Effective potential, renormalization, and nontrivial ultraviolet fixed point in D-dimensional four-fermion theories ($2 < D < 4$) to order $1/N$ in $1/N$ expansion

Hong-Jian He
*China Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, China and Department of Physics, Tsinghua University, Beijing 100084, China**

Yu-Ping Kuang, Qing Wang, and Yu-Ping Yi
China Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, China; Department of Physics, Tsinghua University, Beijing 100084, China; and Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China*

(Received 22 April 1991)

A systematic calculation of the effective potential, renormalization, the nontrivial ultraviolet fixed point, and dynamical symmetry breaking in four-fermion theories in space-time dimension $2 < D < 4$ is presented up to next-to-the-leading order in the $1/N$ expansion. It is shown that the order-$1/N$ correction definitely increases the anomalous dimension of the composite operator $\bar{\Psi}\Psi$ at the fixed point. Some general conclusions on the relation between nontrivial fixed points and the phase transitions in dynamical symmetry breaking are also given.

PACS number(s): 11.10.Gh, 11.15.Pg

I. INTRODUCTION

In recent years, the study of theories with a nontrivial ultraviolet (UV) fixed point and a large anomalous dimension has become one of the attractive subjects in quantum field theory. From the practical point of view, gauge theories with a nontrivial UV fixed point and large anomalous dimension give rise to the possibility of constructing walking technicolor theories which may avoid some fundamental difficulties of technicolor theories (the large anomalous dimension reduces the flavor-changing neutral current and increases the pseudo-Goldstone-boson masses) [1] and constructing the top-mode standard model (dynamical Higgs model based on the top-quark condensate) [2]. Theoretically, a nonperturbative study of field theories with a nontrivial UV fixed point is interesting in its own right. As a field-theory model the four-fermion theory is of special interest since it provides a simple and clear scenario of dynamical symmetry breaking (DSB) [3,4] and it is an important ingredient in walking technicolor dynamics and the top-mode standard model. Furthermore, it is also closely related to condensed matter physics. In perturbation theory, it is well known that the four-fermion theory is renormalizable only if the space-time dimension D is $D \le 2$. However, in an interesting paper, Wilson [5] showed, in the large-N limit, that even if $2 < D < 4$ the four-fermion theory may have a finite continuous limit with a special choice of the bare coupling constant. Moreover, Rosenstein, Warr, and Park [6] examined the broken-symmetry phase in the case of $D = 3$ and proved, in the framework of effective field theory, that the 3-dimensional four-fermion theory is renormalizable to any order in the $1/N$ expansion. This is interesting because it provides an example of nonperturbative renormalization which exceeds the perturbative renormalizability limit $D \le 2$, and, moreover, the $1/N$ expansion is a widely used nonperturbative approach so that much interesting physics can be discussed in this model (especially pions, as pseudo Goldstone bosons, exist when the space-time dimension is greater than two) [7]. Recently, a more transparent description has been given by Kikukawa and Yamawaki [8] and the space-time dimension is generalized to $2 < D < 4$ so that the case of D very close to 4 can be examined. In Ref. [8], the calculation is given in the $N \to \infty$ limit with a particular choice of the renormalization scheme. They have studied explicitly the nontrivial UV fixed point and the anomalous dimension of the composite operator $\bar{\Psi}\Psi$ which may give important hints to the study of walking technicolor theories and the top-mode standard model. The obtained anomalous dimension $\gamma_{\bar{\Psi}\Psi}$ at the critical point is $D-2$ which is really large when D is close to 4 from below. Since in the real physical problems N is not so large, a further study up to next-to-leading order in the $1/N$ expansion is actually needed for examining whether the largeness of $\gamma_{\bar{\Psi}\Psi}$ can still be maintained when order-$1/N$ corrections are taken into account.

In this paper we give a systematic study of the effective potential, renormalization, nontrivial UV fixed point, DSB and the anomalous dimension of the composite operator in four-fermion theories with discrete and continuous chiral symmetries in space-time dimension $2 < D < 4$ up to order $1/N$ in the $1/N$ expansion. We first give general results without specifying the renormalization scheme and then take a convenient subtraction scheme which is easy to implement to higher orders in $1/N$ expansion to obtain the order-$1/N$ corrections to the UV fixed point and $\gamma_{\bar{\Psi}\Psi}$. An interesting result of our cal-

*Mailing address.

culation is that the order-$1/N$ contribution definitely *increases* the anomalous dimension of $\bar{\Psi}\Psi$ at the fixed point. This supports the study of models based on the idea of a large anomalous dimension of $\bar{\Psi}\Psi$ [1,2]. We will also give, in this paper, some general conclusions on the relation between the nontrivial fixed points and the phase transitions in dynamical symmetry breaking.

If only the simplest composite operator $\bar{\Psi}\Psi$ is concerned, the effective potential can be calculated either in the conventional auxiliary field formalism [4] or in a new formalism dealing directly with $\bar{\Psi}\Psi$ [9,10]. However, if we want to consider further the vacuum expectation values (VEV's) of more complicated composite operators, the latter formalism is more convenient than the former one [10,11] since introducing an auxiliary field for a higher-dimensional composite operator will lead to the appearance of higher-dimensional interactions (superficially nonrenormalizable) which causes unnecessary complications. For the sake of further applications, we will take, in this paper, the formalism given in Ref. [10].

This paper is organized as follows. In Sec. II we consider the four-fermion theory with discrete chiral symmetry in space-time dimension $2<D<4$. We first give (in the $N\to\infty$ limit) a systematic way of determining the renormalization constants based on the property of the effective potential. This shows explicitly the renormalizability of the theory. From the obtained renormalization constants we calculate the UV fixed point and the anomalous dimension of $\bar{\Psi}\Psi$. Then we calculate the order-$1/N$ contributions. Section III deals with the four-fermion theory with a continuous chiral symmetry in dimension $2<D<4$. Some general conclusions on the relation between the nontrivial fixed points and the phase transitions in dynamical symmetry breaking will be given in Sec. IV. The final section (Sec. V) is a concluding remark.

II. FOUR-FERMION THEORY WITH DISCRETE CHIRAL SYMMETRY

Consider the simplest four-fermion theory in space-time dimension $2<D<4$:

$$\mathcal{L}(\bar{\Psi},\Psi)=\bar{\Psi}_a i\partial\!\!\!/\Psi_a+\frac{\hat{g}}{2N}(\bar{\Psi}_a\Psi_a)^2 , \tag{1}$$

where $a=1,2,\ldots,N$ is the subscript for the internal degree of freedom. $\Psi_a(x)$ is a four-component Dirac spinor [8], and the coupling constant $\hat{g}$ has dimension $2-D$. This theory is symmetric under the discrete chiral transformation $\Psi_a(x)\to\gamma_5\Psi_a(x)$ [12]. To study the dynamical breaking of this discrete chiral symmetry, we consider the generating functional

$$\begin{aligned}Z[I,\bar{I},K]&=\exp(iW[I,\bar{I},K])\\&=\int\mathcal{D}\Psi\,\mathcal{D}\bar{\Psi}\\&\quad\times\exp\left\{i\int d^Dx[\mathcal{L}(\bar{\Psi},\Psi)+\bar{I}\Psi+\bar{\Psi}I\right.\\&\qquad\left.+K(\hat{g}/N)\bar{\Psi}\Psi+P(K)]\right\} ,\end{aligned} \tag{2}$$

where I, $\hat{I}$, and K are external sources. $P(K)$ is a pure source term which should be

$$P(K)=\frac{\hat{g}}{2N}K^2 \tag{3}$$

in order to satisfy the consistency condition [9]. The classical fields ψ, $\bar{\psi}$, and Σ are defined by

$$\frac{\delta W}{\delta I}\equiv -i\bar{\psi},\quad \frac{\delta W}{\delta \bar{I}}\equiv\psi,\quad \frac{\delta W}{\delta K}\equiv\frac{\hat{g}}{N}\bar{\psi}\psi+\Sigma , \tag{4}$$

where Σ is the connected part of the classical field of $(\hat{g}/N)\bar{\Psi}\Psi$ and its value at $I,\bar{I},K=0$ gives the VEV of $(\hat{g}/N)\bar{\Psi}\Psi$. We first make a partial Legendre transformation

$$\Gamma^P[\psi,\bar{\psi},K]=W[I,\bar{I},K]-\int d^Dx(\bar{I}\psi+\bar{\psi}I) . \tag{5}$$

The formula for Γ^P given in Ref. [10] for the present model is

$$\begin{aligned}\Gamma^P[\psi,\bar{\psi},K]=&\int d^Dx[\mathcal{L}(\bar{\psi},\psi)+K(\hat{g}/N)\bar{\psi}\psi+(\hat{g}/2N)K^2]-i\,\mathrm{Tr}\ln iG_s^{-1}\\&-\frac{N}{2\hat{g}}\int d^Dx\,\Delta^2-i\left\langle 0\left|T\exp\left[i\int d^Dx\,\mathcal{L}_I\right]\right|0\right\rangle_{\mathrm{P2PI}(\Pi_s)} ,\end{aligned} \tag{6}$$

in which the trace Tr is in the functional sense, the quantity Δ is defined by

$$\Delta\equiv-\frac{\delta\Gamma_L^P}{\delta K}=\frac{\hat{g}}{N}\mathrm{Tr}G , \tag{7}$$

where Γ_L^P is the loop contribution to Γ^P and G is the physical propagator, Π_s in (6) is a specially chosen part of the proper self-energy independent of the external momentum

$$\Pi_s=-i\Delta ;$$

G_s in (6) is a special propagator with Π_s as the self-energy which is now

$$iG_s^{-1}=i\partial\!\!\!/+\frac{\hat{g}}{N}K-\Delta ;$$

the interaction Lagrangian $\mathcal{L}_I$ is defined as the terms trilinear and quadrulinear in Ψ and $\bar{\Psi}$ in

$\mathcal{L}(\bar{\psi}+\bar{\Psi},\psi+\Psi)+K(\hat{g}/N)(\bar{\psi}+\bar{\Psi})(\psi+\Psi)$ and the last term in (6) means the sum of all partially two-particle-irreducible vacuum diagrams with respect to Π_s [P2PI(Π_s)] defined in Ref. [10] except the P2PI(Π_s) diagram $(\hat{g}/2N)[\mathrm{Tr}G_s]^2$ (in the diagram, the propagator is G_s and the vertex is determined by $\mathcal{L}_I$).

From (3), (4), and (7) we have

$$\frac{\hat{g}}{N}K-\Delta=\Sigma \tag{8}$$

with which the propagator G_s can be written as

$$iG_s^{-1}=i\partial\!\!\!/+\Sigma\ .$$

The effective action Γ is then obtained via a further Legendre transformation

$$\Gamma[\psi,\bar{\psi},\Sigma]=\Gamma^P[\psi,\bar{\psi},K]-\int d^Dx\, K\left[\frac{\hat{g}}{N}\bar{\psi}\psi+\Sigma\right], \tag{9}$$

from which we get the effective potential

$$V_{\rm eff}[\psi,\bar{\psi},\Sigma]=-\Gamma[\psi,\bar{\psi},\Sigma]\Big/\int d^Dx\ . \tag{10}$$

Since the VEV's $\langle\Psi\rangle$ and $\langle\bar{\Psi}\rangle$ vanish eventually, we shall ignore ψ and $\bar{\psi}$ in the following calculation, i.e., $V_{\rm eff}=V_{\rm eff}[\Sigma]$.

Our system of doing the nonperturbative renormalization is based on the property of the effective potential. Let us define the renormalization constants Z_Σ, $Z_{\hat{g}}$ and the renormalized quantities Σ_R, $\hat{g}_R$ as

$$\Sigma=Z_\Sigma\Sigma_R,\ \hat{g}=Z_{\hat{g}}\hat{g}_R\ . \tag{11}$$

We know that $V_{\rm eff}$ (or Γ) is the generating functional for proper vertices which are related to the S-matrix elements. If the theory is renormalizable, $V_{\rm eff}$ should be independent of the momentum cutoff Λ after the renormalization of Σ and $\hat{g}$ [9], i.e.,

$$V_{\rm eff}[\Sigma(\Lambda),\hat{g}(\Lambda),\Lambda]=V_{\rm eff}[Z_\Sigma(\Lambda,\mu)\Sigma_R(\mu),Z_{\hat{g}}(\Lambda,\mu)\hat{g}_R(\mu),\Lambda]$$
$$=V_{\rm eff}[\Sigma_R(\mu),\hat{g}_R(\mu),\mu]\ , \tag{12}$$

where μ is an arbitrary scale parameter serving as the subtraction point in the renormalization. We shall see that Eq. (12) will determine the renormalization constants Z_Σ and $Z_{\hat{g}}$. For convenience, we introduce the dimensionless coupling constants [5,8]

$$g\equiv\hat{g}\Lambda^{D-2},\quad g_R\equiv\hat{g}_R\mu^{D-2}\ . \tag{13}$$

Then the renormalization constant Z_g defined as

$$g=Z_g g_R \tag{14a}$$

is

$$Z_g=\left[\frac{\Lambda}{\mu}\right]^{D-2}Z_{\hat{g}}\ . \tag{14b}$$

Now we show the details of the calculation.

A. Leading order

To leading order of the $1/N$ expansion, the last term in (6) does not contribute. So we have

$$V_{\rm eff}[\Sigma]=\tfrac{1}{2}N\hat{g}^{-1}\Sigma^2+iN\,\mathrm{tr}\int\frac{d^Dk}{(2\pi)^D}\ln(-k\!\!\!/+\Sigma)$$
$$=\tfrac{1}{2}N\Sigma^2[\hat{g}^{-1}-F(D)\Lambda^{D-2}+G(D)|\Sigma|^{D-2}]\ , \tag{15a}$$

where the trace tr is taken with respect to the spin degree of freedom, and

$$F(D)\equiv\frac{8}{(4\pi)^{D/2}\Gamma(D/2)(D-2)}\ ,$$
$$G(D)\equiv\frac{16\Gamma(2-D/2)}{(4\pi)^{D/2}D(D-2)}\ . \tag{15b}$$

From (11) and (15a) we have

$$V_{\rm eff}=\tfrac{1}{2}NZ_\Sigma^2\Sigma_R^2[Z_{\hat{g}}^{-1}\hat{g}_R^{-1}-F(D)\Lambda^{D-2}+G(D)Z_\Sigma^{D-2}|\Sigma_R|^{D-2}]\ . \tag{16}$$

We see that the requirement (12) can be satisfied if Z_Σ and $[Z_{\hat{g}}^{-1}\hat{g}_R^{-1}-F(D)\Lambda^{D-2}]$ are Λ independent, and this is certainly possible. This explicitly illustrates that the theory is renormalizable. Actually a finite Z_Σ (μ independent) may be absorbed into the definition of Σ_R so that we can take, to this order,

$$Z_\Sigma=1\ . \tag{17}$$

Indeed, to leading order, Σ is just the physical mass of the fermion which should be renormalization-group invariant so that it is not renormalized, just as (17) shows. The Λ independence of $[Z_{\hat{g}}^{-1}\hat{g}_R^{-1}-F(D)\Lambda^{D-2}]$ can be written as

$$\hat{g}^{-1}-F(D)\Lambda^{D-2}=\hat{g}_R^{-1}-A_0\mu^{D-2}\ , \tag{18a}$$

where A_0 is a finite dimensionless constant to be determined by the renormalization condition defining $\hat{g}_R$ at the subtraction point; i.e., A_0 depends on the renormalization scheme. The determination of A_0 will be considered later. (18a) gives

$$Z_{\hat{g}}=1-\hat{g}F(D)\Lambda^{D-2}+\hat{g}A_0\mu^{D-2}\ , \tag{18b}$$

or

$$Z_g=[1-gF(D)](\Lambda/\mu)^{D-2}+gA_0\ . \tag{18c}$$

The renormalization-group β function $\beta(g_R)$ can be calculated directly from (18c):

$$\beta(g_R)\equiv\mu\frac{\partial}{\partial_\mu}g_R\bigg|_{g,\Lambda}=(D-2)g_R(1-g_R/g_R^*)\ , \tag{19}$$

where

$$g_R^*\equiv A_0^{-1}\ . \tag{20}$$

We see that g_R^* is a nontrivial UV fixed point of the theory. (18b) and (19) coincide with the forms obtained in Ref. [8], while, instead of taking a particular subtraction scheme, we give a general scheme-dependent expression for g_R^* here [cf. (20)].

Using (17) and (18) we can express the effective potential (15a) in terms of the renormalized quantities Σ_R and g_R:

$$V_{\rm eff}(\Sigma_R)=\tfrac{1}{2}N\mu^D g_R^{-1}(\Sigma_R/\mu)^2 \times[1+g_R G(D)|\Sigma_R/\mu|^{D-2}-g_R A_0]\ . \tag{21}$$

(21) holds for arbitrary values of g_R. The physical vacuum should be determined by the minimum of $V_{\rm eff}|\Sigma_R|$, i.e., the solution of $dV_{\rm eff}/d\Sigma_R=0$ which is now

$$|\Sigma_R/\mu|[1-g_R/g_R^*+\tfrac{1}{2}g_R DG(D)|\Sigma_R/\mu|^{D-2}]=0\ . \tag{22}$$

(22) has two possible solutions. It is easy to check from (21) that the true minimum is

$$|\Sigma_R|=\begin{cases}0, & g_R\le g_R^*\ ,\\ \mu\left[\dfrac{2(g_R^{*\,-1}-g_R^{-1})}{DG(D)}\right]^{1/(D-2)}, & g_R>g_R^*\ .\end{cases} \tag{23}$$

Therefore the discrete chiral symmetry is unbroken if $g_R<g_R^*$, while it is dynamically broken if $g_R>g_R^*$. g_R^* is just the critical point separating the two phases. The renormalized critical coupling constant obtained in Ref. [8] is different from the bare critical coupling constant g_c. Actually this is not essential since g_R^* is renormalization scheme dependent.

The renormalization of the external source K and the composite operator $\bar\Psi\Psi$ can be written as

$$K=Z_K K_R,\quad (\bar\Psi\Psi)=Z_{\bar\Psi\Psi}(\bar\Psi\Psi)_R\ . \tag{24}$$

Since K couples to $\bar\Psi$ and Ψ in exactly the same way as the composite operator $\bar\Psi\Psi$ does, we must have

$$Z_K=Z_{\bar\Psi\Psi}\ . \tag{25}$$

We know that Σ is the connected piece of $\langle(\hat g/N)\bar\Psi\Psi\rangle$. Since Σ is now independent of Λ [cf. (17)], we have, apart from an irrelevant constant factor,

$$Z_{\bar\Psi\Psi}=Z_{\hat g}^{-1}=(\Lambda/\mu)^{D-2}Z_g^{-1}\ . \tag{26}$$

From (26) we obtain the anomalous dimension of the composite operator $\bar\Psi\Psi$:

$$\gamma_{\bar\Psi\Psi}=-\mu\frac{\partial}{\partial\mu}\ln Z_{\bar\Psi\Psi}\bigg|_{g,\Lambda}=(D-2)g_R/g_R^*\ . \tag{27}$$

This coincides with the result in Ref. [8]. At the critical point, $\gamma_{\bar\Psi\Psi}=D-2$, which is independent of the renormalization scheme as it should be [4]. Note that the results (19), (21), and (27) hold for both $g_R>g_R^*$ and $g_R\le g_R^*$.

Now we consider the determination of A_0. A natural and simple way of normalizing $\hat g_R$ at the subtraction point μ is to refer to the tree-level relation

$$\frac{d^2V_{\rm eff}}{d\Sigma_R^2}\bigg|_{\Sigma_R=\mu}=N\hat g_R^{-1}\ . \tag{28}$$

From (21) and (28) we obtain

$$A_0=\frac{8(D-1)\Gamma(2-D/2)}{(4\pi)^{D/2}(D-2)}\ . \tag{29}$$

The critical coupling constant g_R^* is then given by (20) and (29). The subtraction scheme in Ref. [8] is different from (28). It corresponds to

$$A_0=\frac{8(D-1)\Gamma(2-D/2)B(D/2,D/2)}{(4\pi)^{D/2}(D-2)}\ , \tag{30}$$

which differs from ours by a finite renormalization. Since (28) is simple, we shall take this scheme throughout the calculation to determine the order-$1/N$ corrections.

From (15b) and (16) we see that the coefficient $G(D)$ blows up when $D=4$ so that the renormalized theory does not hold in the exact 4-dimensional case unless $\hat g_R=0$ (a trivial theory). However, we can examine the behavior of the theory when D is close to 4 from below. It is interesting to notice that g_R^* [cf. (20) and (29)], $\gamma_{\bar\Psi\Psi}|_{g_R=g_R^*}$ [cf. (27)], and $|\Sigma_R|$ [cf. (23)] are smooth functions of D when D is close to 4. In particular, when D is close to 4, the renormalization-scheme-independent quantity $\gamma_{\bar\Psi\Psi}|_{g_R=g_R^*}$ is close to 2.

B. Next-to-leading order

To order $1/N$, the effective potential is

$$V_{\rm eff}[\Sigma]=\tfrac{1}{2}N\Sigma^2[\hat g^{-1}F(D)\Lambda^{D-2}+G(D)|\Sigma|^{D-2}]+U\ , \tag{31}$$

where U is the order-$1/N$ correction to $V_{\rm eff}$ contributed by the P2PI(Π_s) diagrams shown in Fig. 1, and it is

$$U=-\frac{i}{2}\int\frac{d^Dk}{(2\pi)^D}\ln[1+iB(k)] \tag{32a}$$

with

$$B(k)\equiv 4i\hat g\int\frac{d^Dp}{(2\pi)^D}\frac{p^2-p\cdot k-\Sigma^2}{(p^2+\Sigma^2)[(p-k)^2+\Sigma^2]}\ . \tag{32b}$$

In (32) the momenta are Euclidean. The integration (32b) can be carried out with the standard technique and the result is

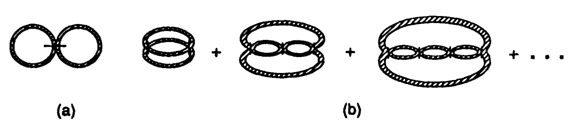

FIG. 1. P2PI(Π_s) diagrams contributed to the last term in (6). The shaded line stands for the propagator G_s, and the bar at the vertex indicates that the indices of the two fermion lines on the same side are paired.

$$B(k)=i\hat{g}\left\{F(D)\Lambda^{D-2}-\frac{1}{2}DG(D)|\Sigma|^{D-2}-\frac{1}{8}D(D-2)G(D)[Q(k,\Sigma)]^{D-1}k^{D-2}\right.$$
$$\left.\times\left[B\left(\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1+Q^{-1}(k,\Sigma)]\right)-B\left(\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1-Q^{-1}(k,\Sigma)]\right)\right]\right\}, \quad (33)$$

where

$$Q(k,\Sigma)\equiv\left[1+\frac{4\Sigma^2}{4k}\right]^{1/2}.$$

Here we have introduced the incomplete β function

$$B(p,q;z)\equiv\int_0^z t^{p-1}(1-t)^{q-1}dt\ . \quad (34)$$

Evidently

$$B(p,q;1)=B(p,q),\quad B(p,q;0)=0\ .$$

To order $1/N$ the renormalization constants Z_Σ and Z_g will now have the form

$$Z_\Sigma=1+b/N\ , \quad (35a)$$

$$Z_g=[1-gF(D)](\Lambda/\mu)^{D-2}+g[A_0+A_1/N]+gc/N\ , \quad (35b)$$

where b and c are Λ-dependent constants which will be determined by the requirement (12), and A_1 is a finite constant to be determined by the normalization condition (28). With (11) and (35), we can write V_{eff} and U as

$$V_{\text{eff}}[\Sigma_R]=\frac{1}{2}N\mu^D g_R^{-1}\left|\frac{\Sigma_R}{\mu}\right|^2\left[1+\frac{2b}{N}\right]\left[1+g_R G(D)\left|\frac{\Sigma_R}{\mu}\right|^{D-2}\left[1+\frac{b}{N}\right]^{D-2}-g_R\left[A_0+\frac{A_1}{N}\right]-g_R\frac{c}{N}\right]+U\ , \quad (36a)$$

$$U=\frac{1}{(4\pi)^{D/2}\Gamma(D/2)}\int_0^\Lambda dk\, k^{D-1}\ln\left\{1-\frac{g_R}{1+g_R[F(D)(\Lambda/\mu)^{(D-2)}-A_0]}\right.$$
$$\times\left\{F(D)\left[\frac{\Lambda}{\mu}\right]^{D-2}-\frac{1}{2}DG(D)\left|\frac{\Sigma_R}{\mu}\right|^{D-2}\right.$$
$$-\frac{1}{8}D(D-2)G(D)\left[\frac{k}{\mu}\right]^{D-2}[Q(k,\Sigma_R)]^{D-1}$$
$$\times\left[B\left(\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1+Q^{-1}(k,\Sigma_R)]\right)\right.$$
$$\left.\left.\left.-B\left(\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1-Q^{-1}(k,\Sigma_R)]\right)\right]\right\}\right\} \quad (36b)$$

To do the renormalization, we need to extract the divergent part from U. First we notice that U can be written as

$$U=U_1+U_2\ , \quad (37)$$

$$U_1\equiv\frac{1}{(4\pi)^{D/2}\Gamma(D/2)}\int_0^\Lambda dk\, k^{D-1}\ln\left[1-\frac{g_R F(D)(\Lambda/\mu)^{D-2}}{1+g_R[F(D)(\Lambda/\mu)^{D-2}-A_0]}\right], \quad (37a)$$

$$U_2\equiv\frac{1}{(4\pi)^{D/2}\Gamma(D/2)}\int_0^\Lambda dk\, k^{D-1}\ln\left\{1+\frac{g_R}{2(1-g_R A_0)}\left\{DG(D)\left|\frac{\Sigma_R}{\mu}\right|^{D-2}\right.\right.$$
$$+\frac{1}{4}D(D-2)G(D)\left[\frac{k}{\mu}\right]^{D-2}[Q(k,\Sigma_R)]^{D-1}$$
$$\times\left[B\left(\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1+Q^{-1}(k,\Sigma_R)]\right)\right.$$
$$\left.\left.\left.-B\left(\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1-Q^{-1}(k,\Sigma_R)]\right)\right]\right\}\right\}. \quad (37b)$$

U_1 is a Σ_R-independent constant, so that it can be simply subtracted from $V_{\rm eff}$ without affecting physics.

In the special case of $D=3$, (37b) reduces to

$$U_2=\frac{1}{4\pi^2}\int_0^\Lambda dk\, k^2 \ln\left[\mu+\frac{g_R}{1-g_R A_0}|\Sigma_R|+\frac{g_R}{1-g_R A_0}\frac{k^2+4|\Sigma_R|^2}{2k}\arctan\frac{k}{2|\Sigma_R|}\right],$$

which is of the same form as the result given in Ref. [13] but with A_0 unspecified.

To extract further the divergent part from U_2 [cf. (37b)] it is sufficient to put

$$U_2=\int\frac{dU_2}{d\Lambda}d\Lambda+(\Lambda-\text{independent terms})$$

and work out only the part $\int(dU_2/d\Lambda)d\Lambda$. After lengthy calculations, we get

$$U_2=U_2^{(1)}+U_2^{(2)}+(\text{finite terms})\,, \tag{38a}$$

$$U_2^{(1)}\equiv\frac{1}{(4\pi)^{D/2}\Gamma(D/2)}\int d\Lambda\,\Lambda^{D-1}\ln[1+H(g_R,D)\Lambda^{D-2}]\,, \tag{38b}$$

$$U_2^{(2)}\equiv\frac{1}{4}(D-1)F(D)\mu^D\left|\frac{\Sigma_R}{\mu}\right|^2\left[\left[\frac{\Lambda}{\mu}\right]^{D-2}-\left[\frac{D-2}{H(g_R,D)}+\frac{8}{DB(D/2-1,D/2-1)}\left|\frac{\Sigma_R}{\mu}\right|^{D-2}\right]\ln\frac{\Lambda}{\mu}\right], \tag{38c}$$

where

$$H(g_R,D)\equiv\frac{2g_R\Gamma(2-D/2)B(D/2-1,D/2-1)}{(4\pi)^{D/2}(1-g_R A_0)}\,. \tag{39}$$

The term $U_2^{(1)}$ is also a Σ_R-independent constant and can be simply subtracted from $V_{\rm eff}$. The only relevant divergent terms are those in $U_2^{(2)}$ which have exactly the same type of $|\Sigma_R/\mu|$ dependence as those in the leading order $V_{\rm eff}$ [cf. (36a)]. Therefore the divergences can be completely eliminated by suitable choices of b and c which are

$$b=\frac{\ln(\Lambda/\mu)}{DB(D/2-1,2-D/2)B(D/2,D/2)}\,,$$
$$c=\tfrac{1}{2}(D-1)F(D)\left[\frac{\Lambda}{\mu}\right]^{D-2}-\frac{D(-2)\ln(\Lambda/\mu)}{g_R DB(D/2-1,2-D/2)B(D/2,D/2)}[1-g_R A_0]\,. \tag{40}$$

Similar to (17), we have ignored in b a possible finite constant which is irrelevant in the calculation of $\beta(g_R)$ and $\gamma_{\bar\Psi\Psi}$. The finite $V_{\rm eff}[\Sigma_R]$ can then be written as

$$V_{\rm eff}[\Sigma_R]=\frac{1}{2}N\mu^D g_R^{-1}\left|\frac{\Sigma_R}{\mu}\right|^2\left[1+g_R G(D)\left|\frac{\Sigma_R}{\mu}\right|^{D-2}-g_R\left[A_0+\frac{A_1}{N}\right]\right]+U_f\,, \tag{41a}$$

where U_f is obtained from subtracting the Λ-dependent part from U; i.e.,

$$\begin{aligned}U_f&\equiv U-U_1-U_2^{(1)}-U^2\\&=\frac{1}{(4\pi)^{D/2}\Gamma(D/2)}\Bigg\{\int_0^\Lambda dk\,k^{D-1}\ln\Bigg[\Bigg[1+\frac{g_R}{2(1-g_R A_0)}\Bigg\{DG(D)\left|\frac{\Sigma_R}{\mu}\right|^{D-2}\\&\qquad+\frac{1}{4}D(D-2)G(D)\left[\frac{k}{\mu}\right]^{D-2}[Q(k,\Sigma_R)]^{D-1}\\&\qquad\times\left[B\left[\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1+Q^{-1}(k,\Sigma_R)]\right]\right.\\&\qquad\left.-B\left[\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1-Q^{-1}(k,\Sigma_R)]\right]\right]\Bigg\}\Bigg]\\&\qquad\times\left[1+H(g_R,D)\left[\frac{k}{\mu}\right]^{D-2}\right]^{-1}\Bigg]\\&\qquad-\frac{2(D-1)\mu^D}{D-2}\left[\left[\frac{\Lambda}{\mu}\right]^{D-2}-\frac{(D-2)\ln\Lambda/\mu}{H(g_R,D)}\right]\left|\frac{\Sigma_R}{\mu}\right|^2+\frac{4\mu^D\ln\frac{\Lambda}{\mu}}{DB(D/2,D/2)}\left|\frac{\Sigma_R}{\mu}\right|^D\Bigg\}\,.\end{aligned} \tag{41b}$$

The constant A_1 determined from (41) and (28) is

$$A_1=-2^{D-3}(D-1)A_0=-\frac{2^D(D-1)^2\Gamma(2-D/2)}{(4\pi)^{D/2}(D-2)} . \tag{42}$$

The above calculation shows explicitly how the four-fermion theory is renormalized up to order $1/N$. From (35) and (40) we get the explicit formulas for Z_Σ and Z_g:

$$Z_\Sigma=1+\frac{\ln\frac{\Lambda}{\mu}}{NDB(D/2-1,2-D/2)B(D/2,D/2)} , \tag{43a}$$

$$Z_g=\left[1-gF(D)\left[1-\frac{D-1}{2N}\right]-\frac{(D-2)\ln\frac{\Lambda}{\mu}}{NDB(D/2-1,2-D/2)B(D/2,D/2)}[1-gF(D)]\right]\left[\frac{\Lambda}{\mu}\right]^{D-2}$$
$$+g\left[A_0+\frac{A_1}{N}\right] . \tag{43b}$$

The renormalization-group β function up to order-$1/N$ can be directly calculated from (43b), and it is

$$\beta(g_R)=(D-2)g_R\left[1-\frac{g_R}{g_R^*}\right]\left[1-\frac{1}{NDB(D/2-1,2-D/2)B(D/2,D/2)}\right] , \tag{44}$$

where

$$g_R^*=A_0^{-1}\left[1+\frac{2^{D-3}(D-1)}{N}\right] . \tag{45}$$

Note that the correction $-[NDB(D/2-1,2-D/2)B(D/2,D/2)]^{-1}$ is *independent* of the renormalization scheme which determines A_0+A_1/N.

From (41) we can calculate the extremum condition $dV_{\rm eff}/D\Sigma_R=0$ which is

$$\left|\frac{\Sigma_R}{\mu}\right|\left[\left[1-\frac{g_R}{g_R^*}+\frac{1}{2}g_R DG(D)\left[1-\frac{2^{D-3}}{N}(D-1)\right]\right]\left|\frac{\Sigma_R}{\mu}\right|^{D-2}\right]=0 .$$

It is easy to check that the true minimum of $V_{\rm eff}$ is

$$\Sigma_R=\begin{cases}0, & g_R\le g_R^* ,\\ \mu\left[\dfrac{2(g_R^{*-1}-g_R^{-1})}{DG(D)[1-2^{D-3}(D-1)/N]}\right]^{1/(D-1)} , & g_R>g_R^* .\end{cases} \tag{46}$$

We see that there are still two phases separated by the nontrivial UV fixed point g_R^* which is increased by an amount of $2^{D-3}(D-1)/NA_0$ by the order-$1/N$ correction.

The renormalization constant $Z_{\bar\Psi\Psi}$ can still be determined from the relation between Σ and $\langle(\hat g/N)\bar\Psi\Psi\rangle$. Now the effect of Z_Σ [cf. (43a)] should be taken into account. The result is

$$Z_{\bar\Psi\Psi}=\left[1+\frac{\ln\frac{\Lambda}{\mu}}{NDB(D/2-1,2-D/2)B(D/2,D/2)}\right]\left[\frac{\Lambda}{\mu}\right]^{D-2}Z_g^{-1} , \tag{47}$$

from which we obtain

$$\gamma_{\bar\Psi\Psi}=(D-2)\frac{g_R}{g_R^*}+\frac{1}{NDB(D/2-1,2-D/2)B(D/2,D/2)}\left[(D-2)\left[1-\frac{g_R}{g_R^*}\right]+1\right] \tag{48}$$

At the critical point $g_R=g_R^*$, the anomalous dimension is *increased* by an amount of $[NDB(D/2-1,2-D/2)B(D/2,D/2)]^{-1}$ by the order-$1/N$ correction which is *independent* of the renormalization scheme. This comes from the term b/N in Z_Σ [Eq. (35a)]. The fact that the order-$1/N$ correction *increases* but *does not decrease* $\gamma_{\bar\Psi\Psi}$ is interesting. It supports the study of models based on the effect of large $\gamma_{\bar\Psi\Psi}$ at the critical point [1,2]. It is interesting to notice that when D is close to 4 from below, the order-$1/N$ corrections to $\beta(g_R)$ and

$\gamma_{\bar{\Psi}\Psi}|_{g_R=g_R^*}$ [cf. (44) and (48)] decrease quickly with increasing D [due to the increase of $B(D/2-1,2-D/2)$], so that we still have $\gamma_{\bar{\Psi}\Psi}|_{g_R=g_R^*}\sim 2$ when D is close to 4 from below.

III. FOUR-FERMION THEORY WITH CONTINUOUS CHIRAL SYMMETRY

The above calculations can be generalized to the four-fermion theory [4]

$$\mathcal{L}(\Psi,\bar{\Psi})=\bar{\Psi}_a i\,\partial\!\!\!/\Psi_a+\frac{\hat{g}}{2N}[(\bar{\Psi}_a\Psi_a)^2-(\bar{\Psi}_a\gamma_5\Psi_a)^2]\ , \tag{49}$$

which is symmetric under a chiral U(1)×U(1) transformation. Now we are going to consider two composite operators $(\hat{g}/N)\bar{\Psi}\Psi$ and $(\hat{g}/N)\bar{\Psi}\gamma_5\Psi$. The generating functional is now

$$\begin{aligned}Z[I,\bar{I},K,K_5]&=\exp(iW[I,\bar{I},K,K_5])\\&=\int\mathcal{D}\Psi\,\mathcal{D}\bar{\Psi}\exp\left[i\int d^Dx\left\{\mathcal{L}(\Psi,\bar{\Psi})+\bar{I}\Psi+\bar{\Psi}I+K\frac{\hat{g}}{N}\bar{\Psi}\Psi+K_5\frac{\hat{g}}{N}\bar{\Psi}i\gamma_5\Psi+P(K,K_5)\right\}\right]\ ,\end{aligned} \tag{50}$$

where K_5 is the external source coupling to $(\hat{g}/N)\bar{\Psi}i\gamma_5\Psi$. The pure source term $P(K,K_5)$ determined by the consistency condition [9] is now

$$P(K,K_5)=\frac{\hat{g}}{2N}(K^2+K_5^2)\ . \tag{51}$$

The classical fields $\psi,\bar{\psi},\Sigma,\Sigma_5$ are defined by

$$\begin{aligned}&\frac{\delta W}{\delta I}\equiv-\bar{\psi},\quad\frac{\delta W}{\delta\bar{I}}\equiv\psi\ ,\\&\frac{\delta W}{\delta K}\equiv\frac{\hat{g}}{N}\bar{\psi}\psi+\Sigma\ ,\\&\frac{\delta W}{\delta K_5}\equiv\frac{\hat{g}}{N}\bar{\psi}i\gamma_5\psi+\Sigma_5\ .\end{aligned} \tag{52}$$

To calculate $V_{\rm eff}$ with the method in Ref. [10], we choose

$$\Pi_s=-i(\Delta+i\gamma_5\Delta_5)\ , \tag{53}$$

where

$$\Delta_5\equiv-\frac{\delta\Gamma_L^P}{\delta K_5}\ , \tag{54}$$

and Γ_L^P is the loop contribution to Γ^P defined by

$$\Gamma^P[\psi,\bar{\psi},K,K_5]=W[I,\bar{I},K,K_5]-\int d^Dx(\bar{I}\psi+\bar{\psi}I)\ . \tag{55}$$

The relation (8) still holds now and in addition we have

$$\frac{\hat{g}}{N}K_5-\Delta_5=\Sigma_5\ . \tag{56}$$

Thus the propagator G_s is now

$$iG_s^{-1}=i\partial\!\!\!/+\Sigma+i\gamma_5\Sigma_5\ . \tag{57}$$

To leading order in $1/N$ expansion, there is no contribution from the P2PI(Π_s) diagrams and we get the effective potential

$$V_{\rm eff}(M)=\tfrac{1}{2}NM^2[\hat{g}^{-1}F(D)\Lambda^{D-2}+G(D)M^{D-2}]\ , \tag{58a}$$

$$M\equiv\sqrt{\Sigma^2+\Sigma_5^2}\ . \tag{58b}$$

We see that (58a) is exactly of the same form as (15a) provided Σ is replaced by M. The renormalization will then be the same as (15) and $|\Sigma|$ and $|\Sigma_R|$ replaced by M and M_R, respectively. Therefore all the above large-N limit results hold with $|\Sigma_R|$ replaced by M_R. Since $V_{\rm eff}$ is only a function of M, independent of the direction on the $\Sigma-\Sigma_5$ plane, we can always choose $|\Sigma_R|=M_R$, $|\Sigma_{5R}=0$ without the loss of generality. The nonvanishing VEV in the broken-symmetry phase breaks the chiral U(1)×U(1) symmetry into a diagonal U(1) symmetry and $(\hat{g}/N)\Psi i\gamma_5\Psi$ is the Goldstone-boson field. Indeed, it is easy to check that

$$\left.\frac{\partial^2V_{\rm eff}}{\partial\Sigma_{5R}^2}\right|_{\substack{|\Sigma_R|=M_R\\|\Sigma_{5R}|=0}}=0\ .$$

The existence of this Goldstone boson is consistent with Coleman's theorem [14] for $2<D<4$. The present case is much simpler than the case of $D=2$ in which a Goldstone pole [4] in the $N\to\infty$ limit is in contradiction with Coleman's theorem. A proper explanation for the $D=2$ case has been given by Witten [15].

The calculation of the order-$1/N$ correction to $V_{\rm eff}$ is also similar to that in the previous model. Now

$$\begin{aligned}V_{\rm eff}(M)=&\tfrac{1}{2}NM^2[\hat{g}^{-1}-F(D)\Lambda^{D-2}+G(D)M^{D-2}]\\&+U+U_5\ ,\end{aligned} \tag{59}$$

where U is given in (32) with Σ replaced by M, and U_5 is the contribution by the P2PI(Π_s) diagrams shown in Fig. 1 with an extra $i\gamma_5$ attached to each vertex. The result is

$$U_5=-\frac{i}{2}\int\frac{d^Dk}{(2\pi)^D}\ln[1+iB_5(k)]\ . \tag{60a}$$

where

$$B_5(k)=4i\hat{g}\int\frac{d^Dp}{(2\pi)^D}\frac{p^2-p\cdot k-M^2}{(p^2+M^2)[(p-k)^2+M^2]}$$
$$=i\hat{g}\left\{F(D)\Lambda^{D-2}-\frac{1}{2}DG(D)M^{D-2}-\frac{1}{8}D(D-2)G(D)k^{D-2}[Q(k,M)]^{D-3}\right.$$
$$\left.\times\left[B\left[\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1+Q^{-1}(k,M)]\right]-B\left[\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1-Q^{-1}(k,M)]\right]\right]\right\}. \tag{60b}$$

The presence of U_5 in (59) makes coupling constant renormalization different from (43b). It is now

$$Z_g=\left[1-gF(D)\left[1-\frac{D-2}{N}\right]-\frac{2(D-2)\ln\frac{\Lambda}{\mu}}{NDB(D/2-1,2-D/2)B(D/2,D/2)}[1-gF(D)]\right]\left[\frac{\Lambda}{\mu}\right]^{D-2}$$
$$+g\left[A_0+\frac{A_1}{N}\right]. \tag{61}$$

Correspondingly $\beta(g_R)$ and $\gamma_{\bar{\Psi}\Psi}$ are all different from (44) and (48). They are now

$$\beta(g_R)=(D-2)g_R\left[1-\frac{g_R}{g_R^*}\right]\left[1-\frac{2}{NDB(D/2,-1,2-D/2)B(D/2,D/2)}\right]. \tag{62}$$

and

$$\gamma_{\bar{\Psi}\Psi}=(D-2)\frac{g_R}{g_R^*}+\frac{D-2}{ND(D-1)B(D/2-1,2-D/2)B(D/2,D/2)}\left[2(D-1)\left[1-\frac{g_R}{g_R^*}\right]+1\right], \tag{63}$$

where g_R^* is given in (45). At the critical point $g_R=g_R^*$, the renormalization-scheme independent order-$1/N$ correction to $\gamma_{\bar{\Psi}\Psi}$ is

$$\frac{D-2}{ND(D-1)B(D/2-1,2-D/2)B(D/2,D/2)}$$

which is also *positive*. In the case when D is close to 4 from below, this correction also becomes small quickly.

After renormalization, the finite effective potential is

$$V_{\text{eff}}(M_R)=\frac{1}{2}N\mu^Dg_R^{-1}\left[\frac{M_R}{\mu}\right]^2\left[1+g_RG(D)\left[\frac{M_R}{\mu}\right]^{D-2}-g_R\left[A_0+\frac{A_1}{N}\right]\right]+U_f+U_{5f}, \tag{64a}$$

where

$$U_f+U_{5f}=\frac{1}{(4\pi)^{D/2}\Gamma(D/2)}\left\{\int_0^\Lambda dk\,k^{D-1}\ln\left[\left[1+\frac{g_R}{2(1-g_RA_0)}\left\{DG(D)\left[\frac{M_R}{\mu}\right]^{D-2}\right.\right.\right.\right.$$
$$+\frac{1}{4}D(D-2)G(D)\left[\frac{k}{\mu}\right]^{D-2}[Q^{D-3}(k,M_R)+Q^{D-1}(k,M_R)]$$
$$\times\left[B\left[\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1+Q^{-1}(k,M_R)]\right]\right.$$
$$\left.\left.\left.\left.-B\left[\frac{D}{2}-1,\frac{D}{2}-1;\frac{1}{2}[1-Q^{-1}(k,M_R)]\right]\right]\right\}\right]$$
$$\times\left[1+H(g_R,D)\left[\frac{k}{\mu}\right]^{D-2}\right]^{-1}\right]-4\left[\left[\frac{\Lambda}{\mu}\right]^{D-2}-\frac{D(-2)\ln\frac{\Lambda}{\mu}}{H(g_R,D)}\right]\mu^D\left[\frac{M_R}{\mu}\right]^2$$
$$\left.+\frac{4(D-2)\ln\frac{\Lambda}{\mu}}{D(D-1)B(D/2,D/2)}\mu^D\left[\frac{M_R}{\mu}\right]^D\right\}. \tag{64b}$$

The qualitative conclusions are not violated by the order-$1/N$ correction.

With the obtained $B(k)$ *and* $B_5(k)$ we can calculate the four-point Green's function and the $2\to 2$ scattering amplitude. The amputated four-point Green's function is simply

$$G^{(4)}_{abcd}(k_1,k_2,k_3,k_4)=\frac{\hat{g}}{N}\delta_{ab}\delta_{cd}\left[\frac{1}{1+iB(\sqrt{s}\,)}+\frac{1}{1+iB_5(\sqrt{s}\,)}\right]+\text{crossed terms} , \tag{65}$$

where $s=-(k_1+k_2)^2$. After analytic continuation to Minkowskian momenta $[s=(k_1+k_2)^2]$, we get the $2\to 2$ scattering amplitude. We give here the explicit form of the scattering amplitude for $D=3$ as an example:

$$A_{abcd}=-\frac{4\pi i}{N}\delta_{ab}\delta_{cd}\left[\frac{\sqrt{s}}{(s-4M_R^2)\left[\ln\left|\dfrac{\sqrt{s}+2M_R}{\sqrt{s}-2M_R}\right|-i\pi\theta(s-4M_R^2)\right]}+\frac{1}{\sqrt{2}\left[\ln\left|\dfrac{\sqrt{s}+2M_R}{\sqrt{s}-2M_R}\right|-i\pi\theta(s-4M_R^2)\right]}\right] . \tag{66}$$

This coincides with the result given in Ref. [16].

IV. RELATION BETWEEN THE NONTRIVIAL FIXED POINTS AND PHASE TRANSITION IN DYNAMICAL SYMMETRY BREAKING

We give here a general discussion on the relation between the nontrivial fixed points and the phase transitions in DSB (phase transition of the second kind). Suppose there is a DSB phase transition and the broken- and unbroken-symmetry phases are separated by the critical point g_R^c. Let $\Omega(g_R,\mu)$ be a dynamically generated physical observable of dimension d serving as an order parameter (for instance, the physical mass). $\Omega(g_R,\mu)$ exists in the broken-symmetry phase and goes to zero at $g_R=g_R^c$. In the following discussion $d>0$ will be assumed as what it is in the usual cases. Since $\Omega(g_R,\mu)$ is a physical observable, it should be independent of the arbitrary scale parameter μ. Therefore it satisfies the well-known renormalization-group equation [4,17]

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g_R)\frac{\partial}{\partial g_R}\right]\Omega(g_R,\mu)=0 , \tag{67}$$

and the solution is

$$\Omega(g_R,\mu)=\text{const}\times\mu^d\exp\left[-d\int^g\frac{dx}{\beta(x)}\right] . \tag{68}$$

Suppose there is a fixed point g_R^* in the theory. For specific discussions, let us consider here a special class of models in which the β function, in the vicinity of g_R^*, is of the simple form

$$\beta(g_R)=C(g_R-g_R^*)^P , \tag{69}$$

where C is a constant, and P is an integer, and we take $P\geq 1$ to guarantee that the running coupling constant approaches g_R^* in the ultraviolet or infrared limit. From (68) and (69) we obtain, in the broken-symmetry phase,

$$\Omega(g_R,\mu)=\begin{cases}\text{const}\times\mu^d(g_R-g_R^*)^{d/C}, \quad P=1 , & \text{(70a)}\\[2ex] \text{const}\times\mu^d\exp\left[\dfrac{d}{C(P-1)}(g_R-g_R^*)^{1-P}\right], & \\ \qquad\qquad P>1 . & \text{(70b)}\end{cases}$$

The first conclusion we can draw from (70) is that the critical point g_R^c separating the two phases must be the fixed point g_R^* since $\Omega(g_R,\mu)$ depends on g_R as a function of $g_R-g_R^*$ and $\Omega(g_R,\mu)$ can go to zero only at $g_R=g_R^*$.

We first consider models with $P=1$. If $C<0$, g_R^* is an UV fixed point $g_R^*=g_{\text{UV}}^*$; if $C>0$, g_R^* is an infrared (IR) fixed point $g_R^*=g_{\text{IR}}^*$. We see from (70a) that $\Omega(g_R,\mu)$ can go to zero at $g_R=g_R^*$ only if $C<0$, i.e., $g_R^*=g_{\text{UV}}^*$. Therefore the critical point can only be an UV fixed point rather than an IR one.

For models with $P>1$, g_R^* is not a simple zero of $\beta(g_R)$, g_R^* is an UV or IR fixed point only if $\beta(g_R<g_R^*)$ and $\beta(g_R>g_R^*)$ are of opposite signs. This implies that P is an odd integer. In this case, we have $g_R^*=g_{\text{UV}}^*$ if $C<0$ or $g_R^*=g_{\text{IR}}^*$ if $C>0$. We see from (70b) that $g_R^*=g_{\text{UV}}^*$ can always be a critical point of DSB with $\Omega=0$ at $g_R=g_{\text{UV}}^*$. If $g_R^*=g_{\text{IR}}^*$, we see from (70b) that it can be a critical point only if $(-1)^{1-P}$ is negative, but this cannot be satisfied if P is an odd integer. Therefore, g_{IR}^* can never be a critical point.

We conclude that in both the $P=1$ and $P>1$ cases the critical point of the DSB phase transition can only be an UV pixed point. Gross and Neveu [4], pointed out, in the case of $g_R^*=0$, that *"infrared-stable theories cannot produce masses dynamically." Our present conclusion is a generalization of this point in* the case of $g_R^*\neq 0$.

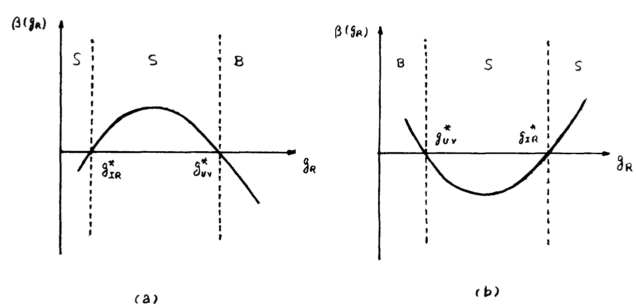

FIG. 2. Theory with two fixed points: (a) $g_{\text{UV}}^*>g_{\text{IR}}^*$ (b) $g_{\text{UV}}^*<g_{\text{IR}}^*$. S stands for the unbroken-symmetry; B stands for the broken-symmetry phase.

The above conclusions can be easily generalized to theories with more than one fixed point. For instance, if there are two fixed points $g^*_{\rm UV}$ and $g^*_{\rm IR}$ only $g^*_{\rm UV}$ can be a critical point of the DSB phase transition. If $g^*_{\rm UV} > g^*_{\rm IR}$ DSB can always take place in the $g_R > g^*_{\rm UV}$ region but never in the region $g_R < g^*_{\rm UV}$ since otherwise $\Omega(g_R, \mu)$ will go to infinity at $g_R = g^*_{\rm IR}$. If $g^*_{\rm UV} < g^*_{\rm IR}$, DSB can only take place in the $g_R < g^*_{\rm UV}$ region, while the $g_R > g^*_{\rm UV}$ region can only be the unbroken-symmetry phase. These are shown in Fig. 2. The D-dimensional four-fermion theories $(2 < D < 4)$ discussed in the previous sections belong to the $P=1$ case in Fig. 2(a) with $g^*_{\rm IR}=0$, which are concrete examples of the above general conclusions.

V. CONCLUSIONS

We have given a systematic calculation of the effective potential, renormalization, nontrivial UV fixed point, dynamical symmetry breaking, and the anomalous dimension $\gamma_{\bar{\Psi}\Psi}$ for four-fermion theories with discrete and continuous chiral symmetries in space-time dimension $2 < D < 4$ up to order $1/N$ in $1/N$ expansion. Our system of doing the renormalization is based on the cutoff independence of the effective potential. This renormalization procedure has been performed in the leading-order calculation for the coupling constant and the Σ renormalization. A simple subtraction scheme is taken throughout the calculation for obtaining the finite order-$1/N$ corrections. The explicit results for the effective potential, dynamical symmetry breaking, the nontrivial UV fixed point g^*_R, and the anomalous dimension $\gamma_{\bar{\Psi}\Psi}$ are all given up to order $1/N$. The interesting result is that the order-$1/N$ contribution definitely *increases* (*does not decrease*) $\gamma_{\bar{\Psi}\Psi}$ at the critical point. This supports the study of models based on the effect of large $\gamma_{\bar{\Psi}\Psi}$ [1,2].

We have also given some general conclusions on the relation between the nontrivial fixed points and the phase transitions in DSB for a certain class of models with $\beta(g_R)$ of the simple form (69). For instance, the critical point of the DSB phase transition can only be an UV fixed point $g^*_{\rm UV}$. The case of more than one fixed point has also been discussed (cf. Fig. 2).

ACKNOWLEDGMENTS

We would like to thank X. Y. Li, B. H. Warr, S.-H. H. Tye, and T. M. Yan for interesting discussions. Two of us (Y.P.K. and Y.P.Y.) are grateful to the Theoretical Physics Institute of the University of Minnesota and the Particle Theory Group of Cornell University for the kind hospitality extended to them as part of the calculation was done during their visit to the institutes.

[1] See, for example, K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986); T. Akiba and T. Yanagida, Phys. Lett. **169B**, 432 (1986); T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D **36**, 563 (1987); B. Holdom, Phys. Lett. **150B**, 301 (1985); Phys. Lett. B **198**, 535 (1987); C. N. Leung, S. T. Love, and W. A. Bardeen, Nucl. Phys. **B273**, 649 (1986); W. A. Bardeen, C. N. Leung, and S. T. Love, *ibid.* **B323**, 493 (1989). For recent reviews, see K. Yamawaki, in *New Trends in Strong Coupling Gauge Theories,* Proceedings of the 1988 International Workshop, Nagoya, Japan, edited by M. Bando, T. Muta, and K. Yamawaki (World Scientific, Singapore, 1989); in *Proceedings of the 1989 Workshop on Dynamical Symmetry Breaking,* Nagoya, Japan, 1989, edited by T. Muta and K. Yamawaki (Nagoya University, Nagoya, Japan, 1990).

[2] Y. Nambu, Enrico Fermi Institute Report No. 89-08, 1989 (unpublished); in *Proceedings of the 1989 Workshop on Dynamical Symmetry Breaking* [1]; V. A. Miransky, M. Tanabashi, and K. Yamawaki, Phys. Lett. B **221**, 177 (1989); Mod. Phys. Lett. A **4**, 1043 (1989); W. A. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. D **41**, 1647 (1990); W. J. Marciano, Phys. Rev. Lett. **62**, 2793 (1989); Phys. Rev. D **41**, 219 (1990); M. Suzuki, *ibid.* **41**, 3457 (1990); Report No. UCB-PTH-89/37 (unpublished).

[3] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[4] D. J. Gross and A. Neveu, Phys. Rev. D **10**, 3235 (1974); D. J. Gross, in *Methods in Field Theory,* Proceedings of the Les Houches Summer School, Les Houches, France, 1975, edited by R. Balian and J. Zinn-Justin, Les Houches Summer School Proceedings Vol. XXVIII (North-Holland, Amsterdam, 1976).

[5] K. G. Wilson, Phys. Rev. D **7**, 2911 (1973).

[6] B. Rosenstein, B. J. Warr, and S. H. Park, Phys. Rev. Lett. **62**, 1433 (1989).

[7] B. Rosenstein and B. J. Warr, Phys. Lett. B **218**, 465 (1989); B. Rosenstein, B. J. Warr, and S. H. Park, *ibid.* **219**, 469 (1989); University of Texas Report No. UTTG-26-89 (unpublished); G. Gat, A. Kovner, B. Rosenstein, and B. J. Warr, Phys. Lett. B **240**, 158 (1990).

[8] Y. Kikukawa and K. Yamawaki, Phys. Lett. B **234**, 497 (1990).

[9] Y. P. Kuang, X. Y. Li, and Z. Y. Zhao, Commun. Theor. Phys. **3**, 251 (1984); H. J. He, Q. Wang, and Y. P. Kuang, Z. Phys. C **45**, 427 (1990).

[10] H. J. He and Y. P. Kuang, Z. Phys. C **47**, 565 (1990).

[11] Y. P. Yi and Y. P. Kuang, Z. Phys. C **36**, 489 (1987).

[12] Here γ_5 is defined by $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ as in the $D=4$ case. The difficulty with this definition when D is nonintegral is the calculation of ${\rm tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\sigma)$ [cf. S. Narison, Phys. Rep. **84**, 263 (1982)]. However, this does not happen in the up to order-$1/N$ calculations in this paper.

[13] G. A. Kovner, B. Rosenstein, and B. J. Warr, Phys. Lett. B **240**, 158 (1990).

[14] S. Coleman, Commun. Math. Phys. **31**, 259 (1973).

[15] E. Witten, Nucl. Phys. **B145**, 110 (1978).

[16] B. Rosenstein and B. J. Warr, Nucl. Phys. **B335**, 288 (1990).

[17] K. Lane, Phys. Rev. D **10**, 1353 (1974).