# $N=1$ supergravity as a nonlinear realization

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The $N=1$ supergravity in superspace (more precisely the minimal Einstein version of it) is consistently reformulated as a simultaneous nonlinear realization of two complex finite-dimensional supergroups generating via their closure the whole infinite-dimensional $N=1$ supergravity group and having in their intersection the rigid $N=1$ Poincaré supergroup chosen as the vacuum-stability subgroup. Thus $N=1$ supergravity is found to be a kind of nonlinear $\sigma$ model describing a partial spontaneous breaking of the infinite-dimensional supersymmetry down to the rigid $N=1$ supersymmetry. The only independent Goldstone superfield accompanying this breaking appears to be an axial-vector superfield $H^{\mu\dot{\mu}}(x,\theta,\bar{\theta})$ identified with the $N=1$ supergravity prepotential. All the other Goldstone superfields are expressed in terms of $H^{\mu\dot{\mu}}$ by imposing appropriate covariant constraints on the corresponding Cartan superforms (the inverse Higgs effect). Thereby, the 15-year-old result of Borisov and Ogievetsky who interpreted Einstein gravity as a nonlinear $\sigma$ model is generalized to the $N=1$ supergravity case. Possible implications of the proposed formulation are discussed. In particular, the intriguing analogy between $N=1$ supergravity and the (super) $p$-brane theories is pointed out.

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## I. INTRODUCTION

It is becoming more and more clear that most of the theories of current interest such as various (super)particle, (super)string and (super)membrane theories can be viewed as generalized nonlinear $\sigma$ models with appropriate (super)group coset spaces playing the role of target manifolds (see, e.g., Refs. [1–4]). As any theory possessing spontaneously broken symmetry, they can be universally constructed in terms of the corresponding nonlinear realizations [5,6]. This gives new geometrical insights into these theories and indicates their profound relations with more customary $\sigma$ models (associated with spontaneously broken internal symmetry, e.g., those describing a chiral dynamics of pions [7]).

It is worth mentioning that an analogous interpretation of the Einstein gravitation theory as well as of the Yang-Mills theory was given much earlier [8,9]. The basic feature of these theories consists of the fact that the underlying nonlinearly realized symmetry is *infinite*-dimensional, namely it is the group Diff$R^{1,3}$ in the Einstein case and the local internal symmetry group in the Yang-Mills theory. However, the infinite dimensionality of these symmetry groups does not represent a principal obstacle since it is possible to formulate the corresponding nonlinear realizations in terms of a *finite* number of Goldstone fields that are identified with the graviton and the Yang-Mills field quanta, respectively. This happens due to the inverse Higgs effect [10] which allows one to reduce the number of Goldstone fields to a subset of the essential ones by imposing appropriate covariant constraints on the corresponding Cartan forms, i.e., which allows one to express most of the Goldstone fields in terms of a few, independent, unremovable Goldstone fields and their derivatives.

The treatment of the gravity theory can be further simplified by the Ogievetsky theorem [11] according to which the group Diff$R^{1,3}$ can be obtained as a closure of two *finite*-dimensional groups—the 15-parameter conformal group and the 20-parameter affine group having the Poincaré group as a common subgroup. Then, as proved by Borisov and Ogievetsky in [8], the Einstein theory appears to be a simultaneous nonlinear realization of these two groups. Thus, in contrast to the Yang-Mills case [9], no need to introduce infinite sets of Goldstone fields arises. The only Goldstone field involved is the one associated with spontaneously broken affine transformations and it is identified quite naturally with the gravitational field [12].

The $\sigma$-model interpretation of Yang-Mills theories has been extended to the $N=1$ super Yang-Mills theory by one of the authors [13]. In contradistinction to the purely bosonic case, the self-consistent nonlinear-realization treatment of the $N=1$ Yang-Mills theory is only possible if one takes the fundamental symmetry to be a complex extension $G^c$ of the Yang-Mills group $G$.

For $N=1$ supergravity, which is very similar to the $N=1$ Yang-Mills theory by its intrinsic geometry, no consistent nonlinear $\sigma$-model formulation has been constructed so far. Only some preliminary steps have been given in [14,15]. It was shown there that the $N=1$ supergravity *infinite*-dimensional superspace groups (for both minimal and nonminimal cases) can be again represented as closures of two appropriate finite-dimensional supergroups but in a more complicated way compared to the gravity case.

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The purpose of the present paper is to fill this gap, i.e., to construct the nonlinear $\sigma$-model formulation of $N=1$ supergravity. We demonstrate that the superspace $N=1$ supergravity can indeed be consistently rederived by the nonlinear realization techniques. For simplicity and without losing generality we shall restrict ourselves to the minimal Einstein $N=1$ supergravity [16–18] since the nonminimal versions can be treated analogously.

In Sec. II we shall briefly review the gauge supergroup of the $N=1$ minimal Einstein supergravity and specify its structure in terms of its two important finite-dimensional subgroups. Both of them are complex and are realized by holomorphic transformations in the complex $N=1$ superspace $C^{4/2}\equiv\{x_L^m,\theta_L^\mu\}$. The first one consists of arbitrary $\theta_L$-dependent translations of $x_L^m$ whereas the other is the special linear group of $C^{4/2}$ (consisting of all linear transformations in $C^{4/2}$ with the Berezinean equal to 1).

Then Secs. III and IV contain nonlinear realizations of these two subgroups (treated as abstract ones) and eliminations of various Goldstone superfield by the inverse Higgs effect. It is shown that the $N=1$ minimal Einstein supergravity can be regarded as a simultaneous nonlinear realization of these two finite-dimensional subgroups of the complex Ogievetsky-Sokatchev supergroup [16] with the rigid $N=1$ Poincaré supergroup as the vacuum-stability subgroup and the axial superfield $H^{\mu\dot\mu}(x,\theta,\bar\theta)$ as the only essential Goldstone superfield [the other Goldstone superfields are eliminated from the theory by the inverse Higgs effect; they are expressed in terms of $H^{\mu\dot\mu}(x,\theta,\bar\theta)$ and its derivatives]. The corresponding Lagrangian turns out to be the simplest invariant with respect to both above-mentioned subgroups.

Finally, in Sec. V the derived results are compared with those of Ogievetsky and Sokatchev [16,17] and the complete correspondence is found.

Note that a nonlinear realization treatment of $N=1$ supergravity along similar lines was discussed recently also by Pletnev [19]. However, it remained incomplete, especially concerning the role of the supergroup of $\theta_L$-dependent translations.

## II. GAUGE SUPERGROUP OF $N=1$ MINIMAL EINSTEIN SUPERGRAVITY AND ITS STRUCTURE

The Ogievetsky-Sokatchev formulation [16] of $N=1$ minimal Einstein supergravity is based on the (4+2)-dimensional complex superspace

$$\begin{aligned} C^{4/2}=C^{4/4}/C^{0/2}&=\{(x_L^{\rho\dot\rho},\theta_L^\mu)\}\\ &=\{(x_R^{\rho\dot\rho},\bar\theta_R^{\dot\mu})\} \end{aligned} \tag{2.1}$$

with $(x_L^{\rho\dot\rho},\theta_L^\mu)$ and $(x_R^{\rho\dot\rho},\bar\theta_R^{\dot\mu})=(x_L^{\rho\dot\rho},\theta_L^\mu)^\dagger$ being its left- and right-handed parametrizations. In superspace $C^{4/2}$ the infinite-parameter complex gauge supergroup $G$ acts. It is infinitesimally defined by

$$\begin{aligned} \delta x_L^{\rho\dot\rho}&=\lambda^{\rho\dot\rho}(x_L,\theta_L)\ ,\\ \delta\theta_L^\mu&=\lambda^\mu(x_L,\theta_L)\ . \end{aligned} \tag{2.2}$$

Here, $\lambda^{\rho\dot\rho}$ and $\lambda^\mu$ are arbitrary superfunction parameters satisfying the condition

$$\frac{\partial\lambda^{\rho\dot\rho}}{\partial x_L^{\rho\dot\rho}}-\frac{\partial\lambda^\mu}{\partial\theta_L^\mu}=0\ , \tag{2.3}$$

which expresses infinitesimally the preservation of the supervolume in $C^{4/2}$ corresponding to the minimal version of the $N=1$ supergravity (for details see [16–18,15]).

Using the results of our previous paper [15] the following theorem is true.

*Theorem.* The infinite-parameter gauge superalgebra $a$ corresponding to transformations (2.2) restricted by (2.3) can be obtained by taking the closure of two finite-dimensional subalgebras:

$$a_{\rm I}=\left\{\begin{aligned} &Q_\mu=-i\frac{\partial}{\partial\theta_L^\mu}\equiv-i\partial_\mu;\ P_{L\rho\dot\rho}=-i\frac{\partial}{\partial x_L^{\rho\dot\rho}}\equiv-i\partial_{L\rho\dot\rho}\ ;\\ &\quad Q^\mu_{\rho\dot\rho}=\theta^\mu\partial_{L\rho\dot\rho};\ K_{\beta\dot\beta}=-i(\theta_L^\mu\theta_{L\mu})\partial_{\beta\dot\beta} \end{aligned}\right\} \tag{2.4}$$

and

$$\begin{aligned} a_{\rm II}=\{&Q_\mu;P_{L\rho\dot\rho};Q^\mu_{\rho\dot\rho};\ R^{\alpha\dot\alpha}{}_{\beta\dot\beta}=-i[x^{\alpha\dot\alpha}\partial_{\beta\dot\beta}-\tfrac14\delta^\alpha{}_\beta\delta^{\dot\alpha}{}_{\dot\beta}(x\partial)]\ ;\\ &T_{(\mu\nu)}=\tfrac12\theta_{(\mu}\partial_{\nu)};\ I^{\rho\dot\rho}{}_\mu=x^{\rho\dot\rho}\partial_\mu;\\ &D=-i(x^{\rho\dot\rho}\partial_{\rho\dot\rho}+2\theta^\mu\partial_\mu)\}\ . \end{aligned} \tag{2.5}$$

The closure is as follows:

$$\underbrace{K_{\rho\dot\rho},\ \overbrace{P_{L\rho\dot\rho},\ Q^\mu_{\rho\dot\rho},\ Q^\mu}^{}}_{a_{\rm I}}\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\phantom{K_{\rho\dot\rho},\ }\overbrace{\phantom{P_{L\rho\dot\rho},\ Q^\mu_{\rho\dot\rho},\ Q^\mu},\ R^{\alpha\dot\alpha}{}_{\beta\dot\beta},\ T_{(\mu\nu)},\ I^{\rho\dot\rho}{}_\mu,\ D}^{a_{\rm II}}\ .$$

The structure relations (nonzero only) of superalgebras $a_{\rm I}$ and $a_{\rm II}$ are given by

$a_{\rm I}$ :

$$\{Q_\mu,Q^\nu_{\rho\dot\rho}\}=\delta^\nu_\mu P_{L\rho\dot\rho},\quad [Q_\mu,K_{\rho\dot\rho}]=-2Q_{\mu\rho\dot\rho}\ , \tag{2.6}$$

$a_{\rm II}$ :

$$\begin{aligned} &[T_{(\alpha\beta)},T_{(\rho\lambda)}]=i(\varepsilon_{\alpha\rho}T_{(\beta\lambda)}+\varepsilon_{\alpha\lambda}T_{(\beta\rho)}+\varepsilon_{\beta\rho}T_{(\alpha\lambda)}\\ &\qquad\qquad+\varepsilon_{\beta\lambda}T_{(\alpha\rho)})\ ,\\ &[R_{\alpha\dot\alpha\beta\dot\beta}\ ,R_{\gamma\dot\gamma\delta\dot\delta}]=i(\varepsilon_{\alpha\delta}\varepsilon_{\dot\alpha\dot\delta}R_{\gamma\dot\gamma\beta\dot\beta}-\varepsilon_{\beta\gamma}\varepsilon_{\dot\beta\dot\gamma}R_{\alpha\dot\alpha\delta\dot\delta})\ ,\\ &[T_{(\alpha\beta)},Q^\mu_{\nu\dot\nu}]=-i(\delta^\mu{}_\beta Q_{\alpha\nu\dot\nu}+\delta_\alpha{}^\mu Q_{\beta\nu\dot\nu})\ ,\\ &[T_{(\alpha\beta)},Q_\mu]=i(\varepsilon_{\alpha\mu}Q_\beta+\varepsilon_{\beta\mu}Q_\alpha)\ ,\\ &[T_{(\alpha\beta)},I_\mu{}^{\rho\dot\rho}]=i(\varepsilon_{\alpha\mu}I_\beta{}^{\rho\dot\rho}+\epsilon_{\beta\mu}I_\alpha{}^{\rho\dot\rho})\ ,\\ &[R_{\alpha\dot\alpha\beta\dot\beta},P_{L\rho\dot\rho}]=i(\varepsilon_{\alpha\rho}\varepsilon_{\dot\alpha\dot\rho}P_{L\beta\dot\beta}-\tfrac14\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta}P_{L\rho\dot\rho})\ ,\\ &[R_{\alpha\dot\alpha\beta\dot\beta},Q^\mu_{\rho\dot\rho}]=i(\varepsilon_{\alpha\rho}\varepsilon_{\dot\alpha\dot\rho}Q^\mu_{\beta\dot\beta}-\tfrac14\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta}Q^\mu_{\rho\dot\rho})\ ,\\ &[R_{\alpha\dot\alpha\beta\dot\beta},I^{\rho\dot\rho}_\mu]=-i(\delta_\beta{}^\rho\delta_{\dot\beta}{}^{\dot\rho}I_{\mu(\alpha\dot\alpha)}-\tfrac14\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta}I_\mu{}^{\rho\dot\rho})\ ,\\ &[D,Q^\mu_{\alpha\dot\alpha}]=-iQ^\mu_{\alpha\dot\alpha},\quad [D,Q_\mu]=2iQ_\mu\ ,\\ &[D,P_{L\rho\dot\rho}]=iP_{L\rho\dot\rho},\quad [D,I_\mu{}^{\rho\dot\rho}]=iI_\mu{}^{\rho\dot\rho}\ ,\\ &\{Q^\mu_{\alpha\dot\alpha},I_\nu{}^{\beta\dot\beta}\}=\frac{i}{2}(\delta^\beta{}_\alpha\delta_{\dot\alpha}{}^{\dot\beta}T^{(\mu}{}_{\nu)}+2\delta^\mu{}_\nu R^{\beta\dot\beta}{}_{\alpha\dot\alpha}\\ &\qquad\qquad+\tfrac12\delta_\alpha{}^\beta\delta_{\dot\alpha}{}^{\dot\beta}\delta^\mu{}_\nu D)\ . \end{aligned} \tag{2.7}$$

The meaning of $a_{\rm I}$ and $a_{\rm II}$ is rather simple. $a_{\rm I}$ is the superalgebra of all Grassmann vector fields (i.e., each element of $a_{\rm I}$ can be obtained by gauging ordinary four-translations in purely Grassmann directions) and $a_{\rm II}$ is the special linear superalgebra in $C^{4/2}$.

*Remarks*

(i) The theorem can be proved by taking into account the following facts: First, all the generators of superalgebras $a_{\rm I}$ and $a_{\rm II}$ belong to the five types of generators listed in [15] which yield transformations (2.2) satisfying (2.3). Second, all the lowest-dimensional generators of superalgebra $a$ can be obtained from the generators of $a_{\rm I}$ and $a_{\rm II}$ by using the relations (2.6), (2.7) and relations of the type $[\![a_{\rm I},a_{\rm II}]\!]$. Finally, the higher-dimensional generators of $a$ can be derived step by step from the previous ones by successively commuting the latter with each other and using the induction technique.

(ii) In [15] the superalgebra $a$ was obtained by taking the closure of two superalgebras that differ from $a_{\rm I}$ and $a_{\rm II}$. The choice of $a_{\rm I}$, $a_{\rm II}$ has the advantage for constructing nonlinear realizations in the next sections since $a_{\rm II}$ (in contradistinction to the superalgebras used in [15]) involves the generators $R^{\alpha\dot\alpha}{}_{\beta\dot\beta}$ and $I_\mu{}^{\rho\dot\rho}$ which will be shown to have as their associate Goldstone superfields those including the lowest components gauge fields of graviton and gravitino.

(iii) The superalgebra $a_{\rm II}$ contains the Lorentz generators $M_{\alpha\beta}$ and $M_{\dot\alpha\dot\beta}$ given by

$$M_{\alpha\beta}=R_{(\alpha\beta)}+T_{(\alpha\beta)},\quad M_{\dot\alpha\dot\beta}=R_{(\dot\alpha\dot\beta)}\ , \tag{2.8}$$

where

$$R_{(\alpha\beta)}=-ix_{L(\alpha}{}^{\dot\rho}\partial_{L\beta)\dot\rho},\quad R_{(\dot\alpha\dot\beta)}=-ix_{L(\dot\alpha}{}^{\rho}\partial_{L\dot\beta)\rho}\ ,$$

$$R_{\alpha\dot\alpha\beta\dot\beta}=\tfrac{1}{4}\{R_{(\alpha\beta)(\dot\alpha\dot\beta)}+\varepsilon_{\alpha\beta}R_{(\dot\alpha\dot\beta)}+\varepsilon_{\dot\alpha\dot\beta}R_{(\alpha\beta)}\}\ .$$

These Lorentz generators form a semidirect sum with superalgebra $a_{\rm I}$. Thus, without losing generality, they can be added to $a_{\rm I}$ and included into the intersection in Fig. 1.

(iv) The generators (2.4), (2.5) are essentially complex and so the corresponding group elements will in general be defined on the complex parameter manifold. Factorizations over one or another real subgroup (i.e., passing to a coset) will then amount to leaving only imaginary parts in the corresponding group parameters. For instance, factorization over physical (real) Lorentz group will mean that the generators (2.8) enter into the coset elements in the combination

$$i(\gamma^{\alpha\beta}M_{\alpha\beta}-\bar\gamma^{\dot\alpha\dot\beta}M_{\dot\alpha\dot\beta})$$

while the combination

$$i(l^{\alpha\beta}M_{\alpha\beta}+\bar l^{\dot\alpha\dot\beta}M_{\dot\alpha\dot\beta})$$

specifies an element of the stability subgroup.

In the next two sections we shall show that the $N=1$ minimal Einstein supergravity is the simultaneous nonlinear realization of the supergroups $G_{\rm I}$ and $G_{\rm II}$ corresponding to superalgebras $a_{\rm I}$ and $a_{\rm II}$, respectively, with the stability subgroup $H$ being the physical Lorentz group generated by $M_{\alpha\beta}$, $M_{\dot\alpha\dot\beta}$ defined in (2.8) [recall the remark (iv)].

## III. NONLINEAR REALIZATION OF $G_{\rm I}$

Let us denote $G_{\rm I}$ the complex supergroup, the superalgebra of which is $a_{\rm I}$ defined in (2.4). In what follows, we shall not need a specific coordinate realization of the $a_{\rm I}$ generators and thus we shall treat them as the abstract ones subjected to Eqs. (2.6). Each element $g_{\rm I}$ of supergroup $G_{\rm I}$ can be parametrized in the following way convenient for constructing the nonlinear realization of $G_{\rm I}$

$$g_{\rm I}=g_1g_2g_3\ , \tag{3.1}$$

where

$$\begin{aligned} g_1&=\exp\{i(\theta^\alpha Q_\alpha+x_L{}^{\alpha\dot\alpha}P_{L\alpha\dot\alpha})\}\ ,\\ g_2&=\exp\{i(\psi_\mu{}^{\rho\dot\rho}Q^\mu{}_{\rho\dot\rho})\}\ ,\\ g_3&=\exp\{ia^{\rho\dot\rho}K_{\rho\dot\rho}\}\ . \end{aligned}\tag{3.2}$$

As has been noted in remark (iii) of Sec. II, group $G_{\rm I}$ can be extended by including physical Lorentz generators. Then (3.1) represents the coset of such extended group over its Lorentz subgroup and $G_{\rm I}$ can be regarded as the corresponding factor group. Transformation properties of the group parameters $\theta^\alpha$, $x_L{}^{\alpha\dot\alpha}$, $\psi_\mu{}^{\rho\dot\rho}$, and $a^{\rho\dot\rho}$ follow from the group multiplication law

$$g_{\rm I}^0g_{\rm I}=g_{\rm I}',\quad g_{\rm I}^0\in G_{\rm I}\ . \tag{3.3}$$

Assuming $g_{\rm I}^0$ of the form

$$g_{\rm I}^0\approx\mathbb{1}+i(\varepsilon^\alpha Q_\alpha+c^{\alpha\dot\alpha}P_{L\alpha\dot\alpha}+\beta_\mu^{\nu\dot\nu}Q^\mu{}_{\nu\dot\nu}+\gamma^{\alpha\dot\alpha}K_{\alpha\dot\alpha}) \tag{3.4}$$

we obtain the infinitesimal transformation laws of the supergroup parameters

$$\begin{aligned} &\delta\theta^\alpha=\varepsilon^\alpha\ ,\\ &\delta x_L{}^{\rho\dot\rho}=c^{\rho\dot\rho}+i\theta^\mu\beta_\mu{}^{\rho\dot\rho}+(\theta^\delta\theta_\delta)\gamma^{\rho\dot\rho}\ ,\\ &\delta\psi_\mu{}^{\rho\dot\rho}=\beta_\mu{}^{\rho\dot\rho}-2i\theta_\mu\gamma^{\rho\dot\rho}\ ,\\ &\delta a^{\rho\dot\rho}=\gamma^{\rho\dot\rho}\ . \end{aligned}\tag{3.5}$$

Now, following the general routines [5,6] we introduce left-invariance Cartan 1-form $\omega_{\rm I}{}^\alpha$, $\omega_{\rm I}{}^{\beta\dot\beta}$, $\omega_{{\rm I}\mu}{}^{\rho\dot\rho}$, and $k_{\rm I}{}^{\beta\dot\beta}$ via

$$g_{\rm I}^{-1}dg_{\rm I}=i\{\omega_{\rm I}{}^\alpha Q_\alpha+\omega_{\rm I}{}^{\beta\dot\beta}P_{L\beta\dot\beta}+\omega_{{\rm I}\mu}{}^{\rho\dot\rho}S^\mu_{\rho\dot\rho}+k_{\rm I}{}^{\beta\dot\beta}K_{\beta\dot\beta}\}\ . \tag{3.6}$$

By a direct computation we obtain

$$\omega_{\rm I}{}^\alpha=d\theta^\alpha\ , \tag{3.7a}$$

$$\omega_{{\rm I}L}{}^{\beta\dot\beta}=dx_L{}^{\beta\dot\beta}+i\psi_\mu{}^{\beta\dot\beta}d\theta^\mu\ , \tag{3.7b}$$

$$\omega_{{\rm I}\mu}{}^{\rho\dot\rho}=d\psi_\mu{}^{\rho\dot\rho}+2ia^{\rho\dot\rho}d\theta_\mu\ , \tag{3.7c}$$

$$k_{\rm I}{}^{\beta\dot\beta}=da^{\beta\dot\beta}\ . \tag{3.7d}$$

We shall use these left-invariant Cartan 1-forms to eliminate some Goldstone superfields which are associated with the group generators. More precisely, we shall identify the group coordinate associated with $P^{\mu\dot{\mu}}=P_L{}^{\mu\dot{\mu}}+P_R{}^{\mu\dot{\mu}}=P_L{}^{\mu\dot{\mu}}+(P_L{}^{\mu\dot{\mu}})$ with the bosonic coordinates $x^{\mu\dot{\mu}}$ of the real superspace, while that for $P_A{}^{\mu\dot{\mu}}=i(P_L{}^{\mu\dot{\mu}}-P_R{}^{\mu\dot{\mu}})$ with the Goldstone superfield $H^{\mu\dot{\mu}}(x,\theta,\bar{\theta})$. The group parameters $\theta^\mu=\theta^\mu_L, \bar{\theta}^{\dot{\mu}}\equiv\bar{\theta}^{\dot{\mu}}_R=(\theta^\mu_L)^\dagger$ are interpreted as Grassmannian coordinates of the real superspace $R^{4/4}$. The complex $(4+2)$-dimensional superspace (2.1) can be regarded as an $(8+4)$-dimensional real superspace $\{(x_L{}^{\rho\dot{\rho}},x_R{}^{\rho\dot{\rho}},\theta^\mu_L,\bar{\theta}^{\dot{\mu}}_R)\}$. In this superspace a real physical superspace $\{(x^{\rho\dot{\rho}},\theta^\mu,\bar{\theta}^{\dot{\mu}})\}$ is imbedded as a $(4+4)$-dimensional hypersurface determined by four equations $x_L{}^{\rho\dot{\rho}}-x_R{}^{\rho\dot{\rho}}=2iH^{\rho\dot{\rho}}(x,\theta,\bar{\theta})$ [16]. The remaining group parameters $\psi_\mu{}^{\rho\dot{\rho}}$ and $a^{\rho\dot{\rho}}$ will be identified with Goldstone superfields $\Psi_\mu{}^{\rho\dot{\rho}}(x,\theta,\bar{\theta})$ and $A^{\rho\dot{\rho}}(x,\theta,\bar{\theta})$ given on $R^{4/4}$ which will be expressed in terms of $H^{\rho\dot{\rho}}(x,\theta,\bar{\theta})$ by exploiting the inverse Higgs effect [10] and thus eliminated from the theory.

Keeping in mind that

$$\psi_{\dot{\mu}}{}^{\alpha\dot{\alpha}}=\overline{\psi_\mu{}^{\alpha\dot{\alpha}}},\quad \omega_R{}^{\alpha\dot{\alpha}}=\overline{\omega_L{}^{\alpha\dot{\alpha}}},\quad\text{and } x_R{}^{\alpha\dot{\alpha}}=\overline{x_L{}^{\alpha\dot{\alpha}}} \tag{3.8}$$

so that

$$x_L{}^{\alpha\dot{\alpha}}=x^{\alpha\dot{\alpha}}+iH^{\alpha\dot{\alpha}},\quad x_R{}^{\alpha\dot{\alpha}}=x^{\alpha\dot{\alpha}}-iH^{\alpha\dot{\alpha}}\ , \tag{3.9}$$

and using (3.7) we find the following expressions for the covariant differentials $\nabla x^{\alpha\dot{\alpha}}$ and $\nabla H^{\alpha\dot{\alpha}}$:

$$\nabla x^{\alpha\dot{\alpha}}\equiv\mathrm{Re}\,\omega_L{}^{\alpha\dot{\alpha}}=dx^{\alpha\dot{\alpha}}+\frac{i}{2}\psi_\mu{}^{\alpha\dot{\alpha}}d\theta^\mu+\frac{i}{2}\psi_{\dot{\mu}}{}^{\alpha\dot{\alpha}}d\bar{\theta}^{\dot{\mu}} \tag{3.10}$$

and

$$\nabla H^{\alpha\dot{\alpha}}\equiv\mathrm{Im}\,\omega_L{}^{\alpha\dot{\alpha}}=dH^{\alpha\dot{\alpha}}+\frac{1}{2}\psi_\mu{}^{\alpha\dot{\alpha}}d\theta^\mu-\frac{1}{2}\psi_{\dot{\mu}}{}^{\alpha\dot{\alpha}}d\bar{\theta}^{\dot{\mu}}\ . \tag{3.11}$$

Now we shall decompose $\nabla H^{\alpha\dot{\alpha}}$ in the covariant differentials $\nabla x^{\alpha\dot{\alpha}}$, $d\theta^\mu$, $d\bar{\theta}^{\dot{\mu}}$ and calculate spinor covariant derivatives of $H^{\alpha\dot{\alpha}}$, i.e., $\mathcal{D}_\mu H^{\alpha\dot{\alpha}}$ and $\bar{\mathcal{D}}_{\dot{\mu}}H^{\alpha\dot{\alpha}}$ (following the general procedure described in [6]). We get

$$\begin{aligned}\nabla H^{\alpha\dot{\alpha}}&\equiv\nabla x^{\beta\dot{\beta}}\partial_{\beta\dot{\beta}}H^{\alpha\dot{\alpha}}+d\theta^\mu\mathcal{D}_\mu H^{\alpha\dot{\alpha}}-d\bar{\theta}^{\dot{\mu}}\bar{\mathcal{D}}_{\dot{\mu}}H^{\alpha\dot{\alpha}}\\&=dx^{\beta\dot{\beta}}\partial_{\beta\dot{\beta}}H^{\alpha\dot{\alpha}}+d\theta^\mu\partial_\mu H^{\alpha\dot{\alpha}}+d\bar{\theta}^{\dot{\mu}}\partial_{\dot{\mu}}H^{\alpha\dot{\alpha}}-\frac{1}{2}d\theta^\mu\psi_\mu{}^{\alpha\dot{\alpha}}+\frac{i}{2}d\theta^\mu\psi_\mu{}^{\nu\dot{\nu}}\partial_{\nu\dot{\nu}}H^{\alpha\dot{\alpha}}+\frac{1}{2}d\bar{\theta}^{\dot{\mu}}\psi_{\dot{\mu}}{}^{\alpha\dot{\alpha}}+\frac{i}{2}d\bar{\theta}^{\dot{\mu}}\psi_{\dot{\mu}}{}^{\nu\dot{\nu}}\partial_{\nu\dot{\nu}}H^{\alpha\dot{\alpha}}\end{aligned} \tag{3.12}$$

so that

$$\begin{aligned}\mathcal{D}_\mu H^{\alpha\dot{\alpha}}&=\partial_\mu H^{\alpha\dot{\alpha}}-\frac{1}{2}\psi_\mu{}^{\nu\dot{\nu}}(\partial_\nu{}^\alpha\partial_{\dot{\nu}}{}^{\dot{\alpha}}-i\partial_{\nu\dot{\nu}}H^{\alpha\dot{\alpha}})\\&\equiv\partial_\mu H^{\alpha\dot{\alpha}}-\frac{1}{2}\psi_\mu{}^{\nu\dot{\nu}}A_{\nu\dot{\nu}}{}^{\alpha\dot{\alpha}}\end{aligned} \tag{3.13}$$

and

$$\begin{aligned}\bar{\mathcal{D}}_{\dot{\mu}}H^{\alpha\dot{\alpha}}&=-\partial_{\dot{\mu}}H^{\alpha\dot{\alpha}}-\frac{1}{2}\psi_{\dot{\mu}}{}^{\nu\dot{\nu}}(\partial_\nu{}^\alpha\partial_{\dot{\nu}}{}^{\dot{\alpha}}+i\partial_{\nu\dot{\nu}}H^{\alpha\dot{\alpha}})\\&\equiv-\partial_{\dot{\mu}}H^{\alpha\dot{\alpha}}-\frac{1}{2}\psi_{\dot{\mu}}{}^{\nu\dot{\nu}}\bar{A}_{\nu\dot{\nu}}{}^{\alpha\dot{\alpha}}\ .\end{aligned} \tag{3.14}$$

Equating spinor covariant derivatives $\mathcal{D}_\mu H^{\alpha\dot{\alpha}}$ and $\bar{\mathcal{D}}_{\dot{\mu}}H^{\alpha\dot{\alpha}}$ to zero

$$\mathcal{D}_\mu H^{\alpha\dot{\alpha}}=\bar{\mathcal{D}}_{\dot{\mu}}H^{\alpha\dot{\alpha}}=0 \tag{3.15}$$

which is the operation covariant with respect to the left action of $G$, we obtain

$$\psi_\mu{}^{\nu\dot{\nu}}=2(A^{-1})^{\nu\dot{\nu}}{}_{\beta\dot{\beta}}\partial_\mu H^{\beta\dot{\beta}}=2\nabla_\mu H^{\nu\dot{\nu}} \tag{3.16}$$

and

$$\psi_{\dot{\mu}}{}^{\nu\dot{\nu}}=-2(\bar{A}^{-1})^{\nu\dot{\nu}}{}_{\beta\dot{\beta}}\partial_{\dot{\mu}}H^{\beta\dot{\beta}}=2\nabla_{\dot{\mu}}H^{\nu\dot{\nu}} \tag{3.17}$$

where the matrix $A^{\nu\dot{\nu}}{}_{\rho\dot{\rho}}$ is defined in Eq. (3.13).

Analogously, we can define covariant derivatives of $\psi_\mu{}^{\rho\dot{\rho}}$ by expanding the 1-form $\omega_\mu{}^{\rho\dot{\rho}}$ in the covariant differentials $\nabla x^{\alpha\dot{\alpha}}$, $d\theta^\mu$, $d\bar{\theta}^{\dot{\mu}}$, namely

$$\begin{aligned}\omega_\mu{}^{\rho\dot{\rho}}&\equiv\nabla x^{\gamma\dot{\gamma}}\partial_{\gamma\dot{\gamma}}\psi_\mu{}^{\rho\dot{\rho}}+d\theta^\beta\mathcal{D}_\beta\psi_\mu{}^{\rho\dot{\rho}}-d\bar{\theta}^{\dot{\beta}}\bar{\mathcal{D}}_{\dot{\beta}}\psi_\mu{}^{\rho\dot{\rho}}\\&=\nabla x^{\gamma\dot{\gamma}}\partial_{\gamma\dot{\gamma}}\psi_\mu{}^{\rho\dot{\rho}}+d\theta^\beta[\nabla_\beta\psi_\mu{}^{\rho\dot{\rho}}+2i\epsilon_{\mu\beta}a^{\rho\dot{\rho}}]\\&\quad-d\bar{\theta}^{\dot{\beta}}\nabla_{\dot{\beta}}\psi_\mu{}^{\rho\dot{\rho}}\ ,\end{aligned} \tag{3.18}$$

$$\nabla_\beta=\partial_\beta+i(A^{-1})^{\nu\dot{\nu}}{}_{\mu\dot{\mu}}\partial_\beta H^{\mu\dot{\mu}}\partial_{\nu\dot{\nu}}=\partial_\beta+i\nabla_\beta H^{\mu\dot{\mu}}\partial_{\mu\dot{\mu}}\ , \tag{3.19}$$

$$\nabla_{\dot{\beta}}=-\partial_{\dot{\beta}}+i(\bar{A}^{-1})^{\nu\dot{\nu}}{}_{\mu\dot{\mu}}\partial_{\dot{\beta}}H^{\mu\dot{\mu}}\partial_{\nu\dot{\nu}}=-\partial_{\dot{\beta}}-i\nabla_{\dot{\beta}}H^{\mu\dot{\mu}}\partial_{\mu\dot{\mu}},$$

$$\begin{aligned}\nabla x^{\gamma\dot{\gamma}}&=dx^{\gamma\dot{\gamma}}+i(A^{-1})^{\gamma\dot{\gamma}}{}_{\beta\dot{\beta}}\partial_\mu H^{\beta\dot{\beta}}d\theta^\mu\\&\quad-i(\bar{A}^{-1})^{\gamma\dot{\gamma}}{}_{\beta\dot{\beta}}\partial_{\dot{\mu}}H^{\beta\dot{\beta}}d\bar{\theta}^{\dot{\mu}}\ ,\end{aligned} \tag{3.20}$$

$$\nabla H^{\gamma\dot{\gamma}}=\nabla x^{\rho\dot{\rho}}\partial_{\rho\dot{\rho}}H^{\gamma\dot{\gamma}} \tag{3.21}$$

[we have substituted expressions (3.16), (3.19) into Eqs. (3.10), (3.11)].

Equating to zero $\mathcal{D}^\mu\psi_\mu^{\rho\dot{\rho}}$, i.e., $\mathcal{D}^\mu\psi_\mu{}^{\rho\dot{\rho}}=\nabla^\mu\psi_\mu{}^{\rho\dot{\rho}}-4ia^{\rho\dot{\rho}}=0$ one finds

$$a^{\rho\dot{\rho}}=-\frac{i}{4}\nabla^\mu\psi_\mu{}^{\rho\dot{\rho}}=\frac{1}{2i}(\nabla^\mu\nabla_\mu)H^{\rho\dot{\rho}}\ . \tag{3.22}$$

Note that $\nabla_{(\beta}\psi_{\mu)}{}^{\rho\dot{\rho}}$ is also covariant, but it vanishes identically after substituting (3.16), (3.17) and using the property $\{\nabla_\mu, \nabla_\nu\} = \nabla_{(\mu}\nabla_{\nu)} = 0$.

Thus the nonlinear realization of $G_{\rm I}$ can be entirely formulated in terms of the superfield $H^{\rho\dot\rho}(x,\theta,\bar\theta)$, all the other superfield supergroup parameters are covariantly eliminated by the inverse Higgs effect in terms of $H^{\rho\dot\rho}$.

The remaining $G_{\rm I}$ covariants are

$$\partial_{\mu\dot\mu}H^{\beta\dot\beta}, \quad \nabla_{\dot\beta}\nabla_\mu H^{\rho\dot\rho} \quad (\text{and conjugated} - \nabla_\beta \nabla_{\dot\mu} H^{\rho\dot\rho}) \tag{3.23}$$

and their $\partial_{\gamma\dot\gamma}$, $\nabla_\beta$, $\nabla_{\dot\mu}$ derivatives.

To summarize, we have shown that the basic building blocks of the Ogievetsky-Sokatchev formulation of minimal $N=1$ supergravity $[H^{\mu\dot\mu}(x,\theta,\bar\theta), \nabla_\beta H^{\mu\dot\mu}, \bar\nabla_{\dot\beta} H^{\mu\dot\mu}]$ naturally emerge already at the rigid-supersymmetry level in the framework of a nonlinear realization of the supergroup $G_{\rm I}$. The problem now is to select those of the $G_{\rm I}$ covariants (3.23) which are simultaneously covariant with respect to the supergroup $G_{\rm II}$. Then the minimal action constructed out of these covariants can be expected to coincide with the minimal $N=1$ supergravity action. But before, we need to implement a nonlinear realization of $G_{\rm II}$ on the same objects $x^{\mu\dot\mu}$, $\theta^\mu$, $\bar\theta^{\dot\mu}$, $H^{\nu\dot\nu}(x,\theta,\bar\theta)$.

## IV. NONLINEAR REALIZATION OF $G_{\rm II}$

Analogously to the previous case, $G_{\rm II}$ denotes a complex supergroup the superalgebra of which is $\mathscr{a}_{\rm II}$ defined in (2.6). Each element $g_{\rm II}$ of supergroup $G_{\rm II}$ can be parametrized as

$$g_{\rm II} = \tilde g_{\rm II} l \ , \tag{4.1}$$

where $\tilde g_{\rm II}$ denotes the element of the coset space $G_{\rm II}/L$ with $L$ being the Lorentz group

$$L = \exp\{il^{\alpha\beta}M_{\alpha\beta}\}\exp\{i\bar l^{\dot\alpha\dot\beta}M_{\dot\alpha\dot\beta}\}, \quad \bar l^{\dot\alpha\dot\beta} \equiv \overline{(l^{\alpha\beta})} \ . \tag{4.2}$$

Here $M_{\alpha\beta}$ and $M_{\dot\alpha\dot\beta}$ are defined in (2.8) and $l$ is an element of the Lorentz group $L$.

The element $\tilde g_{\rm II}$ of the coset space $G_{\rm II}/L$ can be parametrized in the following way:

$$\tilde g_{\rm II} = g_1 g_2 g_3 g_4 g_5 \ , \tag{4.3}$$

where $g_1$ and $g_2$ are defined in (3.2) and

$$\begin{aligned} g_3 &= \exp\{i\lambda^\mu_{\rho\dot\rho} I_\mu{}^{\rho\dot\rho}\} \ , \\ g_4 &= \exp\{i\pi_{\alpha\dot\alpha}{}^{\beta\dot\beta} R^{\alpha\dot\alpha}{}_{\beta\dot\beta}\} \ , \\ g_5 &= \exp\{i\varphi D\} \ . \end{aligned} \tag{4.4}$$

Actually one could start with the general element of $G_{\rm II}$, adding to (4.3) the one more factor, namely $\exp\{i\pi^{\alpha\beta}T_{(\alpha\beta)}\}$ and requiring the right gauge invariance under $L$. But one may completely fix this right gauge freedom by putting $\pi^{\alpha\beta}=0$ and arrive eventually at $\tilde g_{\rm II}$ as a $G_{\rm II}/L$ coset representative. One might equally choose a different gauge condition that would amount to a different form of $\tilde g_{\rm II}$. The choice $\pi^{\alpha\beta}=0$ is most convenient for our purposes.

The Cartan 1-forms are defined once again according to the general rules of [5,6] by

$$\begin{aligned} \tilde g_{\rm II}^{-1} d\tilde g_{\rm II} = i\{ &\omega^\alpha_{\rm II} Q_\alpha + \omega^{\alpha\dot\alpha}_{L\rm II} P_{L\alpha\dot\alpha} + \omega_{{\rm II}\mu}{}^{\alpha\dot\alpha} Q^\mu_{\alpha\dot\alpha} + \Omega^{\dot\mu}_{\rho\dot\rho} I_{\dot\mu}{}^{\rho\dot\rho} \\ &+ \omega_{R\alpha\dot\alpha}{}^{\beta\dot\beta} R^{\alpha\dot\alpha}{}_{\beta\dot\beta} + \omega^{\alpha\beta}_T T_{(\alpha\beta)} + \omega_D D\} \ . \end{aligned} \tag{4.5}$$

By comparing both sides of (4.5) we obtain

$$\begin{aligned} \omega^\alpha_{\rm II} &= (d\theta^\alpha + i\lambda^\alpha_{\rho\dot\rho}\omega_{L\rm I}{}^{\rho\dot\rho})e^{2\varphi} \ , \\ \omega^{\alpha\dot\alpha}_{L\rm II} &= (dx^{\rho\dot\rho}_L + i\psi_\mu{}^{\rho\dot\rho} d\theta^\mu) B^{\alpha\dot\alpha}{}_{\rho\dot\rho} e^\varphi = \omega^{\rho\dot\rho}_{L\rm I} B^{\alpha\dot\alpha}{}_{\rho\dot\rho} e^\varphi \ , \\ \omega_{{\rm II}\mu}{}^{\rho\dot\rho} &= d\psi_\mu{}^{\lambda\dot\lambda} B_{\lambda\dot\lambda}{}^{\rho\dot\rho} e^{-\varphi} \ , \\ \Omega_{\rho\dot\rho}{}^\mu &= (d\lambda^\mu_{\beta\dot\beta} + \lambda^\sigma_{\beta\dot\beta}\lambda^\mu_{\gamma\dot\gamma} d\psi_\sigma{}^{\gamma\dot\gamma})(B^{-1})^{\beta\dot\beta}{}_{\rho\dot\rho} e^\varphi \ , \\ \omega_{R\alpha\dot\alpha}{}^{\beta\dot\beta} &= -\lambda^\gamma_{\rho\dot\rho} d\psi_\gamma{}^{\nu\dot\nu} (B^{-1})^{\rho\dot\rho}{}_{\alpha\dot\alpha} B^{\beta\dot\beta}{}_{\nu\dot\nu} + (B^{-1})_{\alpha\dot\alpha}{}^{\tau\dot\tau} dB^{\beta\dot\beta}{}_{\tau\dot\tau} \ , \\ \omega^{\alpha\beta}_T &= \tfrac14 \lambda^{(\alpha}_{\gamma\dot\rho} d\psi^{\beta)\gamma\dot\rho} \ , \\ \omega_D &= d\varphi - \frac{i}{4}\lambda^\mu_{\rho\dot\rho} d\psi_\mu{}^{\rho\dot\rho} \ , \end{aligned} \tag{4.6}$$

where $B$ is defined by

$$g_5^{-1} R^{\alpha\dot\alpha}{}_{\beta\dot\beta} g_5 = (B^{-1})^{\alpha\dot\alpha}{}_{\tau\dot\tau} (B)_{\beta\dot\beta}{}^{\varphi\dot\varphi} R^{\tau\dot\tau}{}_{\varphi\dot\varphi} \tag{4.7}$$

and is a function of the Goldstone superfields associated with $R^{\alpha\dot\alpha}{}_{\beta\dot\beta}$.

All these 1-forms, except those related to $M_{\alpha\beta}$, $M_{\dot\alpha\dot\beta}$, which are hidden in $\omega_R$ and $\omega_T$, undergo the induced Lorentz transformation with respect to their spinor indices when $G_{\rm II}$ acts on $\tilde g_{\rm II}$ by left shifts

$$g^0_{\rm II}\tilde g_{\rm II} = \tilde g'_{\rm II} L^{\rm ind} \ , \tag{4.8}$$

where

$$L^{\rm ind} \simeq \mathbb{1} + i\delta h^{\alpha\beta}(x,\theta,\bar\theta)M_{\alpha\beta} + i\delta\bar h^{\dot\alpha\dot\beta}(x,\theta,\bar\theta)M_{\dot\alpha\dot\beta} \tag{4.9}$$

and

$$\begin{aligned} g^0_{\rm II} \simeq \mathbb{1} + i(&\varepsilon^\alpha Q_\alpha + c^{\alpha\dot\alpha}P_{L\alpha\dot\alpha} + \beta^{\rho\dot\rho}_\mu Q^\mu_{\rho\dot\rho} + \sigma_{\alpha\dot\alpha}{}^{\beta\dot\beta} R^{\alpha\dot\alpha}{}_{\beta\dot\beta} \\ &+ \nu^{\alpha\beta}T_{\alpha\beta} + \rho^\mu{}_{\sigma\dot\sigma} I_\mu{}^{\sigma\dot\sigma} + cD) \ . \end{aligned} \tag{4.10}$$

Applying the general formula (4.8) we get the transformation properties of the coset parameters $\theta^\mu$, $x^{\rho\dot\rho}_L$, $\psi_\mu{}^{\gamma\dot\gamma}$, $\lambda^\mu_{\rho\dot\rho}$, $h^{\rho\nu}$, $\bar h^{\dot\rho\dot\nu}$, and $B_{\tau\dot\tau}{}^{\sigma\dot\sigma}$ under $g_{\rm II}$:

$$\begin{aligned} \delta\theta^\mu &= \varepsilon^\mu + 2\theta^\alpha \nu_\alpha{}^\mu - 2\theta^\mu c - ix^{\alpha\dot\alpha}_L \rho^\mu_{\alpha\dot\alpha} \ , \\ \delta x^{\rho\dot\rho}_L &= c^{\rho\dot\rho} + i\theta^\mu B_\mu{}^{\rho\dot\rho} - x^{\gamma\dot\gamma}_L \sigma_{\gamma\dot\gamma}{}^{\rho\dot\rho} - x^{\rho\dot\rho}_L c \ , \\ \delta\psi_\mu{}^{\gamma\dot\gamma} &= \beta_\mu{}^{\gamma\dot\gamma} - 2\psi_\alpha{}^{\gamma\dot\gamma}\nu^\alpha_\mu - \psi_\mu{}^{\beta\dot\beta}\sigma_{\beta\dot\beta}{}^{\gamma\dot\gamma} \\ &\quad + \psi_\mu{}^{\gamma\dot\gamma}c + \psi_\mu{}^{\beta\dot\beta}(\psi_\nu{}^{\gamma\dot\gamma}\rho^\nu_{\beta\dot\beta}) \ , \\ \delta\lambda^\mu{}_{\rho\dot\rho} &= \rho^\mu_{\rho\dot\rho} - c\lambda^\mu_{\rho\dot\rho} + 2\lambda^\nu_{\rho\dot\rho}\nu^\mu_\nu + \lambda^\mu_{\varphi\dot\varphi}\sigma_{\rho\dot\rho}{}^{\varphi\dot\varphi} \\ &\quad + \lambda^\beta_{\rho\dot\rho}\psi_\beta{}^{\sigma\dot\sigma}\rho^\mu_{\sigma\dot\sigma} - \lambda^\mu_{\beta\dot\beta}\psi_\nu{}^{\beta\dot\beta}\rho^\nu_{\rho\dot\rho} \ , \end{aligned} \tag{4.11}$$

$$\delta h^{\rho\nu}=\nu^{\rho\nu}+\tfrac{1}{4}\psi^{(\rho\gamma\dot\gamma}\rho^{\nu)}{}_{\gamma\dot\gamma}\ ,$$
$$\delta\bar h^{\dot\rho\dot\nu}=\nu^{\dot\rho\dot\nu}-\tfrac{1}{4}\psi^{(\dot\rho\gamma\dot\gamma}\rho^{\dot\nu)}{}_{\gamma\dot\gamma}\ ,$$
$$\delta B_{\tau\dot\tau}{}^{\sigma\dot\sigma}=2B_{\tau\dot\tau}{}^{\sigma\dot\rho}\delta\bar h_{\dot\rho}^{\dot\sigma}+2B_{\tau\dot\tau}{}^{\rho\dot\sigma}\delta h_{\rho}^{\sigma}+(\sigma_{\tau\dot\tau}{}^{\gamma\dot\gamma}-\psi_\lambda{}^{\gamma\dot\gamma}\rho^{\lambda}_{\tau\dot\tau})B_{\gamma\dot\gamma}{}^{\sigma\dot\sigma}\ .$$

Now we are prepared to eliminate extra Goldstone superfields and single out the covariants of $G_{\rm II}$ which are simultaneously covariant with respect to $G_{\rm I}$.

Looking at Eqs. (4.6) and (4.7) we see that $\omega^{\alpha\dot\alpha}_{L\rm II}$ is covariant also under $G_{\rm I}$ since $R$ and $D$ can be added to $G_{\rm I}$ as extra automorphism generators, and thus $G_{\rm I}$ does not transform the superfields $B$ and $\varphi$ at all.

Next, one should decompose $\omega^{\alpha\dot\alpha}_{L\rm II}$ into the covariant differentials of $x^{\alpha\dot\alpha}$ and $H^{\alpha\dot\alpha}$ ($\Delta x^{\alpha\dot\alpha}$ and $\Delta H^{\alpha\dot\alpha}$) once again and then extract covariant derivatives of $H^{\alpha\dot\alpha}$ from $\Delta H^{\alpha\dot\alpha}$. It turns out that conditions (3.15) for elimination of $\psi_\mu{}^{\rho\dot\rho}$ in the nonlinear realization of $G_{\rm I}$ are simultaneously covariant under $G_{\rm II}$ so that $\psi_\mu{}^{\alpha\dot\alpha}$ given by Eqs. (3.16) and (3.17) possesses correct transformation properties with respect to both $G_{\rm I}$ and $G_{\rm II}$ (for the proof see [9]).

The proof goes as follows. First, we define the $G_{\rm II}$-covariant differentials of $x^{\alpha\dot\alpha}$ and $H^{\alpha\dot\alpha}$:

$$\Delta x^{\alpha\dot\alpha}=\frac{1}{2}(\omega^{\alpha\dot\alpha}_{L\rm II}+\omega^{\alpha\dot\alpha}_{R\rm II})=\nabla x^{\rho\dot\rho}b^{\alpha\dot\alpha}{}_{\rho\dot\rho}+\nabla H^{\rho\dot\rho}c^{\alpha\dot\alpha}{}_{\rho\dot\rho}\ , \tag{4.12}$$

$$\Delta H^{\alpha\dot\alpha}=\frac{1}{2i}(\omega^{\alpha\dot\alpha}_{L\rm II}-\omega^{\alpha\dot\alpha}_{R\rm II})=\nabla H^{\rho\dot\rho}b^{\alpha\dot\alpha}{}_{\rho\dot\rho}-\nabla x^{\rho\dot\rho}c^{\alpha\dot\alpha}{}_{\rho\dot\rho}\ . \tag{4.13}$$

Here $\nabla x$ and $\nabla H$ are defined in Eqs. (3.10) and (3.11) and

$$b^{\alpha\dot\alpha}{}_{\rho\dot\rho}\equiv\frac{1}{2}(B^{\alpha\dot\alpha}{}_{\rho\dot\rho}e^{\varphi}+\bar B^{\alpha\dot\alpha}{}_{\rho\dot\rho}e^{\bar\varphi})\ ,$$
$$c^{\alpha\dot\alpha}{}_{\rho\dot\rho}\equiv\frac{i}{2}(B^{\alpha\dot\alpha}{}_{\rho\dot\rho}e^{\varphi}-\bar B^{\alpha\dot\alpha}{}_{\rho\dot\rho}e^{\bar\varphi})\ . \tag{4.14}$$

Further, using the representation (3.12)

$$\nabla H^{\rho\dot\rho}=\nabla x^{\beta\dot\beta}\partial_{\beta\dot\beta}H^{\rho\dot\rho}+d\theta^\mu\mathcal{D}_\mu H^{\rho\dot\rho}-d\bar\theta^{\dot\mu}\mathcal{D}_{\dot\mu}H^{\rho\dot\rho}$$

and expressing

$$\nabla x^{\alpha\dot\alpha}=\Delta x^{\rho\dot\rho}(b^{-1})^{\alpha\dot\alpha}{}_{\rho\dot\rho}-\nabla H^{\rho\dot\rho}c_{\rho\dot\rho}{}^{\gamma\dot\gamma}(b^{-1})^{\alpha\dot\alpha}{}_{\gamma\dot\gamma}$$

and, also, expressing $\nabla\theta^\alpha$, $d\bar\theta^{\dot\alpha}$ via the covariant differentials $\omega^{\alpha}_{\rm II}$, $\bar\omega^{\dot\alpha}_{\rm II}$ one finds $\Delta H^{\rho\dot\rho}$ in the following generic form

$$\Delta H^{\rho\dot\rho}=\Delta x^{\gamma\dot\gamma}P^{\rho\dot\rho}{}_{\gamma\dot\gamma}-\tilde B^{\rho\dot\rho}{}_{\lambda\dot\lambda}\mathcal{D}_\mu H^{\lambda\dot\lambda}\omega^{\mu}_{\rm II}e^{-2\varphi}+\tilde B^{\rho\dot\rho}{}_{\lambda\dot\lambda}\bar{\mathcal{D}}_{\dot\mu}H^{\lambda\dot\lambda}\bar\omega^{\dot\mu}_{\rm II}e^{-2\bar\varphi}\ , \tag{4.15}$$

where for our purposes, there is no need to know the explicit structure of matrices $P$, $\tilde B$. The main point is that $\tilde B$ is not singular (its starts with the Kronecker symbols and so the $G_{\rm II}$-covariant spinor derivatives of $H^{\rho\dot\rho}$ differ from the $G_{\rm I}$-covariant ones $\mathcal{D}_\mu H^{\lambda\dot\lambda}$, $\bar{\mathcal{D}}_{\dot\mu}H^{\lambda\dot\lambda}$ only by nonsingular matrix factors). So Eqs. (3.15) are also $G_{\rm II}$ covariant and we have proven the proposition given above.

After eliminating $\psi_\mu{}^{\alpha\dot\alpha}$, $\psi_{\dot\mu}{}^{\alpha\dot\alpha}$ by Eqs. (3.16), (3.17) the covariant differentials of $x^{\alpha\dot\alpha}$ and $H^{\alpha\dot\alpha}$, $\Delta H^{\alpha\dot\alpha}$ and $\Delta x^{\alpha\dot\alpha}$, respectively acquire the forms

$$\Delta x^{\rho\dot\rho}=\nabla x^{\alpha\dot\alpha}(b_{\alpha\dot\alpha}{}^{\rho\dot\rho}+\partial_{\alpha\dot\alpha}H^{\gamma\dot\gamma}c_{\gamma\dot\gamma}{}^{\rho\dot\rho})\equiv\nabla x^{\alpha\dot\alpha}M_{\alpha\dot\alpha}{}^{\rho\dot\rho}\ , \tag{4.16}$$

$$\Delta H^{\rho\dot\rho}=-\nabla x^{\gamma\dot\gamma}(c_{\gamma\dot\gamma}{}^{\rho\dot\rho}-\partial_{\gamma\dot\gamma}H^{\lambda\dot\lambda}b_{\lambda\dot\lambda}{}^{\rho\dot\rho})\equiv\nabla x^{\alpha\dot\alpha}N_{\alpha\dot\alpha}{}^{\rho\dot\rho} \tag{4.17}$$

with $b_{\alpha\dot\alpha}{}^{\rho\dot\rho}$ and $c_{\alpha\dot\alpha}{}^{\rho\dot\rho}$ given in (4.14).

Their structure is completely specified by expressing the remaining Goldstone superfields $\lambda$, $\pi$ (or $B$), and $\varphi$ in terms of $H^{\mu\dot\mu}$.

We begin with $B_{\tau\dot\tau}{}^{\sigma\dot\sigma}$. By inspecting the structure of the Cartan forms (4.6), we conclude that $B_{\tau\dot\tau}{}^{\sigma\dot\sigma}$ can be eliminated by imposing appropriate constraints on one of the spinor covariant derivatives of the Goldstone field $\psi_\mu{}^{\alpha\dot\alpha}$. These are defined as the coefficients in front of $\omega^{\alpha}_{\rm II}$, $\bar\omega^{\dot\alpha}_{\rm II}$ in the $G_{\rm II}$-covariant Cartan form $\omega_{{\rm II}\mu}{}^{\rho\dot\rho}$

$$\omega^{\rho\dot\rho}_{{\rm II}\mu}\equiv\Delta x^{\lambda\dot\lambda}\Delta_{\lambda\dot\lambda}\psi_\mu{}^{\rho\dot\rho}+\omega^{\alpha}_{\rm II}\Delta_\alpha\psi_\mu{}^{\rho\dot\rho}-\bar\omega^{\dot\alpha}_{\rm II}\bar\Delta_{\dot\alpha}\psi_\mu{}^{\rho\dot\rho}\ , \tag{4.18}$$

where

$$\Delta_\alpha\psi_\mu{}^{\rho\dot\rho}=\nabla_\alpha\psi_\mu{}^{\lambda\dot\lambda}B_{\lambda\dot\lambda}{}^{\rho\dot\rho}e^{-3\varphi},$$
$$\bar\Delta_{\dot\alpha}\psi_\mu{}^{\rho\dot\rho}=\bar\nabla_{\dot\alpha}\psi_\mu{}^{\lambda\dot\lambda}B_{\lambda\dot\lambda}{}^{\rho\dot\rho}e^{-\varphi-2\bar\varphi}\ . \tag{4.19}$$

The derivative $\Delta_\alpha\psi_\mu{}^{\rho\dot\rho}$ involves $\nabla_\alpha\psi_\mu{}^{\lambda\dot\lambda}=2\nabla_\alpha\nabla_\mu H^{\lambda\dot\lambda}$ which is not $G_{\rm I}$ covariant. On the other hand, $\bar\nabla_{\dot\alpha}\psi_\mu{}^{\lambda\dot\lambda}=2\bar\nabla_{\dot\alpha}\nabla_\mu H^{\lambda\dot\lambda}$ belongs to the set of the $G_{\rm I}$ covariants (3.23) and so $\bar\Delta_{\dot\alpha}\psi_\mu{}^{\rho\dot\rho}$ is covariant both under $G_{\rm I}$ and $G_{\rm II}$ (and hence under the whole infinite-dimensional $N=1$ supergravity group). Thus, $\bar\Delta_{\dot\alpha}\psi_\mu{}^{\rho\dot\rho}$ can be used for implementing the sought after covariant constraint. It is meaningless to equate $\bar\Delta_{\dot\alpha}\psi_\mu{}^{\rho\dot\rho}$ to zero because it would contradict the flat-superspace limit

$$\varphi=0,\quad \lambda^{\nu}_{\rho\dot\rho}=0,\quad B_{\lambda\dot\lambda}{}^{\rho\dot\rho}=\delta_\lambda{}^\rho\delta_{\dot\lambda}{}^{\dot\rho},\quad H^{\mu\dot\mu}=\tfrac{1}{2}\theta^\mu\bar\theta^{\dot\mu}\ ,$$
$$\psi_\mu{}^{\lambda\dot\lambda}=2\nabla_\mu H^{\lambda\dot\lambda}=\delta_\mu{}^\lambda\bar\theta^{\dot\lambda}\ . \tag{4.20}$$

Thus, one should equate $\bar\Delta_{\dot\alpha}\psi_\mu{}^{\rho\dot\rho}$ to a proper Lorentz-covariant constant matrix. The only such constraint consistent with the flat limit (4.20) is the following:

$$\bar\Delta_{\dot\alpha}\psi_\mu{}^{\lambda\dot\lambda}=-\delta_{\dot\alpha}{}^{\dot\lambda}\delta_\mu{}^\lambda \tag{4.21}$$

or

$$\bar\nabla_{\dot\alpha}\nabla_\mu H^{\rho\dot\rho}B_{\rho\dot\rho}{}^{\lambda\dot\lambda}=-\tfrac{1}{2}\delta_{\dot\alpha}{}^{\dot\lambda}\delta_\mu{}^{\lambda}e^{\varphi+2\bar\varphi}\ . \tag{4.22}$$

From here

$$B^{\gamma\dot\gamma}{}_{\beta\dot\beta}=-\tfrac{1}{2}e^{\varphi+2\bar\varphi}(\hat e^{-1})^{\gamma\dot\gamma}{}_{\beta\dot\beta}\ , \tag{4.23}$$

where

$$\hat e_{\gamma\dot\gamma}{}^{\beta\dot\beta}\equiv\bar\nabla_{\dot\gamma}\nabla_\gamma H^{\beta\dot\beta}\ . \tag{4.24}$$

Taking into account the $\det B=1$, one also finds

$$e^{2\varphi}=(2)^{-2/3}(\det\hat{e})^{-1/6}(\det\hat{r})^{1/3}\ ,$$
$$e^{2\bar{\varphi}}=(2)^{-2/3}(\det\hat{r})^{-1/6}(\det\hat{e})^{1/3}\ , \tag{4.25}$$

where $\hat{r}=-\overline{(\hat{e})}=\nabla\bar{\nabla}H$. Comparing (4.24), (4.25) with the set (3.23), we see that $B^{\gamma\dot{\gamma}}{}_{\beta\dot{\beta}}$ and $\varphi$, $\bar{\varphi}$ are indeed $G_{\mathrm{I}}$ scalars.

The objects $e^{-2\varphi}$, $e^{-2\bar{\varphi}}$ can be identified with the quantities $F$, $\bar{F}$ playing a crucial role in the Ogievetsky-Sokatchev approach [17].

Note that the constraint (4.21) can be given a more familiar meaning as the vanishing of a covariant derivative (which is generic for the inverse Higgs phenomenon) [9] after passing to a real basis in the superalgebra $a_{\mathrm{II}}$ and singling out an ordinary real Poincaré subsuperalgebra. Namely, the subset of $a_{\mathrm{II}}$ generators

$$\{P_{L\mu\dot{\mu}},\ P_{R\mu\dot{\mu}}\equiv\overline{(P_{L\mu\dot{\mu}})},\ Q^{\mu}_{\rho\dot{\gamma}},\ \bar{Q}^{\dot{\mu}}_{\dot{\rho}\gamma},\ Q_{\mu},\ \bar{Q}_{\dot{\mu}}=\overline{(Q_{\mu})}\}$$

can be rearranged as

$$\{P^{P}_{\mu\dot{\mu}}=P_{L\mu\dot{\mu}}+P_{R\mu\dot{\mu}},\ Q^{P}_{\gamma}=Q_{\gamma}+\bar{Q}^{\dot{\mu}}_{\dot{\mu}\gamma},\ \bar{Q}^{P}_{\dot{\gamma}}=\bar{Q}_{\dot{\gamma}}+Q^{\mu}_{\mu\dot{\gamma}},\tilde{Q}^{\mu}_{\gamma\dot{\gamma}},\tilde{\bar{Q}}^{\dot{\mu}}_{\dot{\gamma}\gamma}\ ,$$
$$P^{A}_{\mu\dot{\mu}}=P_{L\mu\dot{\mu}}-P_{R\mu\dot{\mu}},\ Q^{A}_{\gamma}=Q_{\gamma}-\bar{Q}^{\dot{\mu}}_{\dot{\mu}\gamma},\ \bar{Q}^{A}_{\dot{\gamma}}=\bar{Q}_{\dot{\gamma}}-Q^{\mu}_{\mu\dot{\gamma}}\}\ ,$$

where $P^{P}_{\mu\dot{\mu}}$, $Q^{P}_{\gamma}$, $\bar{Q}^{P}_{\dot{\gamma}}$ form the real Poincaré superalgebra

$$\{Q^{P}_{\mu},\bar{Q}^{P}_{\dot{\gamma}}\}=P_{\mu\dot{\gamma}},\quad \{Q^{P}_{\mu},Q^{P}_{\lambda}\}=0 \tag{4.26}$$

and $\tilde{Q}^{\mu}_{\gamma\dot{\gamma}}$, $\tilde{\bar{Q}}^{\dot{\mu}}_{\dot{\gamma}\gamma}$ are traceless, $\tilde{Q}^{\mu}_{\mu\dot{\gamma}}=0$. Then the relevant piece of the $a_{\mathrm{II}}$-valued Cartan form (in the real basis) can be written as

$$i[\tfrac{1}{2}\omega^{\mu}_{L\mathrm{II}}+\tfrac{1}{4}\bar{\omega}^{\dot{\rho}\mu}_{\mathrm{II}\dot{\rho}}]Q^{P}_{\mu}+i[\tfrac{1}{2}\bar{\omega}^{\dot{\mu}}_{R\mathrm{II}}+\tfrac{1}{4}\omega_{\mathrm{II}\rho}{}^{\rho\dot{\mu}}]\bar{Q}^{P}_{\dot{\mu}}+i\Delta x^{\mu\dot{\mu}}P_{\mu\dot{\mu}}-\Delta H^{\mu\dot{\mu}}P^{A}_{\mu\dot{\mu}}$$
$$+i[\tfrac{1}{2}\omega^{\mu}_{L\mathrm{II}}-\tfrac{1}{4}\bar{\omega}_{\mathrm{II}\dot{\rho}}{}^{\dot{\rho}\mu}]Q^{A}_{\mu}+i[\tfrac{1}{2}\bar{\omega}^{\dot{\mu}}_{R\mathrm{II}}-\tfrac{1}{4}\omega_{\mathrm{II}\rho}{}^{\rho\dot{\mu}}]\bar{Q}^{A}_{\dot{\mu}}+i\omega_{\mathrm{II}(\mu}{}^{\gamma)\dot{\gamma}}\tilde{Q}^{\mu}_{\gamma\dot{\gamma}}+i\bar{\omega}_{\mathrm{II}(\dot{\mu}}{}^{\dot{\gamma})\gamma}\tilde{\bar{Q}}^{\dot{\mu}}_{\dot{\gamma}\gamma}\ . \tag{4.27}$$

Now Eq. (4.21) is easily recognized as the condition that the covariant $\bar{\theta}$ projections of the Cartan form before the generators $Q^{A}_{\mu}$, $\bar{Q}^{A}_{\dot{\mu}}$, $\tilde{Q}^{\mu}_{\gamma\dot{\gamma}}$, $\tilde{\bar{Q}}^{\dot{\mu}}_{\dot{\gamma}\gamma}$ vanish. It is worth mentioning here that in the flat limit (4.20) the expression (4.27) goes over to the familiar Cartan forms of $N=1$ Poincaré supersymmetry [taking account of Eqs. (3.16), (3.17), (4.18), and (4.19)]

$$[\mathrm{Eq.}\ (4.21)]\Longrightarrow i d\theta^{\mu}Q^{P}_{\mu}+i d\bar{\theta}^{\dot{\mu}}Q^{P}_{\dot{\mu}}+i\left(dx^{\mu\dot{\mu}}+\frac{i}{2}(\theta^{\mu}d\bar{\theta}^{\dot{\mu}}+\bar{\theta}^{\dot{\mu}}d\theta^{\mu})\right)P_{\mu\dot{\mu}}\ .$$

Let us now explain how to eliminate the Goldstone superfield $\lambda^{\mu}_{\rho\dot{\rho}}$. The corresponding constraints arise from the requirement that in the $d\bar{\theta}$ projections of forms before the generators $R$, $D$, and $T$, only the inhomogeneously transformed components associated with the Lorentz generators $M_{\alpha\beta}$, $M_{\dot{\alpha}\dot{\beta}}$ survive. The resulting equations are again manifestly $G_{\mathrm{II}}$ and $G_{\mathrm{I}}$ covariant. They are of the form

$$\delta^{(\dot{\sigma}}_{\dot{\gamma}}\lambda^{(\sigma}_{\rho\dot{\rho}}\nabla^{\dot{\lambda})}\bar{\nabla}^{\lambda)}H^{\rho\dot{\rho}}+\tfrac{1}{2}(B^{-1})^{(\lambda(\dot{\lambda}\tau\dot{\tau}}\bar{\nabla}_{\dot{\gamma}}B^{\sigma)\dot{\sigma})}{}_{\tau\dot{\tau}}=0\ , \tag{4.28}$$

$$\lambda^{\nu}_{\beta\dot{\rho}}\nabla_{\dot{\gamma}}\bar{\nabla}^{\rho}H^{\beta\dot{\rho}}=-\frac{1}{2}(B^{-1})^{\nu\dot{\omega}\tau\dot{\tau}}\bar{\nabla}_{\dot{\gamma}}B^{\rho}{}_{\tau\dot{\tau}\dot{\omega}}+\varepsilon^{\nu\rho}\bar{\nabla}_{\dot{\gamma}}\varphi\ . \tag{4.29}$$

The first equation originates from the Cartan form standing before the generator $R$ while the second from the forms associated with the $D$ and $T^{\alpha\beta}$. From Eq. (4.29) one gets

$$\lambda^{\nu}_{\beta\dot{\rho}}=-[\tfrac{1}{2}(B^{-1})^{\nu\dot{\omega}\tau\dot{\tau}}\bar{\nabla}_{\dot{\gamma}}B_{\tau\dot{\tau}\rho\dot{\omega}}+\delta_{\rho}{}^{\nu}\bar{\nabla}_{\dot{\gamma}}\varphi](\hat{e}^{-1})^{\dot{\gamma}\rho}{}_{\beta\dot{\rho}}\ . \tag{4.30}$$

Note that Eq. (4.28) is satisfied automatically by substituting (4.20) into it and that $\lambda^{\nu}_{\beta\dot{\rho}}$ turns out to be constructed from the $G_{\mathrm{I}}$ invariants.

The Lorentz connections are just the surviving pieces of the $d\bar{\theta}$ projections of the form $\omega_R,\omega_T$: $-i\omega_{\dot{\alpha}}{}^{(\alpha\beta)}M_{(\alpha\beta)}-i\omega_{\dot{\alpha}}{}^{(\dot{\rho}\dot{\rho})}M_{(\dot{\rho}\dot{\rho})}$, namely

$$\omega_{\dot{\gamma}}{}^{(\rho\nu)}=-\tfrac{1}{4}(\hat{e})^{(\rho\dot{\omega}\tau\dot{\tau}}\bar{\nabla}_{\dot{\gamma}}(\hat{e}^{-1})^{\nu)}{}_{\tau\dot{\tau}\dot{\omega}}e^{-2\bar{\varphi}}\ ,$$
$$\omega_{\dot{\gamma}}{}^{(\dot{\rho}\dot{\nu})}=-\tfrac{1}{4}\{(\hat{e})^{\omega(\dot{\rho}\tau\dot{\tau}}\bar{\nabla}_{\dot{\gamma}}(\hat{e}^{-1})_{\tau\dot{\tau}\omega}{}^{\dot{\nu})}$$
$$+2\delta_{\dot{\gamma}}{}^{(\dot{\rho}}\bar{\nabla}^{\dot{\nu})}(2\varphi+2\bar{\varphi})\}e^{-2\bar{\varphi}} \tag{4.31}$$

and their conjugates. They coincide with the connections found by Ogievetsky and Sokatchev in [17].

To bring the second expression into the form given in [17], one must use the identity

$$\varepsilon_{\dot{\rho}\dot{\lambda}}\bar{\nabla}_{\dot{\gamma}}(\varphi+2\bar{\varphi})+\epsilon_{\dot{\rho}\dot{\gamma}}\bar{\nabla}_{\dot{\lambda}}(\varphi+2\bar{\varphi})$$
$$=-\frac{1}{4}\{(\hat{e})_{\nu\dot{\gamma}}{}^{\tau\dot{\tau}}\bar{\nabla}_{\dot{\rho}}(\hat{e}^{-1})_{\tau\dot{\tau}}{}^{\nu}{}_{\dot{\lambda}}+(\hat{e})_{\nu\dot{\lambda}}{}^{\tau\dot{\tau}}\bar{\nabla}_{\dot{\rho}}(\hat{e}^{-1})_{\tau\dot{\tau}}{}^{\nu}{}_{\dot{\gamma}}\}\ .$$

Then one gets

$$\omega_{\dot{\gamma}}{}^{(\dot{\rho}\dot{\nu})}=\delta_{\dot{\gamma}}{}^{\dot{\rho}}\bar{\nabla}^{\dot{\nu}}\bar{\varphi}+\delta_{\dot{\gamma}}{}^{\dot{\nu}}\bar{\nabla}^{\dot{\rho}}\bar{\varphi} \tag{4.32}$$

that coincides (up to the factor 2) with the expression in [17].

Thus we have shown that all the Goldstone superfields of nonlinear realization of supergroup $G_{\rm II}$, except $H^{\mu\dot\mu}(x,\theta,\bar\theta)$, can be eliminated in a manifestly covariant way with respect to both $G_{\rm II}$ and $G_{\rm I}$, hence to the whole $N=1$ supergravity group. We are eventually left with a single Goldstone superfield $H^{\mu\dot\mu}(x,\theta,\bar\theta)$ which alone supplies nonlinear realizations of $G_{\rm I}$ and $G_{\rm II}$. This confirms, from another point of view than in [16–18], its role as the fundamental geometric object of the minimal $N=1$ supergravity.

It remains to see how the minimal $N=1$ supergravity action reappears within the present framework.

## V. THE INVARIANT ACTION

After employing the inverse-Higgs-effect constraints, the remaining simultaneous $G_{\rm I}$ and $G_{\rm II}$ covariants are reduced to the following set:

(i) The covariant differentials of the $N=1$ superspace coordinates:

$$\Delta x^{\mu\dot\mu}=\tfrac{1}{2}(\omega^{\mu\dot\mu}_{L\,\rm II}+\omega^{\mu\dot\mu}_{R\,\rm II})\ ,\qquad \Delta\theta^{\mu}=\omega^{\mu}_{\rm II},\quad \Delta\bar\theta^{\dot\mu}=\bar\omega^{\dot\mu}_{\rm II}=\overline{(\omega^{\mu}_{\rm II})}\ . \tag{5.1}$$

(ii) The covariant differential of $H^{\mu\dot\mu}(x,\theta,\bar\theta)$:

$$\Delta H^{\mu\dot\mu}=\frac{1}{2i}(\omega^{\mu\dot\mu}_{L\,\rm II}-\omega^{\mu\dot\mu}_{R\,\rm II})\ . \tag{5.2}$$

(iii) The $\bar\theta$-covariant derivative of the Goldstone field $\lambda^{\mu}_{\rho\dot\rho}$ (the projection of the Cartan form $\Omega^{\mu}_{\rho\dot\rho}$ onto the covariant differential $\Delta\theta^{\mu}$):

$$\mathcal{D}_{\dot\mu}\lambda^{\mu}_{\rho\dot\rho}=(\Delta_{\dot\mu}\lambda^{\mu}_{\beta\dot\beta}+\lambda^{\omega}_{\beta\dot\beta}\lambda^{\mu}_{\gamma\dot\gamma}\Delta_{\dot\mu}\psi^{\gamma\dot\gamma}_{\omega})(B^{-1})^{\beta\dot\beta}{}_{\rho\dot\rho}e^{\varphi}\ . \tag{5.3}$$

The $\bar\theta$ projections of the rest of the Cartan forms are either zero by the inverse Higgs effect or are the components of the inhomogeneously transforming Lorentz connection [see Eqs. (4.31)]. Concerning the $G_{\rm II}$ covariant $\theta$ projections, they, as was already mentioned, essentially involve a $G_{\rm I}$-noncovariant quantity $\nabla_\rho\psi_\mu{}^{\gamma\dot\rho}$ and so are not tensors with respect to the $N=1$ supergravity group. Thus the objects (5.1)–(5.3) are the only obvious building blocks for constructing the mutual invariants of $G_{\rm II}$ and $G_{\rm I}$.

In fact, only the covariant differentials (5.1) actually matter after substitution of the inverse-Higgs-effect expressions (3.16), (3.17), (4.23), (4.25), (4.27) for the Goldstone superfields $\psi_\mu{}^{\rho\dot\rho}$, $B^{\tau\dot\tau}{}_{\rho\dot\rho}$, $\varphi$, $\lambda^{\mu}_{\beta\dot\beta}$. The expression (5.3) gives rise to a higher-derivative invariant (it is proportional to one of the basic supertensors of minimal $N=1$ supergravity, the superfield $R$ [3]), while $\Delta H^{\rho\dot\rho}$ (5.2) identically vanishes. Indeed, after some algebra the matrix $N_{\alpha\dot\alpha}{}^{\rho\dot\rho}$ in Eq. (4.19) can be represented as

$$N_{\alpha\dot\alpha}{}^{\rho\dot\rho}=-\frac{i}{4}e^{2(\varphi+\bar\varphi)}A_{\alpha\dot\alpha}{}^{\lambda\dot\lambda}(\hat e^{-1})^{\mu\dot\mu}{}_{\lambda\dot\lambda}[\hat e_{\mu\dot\mu}{}^{\nu\dot\nu}A_{\nu\dot\nu}{}^{\sigma\dot\sigma}+\hat r_{\mu\dot\mu}{}^{\nu\dot\nu}\bar A_{\nu\dot\nu}{}^{\sigma\dot\sigma}](\bar A^{-1})_{\sigma\dot\sigma}{}^{\gamma\dot\gamma}(\hat r^{-1})_{\gamma\dot\gamma}{}^{\rho\dot\rho}\ . \tag{5.4}$$

The expression within the square brackets equals zero:

$$[\hat e_{\mu\dot\mu}{}^{\nu\dot\nu}A_{\nu\dot\nu}{}^{\sigma\dot\sigma}+\hat r_{\mu\dot\mu}{}^{\nu\dot\nu}\bar A_{\nu\dot\nu}{}^{\sigma\dot\sigma}]=\{\nabla_\beta,\bar\nabla_{\dot\beta}\}H^{\sigma\dot\sigma}-i[\bar\nabla_{\dot\beta},\nabla_\beta]H^{\nu\dot\nu}\partial_{\nu\dot\nu}H^{\sigma\dot\sigma}=0\ , \tag{5.5}$$

where we have used the relation [17]

$$\{\nabla_\beta,\bar\nabla_{\dot\beta}\}=-i[\nabla_\beta,\bar\nabla_{\dot\beta}]H^{\nu\dot\nu}\partial_{\nu\dot\nu}\ .$$

Thus we have

$$\Delta H^{\rho\dot\rho}=0\ . \tag{5.6}$$

Note that the vanishing of the covariant differential $\Delta H^{\rho\dot\rho}$ has an analogue in the case of pure gravity [8] where the covariant derivative of the symmetric Goldstone field (corresponding to spontaneously broken affine transformations) is also zero.

Finally, we are left with the covariant differentials $\Delta x^{\mu\dot\mu}$, $\Delta\theta^{\mu}$, $\Delta\bar\theta^{\dot\mu}$. An obvious simplest invariant is the supervolume of $N=1$ superspace $(x^{\mu\dot\mu},\theta^{\mu},\bar\theta^{\dot\mu})$ constructed as an integral of the Berezinean of the corresponding vielbeins over $d^4x\,d^2\theta\,d^2\bar\theta$:

$$\Delta z^{N}\equiv(\Delta x^{\beta\dot\beta},\Delta\theta^{\alpha},\bar\Delta\,\bar\theta^{\dot\alpha})\equiv dz^{M}E^{N}{}_{M}=dx^{\alpha\dot\alpha}E^{N}_{\alpha\dot\alpha}+d\theta^{\mu}E^{N}_{\mu}+d\bar\theta^{\dot\mu}E^{N}_{\dot\mu}\ . \tag{5.7}$$

We are led to find explicit expressions for $E^{N}{}_{M}$. This can be done by substituting the expressions of Goldstone superfields into the explicit formulas for $\Delta x^{\rho\dot\rho}$, $\Delta\theta^{\mu}$, $\Delta\bar\theta^{\dot\mu}$, Eqs. (4.6), (4.17), (4.19), and (3.20). One gets

$$E^{\gamma}_{\mu}=\delta_\mu{}^{\gamma}e^{2\varphi}-i\nabla_\mu H^{\rho\dot\rho}E_{\rho\dot\rho}{}^{\gamma},\quad E^{\gamma}_{\alpha\dot\alpha}=i\bar A_{\alpha\dot\alpha}{}^{\beta\dot\beta}\lambda^{\gamma}_{\beta\dot\beta}e^{2\varphi}\ , \tag{5.8}$$

$$E^{\gamma}_{\dot\mu}=-i\bar\nabla_{\dot\mu}H^{\rho\dot\rho}E_{\rho\dot\rho}{}^{\gamma},\quad E_{\dot\mu}{}^{\dot\gamma}=\overline{(E^{\gamma}_{\mu})},$$

$$E^{\dot\gamma}_{\rho\dot\rho}=\overline{(E^{\gamma}_{\rho\dot\rho})},\quad E^{\dot\gamma}_{\mu}=\overline{(E^{\gamma}_{\dot\mu})}\ ;$$

$$E^{\alpha\dot\alpha}_{\rho\dot\rho}=-2^{-4/3}(\bar A A)_{\rho\dot\rho}{}^{\gamma\dot\gamma}(\hat e^{-1})_{\gamma\dot\gamma}{}^{\nu\dot\nu}\times\hat e_{\nu\dot\nu}{}^{\sigma\dot\sigma}(\hat r^{-1})_{\sigma\dot\sigma}{}^{\alpha\dot\alpha}\det(\hat e\,\hat r)\ , \tag{5.9}$$

$$E_{\mu}{}^{\alpha\dot\alpha}=-i\nabla_\mu H^{\rho\dot\rho}E_{\rho\dot\rho}{}^{\alpha\dot\alpha},\quad E_{\dot\mu}{}^{\alpha\dot\alpha}=\overline{(E_{\mu}{}^{\dot\alpha\alpha})}\ ,$$

where

$$\hat{c}_{\rho\dot{\rho}}{}^{\mu\dot{\mu}}=\tfrac{1}{2}[\nabla_\rho,\overline{\nabla}_{\dot{\rho}}]H^{\mu\dot{\mu}}=\tfrac{1}{2}[(\hat{r})-(\hat{e})]_{\rho\dot{\rho}}{}^{\mu\dot{\mu}} .$$

Now one might immediately calculate $\mathrm{Ber}E^N{}_M$. However, it is more instructive to find first the components of the inverse vielbein $\widetilde{E}^M{}_N$. These are introduced most directly through the differential of some Lorentz-scalar $N=1$ superfield $\phi$

$$\begin{aligned} d\phi &\equiv dz^M\partial_M\phi=\Delta z^M\mathcal{D}_M\phi=\Delta z^N\widetilde{E}_N{}^M\partial_M\phi , \\ \mathcal{D}_N &=\widetilde{E}_N{}^M\partial_M, \quad E_M{}^N\widetilde{E}_N{}^{M'}=\delta_M{}^{M'} . \end{aligned} \tag{5.10}$$

One gets

$$\begin{aligned} &\widetilde{E}_\alpha{}^\mu=\delta_\alpha{}^\mu e^{-2\varphi}, \quad \widetilde{E}_{\dot{\alpha}}{}^{\dot{\mu}}=\delta_{\dot{\alpha}}{}^{\dot{\mu}}e^{-2\overline{\varphi}}, \quad \widetilde{E}_\alpha{}^{\dot{\mu}}=\widetilde{E}_{\dot{\alpha}}{}^\mu=0 , \\ &\widetilde{E}_\alpha{}^{\mu\dot{\mu}}=ie^{-2\varphi}\nabla_\alpha H^{\mu\dot{\mu}}, \quad \widetilde{E}_{\dot{\alpha}}{}^{\rho\dot{\rho}}=ie^{-2\overline{\varphi}}\overline{\nabla}_{\dot{\alpha}}H^{\rho\dot{\rho}} , \\ &\widetilde{E}^\mu_{\beta\dot{\beta}}=ie^{-2(\varphi+\overline{\varphi})}[\omega_{\dot{\beta}}{}^{(\varphi\mu)}\varepsilon_{\beta\varphi}-\delta_\beta{}^\mu\nabla_{\dot{\beta}}\varphi] , \\ &\widetilde{E}_{\beta\dot{\beta}}{}^{\rho\dot{\rho}}=(E^{-1})_{\beta\dot{\beta}}{}^{\rho\dot{\rho}}+i\widetilde{E}^{\mu}_{\beta\dot{\beta}}\nabla_\mu H^{\rho\dot{\rho}}+i\widetilde{E}^{\dot{\mu}}_{\beta\dot{\beta}}\overline{\nabla}_{\dot{\mu}}H^{\rho\dot{\rho}} , \end{aligned} \tag{5.11}$$

where $(E^{-1})_{\beta\dot{\beta}}{}^{\rho\dot{\rho}}$ is the inverse of $E_{\beta\dot{\beta}}{}^{\rho\dot{\rho}}$,

$$(E^{-1})_{\beta\dot{\beta}}{}^{\rho\dot{\rho}}=e^{-2(\varphi+\overline{\varphi})}\hat{c}_{\beta\dot{\beta}}{}^{\rho\dot{\rho}} . \tag{5.12}$$

Now, using the standard definitions and Eqs. (4.25), we have

$$\begin{aligned} \mathrm{Ber}E^N{}_M=\mathrm{Ber}^{-1}\widetilde{E}_N{}^M &=\det{}^{-1}[\widetilde{E}_{\beta\dot{\beta}}{}^{\rho\dot{\rho}}+\widetilde{E}_\alpha{}^{\rho\dot{\rho}}(\widetilde{E}^{-1})^\alpha{}_\mu\widetilde{E}^{\mu}_{\beta\dot{\beta}}+\widetilde{E}_{\dot{\alpha}}{}^{\rho\dot{\rho}}(\widetilde{E}^{-1})_{\dot{\mu}}{}^\alpha\widetilde{E}^{\dot{\mu}}_{\beta\dot{\beta}}]\det\widetilde{E}^{\mu}_{\alpha}\det\widetilde{E}^{\dot{\mu}}_{\dot{\alpha}} \\ &=\det{}^{-1}[(E^{-1})_{\beta\dot{\beta}}{}^{\rho\dot{\rho}}]e^{-4(\varphi+\overline{\varphi})} \\ &=2^{-8/3}(\det\hat{e}\,\hat{r})^{1/3}\det{}^{-1}\hat{c} \\ &=2^{-8/3}(\det\hat{e})^{-1/6}(\det\hat{r})^{-1/6}\det{}^{1/2}A\ \det{}^{1/2}\overline{A} , \end{aligned} \tag{5.13}$$

which coincides, up to a renormalization factor, with the minimal Einstein $N=1$ supergravity superspace Lagrangian in the form given by Ogievetsky and Sokatchev [17]. The expressions for the inverse vielbeins (5.11) are also in one-to-one correspondence with those presented in [17]. Thus we have constructed, following the standard nonlinear realization prescriptions, the minimal invariant action for the nonlinear realization of $\mathcal{G}_{\mathrm{II}}$ in the coset $G_{\mathrm{II}}/L$ and have demonstrated that it is just the action of minimal $N=1$ supergravity:

$$S=\frac{1}{\kappa^2}\int d^4x\, d^2\theta\, \mathrm{Ber}E^N_M . \tag{5.14}$$

## VI. CONCLUDING REMARKS

We finish by listing the basic peculiarities of the nonlinear realization treatment of $N=1$ supergravity, discussing its analogies with the $p$-brane–type theories, and indicating some possible directions in which it could be further elaborated.

First of all, the nonlinear realization approach allows an algorithmic construction of $N=1$ supergravity based on the universal method of Cartan forms augmented with the inverse Higgs phenomenon. The $N=1$ supergravity prepotential $H^{\mu\dot{\mu}}(x,\theta,\overline{\theta})$ appears from the beginning as a Goldstone superfield describing the simultaneous spontaneous breaking of $G_{\mathrm{I}}$ and $G_{\mathrm{II}}$ supersymmetries. Many objects and relations introduced "by hand" or postulated in the Ogievetsky-Sokatchev approach acquire a clear group-theoretical meaning. For instance, the objects $F$ and $\overline{F}$ playing the crucial role in the Ogievetsky-Sokatchev formulation [16,17] turn out to be related to the Goldstone superfield associated with the spontaneously broken generator $D_{\mathrm{II}}$ of the supergroup $G_{\mathrm{II}}$. The relations (4.25) postulated in [17] prove to be a particular case of the inverse Higgs effect. It is worth mentioning that the inverse-Higgs-effect constraints are purely algebraic, in contradistinction to the standard $N=1$ supergravity constraints which are reduced to certain differential equations (vanishing of some components of the torsion), the prepotential being a solution of the latter. In the present formulation these latter constraints are secondary, they can be shown to be a consequence of the Maurer-Cartan structure equations for $G_{\mathrm{I}}$ and $G_{\mathrm{II}}$.

It is interesting to see how the complex geometry of $N=1$ supergravity [16] (the preservation of chirality) reappears in the framework of the nonlinear realization description. Primarily, it manifests itself in that one deals with the complex supergroups $G_{\mathrm{I}}$ and $G_{\mathrm{II}}$ in a holomorphic parametrization (cf. the $N=1$ super Yang-Mills theory which can be interpreted as a nonlinear realization of complex extension of local internal symmetry [13]). The $C^{4/2}$ coordinates $x_L^{\mu\dot{\mu}},\theta^\mu$ naturally arise as the parameters of the relevant complex coset spaces. The constraints of the inverse Higgs effects in the present case can also be interpreted as a kind of covariant chirality conditions starting the absence of the $d\overline{\theta}$ projections in the corresponding Cartan forms.

Let us stress the defining role of the nonlinear realization of linear supergroup $G_{\mathrm{II}}$. The structure of the basic building blocks of $N=1$ supergravity, the covariant differentials $\Delta x^{\mu\dot{\mu}}$, $\Delta\theta^\mu$, $\overline{\Delta}^{\dot{\mu}}$, is completely specified by this nonlinear realization (together with the inverse Higgs effect). The role of $G_{\mathrm{I}}$ is in a sense subsidiary: it provides very simple criteria for determining in what cases the $G_{\mathrm{II}}$-covariant quantities and relations are covariant under the whole $N=1$ supergravity group. This concerns, e.g., the equations of the inverse Higgs effect. Recall that in the case of Einstein gravity treated as a nonlinear realization [8] the role of the conformal group (which is the analogues of $G_{\text{I}}$) is more essential: Conformal invariance alone picks out the Einstein Lagrangian among several appropriate invariants of the nonlinearly realized affine symmetry.

The construction of $N=1$ supergravity as a nonlinear realization of the complex supergroup $G_{\text{II}}$ in the coset supermanifold $G_{\text{II}}/L$, with $N=1$ superspace $(x^{\mu\dot{\mu}},\theta^{\mu},\bar{\theta}^{\dot{\mu}})$ as a real subspace and the $N=1$ supergravity action as a $G_{\text{II}}$-invariant supervolume of this subspace suggests an interesting analogy of $N=1$ supergravity with the (super)$p$-branes (strings, membranes, etc.) in the treatment of Refs. [2]. Actually, the minimal $N=1$ supergravity is recognized as a kind of "spinning" super $p$-brane of dimension (4/4) moving in the complex coset $G_{\text{II}}/L$ as the target space. The Goldstone superfields eliminated by the inverse Higgs effect are direct analogues of the Goldstone field which parametrize in ordinary $p$ branes the cosets of the relevant Lorentz groups and are expressed there in terms of the translation Goldstone fields by the same procedure [2]. This similarity raises some interesting questions, in particular, whether $N=1$ supergravity can be reproduced as an effective "low-energy" limit of some higher-dimensional superfield supersymmetric theories, by analogy with condensation of (super)$p$-branches in a field theory [20].

Closely related to the latter remark is the problem of existence of theories with a "linearity realized" $N=1$ supergravity group. Such theories could be related to the nonlinear realization formulation of $N=1$ supergravity much like linear $\sigma$ models with associated internal symmetries related to the corresponding nonlinear $\sigma$ models, via appearance of nonzero vacuum expectation values of some fields. Our constructions give a hint that these linear realizations should operate with linear representations of supergroup $G_{\text{II}}$. An analogous problem for the Einstein gravity has been settled in [8]. As was suggested by Witten [21], the linear $\sigma$ model of this kind describes the phases with unbroken local symmetries in gauge theories and can be presumably understood as topological field theories.

Finally, we note that the nonlinear realization treatment of the nonminimal $N=1$ supergravity theories can seemingly be constructed in an analogous way; however, owing to technical complications such a construction does not seem too enlightening. It is a much more ambitious problem to find a general principle allowing us to construct higher-$N$ supergravities by the nonlinear realization techniques. One might hope to obtain in this way the geometric prepotential formulations of supergravities with $N \geq 3$ which are unknown at present.

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