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# Baryogenesis via leptogenesis

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If right-handed Majorana neutrinos are added to the standard model, then lepton-number-violating out-of-equilibrium decays of right-handed neutrinos combined with anomalous electroweak processes can generate the baryon number of the Universe. We analyze this mechanism in detail, and determine the ranges of parameters for which the correct baryon number is generated. We find that the scenario works for a wide range of parameters in the neutrino sector, including right-handed neutrino masses ranging from $\sim 1$ TeV to $\sim 10^{19}$ GeV, depending on the assumptions made about the structure of the neutrino mass matrices.

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## I. INTRODUCTION

The basic ingredients necessary for generating a nonzero baryon number $B$ in the Universe from initial conditions in which $B=0$ were discovered long ago [1]. Since then, many specific baryogenesis scenarios have been proposed [2]. All of these models seek to explain the small ratio

$$\frac{n_B}{n_\gamma}=(4-7)\times 10^{-10} \tag{1.1}$$

required by the standard model of big-bang nucleosynthesis [3].

In this paper, we examine in detail a scenario in which out-of-equilibrium lepton-number-violating decays can generate a nonzero lepton number $L$. This lepton number is then partially converted into a baryon number by electroweak processes which anomalously violate $B+L$, thus generating the observed baryon number of the Universe. These ideas have been discussed in Refs. [4] and [5], but a detailed quantitative analysis was not performed.

There are several motivations for studying this scenario in detail. First, in contrast with many other baryogenesis scenarios, it requires only a rather modest extension of the standard model, namely, the addition of right-handed neutrinos with large Majorana masses. Such models are very natural and interesting in their own right, and have been studied by many authors. Models of this kind incorporate the seesaw mechanism [6] which naturally explains why the observed left-handed neutrinos are light, and allows the standard model to be embedded in theories with higher gauge symmetries, such as left-right-symmetric models and SO(10) grand unified models. On the phenomenological side, small neutrino masses are interesting for neutrino mixing experiments and may explain the apparent deficit of neutrinos from the Sun. It is an interesting question to ask what range of parameters (if any) in this extended neutrino sector can also explain the observed baryon number of the Universe, and compare this range to theoretical prejudice and experimental observation.

## II. THE MODEL

We consider the standard model with the addition of three generations of right-handed neutrinos which are singlets under $SU(2)_W\times U(1)_Y$. They are assumed to have Majorana masses of order $M_R \gg 100$ GeV. (We will see that we can actually generate sufficient baryon number for $M_R \gtrsim 1$ TeV.) The right-handed neutrinos interact with ordinary leptons through the Higgs Yukawa coupling

$$\mathcal{L}_{\rm int}=h_{jk}(\bar{L}_{Lj}H)N_{Rk}+\text{H.c.}\ , \tag{2.1}$$

where $L_L$ is the usual lepton doublet and $H$ is the Higgs doublet. Unless we specify otherwise, we will work in a basis where both the $\nu$ and $N$ Majorana mass matrices are diagonal. The Dirac mass matrix is then $m_{Djk}=h_{jk}v/\sqrt{2}$, and is not diagonal in this basis. The interaction (2.1) is the only renormalizable coupling which connects the $N$'s with the known particles of the standard model. In most models incorporating $N$'s, there will be additional interactions involving the $N$'s, such as exchange of heavy $W_R$ bosons. We will see that these interactions are constrained by the requirement that the $N$'s be out of equilibrium when they decay, but that realistic models are possible.

We begin by explaining how the three basic ingredients needed for baryogenesis [1] arise naturally in this model. The first necessary ingredient is clearly baryon-number violation. In our model, this comes about through the combination of $L$ violation due to the $N$ Majorana mass terms and the anomalous $B+L$ violation in the standard model. The idea that $B+L$ could be violated rapidly at high temperatures was proposed in Ref. [4]. At temperatures $T$ satisfying $M_W(T) \ll T \ll M_W(T)/\alpha_W$, the rate can be calculated reliably using semiclassical methods, and it is found that the rate exceeds the expansion rate of the Universe for $T \gtrsim 200$ GeV [7], [8]. At temperatures much larger than $M_W$, qualitative arguments suggest that the rate of $B+L$ violation is given by [7], [9]

$$\Gamma_{B+L}\sim\alpha_W^4 T\ . \tag{2.2}$$

This rate is larger than the universal expansion rate up to temperatures $T \sim 10^{12}$ GeV. Further, the hypothesis that the rate of $B+L$ violation is suppressed at high temperatures has received further support from analytic computations in $(1+1)$-dimensional models [10] and numerical simulations [11]. We conclude that there is no strong reason to doubt that $(B+L)$-violating processes are effectively in equilibrium for temperatures 200 GeV $\lesssim T \lesssim 10^{12}$ GeV.

The second necessary ingredient for baryogenesis is $C$ and $CP$ violation. The reason is simply that baryon number is odd under both $C$ and $CP$, and so if either $C$ or $CP$ were conserved, there would be as many baryons as antibaryons produced on average. In our model, $C$ is violated by chiral couplings and $CP$ is violated by phases in the Yukawa couplings of Eq. (2.1).

The final necessary ingredient is that $B$-violating processes be out of equilibrium. (In the presence of $B+L$ violation, this will be true if $L$-violating processes are out of equilibrium.) The reason that out-of-equilibrium processes are necessary is that the phase-space distribution of particles in equilibrium is given by either a Fermi-Dirac or Bose-Einstein distribution, possibly with a chemical potential. [This still holds in the presence of $CP$ (or $T$) violation, as discussed for example in [12].] If all allowed reactions are in equilibrium, processes such as $\bar{\nu}\nu \to e^+e^-$ impose $\mu_\nu + \nu_{\bar{\nu}} = 0$, while $L$-violating processes such as $\nu\nu \to e^+e^-$ impose $\mu_\nu = 0$. Thus, in equilibrium there can be no net $L$, and hence no baryogenesis via the mechanism we are discussing.

In the context of our model, the lightest $N$ (which we will refer to as $N_1$) is naturally out of equilibrium when it decays. If we assume that there are no nonstandard interactions other than those of Eq. (2.1), then the rates for processes which can deplete the number of $N_1$'s is proportional to the Yukawa couplings $h$. If we make the reasonable assumption that the neutrino Dirac masses are of the same order as the Dirac masses of their charged-lepton partners, then $h_{jk} \ll 1$. Thus, the $N_1$'s are naturally overabundant when they decay. The decay rate at temperatures $T \sim M_{N_1}$ is roughly

$$\Gamma_{N_1} \sim \frac{m_{D_1}^2 M_{N_1}}{16\pi v^2} . \tag{2.3}$$

Here, we assume that $N_1$ couples dominantly to the left-handed neutrino $\nu_1$ with Dirac mass $m_{D_1}$. Demanding that the rate (2.3) be smaller than the expansion rate of the Universe at temperatures $T \sim M_{N_1}$ gives the constraint

$$M_{N_1} \gtrsim (10^{11}\ \text{GeV}) \left[ \frac{m_{D_1}}{1\ \text{GeV}} \right]^2 . \tag{2.4}$$

As we will explain below, sufficient $B$ can be generated even if $M_{N_1}$ is significantly lower than suggested by this estimate.

Of course, in order that $N_1$ be out of equilibrium, there can be no other processes occurring rapidly which can change the number of $N_1$'s. For example, in a left-right-symmetric model, processes such as $N\bar{e}_R \to u_R \bar{d}_R$ mediated by $W_R$ exchange will deplete the $N$ number density unless the $W_R$ is sufficiently heavy. The reaction rate for such processes at temperatures $T \ll M_{W_R}$ is roughly

$$\Gamma \sim \frac{g_R^4 T^5}{16\pi M_{W_R}^4} , \tag{2.5}$$

where $g_R$ is the right-handed gauge coupling. In order for this rate to be smaller than the $N_1$ decay rate for temperatures $T \gtrsim M_{N_1}$, we require

$$\frac{M_{W_R}}{M_{N_1}} \gtrsim 10^2 \left[ \frac{M_{N_1}}{1\ \text{TeV}} \right]^{-1/4} . \tag{2.6}$$

assuming $g_R \sim g_2$. Since the natural scale for $M_{W_R}$ and $M_{N_1}$ is the same, this is not an overly restrictive bound.

Another popular model of right-handed neutrino masses is the singlet Majoron model [13]. In this model, the $N$'s couple to a gauge-singlet scalar field $\Phi$ which carries lepton number 2:

$$\mathcal{L} = \kappa_{jk} \bar{N}_j \Phi N_k^c + \text{H.c} . \tag{2.7}$$

When $\Phi$ acquires a vacuum expectation value, the lepton number is broken and the $N$'s get Majorana masses. Since the lepton number is a global symmetry in this model, there will be a Nambu-Goldstone boson, the Majoron. The Majoron couplings to ordinary matter are very weak [13], and it will not mediate any interactions strong enough to keep the $N$'s in equilibrium as long as $\langle \Phi \rangle \gg v$.

## III. THE RATES

In this section, we present the results for the various reaction rates of interest. As already mentioned, we will be interested in temperatures $T \gg v$, so it is certainly a good approximation to neglect all masses except those of the $N$'s. We can therefore compute amplitudes in the symmetric phase, where the fermions are in states of definite helicity, gauge bosons have two polarization states, and all four components of the Higgs doublet are considered physical. For the purposes of doing thermal averages, it is then convenient to regard all of the particles in a $SU(2)_W$ doublet as a single particle with twice the number of internal degrees of freedom.

The $\nu$'s have Majorana masses, so that strictly speaking they cannot carry lepton number. However, at high temperatures, the neutrinos are approximately in eigenstates of definite helicity, so we can assign $L = \pm 1$ to left- and right-helicity neutrino states. For notational convenience, we will denote left-helicity neutrinos as $\nu$ and right-helicity neutrinos as $\bar{\nu}$.

The lightest right-handed neutrino can decay via the diagram shown in Fig. 1. The total decay rate at the tree level is

$$\Gamma_D = \frac{(h^\dagger h)_{11}}{16\pi} M_{N_1} . \tag{3.1}$$

When loop effects are taken into account, there will be a $CP$ asymmetry in this decay. We parametrize this asym-

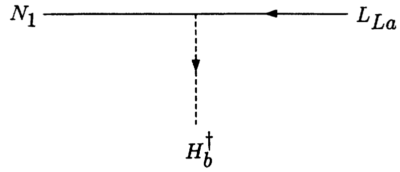


FIG. 1. The tree-level diagram responsible for $N_1$ decay. Here and in the following figures, $a,b,\ldots$ are $SU(2)_W$-doublet indices.

metry by a small parameter $\epsilon$, defined by

$$|\mathcal{A}(N_1\to\nu_1 h^{0*})|^2=|\mathcal{A}(N_1\to\bar{\nu}_1 h^0)|^2=\epsilon|\mathcal{A}|^2 . \tag{3.2}$$

The leading contribution to $\epsilon$ comes from interference between the tree-level amplitude with loop amplitudes which contain on-shell intermediate states. At one loop, the only $CP$-violating contribution comes from the diagram shown in Fig. 2. We find

$$\epsilon=\frac{1}{\pi(h^\dagger h)_{11}}\sum_j \mathrm{Im}[(h^\dagger h)_{1j}(h^T h^*)_{j1}]f(M_{Nj}^2/M_{N1}^2) , \tag{3.3}$$

where

$$f(x)=\sqrt{x}\left[1-(1+x)\ln\frac{1+x}{x}\right]$$
$$\simeq\frac{1}{2\sqrt{x}} \quad \text{for } x\gg 1 . \tag{3.4}$$

(A different result for $\epsilon$ was obtained in Ref. [5]. However, the discrepancy does not affect our results because of the large uncertainty in the neutrino mass matrix.) To reassure ourselves that $\epsilon\neq 0$, we note that in the $N$ mass eigenbasis, the only freedom to change basis is to redefine the left-handed neutrino fields, corresponding to $h\mapsto U_\nu^\dagger h$. However, it is easy to see that $\epsilon$ is independent of $U_\nu$, so that there is no way to rephase the fields to set $\epsilon=0$.

The interaction (2.1) will also mediate $L$-violating scattering processes via Higgs-boson exchange. The $s$-channel processes shown in Fig. 3 give rise to the reduced cross section

$$\hat{\sigma}_{Hs}(s)=\frac{m_t^2(h^\dagger h)_{11}}{2\pi v^2}\left[\frac{s-M_{N_1}^2}{s}\right]^2 , \tag{3.5}$$

where we have summed over flavors. (The dimensionless reduced cross section $\hat{\sigma}$ is essentially the square of the amplitude summed over final states, and is the natural quantity which enters into the thermally averaged rates. Precise formulas are given in Appendix A.) Of course, there are also processes involving other quarks, but these are negligible because of their small Yukawa couplings to the Higgs field.

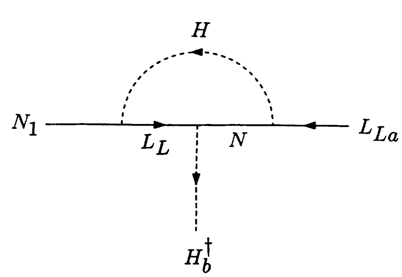


FIG. 2. The one-loop diagram which contributes to the $CP$ asymmetry in $N_1$ decay.

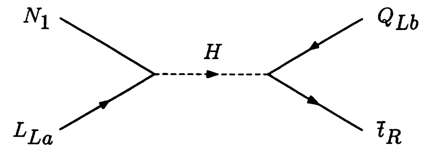


FIG. 3. The diagram contributing to $L$-violating scattering processes via $s$-channel Higgs-boson exchange.

The $t$-channel Higgs-boson-exchange processes shown in Fig. 4 give rise to the reduced cross section

$$\hat{\sigma}_{Ht}(s)=\frac{m_t^2(h^\dagger h)_{11}}{\pi v^2 s}\left[s-M_{N_1}^2+M_{N_1}^2\ln\frac{s-M_{N_1}^2+m_H^2}{m_H^2}\right] , \tag{3.6}$$

where $m_H$ is the mass of the physical Higgs scalar. We have kept only the leading-logarithmic dependence on $m_H$. The results are very insensitive to the precise value of $m_H$, and we use $m_H=800$ GeV for definiteness.

There are also $L$-violating scattering processes involving $N$ exchange, shown in Fig. 5. The full flavor structure of this amplitude is quite complex, and will not be needed, since we must guess at the flavor structure in the neutrino sector in any case. Therefore, we assume that the dominant contribution comes from $N_j$ exchange (for some fixed $j$). The expression then becomes

$$\hat{\sigma}_{N_j}\simeq\frac{|(h^\dagger h)_{jj}|^2 M_{N_j}^2}{2\pi s}g_j(s/M_{N_j}^2) , \tag{3.7}$$

where

$$g_j(x)\equiv x+\frac{x}{D_j(x)}+\frac{x^2}{2D_j^2(x)}\left[1+\frac{1+x}{D_j(x)}\right]\ln(1+x)$$
$$\simeq\begin{cases}\frac{3}{2}x^2 & \text{for } x\ll 1 ,\\ x & \text{for } x\gg 1 ,\end{cases} \tag{3.8}$$

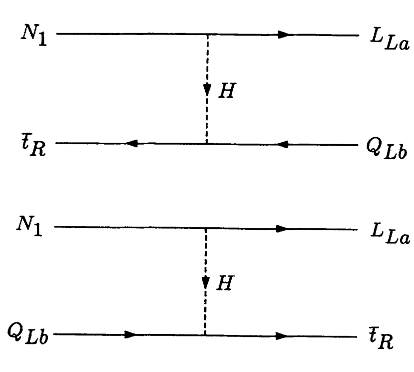


FIG. 4. Diagrams contributing to $L$-violating scattering processes via $t$-channel Higgs-boson exchange.

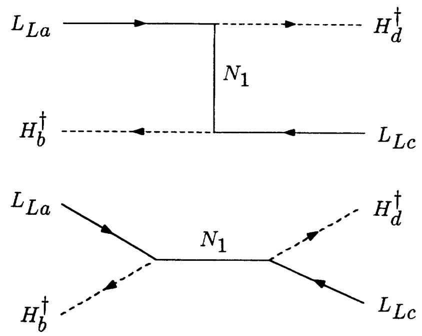


FIG. 5. Diagrams contributing to $L$-violating scattering processes via $N_1$ exchange.

and

$$\frac{1}{D_j(x)} \equiv \frac{x-1}{(x-1)^2+\Gamma_{N_j}^2/M_{N_j}^2} . \tag{3.9}$$

To obtain the result (3.7), we have removed the contribution due to real intermediate state $N_j$'s. (Note that there is no enhancement at $s \simeq M_{N_j}^2$.) This contribution is not correctly taken into account by the usual (zero-temperature) Feynman rules, since the physical intermediate state in the early Universe is a quantum state containing many identical $N$'s. The contribution to the Boltzmann equations coming from real intermediate state $N$'s is taken into account via the decay and inverse decay processes already discussed.

Note that for $s \ll M_{N_j}^2$, we have

$$\hat{\sigma}_N = \frac{3s}{4\pi v^4} \sum_j m_{\nu j}^2 , \tag{3.10}$$

where $m_{\nu j}$ are the physical $\nu$ masses. This shows that at low energies, the lepton-number-violating scattering rates are controlled directly by the left-handed neutrino masses.

## IV. BOLTZMANN EQUATIONS

In this section, we write down the relevant Boltzmann equations describing the evolution of $B$ and $N_1$. We make the approximations of kinetic equilibrium and Maxwell-Boltzmann statistics. The derivation of such Boltzmann equations and the thermal averaging used is reviewed, e.g., in Refs. [12] and [14]. Some relevant formulas and brief remarks are collected in Appendix A.

Using the formalism of Appendix A, we can write

$$\dot{n}_{N_1} + 3Hn_{N_1} = \left[ \frac{n_{N_1}}{n_{N_1}^{\rm eq}} - 1 \right] (\gamma_D + \gamma_{H_t} + \gamma_{H_s}) \equiv -C_N , \tag{4.1}$$

$$\dot{n}_{B-L} + 3Hn_{B-L} = \epsilon \left[ \frac{n_{N_1}}{n_{N_1}^{\rm eq}} - 1 \right] \gamma_D - \frac{n_{B-L}}{n_l^{\rm eq}} \left[ \gamma_N + \frac{n_{N_1}}{n_{N_1}^{\rm eq}} \gamma_{H_s} + \gamma_{H_t} \right] \equiv -C_{B-L} . \tag{4.2}$$

Here, $H$ is the expansion rate of the Universe, the $n$'s are number densities, and the $\gamma$'s are the rate factors defined in Appendix A. The rate factors are labeled by the same subscripts used in the previous section. $n_l$ is the number density for a left-handed doublet, i.e., $g_l = 2$. We have written the Boltzmann equation for $B-L$, since this quantity is conserved by the anomalous $(B+L)$-violating processes.

We will not discuss the derivation of Eqs. (4.1) and (4.2) in detail, since a detailed discussion is given in [12]. We will limit ourselves to some brief remarks. First of all, the equations are physically sensible and have a very simple interpretation. The $3Hn$ terms describe the depletion of the number densities due to the expansion of the Universe. Equation (4.1) describes the depletion of $N_1$ due to their decay and annihilation processes. The first term in Eq. (4.2) describes the creation of lepton number from the $L$ asymmetry in $N_1$ decay, while the second term describes the dissipation of lepton number due to $L$-violating scattering processes.

Also, note that in equilibrium, the right-hand side of Eq. (4.2) vanishes, in accordance with the general result that no particle-antiparticle asymmetry can exist in equilibrium. In fact, there are some subtleties which must be taken into account to ensure that this happens: If the $CP$ asymmetry in $\gamma_D$ were the only source of $CP$ violation, then the Boltzmann equation would predict a nonzero $L$ even in equilibrium. Also, using unitarity, it can be shown that the zero-temperature cross sections for the scattering processes we have considered do not have a net $CP$ asymmetry, so it appears that they cannot resolve the dilemma. However, when the contribution from real intermediate $N_1$'s is subtracted out from the scattering processes, it is found that the resulting scattering rates *do* have a $CP$ asymmetry which combines with the asymmetry from $N_1$ decay to give the first term in the Boltzmann equation (4.2). This is discussed in detail in [12].

When the anomalous $(B+L)$-violating processes are in equilibrium, the baryon number $n_B$ is related to $n_{B-L}$ by [9,15]

$$n_B = \tfrac{28}{79} n_{B-L} \tag{4.3}$$

for three generations and one Higgs doublet. (The reason that the factor is not simply $\frac{1}{2}$ is that the anomalous electroweak processes only act on the left-handed fields, and so do not directly couple to $B$ and $L$.) Note that it is not necessary that $B+L$ violation be in equilibrium during the era that a $B-L$ asymmetry is being generated. As long as $B+L$ violation is in equilibrium at some time afterwards, $B-L$ will be converted to $B$ according to Eq. (4.3).

It is convenient to scale out the expansion of the Universe by using rescaled variables

$$N_{N_1} \equiv \frac{n_{N_1}}{s} , \tag{4.4}$$

etc., where $s$ is the entropy density [3]. As long as the Universe expands isentropically, $s \propto R^{-3}$ where $R$ is the scale factor, and we have

$$\frac{dN_{N_1}}{dx} = -\frac{x}{H(M_1)s} C_N , \tag{4.5}$$

etc., where we have defined

$$x \equiv \frac{M_{N_1}}{T} . \tag{4.6}$$

Equation (4.5) no longer holds if there are significant deviations from equilibrium and the Universe does not expand isentropically. In particular, the $N_1$'s can be far out of equilibrium and can create substantial entropy when they decay. (Physically, what happens is that the decay products rapidly thermalize and heat up the thermal bath of radiation, which causes the Universe to expand faster, which in turn dilutes any out-of-equilibrium number densities such as $n_{B-L}$. At the end of the out-of-equilibrium era, the entropy is again proportional to the equilibrium number density of radiation, and so measures the dilution of quantities such as $n_{B-L}/n_\gamma$.) This effect is easily estimated: If the $N_1$'s decay at a temperature $T_{\rm decay}$, then by the second law of thermodynamics the entropy generated is

$$\Delta s \simeq \frac{\rho_{N_1}(T_{\rm decay})}{T_{\rm decay}} . \tag{4.7}$$

If the $N_1$'s are far out of equilibrium when they decay, they will be nonrelativistic and $\rho_N = M_N n_N \simeq M_N n_\gamma$, so that

$$\frac{\Delta s}{s} \sim \frac{M_{N_1}}{g_* T_{\rm decay}} . \tag{4.8}$$

This shows that entropy generation by $N_1$ decays can be significant if the decay takes place at $x \gtrsim g_* \sim 100$. Perhaps surprisingly, it has been shown [16] that the simple estimate (4.7) is accurate to within factors of order unity when entropy generation is important. We will therefore use Eq. (4.7) to estimate the effect of entropy generation, defining the instant of decay as the time when $N_{N_1} = N_{N_1}^{\rm eq}/e$.

## V. NEUTRINO FLAVOR SCENARIOS

We will now make some assumptions about the flavor structure in the neutrino sector in order to reduce the number of free parameters in the model. We will assume that the neutrino Dirac masses fall into a hierarchical pattern qualitatively similar to that of the leptons and quarks. This is certainly a reasonable assumption, since in most seesaw models, all of the Dirac mass matrices have a similar origin. For purposes of making estimates, we will assume that the neutrino Dirac matrix in the weak basis has the "hierarchical texture"

$$m_D \sim \begin{bmatrix} \eta^2 & \eta^2 & \eta^2 \\ \eta^2 & \eta & \eta \\ \eta^2 & \eta & 1 \end{bmatrix} m_{D3} , \tag{5.1}$$

where $\eta$ is a small parameter which controls both the mass hierarchy and mixing angles.

We assume that the right-handed neutrino mass matrix $M_N$ has a texture similar to that of Eq. (5.1). We now consider two different mass scenarios.

In the "democratic" mass scenario, we assume that the basis which diagonalizes $M_N$ is completely uncorrelated with the basis which diagonalizes $m_D$. Then, we expect

$$m_{\nu j} \simeq \frac{m_{Dj}^2}{M_{N_1}} , \tag{5.2}$$

where $m_{Dj}$ are the eigenvalues of $m_D$. This is the case most often considered in the literature. Note that in this case, the physical left-handed neutrino masses display a hierarchy $m_{\nu j+1}/m_{\nu j} \sim \eta^2$.

However, there is another possibility, which we call the "correlated" mass scenario, in which there is a basis which approximately diagonalizes both $m_D$ and $M_N$. In this case, the physical left-handed neutrino Majorana masses are given by

$$m_{\nu j} \simeq \frac{m_{Dj}^2}{M_{Nj}} . \tag{5.3}$$

This scenario may be appealing if the scale of the right-handed neutrino masses are not far above the weak scale, so that one may reasonably speculate that $m_D$ and $M_N$ have a similar origin. In this case, $m_{\nu j+1}/m_{\nu j} \sim \eta$.

In the democratic case, we estimate

$$(h^\dagger h)_{11} \simeq \frac{2m_{D3}^2}{v^2} \quad \text{(democratic)} , \tag{5.4}$$

which controls the rate for scattering due to Higgs-boson exchange. $N$-exchange scattering is assumed to be controlled by $N_1$ exchange, so that Eq. (5.4) controls this rate as well. The expression (3.3) for the $CP$-violating parameter $\epsilon$ is assumed to be dominated by the contribution from $j=2$. (Note that the term with $j=1$ vanishes identically.) Thus, we estimate

$$\epsilon \simeq \frac{m_{D3}^2}{\pi v^2} \frac{M_{N_1}}{M_{N_2}} \sin\delta \quad \text{(democratic)} , \tag{5.5}$$

where $\delta$ is a $CP$-violating phase. (In what follows, we will assume maximal $CP$ violation: $\sin\delta \simeq 1$.) The important parameters in this scenario are thus $M_{N_1}, m_{D3}$ and the ratio $M_{N_1}/M_{N_2}$.

In the correlated case, we estimate

$$(h^\dagger h)_{11} \simeq \frac{2m_{D1}^2}{v^2} \quad \text{(correlated)} . \tag{5.6}$$

$N$-exchange scattering is assumed to be controlled by $N_3$ exchange, and we estimate

$$(h^\dagger h)_{33} \simeq \frac{2m_{D3}^2}{v^2} \quad \text{(correlated)} . \tag{5.7}$$

If the hierarchy of $N$ masses is the same as that of the $\nu$ masses, then $\epsilon$ will be dominated by the contribution from $j=3$, and we estimate

$$\epsilon \simeq \frac{m_{D3}^2}{\pi v^2} \frac{M_{N_1}}{M_{N_3}} \sin\delta \quad \text{(correlated)} . \tag{5.8}$$

The important parameters in this scenario are thus $M_{N_1}$, $m_{D1}$, $m_{D3}$, and the ratio $M_{N_1}/M_{N_3}$.

The assumptions we have made are intended to be crude approximations, and are not expected to be realistic in detail. In particular, the value of $\epsilon$ depends on ratios of masses of right-handed neutrinos, which we cannot estimate with any real confidence. This is important to keep in mind, since the baryon asymmetry generated in this model is essentially proportional to $\epsilon$.

## VI. RESULTS

The Boltzmann equations (4.1) and (4.2) were numerically integrated to obtain the present baryon asymmetry. Typical results are shown in Figs. 6 and 7.

We can get an approximate analytical understanding of the solutions as follows. We assume for simplicity that only the decay terms are important. For $x \ll 1$, we then have

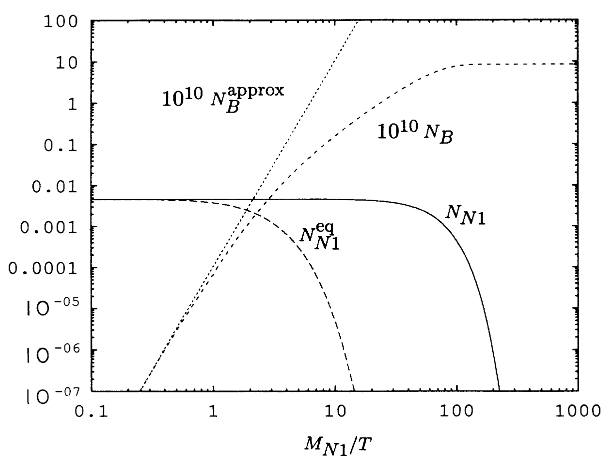


FIG. 6. $N_{N_1}$, $N_{N_1}^{\rm eq}$, and $N_B(\times 10^{10})$ as a function of temperature for the "democratic" neutrino mass scenario with $M_{N_1} = 5\times 10^{15}$ GeV and $m_{D3} = 1$ GeV. The straight dashed line corresponds to the approximate solution for $N_B$ discussed in the text. This is meant to illustrate a "typical" case where the baryogenesis scenario works well. For larger values of $M_{N_1}$, $N_{N_1}$ goes farther out of equilibrium, causing entropy generation which dilutes the final value of $N_B$.

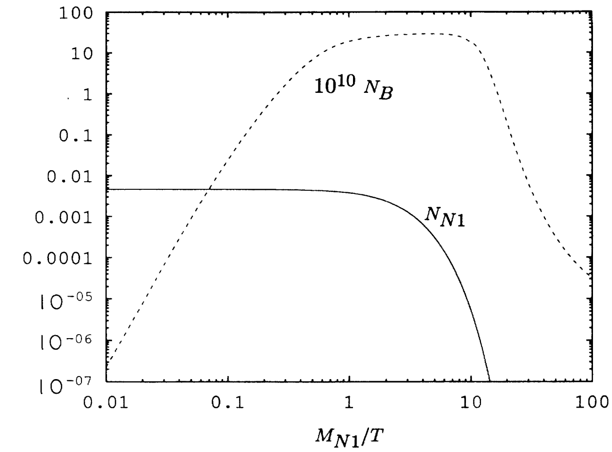


FIG. 7. $N_{N_1}$ and $N_B(\times 10^{10})$ as a function of temperature for the "democratic" neutrino mass scenario with $M_{N_1} = 10^{11}$ GeV and $m_{D3} = 100$ GeV. Note that a large baryon asymmetry is generated at temperatures $T \sim M_{N_1}$, but that it is dissipated (by scattering processes) at lower temperatures. For smaller values of $M_{N_1}$, the scattering processes become important at higher values of $T/M_{N_1}$, and there is no baryon number generated at $T \sim M_{N_1}$.

$$\frac{\langle \Gamma_D \rangle}{H(M_{N_1})} = Kx , \tag{6.1}$$

where

$$K = \frac{(h^\dagger h)_{11}}{32\pi} \frac{M_{N_1}}{H(M_{N_1})} . \tag{6.2}$$

If $K \ll 1$, $N_1$ will be far out of equilibrium when it decays, and we expect our baryogenesis mechanism to be effective. However, we will see that the mechanism works even if $K \sim 100$. The Boltzmann equations can be written as

$$\frac{dN_N}{dx} = -(N_N - N_N^{\rm eq}) K x^2 , \tag{6.3}$$

$$\frac{dN_{B-L}}{dx} = (N_N - N_N^{\rm eq}) \epsilon K x^2 . \tag{6.4}$$

We solve the Boltzmann equations by writing

$$N_{N_1} = N^{\rm eq}(m=0)(1+\Delta) . \tag{6.5}$$

$\Delta$ measures the deviation of $N_{N_1}$ from the equilibrium value appropriate for a massless (nondecaying) particle. Then

$$\frac{d\Delta}{dx} = -\left[\Delta + \frac{x^2}{4}\right] K x^2 , \tag{6.6}$$

$$\frac{dN_{B-L}}{dx} = N^{\rm eq}(m=0)\left[\Delta + \frac{x^2}{4}\right] \epsilon K x^2 . \tag{6.7}$$

The $x^2/4$ term comes from the difference $N_1^{\rm eq} - N^{\rm eq}(m=0)$. For $\Delta \ll x^2/4$, we have the solution

$$\Delta(x) = -\frac{Kx^5}{20}\,, \tag{6.8}$$

which remains valid as long as $x^3 \ll 5/K$. Substituting this into Eq. (6.4), we obtain the solution

$$N_{B-L}(x) = \frac{9}{4\pi^4 g_*}\epsilon K x^5\,. \tag{6.9}$$

The physics of this solution is very simple: At high temperatures, the decay rate is so small that $N_{N_1}$ is nearly constant, while $N_{N_1}^{\rm eq}$ is changing due to effects of the nonzero $N_1$ mass. This small deviation from equilibrium is enough to drive the $B-L$ generation given by Eq. (6.9).

This approximate solution is plotted along with numerical solutions in Fig. 6. Note that $N_B$ becomes constant near the temperature where the approximation breaks down. This occurs when $Kx^5 \sim 20$, so the $B-L$ generated is (very roughly)

$$N_{B-L} \sim \frac{\epsilon}{g_*}\,, \tag{6.10}$$

which is independent of $K$. This explains why the baryon asymmetry generated is insensitive to $K$ over a large range of $K$ values. Also, since the solution is valid as long as $x \ll (5/K)^{1/3}$, it at least partially explains why baryogenesis is effective for large values of $K$.

In Figs. 8–10, we show the generated baryon asymmetry for various parameter choices for both the democratic and correlated neutrino mass scenarios. In the democratic scenario, the most important parameters are $M_{N_1}$, $m_{D3}$, and the ratio $M_{N_1}/M_{N_2}$ which enters into $\epsilon$. In Fig. 8 we plot the generated baryon number as a function of $M_{N_1}$ for several choices of $m_{D3}$. We have assumed

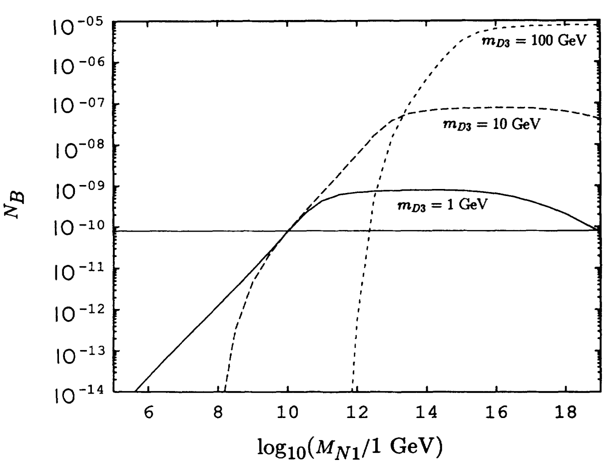


FIG. 8. The generated baryon asymmetry as a function of $M_{N_1}$ for the "democratic" neutrino mass scenario for various values of $m_{D3}$. The value of $N_B$ required by big-bang nucleosynthesis is given by the solid horizontal line. Note that $N_B$ is essentially proportional to the $CP$-violating parameter $\epsilon$, and our estimate of $\epsilon$ is rather uncertain, as explained in the text. Therefore, we can conclude that the scenario should probably be considered viable for $M_{N_1} \gtrsim 10^9$ GeV and $m_{D3} \lesssim 10$ GeV.

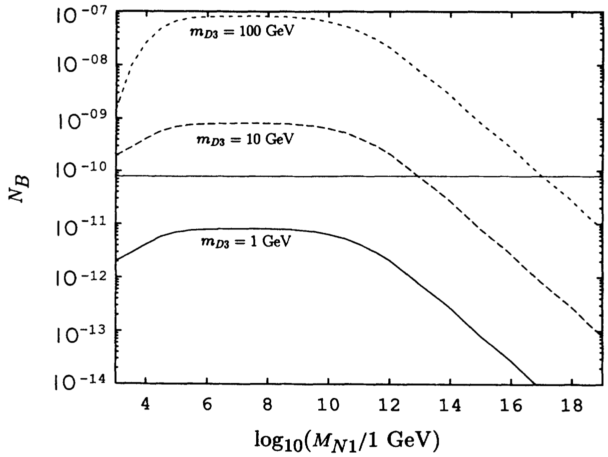


FIG. 9. The generated baryon asymmetry as a function of $M_{N_1}$ for the "correlated" neutrino mass scenario for $M_{D_1}=1$ MeV and various values of $m_{D3}$. We see that the scenario can work well for $M_{N_1}$ as low as $\sim 1$ TeV.

that $M_{N_1}/M_{N_2}=0.1$ for definiteness. Since the baryon number generated in this model is essentially proportional to $\epsilon$, and $\epsilon \propto M_{N_1}/M_{NM_2}$, the actual values of $B$ plotted should be viewed only as "typical" values for the given values of $M_{N_1}$ and $m_{D3}$. However, we expect the dependence of $B$ on $M_{N_1}$ and $m_{D3}$ shown in this figure to be correct.

We note that both for an "intermediate-scale seesaw" ($M_{N_1} \sim 10^8$ GeV) [17] and a "grand-unified-theory-(GUT-) scale seesaw" ($M_{N_1} \sim 10^{15}$ GeV) [6], the model considered here can give rise to the observed baryon asymmetry. These see-saw scenarios are attractive because they are theoretically well motivated by grand unification arguments, and because they give rise to neutrino masses with the correct order of magnitude to resolve the solar-neutrino problem via the Mikheyev-Smirnov-Wolfenstein (MSW) effect [18].

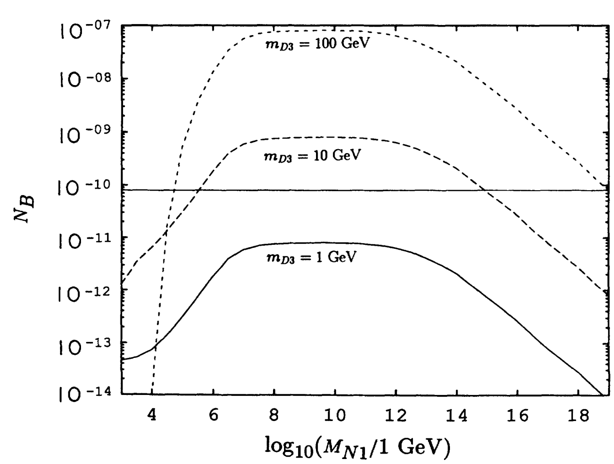


FIG. 10. The generated baryon asymmetry as a function of $M_{N_1}$ for the "correlated" neutrino mass scenario for $m_{D_1}=10$ MeV and various values of $m_{D_3}$.

Even with the large uncertainties present in our estimates, it is clear that sufficient baryon number cannot be generated for much of the parameter space. The value of $\epsilon$ may be much smaller than our estimates, since it is proportional to $\sin\delta$, where $\delta$ is a *CP*-violating phase. However, it is difficult to imagine how $\epsilon$ could be several orders of magnitude *larger* than our estimate. Thus, we can think of our results for $B$ as rough upper bounds. An example of a nontrivial case in which sufficient baryon asymmetry cannot be generated is $M_{N_1} \sim 10^8$ GeV with $m_{D3} \sim 100$ GeV. This corresponds to a simple possibility in the context of SO(10) with an intermediate breaking scale of $10^8$ GeV and where we take $m_{D3} \simeq m_t \gtrsim 100$ GeV.

In the correlated scenario, the most important parameters are $M_{N_1}$, $m_{D1}$, $m_{D3}$, and the ratio $M_{N_1}/M_{N_3}$ which enters into $\epsilon$. In Figs. 9 and 10 we plot the generated baryon number as a function of $M_{N_1}$ for several choices of $m_{D1}$ and $m_{D3}$. We have assumed $M_{N_1}/M_{N_3} = 10^{-3}$ for definiteness.

Note that in this case, the baryon asymmetry can be generated for Majorana masses $\sim 1$ TeV. If we assume that the neutrino Dirac masses are close to those of their charged-lepton partners, then this means that

$$m_{\nu_e} \sim 1 \text{ eV}, \quad m_{\nu_\mu} \sim 1 \text{ keV}, \quad m_{\nu_\tau} \sim 10 \text{ keV} , \tag{6.11}$$

if we assume $M_{N_1}:M_{N_2}:M_{N_3} = 1:10:100$. Thus, this scenario predicts neutrino masses near their direct experimental upper limits. Also, since the lightest right-handed neutrino has mass $\sim 1$ TeV, we might expect to see new lepton flavor physics at high-energy colliders. We find this scenario rather attractive, since it holds out the possibility that the physics responsible both for neutrino mass and baryogenesis in the early Universe is directly experimentally accessible.

## VII. CONCLUSIONS

We have seen that the observed baryon asymmetry in the Universe can be accounted for assuming only the existence of right-handed neutrinos with large Majorana masses. Depending on the assumptions made about the structure of the neutrino mass matrices, the scenario works for right-handed neutrino Majorana masses from 1 TeV all the way to the Planck scale $10^{19}$ GeV. On the one hand, this may be viewed as disappointing, since the model does not make a sharp prediction about the baryon asymmetry. On the other hand, we feel that it is very interesting that a very attractive class of models is capable of generating the baryon number of the Universe, including parameter ranges which are considered to be theoretically attractive for very different reasons.

## ACKNOWLEDGMENTS

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## APPENDIX A: THERMAL AVERAGES

In this Appendix, we review the formalism for thermal averaging of reaction rates and collect some useful formulas and establish notation.

We assume Maxwell-Boltzmann statistics, so that the equilibrium phase-space density of a particle species $a$ is

$$f_a^{\text{eq}}(\mathbf{p}) = e^{-E(\mathbf{p})/T} . \tag{A1}$$

(We assume that there are no net conserved charges present in the early Universe, so that all chemical potentials vanish.) The approximation of Maxwell-Boltzmann statistics is expected to introduce errors of order 10%, which is tolerable for our present purpose. The number density is

$$n_a = g_a \int \frac{d^3p}{(2\pi)^3} f_a(\mathbf{p}) , \tag{A2}$$

where $g_a$ is the number of internal degrees of freedom. In equilibrium, we have

$$n_a^{\text{eq}} = \frac{g_a m_a^2 T}{2\pi^2} K_2(m_a/T) , \tag{A3}$$

where $K_n(x)$ is a modified Bessel function. (We follow the conventions of [19].)

The Boltzmann equation for the phase-space distribution of the particle species $a$ can be written as

$$L f_a = -\tfrac{1}{2} C_a[f] , \tag{A4}$$

where the Liouville operator in a Roberson-Walker spacetime is given by

$$L f_a \equiv E_a \dot{f}_a - H |\mathbf{p}_a|^2 \frac{\partial f_a}{\partial E_a} \tag{A5}$$

and the "collision integral" is given by

$$\begin{aligned} C_a[f] \equiv \sum_{aX \leftrightarrow Y} \int d\pi_X d\pi_Y (2\pi)^4 \delta^4(p_a + p_X - p_Y) \\ \times [f_a f_X |\mathcal{A}(aX \to Y)|^2 \\ - f_Y |\mathcal{A}(Y \to aX)|^2] , \end{aligned} \tag{A6}$$

where the sum runs over all allowed processes $aX \leftrightarrow Y$, where $X$ and $Y$ are multiparticle states. $|\mathcal{A}|^2$ is the transition amplitude (averaged over internal degrees of freedom in both the initial and final states). We have used the abbreviations

$$d\pi_X \equiv \prod_{b \in X} d\pi_b, \quad d\pi_b \equiv g_b \frac{d^3 p_b}{(2\pi)^2} \frac{1}{2E(\mathbf{p}_b)} , \tag{A7}$$

$$p_X \equiv \sum_{b \in X} p_b, \quad f_X \equiv \prod_{b \in X} f_b , \tag{A8}$$

etc. As always, if there are identical particles in the initial or final states, appropriate symmetry factors must be introduced to avoid double-counting initial or final states.

We can write an equation for the number density by integrating Eq. (A4) over $d\pi_a$:

$$2\int d\pi_a L f_a = \dot{n}_a + 3Hn_a = -\int d\pi_a C_a[f] \; . \tag{A9}$$

Note that the right-hand side of this equation depends on the functional form of the phase-space distributions.

There are several assumptions which can be used to greatly simplify the Boltzmann equations. First, we assume that the particle species is in kinetic equilibrium (but not necessarily chemical equilibrium), so that the phase space distribution is

$$f_a(E) = \frac{n_a}{n_a^{\rm eq}} e^{-E/T} \; . \tag{A10}$$

This will be the case if there are reactions occurring rapidly which can change the kinetic energy of the $a$ particles, but the reactions which can change the number of $a$'s are out of equilibrium. Even in cases where this is not strictly true, the assumption of kinetic equilibrium can be viewed as an ansatz that the most important out-of-equilibrium effect is the deviation of the number density from its equilibrium value. We can then write a closed set of equations for the number densities:

$$\dot{n}_a + 3Hn_a = -\sum_{aX\leftrightarrow Y} \left[ \frac{n_a n_X}{n_a^{\rm eq} N_X^{\rm eq}} \gamma(aX\to Y) - \frac{n_Y}{n_Y^{\rm eq}} \gamma(Y\to aX) \right] , \tag{A11}$$

where

$$\gamma(aX\to Y) = \int d\pi_a d\pi_X d\pi_Y (2\pi)^4 \delta^4(p_a + p_X - p_y) \times f_a^{\rm eq} f_X^{\rm eq} |\mathcal{A}(aX\to Y)|^2 \; . \tag{A12}$$

If we neglect $CP$ violation, we have

$$|\mathcal{A}(X\to Y)|^2 = |\mathcal{A}(Y\to X)|^2 \; , \tag{A13}$$

and the Boltzmann equations simplify further. Note that in this case the $\gamma$'s are independent of the direction of the reaction, since the energy-conserving $\delta$ function allows us to make the replacement $f_a^{\rm eq} f_X^{\rm eq} = f_Y^{\rm eq}$ in Eq. (A12).

We now present explicit formulas for the collision terms for decays and two-body scattering, assuming that $CP$ is not violated. For decays and inverse decays $a\leftrightarrow Y$, we have

$$\gamma(a\leftrightarrow Y) = n_a^{\rm eq} \langle \Gamma(a\to Y) \rangle \; , \tag{A14}$$

where

$$\langle \Gamma(a\to Y) \rangle \equiv \frac{K_1(m_a/T)}{K_2(m_a/T)} \Gamma(a\to Y) \; . \tag{A15}$$

Here, $\Gamma$ is the usual (zero-temperature) decay rate, and the prefactor can be interpreted as a time dilation factor.

For two-body scattering $ab\leftrightarrow Y$,

$$\gamma(ab\leftrightarrow Y) = \frac{T}{64\pi^4} \int ds \sqrt{s} K_1(\sqrt{s}/T) \hat{\sigma}(s) \; . \tag{A16}$$

Here, $s$ is the usual Mandelstam variable, and the dimensionless "reduced cross section" $\hat{\sigma}(s)$ is the amplitude summed over final states,

$$\hat{\sigma}(s) \equiv 8\pi \Phi_2(s) \int d\pi_Y (2\pi)^4 \delta^4(p_a + p_b - p_Y) \times |\mathcal{A}(ab\leftrightarrow Y)|^2 \; , \tag{A17}$$

where $\Phi_2(s)$ is two-body phase space for the initial state:

$$\Phi_2(s) \equiv \int d\pi_a d\pi_b (2\pi)^4 \delta^4(p_a + p_b - p_Y) = \frac{g_a g_b}{8\pi s} \{ [s - (m_a + m_b)^2][s - (m_a - m_b)^2] \}^{1/2} \; . \tag{A18}$$

For two-to-two scattering,

$$\frac{d\hat{\sigma}}{dt} = \frac{g_a g_b g_c g_d}{8\pi s} |\mathcal{A}(ab\leftrightarrow Y)|^2 \; , \tag{A19}$$

where $s$ and $t$ are Mandelstam variables. The integral in Eq. (A16) can be efficiently evaluated numerically. We can relate $\gamma$ to more familiar quantities by writing

$$\gamma(ab\leftrightarrow Y) = n_a^{\rm eq} n_b^{\rm eq} \langle \sigma(ab\to Y) |\mathbf{v}| \rangle \; . \tag{A20}$$

In the presence of $CP$ violation, the $\gamma$'s depend on the reaction direction. In addition, there are subtleties regarding the handling of processes in which it is possible to create a real intermediate state. This issue is discussed briefly in the main text, and we refer to [12] for more detail.

## APPENDIX B: AVOIDING MAJORANA CONFUSION

In this Appendix, we give the Feynman rules for Majorana fermions. The graphical rules presented here are much simpler than those given in the literature [20], and are free from subtleties concerning the overall sign of amplitudes. It is highly probable that these rules are known to many practitioners, but to our knowledge they have not been explicitly stated in the literature, so we will state them concisely here.

Consider first the Lagrangian for a free two-component (right-handed) fermion with a Majorana mass term:

$$\mathcal{L} = \bar{\psi}_R i\partial\!\!\!/ \psi_R + \frac{m}{2}(\bar{\psi}_R \psi_R^c + \text{H.c.}) \; . \tag{B1}$$

(Our conventions for charge conjugation are given in Appendix C.) To quantize this, it is convenient to introduce a four-component Majorana field defined by

$$\chi \equiv \psi_R + \psi_R^c \; , \tag{B2}$$

where $\psi_R$ is now viewed as the right-handed projection of a four-component spinor. By definition, $\chi$ satisfies the Majorana condition

$$\chi^c = \chi \; . \tag{B3}$$

We can now rewrite the Lagrangian in terms of $\chi$:

$$\mathcal{L} = \tfrac{1}{2}\bar{\chi}(i\partial\!\!\!/ - m)\chi = -\tfrac{1}{2}\chi^T C^\dagger (i\partial\!\!\!/ - m)\chi \; . \tag{B4}$$

In the last line we have eliminated $\bar{\chi}$ using the Majorana

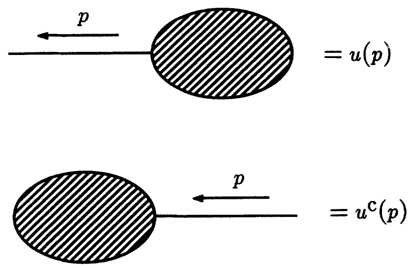


FIG. 11. Majorana spin wave functions.

condition:

$$\bar{\chi}=\bar{\chi}^c=-\chi^T C^\dagger \, . \tag{B5}$$

The propagator is just the inverse of the quadratic form in Eq. (B4):

$$\langle 0|T\chi_\alpha(0)\chi_\beta(x)|0\rangle=\int \frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\left[\frac{-i}{\not{p}-m+i0+}C\right]_{\alpha\beta} . \tag{B6}$$

In contrast with other approaches, there is only one kind of propagator, greatly simplifying the graphical rules. Since there is no conserved charge carried by $\chi$, we denote the propagator by a solid line with no arrows.

The spinor wave functions for external $\chi$ lines are given in Fig. 11. In calculations, it is often useful to use the fact that the standard Dirac spin wave functions can be chosen to satisfy

$$u_s^c=\bar{v}\,^T_{-s}, \quad v_s^c=\bar{u}\,^T_{-s} \, , \tag{B7}$$

where $s=\pm 1$ labels the spin state.

To derive Feynman rules for vertices, one simply eliminates $\bar{\chi}$ in the Lagrangian using Eq. (B5). For example, for the Yukawa coupling

$$\begin{aligned}\mathcal{L}_{\rm int}&=h\phi\bar{\psi}_R\psi_R^c+\text{H.c.}\\ &=h\phi\chi^T CP_L\chi-h^*\phi^\dagger\chi^T C^\dagger P_R\chi \, ,\end{aligned} \tag{B8}$$

the Feynman rules may be easily read off. Again, this approach yields only a single vertex for each physical process, while other approaches yield several.

Note that if the Lagrangian for the Majorana field is derived starting from a Lagrangian for a two-component field as done here, the resulting Lagrangian will contain helicity projection operators which ensure that the proper number of spin states propagate in fermion loops. Thus, there is no need to multiply by $\frac{1}{2}$ for closed Majorana fermion loops, as is required in other approaches. This eliminates confusion which can occur when there are both Dirac and Majorana fermions in the same loop.

## APPENDIX C: CONVENTIONS FOR CHARGE CONJUGATION

We summarize our conventions for charge conjugation here, since there are several different conventions currently in use. We define

$$\psi^c\equiv C\bar{\psi}^T \, , \tag{C1}$$

where $C$ satisfies

$$C^\dagger=C^{-1}, \quad C^T=-C \, , \tag{C2}$$

$$C^\dagger\gamma_\mu C=-\gamma_\mu^T \, . \tag{C3}$$

We use the notation

$$\bar{\psi}^c\equiv\overline{(\psi^c)}=-\psi^T C^\dagger \, , \tag{C4}$$

$$\psi_R^c\equiv\overline{(P_R\psi)}^c \, , \tag{C5}$$

$$\bar{\psi}_R^c\equiv\overline{(P_R\psi)^c} \, , \tag{C6}$$

etc. Then we have

$$(\psi^c)^c=\psi, \quad (\bar{\psi}^c)^c=\bar{\psi} \, . \tag{C7}$$

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