

Here we list the forms of the effective propagators and vertices used for the resummed fermion self-energy, as well as the corresponding Ward identities which provide simple relations among them.

A. Gluon propagator

We begin with the gluon propagator, and assume its bare form is given by

$$D^{(0)}_{\mu\nu}(K) = \frac{1}{K^2}A_{\mu\nu} - \frac{1}{\mathbf{k}^2}\mathcal{F}_\mu\mathcal{F}_\nu + c^2(K)\frac{K_\mu K_\nu}{K^4}, \tag{3.1}$$

where $\mathcal{F}_\mu = \tilde{n}_\mu + \lambda(K)K_\mu$ and n_μ is a fixed vector identified with the velocity of the heat bath. Here $\tilde{n}_\mu = P_{\mu\nu}n^\nu$, $P_{\mu\nu} = g_{\mu\nu} - K_\mu K_\nu/K^2$, and $A_{\mu\nu} = P_{\mu\nu} - \tilde{n}_\mu\tilde{n}_\nu/\tilde{n}^2$, which for $n_\mu = \delta_{0\mu}$ is the spatially transverse projection operator $-\delta_{ij} + k_i k_j/\mathbf{k}^2$. The gauges used in Eq. (3.1) are quite general, and include as special cases covariant gauges [$\lambda = 0$, $c^2 = 1/\alpha$) and regulated axial gauges [13] [$\lambda = k_0\mathbf{k}^2/K_C^2K^2$, $c^2 = -\alpha(1-\alpha)K^4/K_C^4$, where $K_C^2 = (1-\alpha)k_0^2 + \alpha\mathbf{k}^2$]. We note that as the propagator is even under $K \to -K$ the gauge parameters $\lambda(K)$ and $c^2(K)$ are odd and even, respectively.

The effective gluon propagator [2, 5, 6]

$$D_{\mu\nu}(K) = \frac{1}{k_t^2}A_{\mu\nu} - \frac{1}{k_l^2}\mathcal{F}_\mu\mathcal{F}_\nu + c^2(K)\frac{K_\mu K_\nu}{K^4} \tag{3.2}$$

is obtained from the bare propagator of Eq. (3.1) by substituting ${k_t}^2 \equiv K^2 + \Pi_t$ for K^2 and ${k_l}^2 \equiv \mathbf{k}^2 - \Pi_l$ for $\mathbf{k}^2$, where, with $k \equiv |\mathbf{k}|$,

$$\Pi_t(k_0,k) \approx -\frac{3{m_g}^2{k_0}^2}{2k^2}\left[1-\left(1-\frac{k^2}{{k_0}^2}\right)\frac{k_0}{2k}\ln\left(\frac{k_0+k}{k_0-k}\right)\right],$$
$$\Pi_l(k_0,k) \approx -3{m_g}^2\left[1-\frac{k_0}{2k}\ln\left(\frac{k_0+k}{k_0-k}\right)\right], \tag{3.3}$$

arise from hard thermal loops and ${m_g}^2 = g^2T^2(N+N_f/2)/9$. In this and the following $\approx$ denotes equality up to hard thermal loop contributions. The transverse dispersion relation $k_t^2[\omega_t(k),k]=0$ and the corresponding contributions to

$$\Im_k\frac{1}{k_t^2} = R_t(k_0,k)[\delta(k_0-\omega_t(k))-\delta(k_0+\omega_t(k))]+\beta_t(k_0,k)\theta(k^2-k_0^2), \tag{3.4}$$

as well as those for the plasmon mode, are discussed in Refs. [2, 6].

B. Quark propagator

The effective quark propagator $S^{-1}(P) = \not{P} - \delta\Sigma(P)$ includes the hard thermal loop [3, 14]

$$\delta\Sigma(P) \approx g^2C_f\int dK\; D^{(0)}_{\mu\nu}(K)\gamma^\mu S^{(0)}(P-K)\gamma^\nu \approx {m_f}^2\gamma_\mu\int\frac{d\Omega}{4\pi}\frac{\hat{\mathbf{K}}^\mu}{P\cdot\hat{\mathbf{K}}} \equiv \gamma_\mu\xi^\mu(P). \tag{3.5}$$

Here, $m_f^2 = C_fg^2T^2/8$, $\hat{\mathbf{K}}^\mu = (1,\hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|)$, $d\Omega$ is the angular measure, $D^{(0)}_{\mu\nu}(P)$ is the bare gluon propagator of Eq. (3.1), and $S^{(0)}(P)$ is the bare quark propagator

$$S^{(0)}(P) = \frac{1}{\not{P}-m} = \frac{\not{P}+m}{P^2-m^2}. \tag{3.6}$$

Now, $P\cdot\xi = {m_f}^2$ from Eq. (3.5) gives $\xi^i(P) = [p_0\xi_0 - {m_f}^2]\hat{\mathbf{p}}^i/p$, which as shown in Refs. [3, 14] can be used to split the effective propagator into two modes:

$$S(P) = \frac{1}{D_0(P)\,\gamma^0 - D_s(P)\,\hat{\not{\mathbf{p}}}} = \chi_0(P)\gamma_0 - \chi_s(P)\;\hat{\not{\mathbf{p}}}$$
$$= \frac{1}{2}\Delta_+(P)(\gamma_0 - \hat{\not{\mathbf{p}}}) + \frac{1}{2}\Delta_-(P)(\gamma_0 + \hat{\not{\mathbf{p}}}), \tag{3.7}$$

where

$$D_0(P) = p_0 - \xi_0,$$
$$D_s(P) = p - \frac{p_0\xi_0 - {m_f}^2}{p},$$
$$\Delta_+(P) = \frac{1}{D_0(P) - D_s(P)},$$
$$\Delta_-(P) = \frac{1}{D_0(P) + D_s(P)}. \tag{3.8}$$

The dispersion relations of these two modes and their contributions to

$$\Im_p\Delta_+(p_0,p) = R(p_0,p)\,[\delta(p_0-\omega_+(p))+\delta(p_0+\omega_-(p))]+\beta_+(p_0,p)\theta(p^2-p_0^2),$$
$$\Im_p\Delta_-(p_0,p) = R(p_0,p)\,[\delta(p_0-\omega_-(p))+\delta(p_0+\omega_+(p))]+\beta_-(p_0,p)\theta(p^2-p_0^2)\;, \tag{3.9}$$

are discussed in Refs. [3, 14].

C. Three-point vertex

The effective vertex between a quark pair and a gluon can be written as

$$^*\Gamma^\mu(P,Q) = \gamma^\mu + \delta\Gamma^\mu_{(1)} + \delta\Gamma^\mu_{(2)} \equiv \gamma^\mu + \delta\Gamma^\mu(P,Q), \tag{3.10}$$

where P and Q are the incoming and outgoing quark momenta, respectively. The two hard thermal loops are given by

$$\delta\Gamma^{\mu}_{(1)}(P,Q) \approx -g^2\left(\frac{N}{2}-C_f\right)\int dK\; D^{(0)}_{\lambda\rho}(K)\gamma^\rho S^{(0)}(P-K)\gamma^\mu S^{(0)}(Q-K)\gamma^\lambda,$$

$$\delta\Gamma^{\mu}_{(2)}(P,Q) \approx -\frac{g^2N}{2}\int dK\; \gamma^\beta S^{(0)}(K)\gamma^\rho D^{(0)}_{\rho\nu}(K-P)\Gamma^{\mu\nu\alpha}(P-Q,K-P,Q-K)D^{(0)}_{\alpha\beta}(K-Q), \tag{3.11}$$

where $\Gamma^{\alpha\beta\gamma}(P,Q,K)=g^{\alpha\beta}(P-Q)^\gamma+g^{\beta\gamma}(Q-K)^\alpha+g^{\gamma\alpha}(K-P)^\beta$. Since the terms proportional to $N/2$ cancel, we have [7–9]

$$\delta\Gamma^\nu(P,Q)\approx m_f{}^2\gamma_\mu\int\frac{d\Omega}{4\pi}\frac{\hat{\mathbf{K}}^\mu\hat{\mathbf{K}}^\nu}{P\cdot\hat{\mathbf{K}}\;Q\cdot\hat{\mathbf{K}}}, \tag{3.12}$$

which, using the effective quark propagator of Eq. (3.5), gives the Ward identity

$$(Q-P)\cdot{}^*\Gamma(P,Q)\approx S^{-1}(Q)-S^{-1}(P). \tag{3.13}$$

D. Four-point vertex

With the color indices contracted, the effective four-point vertex between a quark pair and two gluons is given by

$$\begin{aligned}{}^*\Gamma^{\mu\nu}(P,Q;S,T) &= \delta\Gamma^{\mu\nu}_{(1)}+\delta\Gamma^{\mu\nu}_{(2)}+\delta\Gamma^{\mu\nu}_{(3)}\\ &\equiv \delta\Gamma^{\mu\nu}(P,Q;S,T),\end{aligned} \tag{3.14}$$

where $P+S=Q+T=R$. The three hard thermal loops are decomposed as

$$\delta\Gamma^{\mu\nu}_{(i)}(P,Q;S,T)=\mathcal{J}^{\mu\nu}_{(i)}(P,Q;S,T)+\mathcal{J}^{\mu\nu}_{(i)}(P,Q;-T,-S) \tag{3.15}$$

for $i=1,2,3$, where

$$\begin{aligned}\mathcal{J}^{\mu\nu}_{(1)}(P,Q;S,T)&\approx g^2C_f\int dK\;\gamma^\lambda S^{(0)}(Q+K)\gamma^\nu S^{(0)}(R+K)\gamma^\mu S^{(0)}(P+K)\gamma^\rho D^{(0)}_{\rho\lambda}(K),\\ \mathcal{J}^{\mu\nu}_{(2)}(P,Q;S,T)&\approx\frac{g^2N}{2}\int dK\;D^{(0)}_{\alpha\beta}(K+S)\gamma^\beta S^{(0)}(Q-S-K)\gamma^\nu S^{(0)}(P-K)\gamma^\lambda\\ &\quad\times D^{(0)}_{\lambda\rho}(K)\Gamma^{\rho\mu\alpha}(K,S,-K-S),\\ \mathcal{J}^{\mu\nu}_{(3)}(P,Q;S,T)&\approx g^2N\int dK\;D^{(0)}_{\alpha\beta}(K)\Gamma^{\mu\beta\gamma}(-S,K,S-K)D^{(0)}_{\gamma\delta}(S-K)\\ &\quad\times\Gamma^{\nu\delta\rho}(T,K-S,Q-P-K)D^{(0)}_{\rho\lambda}(Q-K-P)\gamma^\lambda S^{(0)}(P+K)\gamma^\alpha.\end{aligned} \tag{3.16}$$

Since the second and third terms cancel, we have [7–9]

$$^*\Gamma^{\mu\nu}(P,Q;S,T)\approx m_f{}^2\gamma_\rho\int\frac{d\Omega}{4\pi}\frac{\hat{\mathbf{K}}^\mu\hat{\mathbf{K}}^\nu\hat{\mathbf{K}}^\rho}{(P+S)\cdot\hat{\mathbf{K}}\;(P-T)\cdot\hat{\mathbf{K}}}\left(\frac{1}{P\cdot\hat{\mathbf{K}}}+\frac{1}{Q\cdot\hat{\mathbf{K}}}\right), \tag{3.17}$$

which, using the three-point vertex of Eq. (3.12), leads to the Ward identity

$$T_\mu{}^*\Gamma^{\mu\nu}(P,Q;S,T)\approx{}^*\Gamma^\nu(P-T,Q)-{}^*\Gamma^\nu(P,Q+T). \tag{3.18}$$

IV. HEAVY QUARK

Employing dispersion relation methods, Pisarski has used the effective expansion to calculate the lowest-order damping rate of a heavy quark of mass $m\gg T$ in the long-wavelength limit [3]. We reconsider this calculation as a simple check on the methods of Sec. II. With $m >> T$ only the bare gluon propagator has to be replaced by its effective form, which leads to consideration of the resummed self-energy [3]

$$\Sigma(P)=g^2C_f\int_{\text{soft}}dK\;D_{\mu\nu}(K)\gamma^\mu S^{(0)}(Q)\gamma^\nu, \tag{4.1}$$

where $Q=P-K$ and the integration is carried out only for soft momenta.

The considerations of Sec. II to evaluate the frequency sum and then find the imaginary part of Eq. (4.1) after analytic continuation are simplified by the presence of the bare quark propagator. In the long-wavelength limit $|\mathbf{p}|\to 0$ the mass-shell condition $p_0=m$ imposed on $\delta(p_0+k_0-\omega(q))$ implies that $q_0\simeq m$ and $k_0\simeq\mathbf{k}^2/2m$. The other δ functions that arise have no support in this region of interest. As will be seen in the next section, Ward identities can be used to show that the gauge-dependent contributions to Eq. (4.1) are proportional to the mass-shell condition, and apart from some concerns to be discussed shortly that arise in certain gauges [11], such terms will not contribute to this order. One then finds the discontinuity of the self-energy of Eq. (4.1) in this limit is given by the gauge-independent result

$$\mathrm{Im}_p \Sigma(p) = -\frac{m_g{}^2 T C_f}{2\pi} \int_0^\infty dk \left(2(\gamma^0 - 1)\Im_k \frac{1}{k_t^2} - (\gamma^0 + 1)\Im_k \frac{1}{k_l^2} \right)$$
$$= -(\gamma^0 - 1)\, \frac{3g^2 T C_f m_g}{16\pi \mu^2_{\mathrm{cutoff}}} - (\gamma^0 + 1)\, \frac{g^2 T C_f}{16\pi}. \tag{4.2}$$

The first term with the infrared cutoff contributes to wave-function renormalization, while the second term shifts the pole. The solution $\omega = m - ig^2 T C_f / 8\pi$ to the dispersion relation thus has an imaginary part, which leads to a damping rate [3].

V. MASSLESS QUARK

For a massless quark in the long-wavelength limit, the one-loop approximation to the fermion self-energy generally leads to a gauge-dependent damping constant, as happens for pure gluons [1]. The use of the effective propagators and vertices in the calculation will be seen to formally give a gauge-independent answer, but is significantly more involved in the massless case because all must be included [2, 3, 7].

A. The quark self-energy

The resummed quark self-energy

$$\Sigma(P) = \Sigma_{(1)}(P) + \Sigma_{(2)}(P) \tag{5.1}$$

consists of two distinct terms [3, 7]: one from the quark-gluon loop,

$$\Sigma_{(1)}(P) = g^2 C_f \int_{\mathrm{soft}} dK\, {}^*\Gamma^\mu(P,Q) S(Q)\, {}^*\Gamma^\nu(Q,P) D_{\mu\nu}(K), \tag{5.2}$$

where $Q = P - K$, and the other from the single-gluon loop,

$$\Sigma_{(2)}(P) = \tfrac{1}{2} g^2 C_f \int_{\mathrm{soft}} dK\ D_{\mu\nu}(K)\, {}^*\Gamma^{\mu\nu}(P,P;K,K). \tag{5.3}$$

We first examine the gauge-dependent terms. The term linearly proportional to the gauge parameter $\lambda(K)$ of Eq. (3.2) is given by

$$-g^2 C_f \int \frac{dK}{k_l^2} \lambda(K)(\tilde{n}_\mu K_\nu + \tilde{n}_\nu K_\mu)[{}^*\Gamma^\mu(P,Q) S(Q)\, {}^*\Gamma^\nu(Q,P) + \tfrac{1}{2}\, {}^*\Gamma^{\mu\nu}(P,P;K,K)]$$
$$= -g^2 C_f \int \frac{dK}{k_l^2} \lambda(K) \tilde{n}_\mu [{}^*\Gamma^\mu(P,Q) S(Q) S^{-1}(P) + S^{-1}(P) S(Q)\, {}^*\Gamma^\mu(Q,P)], \tag{5.4}$$

where the Ward identities of Eqs. (3.13) and (3.18) have been used. The remaining gauge-dependent terms, again using Eqs. (3.13) and (3.18), can be written as

$$g^2 C_f \int dK \left(\frac{c^2(K)}{K^4} - \frac{\lambda^2(K)}{k_l^2} \right) K_\mu K_\nu [{}^*\Gamma^\mu(P,Q) S(Q)\, {}^*\Gamma^\nu(Q,P) + \tfrac{1}{2}\, {}^*\Gamma^{\mu\nu}(P,P;K,K)$$
$$= g^2 C_f \int dK \left(\frac{c^2(K)}{K^4} - \frac{\lambda^2(K)}{k_l^2} \right) \left[S^{-1}(P) S(Q) S^{-1}(P) - S^{-1}(P) \right], \tag{5.5}$$

Thus, both gauge-dependent terms of Eqs. (5.4) and (5.5) are proportional to $S^{-1}(P)$, and will vanish when the mass-shell condition is imposed if the associated integrals are not singular in this limit. Recently these integrals have been examined in detail, and questions were raised concerning the neglect of such terms in certain gauges, particularly the usual covariant gauges, in both the fermion and gluon damping rate calculations to lowest order [11]. This problem was subsequently shown to be related to the introduction of an infrared cutoff and the resulting question of when such a cutoff should be taken to zero as the mass-shell limit is approached [15]. We do not discuss this point further but rather assume that, if this problem cannot be resolved, we work in those classes of gauges where these singular integrals do not arise.

We thus drop the gauge-dependent terms of Eqs. (5.4) and (5.5) on mass shell, and consider only the gauge-independent contributions to Eq. (5.1). Two such terms arise. One, from the transverse gluons, is found in the long-wavelength limit to be

$$-g^2C_f\int\frac{dK}{k_t^2}A_{\mu\nu}[{}^*\Gamma^\mu(P,Q)S(Q){}^*\Gamma^\nu(Q,P)+\tfrac{1}{2}{}^*\Gamma^{\mu\nu}(P,P;K,K)]$$

$$=g^2C_f\gamma_0\int\frac{dK}{k_t^2}\left\{2\chi_0(q_0,k)\left[1-\frac{m_f^2q_0}{2k^2p_0}-\frac{1}{2p_0}\left(1-\frac{q_0^2}{k^2}\right)\xi_0(q_0,k)\right]^2+\frac{1}{p_0^2}\left[1-\frac{q_0^2}{k^2}\right]\xi_0(q_0,k)\right\}, \quad (5.6)$$

where only terms relevant to the imaginary part are retained. The other gauge-independent contribution, from the plasmon mode, is in the long-wavelength limit

$$-g^2C_f\int\frac{dK}{k_l^2}\tilde{n}_\mu\tilde{n}_\nu\;[{}^*\Gamma^\mu(P,Q)S(Q){}^*\Gamma^\nu(Q,P)+\tfrac{1}{2}{}^*\Gamma^{\mu\nu}(P,P;K,K)]$$

$$=-g^2C_f\gamma_0\int\frac{dK}{k_l^2}\left\{\left[\frac{k^2}{p_0^2}+\left(2-\frac{k_0}{p_0}\right)^2\right]\chi_0(q_0,k)-\frac{2k}{p_0}\left(2-\frac{k_0}{p_0}\right)\chi_s(q_0,k)\right\}. \quad (5.7)$$

The total contribution to the imaginary part of the quark self-energy then becomes

$$\gamma_0g^2C_f\,\mathrm{Im}_p\int dK\left\{\frac{1}{k_t^2}\left[\frac{1}{4p_0^2k^2}[(2p_0-k_0+k)^2(k_0+k)^2\Delta_+(q_0,k)+(2p_0-k_0-k)^2(k_0-k)^2\Delta_-(q_0,k)]\right.\right.$$
$$\left.+\frac{1}{2p_0^2}\left(1-\frac{q_0^2}{k^2}\right)\xi_0(q_0,k)\right]$$
$$\left.-\frac{1}{2p_0^2k_l^2}\left[\Delta_+(q_0,k)(2p_0-k_0-k)^2+\Delta_-(q_0,k)(2p_0-k_0+k)^2\right]\right\}. \quad (5.8)$$

B. The damping constant

The frequency sum and the extraction of the imaginary part after analytic continuation of Eq. (5.8) can be performed using the considerations of Sec. II. In this case, in addition to the contributions from the poles, there are terms arising from the cuts. On mass shell in the high-temperature, long-wavelength limit, we find, for Eq. (5.8)

$$\mathrm{Im}_p\Sigma(p_0=m_f,p=0)=\gamma_0\frac{g^2TC_f}{2\pi}\int_0^\infty k^2dk\int_0^\infty\frac{dk_0}{k_0}\left\{\frac{(k-k_0+2m_f)^2(k+k_0)^2}{4m_f^2k^2}\Im_k\frac{1}{k_t^2}\Im_p\Delta_+(q_0,k)\right.$$
$$+\frac{(k+k_0-2m_f)^2(k-k_0)^2}{4m_f^2k^2}\Im_k\frac{1}{k_t^2}\Im_p\Delta_-(q_0,k)$$
$$-\frac{(k+k_0-2m_f)^2}{2m_f{}^2}\Im_k\frac{1}{k_l^2}\Im_p\Delta_+(q_0,k)$$
$$-\frac{(k-k_0+2m_f)^2}{2m_f{}^2}\Im_k\frac{1}{k_l^2}\Im_p\Delta_-(q_0,k)$$
$$\left.-\frac{1}{4k^3}(k^2-q_0^2)\Im_k\frac{1}{k_t^2}\theta(k^2-q_0^2)\right\}. \quad (5.9)$$

One can verify that the same result of Eq. (5.9) is obtained using the dispersion relation methods of Refs. [3, 10]. Since in Eq. (5.9) $\Im_k[1/k_t^2]/k_0$ is positive definite while $\Im_k[1/k_l^2]/k_0$, $\Im_p\Delta_+$, and $\Im_p\Delta_-$ are all negative definite, $\mathrm{Im}_p\Sigma(p_0=m_f,p=0)$ is negative definite, which leads to a positive-definite damping constant.

It is convenient in Eq. (5.9) to rescale $k_0\to m_fk_0$ and $k\to m_fk$. If a tilde symbol above a function denotes its form after such a rescaling, then with the damping constant defined as

$$\gamma(p=0)=-\frac{1}{8}\mathrm{Tr}\;[\gamma_0\mathrm{Im}_p\Sigma(p_0=m_f,p=0)]\equiv a(N,N_f)\,\frac{g^2TC_f}{4\pi}, \quad (5.10)$$

one obtains

$$
\begin{aligned}
a(N,N_f) = &-\left.\frac{[k+\tilde{\omega}_t(k)]^2[k-\tilde{\omega}_t(k)+2]^2\tilde{R}_t(\tilde{\omega}_t(k),k)\tilde{R}(\tilde{\omega}_+(k),k)}{4\tilde{\omega}_t(k)|\tilde{\omega}_t'(k)-\tilde{\omega}_+'(k)|}\right|_{\tilde{\omega}_t(k)-\tilde{\omega}_+(k)=1} \\
&-\left.\frac{[k-\tilde{\omega}_t(k)]^2[k+\tilde{\omega}_t(k)-2]^2\tilde{R}_t(\tilde{\omega}_t(k),k)\tilde{R}(\tilde{\omega}_-(k),k)}{4\tilde{\omega}_t(k)|\tilde{\omega}_t'(k)-\tilde{\omega}_-'(k)|}\right|_{\tilde{\omega}_t(k)-\tilde{\omega}_-(k)=1} \\
&+\left.\frac{k^2[k-\tilde{\omega}_l(k)+2]^2\tilde{R}_l(\tilde{\omega}_l(k),k)\tilde{R}(\tilde{\omega}_+(k),k)}{2\tilde{\omega}_l(k)|\tilde{\omega}_l'(k)-\tilde{\omega}_+'(k)|}\right|_{\tilde{\omega}_l(k)-\tilde{\omega}_+(k)=1} \\
&+\left.\frac{k^2[k+\tilde{\omega}_l(k)-2]^2\tilde{R}_l(\tilde{\omega}_l(k),k)\tilde{R}(\tilde{\omega}_-(k),k)}{2\tilde{\omega}_l(k)|\tilde{\omega}_l'(k)-\tilde{\omega}_-'(k)|}\right|_{\tilde{\omega}_l(k)-\tilde{\omega}_-(k)=1} \\
&-\frac{1}{4}\int\frac{dk}{\tilde{\omega}_t(k)}\tilde{R}_t(\tilde{\omega}_t(k),k)\,\theta[k-|\tilde{\omega}_t(k)-1|] \\
&\quad\times\Big([k+\tilde{\omega}_t(k)]^2[k-\tilde{\omega}_t(k)+2]^2\tilde{\beta}_+(1-\tilde{\omega}_t(k),k)+[k-\tilde{\omega}_t(k)]^2[k+\tilde{\omega}_t(k)-2]^2\tilde{\beta}_-(1-\tilde{\omega}_t(k),k) \\
&\qquad-\frac{1}{k}[k^2-(1-\tilde{\omega}_t(k))^2]\Big) \\
&+\frac{1}{2}\int\frac{k^2dk}{\tilde{\omega}_l(k)}\tilde{R}_l(\tilde{\omega}_l(k),k)\,\theta[k-|\tilde{\omega}_l(k)-1|] \\
&\quad\times\{[k+\tilde{\omega}_l(k)-2]^2\tilde{\beta}_+(1-\tilde{\omega}_l(k),k)+[k-\tilde{\omega}_l(k)+2]^2\tilde{\beta}_-(1-\tilde{\omega}_l(k),k)\} \\
&-\frac{1}{4}\int_0^\infty\frac{dk}{1-\tilde{\omega}_+(k)}\tilde{R}(\tilde{\omega}_+(k),k)\{[k+\tilde{\omega}_+(k)+1]^2[k-\tilde{\omega}_+(k)+1]^2\tilde{\beta}_t(1-\tilde{\omega}_+(k),k) \\
&\qquad-2k^2[k-\tilde{\omega}_+(k)-1]^2\tilde{\beta}_l(1-\tilde{\omega}_+(k),k)\} \\
&-\frac{1}{4}\int_0^\infty\frac{dk}{1-\tilde{\omega}_-(k)}\tilde{R}(\tilde{\omega}_-(k),k)\{[k-\tilde{\omega}_-(k)-1]^2[k+\tilde{\omega}_-(k)-1]^2\tilde{\beta}_t(1-\tilde{\omega}_-(k),k) \\
&\qquad-2k^2[k+\tilde{\omega}_-(k)+1]^2\tilde{\beta}_l(1-\tilde{\omega}_-(k),k)\} \\
&-\frac{1}{4}\int_{\frac{1}{2}}^\infty dk\int_{1-k}^k\frac{dk_0}{k_0}\Bigg[\tilde{\beta}_t(k_0,k)\Big([k+k_0]^2[k-k_0+2]^2\tilde{\beta}_+(1-k_0,k) \\
&\qquad+[k-k_0]^2[k+k_0-2]^2\tilde{\beta}_-(1-k_0,k)-\frac{1}{k}[k^2-(1-k_0)^2]\Big) \\
&\qquad-2k^2\tilde{\beta}_l(k_0,k)\,[(k+k_0-2)^2\tilde{\beta}_+(1-k_0,k)+(k-k_0+2)^2\tilde{\beta}_-(1-k_0,k)]\Bigg].
\end{aligned}
\tag{5.11}
$$

Here,

$$
\begin{aligned}
\tilde{R}_t(\omega,k) &= \frac{\omega(\omega^2-k^2)}{3r\omega^2-(\omega^2-k^2)^2}, \\
\tilde{R}_l(\omega,k) &= -\frac{\omega(\omega^2-k^2)}{k^2(3r-\omega^2+k^2)}, \\
\tilde{R}(\omega,k) &= -\frac{\omega^2-k^2}{2}, \\
\tilde{\beta}_t(\omega,k) &= \frac{1}{\pi}\frac{r\tilde{\Pi}_t^I}{(\omega^2-k^2+r\tilde{\Pi}_t)^2+(r\tilde{\Pi}_t^I)^2}, \\
\tilde{\beta}_l(\omega,k) &= \frac{1}{\pi}\frac{r\tilde{\Pi}_l^I}{(k^2-r\tilde{\Pi}_l)^2+(r\tilde{\Pi}_l^I)^2}, \\
\tilde{\beta}_+(\omega,k) &= -\frac{1}{\pi}\frac{\tilde{D}_0^I-\tilde{D}_s^I}{(\tilde{D}_0-\tilde{D}_s)^2+(\tilde{D}_0^I-\tilde{D}_s^I)^2}, \\
\tilde{\beta}_-(\omega,k) &= \tilde{\beta}_+(-\omega,k),
\end{aligned}
\tag{5.12}
$$

where $r = {m_g}^2/{m_f}^2 = (\frac{8}{9})(N+N_f/2)/C_f$, and

$$
\begin{aligned}
&\tilde{\Pi}_t^I(\omega,k) = \frac{3\pi\omega(k^2-\omega^2)}{4k^3}, \quad \tilde{\Pi}_l^I(\omega,k) = -\frac{3\pi\omega}{2k}, \\
&\tilde{D}_0^I(k) = \frac{\pi}{2k}, \quad \tilde{D}_s^I(\omega,k) = \frac{\pi\omega}{2k^2}, \\
&\tilde{\Pi}_t(\omega,k) = -\frac{3\omega^2}{2k^2}\left[1+\left(\frac{k^2}{\omega^2}-1\right)\frac{\omega}{2k}\ln\left(\frac{k+\omega}{k-\omega}\right)\right], \\
&\tilde{\Pi}_l(\omega,k) = -3\left[1-\frac{\omega}{2k}\ln\left(\frac{k+\omega}{k-\omega}\right)\right], \\
&\tilde{D}_0(\omega,k) = \omega-\frac{1}{2k}\ln\left(\frac{k+\omega}{k-\omega}\right), \\
&\tilde{D}_s(\omega,k) = k-\frac{1}{k}\left[\frac{\omega}{2k}\ln\left(\frac{k+\omega}{k-\omega}\right)-1\right],
\end{aligned}
\tag{5.13}
$$

with the rescaled dispersion relations which determine $\tilde{\omega}_t$, $\tilde{\omega}_l$, $\tilde{\omega}_+$, and $\tilde{\omega}_-$ given by

$$\tilde{\omega}_t^2 - k^2 - \frac{3r\tilde{\omega}_t^2}{2k^2}\left[1 - \left(1 - \frac{k^2}{\tilde{\omega}_t^2}\right)\frac{\tilde{\omega}_t}{2k}\ln\left(\frac{\tilde{\omega}_t + k}{\tilde{\omega}_t - k}\right)\right] = 0,$$
$$k^2 + 3r\left[1 - \frac{\tilde{\omega}_l}{2k}\ln\left(\frac{\tilde{\omega}_l + k}{\tilde{\omega}_l - k}\right)\right] = 0,$$
$$\tilde{\omega}_+ = k + \frac{1}{k}\left[1 + \frac{1}{2}\left(1 - \frac{\tilde{\omega}_+}{k}\right)\ln\left(\frac{\tilde{\omega}_+ + k}{\tilde{\omega}_+ - k}\right)\right], \tag{5.14}$$
$$\tilde{\omega}_- = -k + \frac{1}{k}\left[-1 + \frac{1}{2}\left(1 + \frac{\tilde{\omega}_-}{k}\right)\ln\left(\frac{\tilde{\omega}_- + k}{\tilde{\omega}_- - k}\right)\right].$$

The first four terms in Eq. (5.11) are pole–pole contributions, the fifth to eighth are pole–cut terms, and the last is the cut–cut term. The expressions in Eq. (5.11) were evaluated numerically with the following results. For SU(3) with $N_f = 2$ no pole–pole terms contribute and $a(N, N_f) = 1.399$. For SU(2) with $N_f = 2$, one pole–pole term from the transverse-gluon and negative fermion mode contributes and $a(N, N_f) = 1.451$. For SU(2) with $N_f = 4$, all four pole–pole terms contribute and $a(N, N_f) = 1.573$.

VI. SUMMARY

We have examined the calculation of the lowest-order damping rate of heavy and massless fermions in hot gauge theories in the long-wavelength limit using the resummation techniques developed recently in terms of hard thermal loops. An approach was used to evaluate the Matsubara frequency sum and extract the imaginary part of the resulting analytically continued expressions which gave the same final results as those found by dispersion relation methods [3, 10]. Apart from questions raised recently concerning the neglect in some gauges of certain terms proportional to the mass-shell condition [11, 15], gauge-independent results for this rate were obtained in a wide class of gauges. Ward identities between the effective propagators and vertices simplified significantly the calculations, and the answers obtained, both in magnitude and in sign, indicate that these techniques give tractable and reasonable results.

ACKNOWLEDGMENTS

We thank A. Rebhan and J. C. Taylor for discussions, and also E. Braaten and R. Pisarski for discussions and for pointing out an error in an earlier version. This work was supported by the Natural Sciences and Engineering Research Council of Canada.

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