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Phase transitions and formation of bubbles in the early Universe

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We analyze the bubble-formation process that occurs when there is phase coexistence in the early Universe. In this paper we stress the relevance of determining the surface tension in order to compute some quantities that are relevent to cosmology such as the number density of bubbles, the contrast density, and the most probable sizes of bubbles (critical radius). We show how these quantities can be expressed as a function of the surface tension. We propose a method for computing this thermodynamical variable. The surface tension is shown to acquire a very simple dependence in the high-temperature limit and can be easily predicted up to the one-loop approximation.

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I. INTRODUCTION

As one probes the large-scale structure of the Universe some puzzling features, in the distribution of galaxies on large scales, emerges. In particular, there is strong evidence that the distribution of galaxies is made up of thin sheets (or surfaces) in which the galaxies lie. These sheets surround vast voids (regions in space containing few bright galaxies). Furthermore the sheets seem to be surfaces of several adjacent bubbles [1].

The nowadays-accepted picture for the development of structure in the Universe is based upon the growth, due to gravitational instability after the recombination era, of small perturbations in the density. There is a widespread belief that these initial perturbations should result from processes operating in the very early Universe, that is, processes that took place very close to the singularity. Cosmological phase transitions might have played a decisive role in shaping the nowadays-observed Universe. This follows from the fact that the theory of the fundamental interactions is based on gauge symmetries (that are spontaneously broken as the Universe lowers its temperature) and that the appearance of inhomogeneities (or symmetric defects) is a feature of theories whose symmetry is spontaneously broken [2]. Since the appearance of inhomogeneities in cosmological phase transitions is expected on very general grounds it seems natural to think of these inhomogeneities as seeds for galaxy formation. There are, in fact, suggestions that topological defects such as strings and domain walls might generate the contrast density required for giving rise to the observed structures in the Universe [3].

In this paper we deal with the problem of phase coexistence in the Universe. Whenever there is phase coexistence there is formation of bubbles (or droplets) in the system. In order to deal with the problem of bubble formation in phase transitions we have used the droplet picture of phase transitions [4,5]. This method has been developed for dealing with bubble formation in phase transitions that are very familiar to physicists (liquid-vapor phase transition, for instance).

We will explore the possibility that the observed large-scale structure of the Universe emerged from the existence of interfaces separating regions of different phases in the Universe. We imagine that at some stage of the Universe there was phase coexistence, i.e., there was an area in the Universe in which two bulk thermodynamic phases coexisted in such a way that regions of space (bubbles) were separated by a relatively narrow region, the interfacial region, over which the properties of the system must change from those of one phase to those of the second phase. In the case of a magnetic material the interfacial region is planar and is referred to as the Bloch wall. In the case of theories with spontaneous symmetry breakdown the planar interfacial region is referred to as a domain wall.

We will achieve a description of phase transitions from the knowledge of the interfacial free energy per unit area (that will be referred to from now on as the surface tension). The idea is that one can define first the thermodynamics of a single interface and afterwards to extend it to a description of the system as a whole.

In field theory there are two circumstances under which the Universe might have developed bubbles or domains. We will distinguish these two situations and will refer to them as the degenerate and nondegenerate cases. The nondegenerate case occurs when the order parameter has more than one component and the Hamiltonian is different for each value of the order parameter. In cosmology we would say that the two phases would have different cosmological constants. Under these circumstances, below a certain temperature the phase with the order parameter $\rho_0=0$ becomes metastable. The change from a metastable to a stable phase occurs as the result of fluctuations in a homogeneous medium. Within the homogeneous medium there is formation of small quantities (droplets) of the new phase.

The degenerate case occurs when the order parameter has, say, n components ρ_i but the Hamiltonian depends only on the sum of the squares of these components. The Hamiltonian is independent of the direction of the n-dimensional vector ρ. In field theory we would say that the vacuum of the theory is degenerate. A typical and familiar example of a degenerate system is a purely exchange ferromagnet, whose energy is independent of the direction of the magnetization vector.

The plan of the paper is the following: In Sec. II we review the general aspects of surfaces, their thermodynamics, and the formation of bubbles. In Sec. III we establish the general framework and give formal expressions, in field theory at finite temperature, for the free energy per unit area (surface tension) of a domain wall. As an example we find the surface tension and the critical temperature for the minimal SU(5) model. The expressions obtained are fairly simple in the high-temperature limit. In Sec. IV we consider bubbles in both cases of vacua, the nondegenerate and the degenerate one. Conclusions are presented in Sec. V.

II. SURFACES—CLASSICAL RESULTS

A. Surface tension

The thermodynamical properties of an interface can be entirely characterized by the surface tension σ. This thermodynamical variable is defined in terms of the work (dW) needed to vary the surface by an amount dA by

$$dW=\sigma\, dA \; . \tag{2.1}$$

The surface tension depends on the temperature as well as other variables, that we call external variables, such as magnetic fields,

$$\sigma=\sigma(T,x_i,\ldots,x_n)\;, \tag{2.2}$$

where x_i is the ith external variable. x_i accounts for the bulk environmental action over the surface.

In order to take into account surface effects, by taking the volume fixed, one writes

$$dF=dE-T\,dS=\mu\, dN+\sigma\, dA\;, \tag{2.3}$$

where the differentials stand for these elements in the two-phase system. From the equation for the thermodynamic potential Ω, that by definition is given by

$$d\Omega=-S\,dT-N\,d\mu+\sigma\, dA \tag{2.4}$$

one gets, for T and μ constant,

$$d\Omega=\sigma\, dA\;, \tag{2.5}$$

whereas within the canonical ensemble [taking N fixed in (2.3)] one gets

$$dF=\sigma\, dA\; . \tag{2.6}$$

From (2.6) it follows that f (the free energy per unit area) is equal to σ. From (2.5) it follows then that the entropy (per unit area) is given by

$$s=-\frac{d\sigma}{dT} \tag{2.7}$$

and the surface energy is

$$\varepsilon=f+Ts=\sigma-T\frac{d\sigma}{dT}\; . \tag{2.8}$$

If one represents by E^0, S^0, and F^0 the internal energy, entropy, and free energy of the two-phase system without the surface (that is, excluding the interfacial region) then the same quantities when a single surface of area dA is present in the system are given by

$$S=S^0+s(T,x)dA\;, \tag{2.9a}$$

$$E=E^0+\varepsilon(T,x)dA\;, \tag{2.9b}$$

$$F=F^0+\sigma(T,x)dA \tag{2.9c}$$

with $s(T,x)$ and $\varepsilon(T,x)$ defined by (2.7) and (2.8), respectively.

The main conclusion is that, as pointed out earlier, the surface tension, defined in (2.1), is the essential thermodynamic variable of the interface. From it one gets the free energy, entropy, and energy of an interface of area A, as

$$F_S=\sigma A\;, \tag{2.10a}$$

$$S_S=-A\frac{d\sigma}{dT}\;, \tag{2.10b}$$

$$E_S=\left[\sigma-T\left[\frac{d\sigma}{dT}\right]\right]A\; . \tag{2.10c}$$

Furthermore, from the definition (2.1) it follows that σ represents the cost in energy, per unit area, for introducing an interface into the system. This cost in energy can be expressed as a difference in the free energy of the two-phase system.

B. Phase transition

The bubbles with which we will be concerned in this paper are associated with phase coexistence in some stage of the Universe. We imagine two bulk thermodynamic phases separated by a relatively narrow region, the interfacial region, over which the properties of the system change from one phase to the other. Phase coexistence occurs in simple fluids, binary fluids, and in anisotropic magnets. The latter case is a prototype of models in which there is spontaneous symmetry breakdown and the interfaces are referred to as domain walls. There is, then, a strong correlation between the existence of interfaces and the occurrence of phase transitions.

At the critical temperature the surface tension vanishes

$$\sigma(T_c,x_i,\ldots,x_n)=0\; . \tag{2.11}$$

The condition (2.11) implies, for theories in which the vacuum is degenerate, that the cost for introducing a surface of arbitrary size into the system is zero and consequently the system is "insensible" to boundary conditions.

In this paper we will show that in field theory at finite temperature it is possible to account for the vanishing of the surface tension since one can provide a definite scheme for computing this relevant parameter. That allows us to determine many relevant parameters in cosmology. The approach is then, starting from the thermodynamical variable $\sigma(T)$, which is associated with a single bubble, to extend our thermodynamical discussion to the field-theoretical description of the system as a whole.

The dependence of the surface tension on the temperature in some cases is given by a universal function of T/T_c. For example, from the law of corresponding states it follows that the surface tension can be written as [6]

$$\frac{\sigma(T)}{\sigma(T=0)}=f(T/T_c)\ . \tag{2.12}$$

A dependence of the form (2.12) we shall call a corresponding-state dependence.

The surface tension in the high-temperature limit and up to the one-loop approximation, for any renormalizable theory, can be written in the form (2.12), with $f(T/T_c)=1-T^2/T_c^2$, so that the corresponding-state dependence is valid in field theory.

C. Bubble formation and critical sizes

According to the thermodynamic theory of fluctuations the probability for producing a bubble of radius R is given by

$$\omega\sim\exp\left[-\frac{\Delta F(R)}{T}\right], \tag{2.13}$$

where $\Delta F(R)$ is the cost in energy for introducing such an object into the system [6]. Usually, and as it will be done in this paper, the cost in energy can be expressed as a difference of thermodynamical potentials. In order to determine the most probable bubble radius we just look for the value of R that minimizes $\Delta F(R)$, defined in (2.13). This radius (critical radius R_c) is given by

$$\left.\frac{d\Delta F(R)}{dR}\right|_{R=R_c}=0\ . \tag{2.14}$$

As a simple example we consider the formation of bubbles in the case of a liquid-vapor phase transition. The bubbles will be considered as spheres of radius R. Under these circumstances one has to consider the variation in the thermodynamic potential Ω. Before the appearance of a single bubble in the system the potential is given by

$$\Omega^0=-P^0\left[V^0+\frac{4\pi}{3}R^3\right]. \tag{2.15}$$

After the appearance of the bubble in the system whose pressure is now P,

$$\Omega=-P^0V^0-P\frac{4\pi}{3}R^3+\sigma 4\pi R^2\ . \tag{2.16}$$

From (2.15) and (2.16) it follows that

$$\Delta F(R)=\Omega-\Omega^0=-(P-P_0)\frac{4\pi}{3}R^3+\sigma 4\pi R^2\ . \tag{2.17}$$

The probability for producing a bubble of radius R is then, from (2.13),

$$\omega\sim\exp\left[\frac{4\pi}{3}R^3(P-P^0)-4\pi R^2\sigma\right]. \tag{2.18}$$

A dependence of the form (2.18) is known as the capillarity approximation [5]. We shall see that for nondegenerate vacua it is possible to get a dependence of the form (2.18) within the one-loop approximation in the high-T limit. The critical size is then [from (2.14)],

$$R_{\rm cr}=\frac{2\sigma}{P-P^0} \tag{2.19}$$

and the probability for the most favorable bubble radius will be given, following (2.13), by

$$\omega\sim\exp\left[\frac{-16\pi\sigma^3}{3(P-P^0)^2T}\right]. \tag{2.20}$$

As can be seen from (2.19) and (2.20) one can get relevant information on the size of the most probable bubbles (critical bubbles) and their distribution from the knowledge of the surface tension.

Formally, the critical sizes of bubbles tends to zero at the critical temperature,

$$\lim_{T\to T_c}R_{\rm cr}(T)=0\ . \tag{2.21}$$

This is a consequence of the capillarity approximation which, in field theory, follows from the fact that the two phases exhibit different cosmological constants.

III. SURFACES IN FIELD THEORY

A. Surface tension — definitions

We have shown in the preceding section that the relevant quantity, whenever there is phase coexistence, is the surface tension. In field theory at finite temperature one has a well-defined approach for computing this thermodynamical variable.

Let $\phi_D(x)$ represent a field configuration describing a defect (for example, a bubble) in the system. We shall be interested in the thermodynamical properties of the system in the presence of such a background field. This, on the other hand, should be inferred from the partition function $Z(\phi_D)$ defined as

$$Z(\phi_D)=\int[D\phi]e^{-S[\phi+\phi_D]}\ . \tag{3.1}$$

The free energy of the system in the presence of the background field $\phi_D(x)$ is

$$F(\phi_D)=-\beta^{-1}\ln Z(\phi_D)\ . \tag{3.2}$$

One might be interested also in analyzing the free energy associated with a uniform background field that we represent by ϕ_0. The free energy of the system in the presence of this uniform background field ϕ_0 is

$$F^0(\phi_0)=-\beta^{-1}\ln Z^0(\phi_0)\ , \tag{3.3}$$

where $Z^0(\phi_0)$ is obtained from (3.1) by substituting ϕ_D in (3.1) by ϕ_0.

The vacuum of the theory is associated with the field configuration that minimizes $F^0(\phi_0)$:

$$\left.\frac{\delta F^0(\phi_0)}{\delta\phi_0}\right|_{\phi_0=\phi_v}=0\ . \tag{3.4}$$

$F(\phi_D)$ defined in (3.2) can be thought of as a thermodynamical potential associated with a spatially inhomogeneous system. The corresponding equilibrium condition is found by solving the following variational problem:

$$\left.\frac{\delta F(\phi)}{\delta\phi}\right|_{\phi=\phi_D}=0 . \tag{3.5}$$

One is then led to a variational problem which, more generally, can be stated as follows: Let ϕ_D be a solution of the following variational problem:

$$\left.\frac{\delta \Gamma(\phi)}{\delta\phi}\right|_{\phi=\phi_D}=0 , \tag{3.6}$$

where Γ is a group-invariant functional. It is possible then to show, by using the background-field method, that under condition (3.6) one can write the cost in energy for introducing a defect in the system as [9]

$$\Delta F=F(\phi_D)-F(\phi_v)=\Gamma(\phi_D)-\Gamma(\phi_v) , \tag{3.7}$$

where Γ is the effective action defined by

$$\Gamma=\sum_n \frac{1}{n!}\int \cdots \int dx_1 \cdots dx_n \Gamma^{(n)}(x_1,\ldots,x_n) \times\phi(x_1)\cdots\phi(x_n) , \tag{3.8}$$

where $\Gamma^{(n)}$ is the one-particle-irreducible Green's functions of the theory.

If one uses the Fourier transform of $\Gamma^{(n)}$, defined as

$$\Gamma^{(n)}(\tau_1\mathbf{x}_1,\ldots,\tau_n\mathbf{x}_n)=\beta^{-n}\prod_{j=1}^{n}\sum_{n_j=-\infty}^{+\infty}\int\frac{d^3\mathbf{k}_j}{(2\pi)^3}\tilde{\Gamma}^{(n)}(\omega_1\mathbf{k}_1,\ldots,\omega_n\mathbf{k}_n)\exp\left[-i\sum_{l=1}^{n}(\omega_l\tau_l+\mathbf{k}_l\cdot\mathbf{x}_l)\right] , \tag{3.9}$$

where $\omega_l=2\pi l/\beta$, and remembering that translational symmetry allows us to set

$$\tilde{\Gamma}^{(n)}(\{\omega_i\mathbf{k}_i\})=\beta(2\pi)^3\delta\left[\sum_i\omega_i\right]\delta^3\left[\sum_i\mathbf{k}_i\right] \times\tilde{\Gamma}^{(n)}(\{\omega_i\mathbf{k}_i\}) \tag{3.10}$$

then, for static field configurations (those with which we will be concerned in this paper), the general structure of $\Gamma(\phi_D)$ is

$$\Gamma(\phi_D)=\beta\sum_{n=1}^{\infty}\frac{1}{n!}\prod_{j=1}^{n}\int d^3\mathbf{k}_j\tilde{\phi}_D(-\mathbf{k}_j) \times\tilde{\Gamma}^{(n)}(\{\mathbf{k}_j,\omega_j=0\})\delta^3\left[\sum_j\mathbf{k}_j\right] . \tag{3.11}$$

The graphs that contribute to $\tilde{\Gamma}^{(n)}$ will involve sums over the discrete ω_j which, once performed, yield a term independent of temperature plus one which has the full T dependence. This separation can always be implemented [7]. One can then split $\tilde{\Gamma}^{(n)}$ into two parts

$$\tilde{\Gamma}^{(n)}(\{\mathbf{k}_i,\omega_i=0\})=\tilde{\Gamma}_0^{(n)}(\{\mathbf{k}_i\})+\tilde{\Gamma}_T^{(n)}(\{\mathbf{k}_i,\omega_i=0\}) , \tag{3.12}$$

where the second term contains all the T dependence. The general structure of this dependence can be inferred by making a change in all internal-momenta integration variables. This change is just a replacement $\mathbf{p}\to\mathbf{p}'=\mathbf{p}\beta$. After this scaling in the internal momenta one can predict, from pure dimensional analysis, that $\tilde{\Gamma}_T^{(n)}(\{\mathbf{k}_i,\omega_i=0\})$ have the following structure [8]

$$\tilde{\Gamma}_T^{(n)}(\{\mathbf{k}_i,\omega_i=0\})=\sum_{\gamma_n}T^{d(\gamma_n)}G_{\gamma_n}\left[\frac{\mathbf{k}_i}{T},\frac{n}{T}\right] , \tag{3.13}$$

where $d(\gamma_n)$ is the superficial degree of divergence of a graph γ_n contributing to $\tilde{\Gamma}$ and G_{γ_n} is dimensionless. Putting (3.10), (3.12), and (3.13) together, we have

$$\Gamma(\phi_D)=\Gamma_0(\phi_D)+\sum_{n=1}^{\infty}\frac{1}{n!}\prod_{j=1}^{n}\int d^3\mathbf{k}_j\tilde{\phi}_D(-\mathbf{k}_j)\sum_{\gamma_n}T^{d(\gamma_n)}G_{\gamma_n}\left[\frac{\mathbf{k}_i}{T},\frac{n}{T}\right]\delta^3\left[\sum_j\mathbf{k}_j\right] , \tag{3.14}$$

where $\Gamma_0(\phi_D)$ is the effective action computed with the background field ϕ_D at zero temperature.

Using (3.7) and remembering that the surface tension σ can be defined as $\Delta F/L^2$, we can write the following general expression for the surface tension involving a background field ϕ_D

$$\sigma(T)=\frac{1}{L^2}[\Gamma_0(\phi_D)-\Gamma_0(\phi_v)] +\frac{1}{L^2}\left\{\sum_{n=1}^{\infty}\frac{1}{n!}\prod_{j=1}^{n}\int d^3\mathbf{k}_j\tilde{\phi}_D(-\mathbf{k}_j)\delta^3\left[\sum_j\mathbf{k}_j\right]\sum_{\gamma_n}T^{d(\gamma_n)}G_{\gamma_n}\left[\frac{\mathbf{k}_i}{T},\frac{n}{T}\right]-L^3\sum_{n=1}^{\infty}\frac{1}{n!}\phi_v^n\sum_{\gamma_n}T^{d(\gamma_n)}G_{\gamma_n}\left[0,\frac{n}{T}\right]\right\} . \tag{3.15}$$

From (3.15) it follows that the general structure of $\sigma(T)$ is

$$\sigma(T)=\sigma(0)-T^2g^{(2)}(T,m)+Tg^{(1)}(T,m)+\cdots \, , \qquad (3.16)$$

where

$$\sigma(0)=\frac{1}{L^2}[\Gamma_0(\phi_D)-\Gamma_0(\phi_v)] \qquad (3.17)$$

and the terms involving powers of T^2 and T in (3.16) come from graphs having superficial degree of divergence 2 and 1.

From expression (3.15) one can see that, in the high-temperature limit, the leading contributions come from graphs that have higher superficial degrees of divergence. As we will show in the next section, these graphs up to a given order in the semiclassical expansion, are easy to isolate.

B. Domain-wall free energy

Field theories whose gauge symmetry is spontaneously broken might exhibit topologically stable defects. The prediction of the type of defect relies upon topological arguments. Under certain circumstances one can predict the existence of domain walls. These defects corresponds to infinite interfaces separating two vacua configurations (planar interfaces). At the classical level these objects are associated with solutions of (3.5) when one takes Γ computed at the zero-loop level.

In this section we will review the approach for computing the surface tension in field theory [9]. In this case we will be concerned with the computation of free energies associated with domain walls. Let σ_w represent the free energy associated with a domain wall. In the field theory σ_w is given as

$$\sigma_w=-\frac{\beta^{-1}}{L^2}\ln\left[\frac{Z_w}{Z_v}\right] , \qquad (3.18)$$

where Z_w stands for the partition function of the system evaluated when one imposes boundary conditions that force the existence of a domain-wall defect in the system, while Z_v is the partition function obtained using topologically trivial boundary conditions (vacuum sector). L is the size of the system.

We have shown that the various thermodynamical functions can be written in the one-loop approximation, as differences of the effective action of the theory evaluated at certain field configurations. Let $\Gamma(\phi)$ be the effective action of the theory and ϕ_v be the constant-field configuration associated with the vacuum of the theory. Then, in terms of the effective action one writes σ_w as in (3.7), with ϕ_D changed to ϕ_w, the field configuration associated with the wall,

$$\sigma_w=\frac{1}{L^2}[\Gamma(\phi_w)-\Gamma(\phi_v)] \, . \qquad (3.19)$$

The special field-theoretical configurations ϕ_D (ϕ_w for the wall), are the defects associated with the classical solutions of the Euler-Lagrange equations of the model.

The dependence of σ_w on T is given in (3.15). The critical temperature T_c is given from the condition (2.11) i.e., $\sigma_w(T_c)=0$. The interpretation in this case is that above T_c there is symmetry restoration as a result of condensation of domain walls [9].

As an example let us consider the minimal SU(5) grand unified theory at finite temperature. Its Euclidean Lagrangian density is

$$\mathcal{L}=-\frac{1}{4}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]+\frac{1}{2}\mathrm{Tr}[|D_\mu\phi|^2]+V(\phi) \, , \qquad (3.20)$$

where ϕ is the Higgs multiplet belonging to the adjoint representation and

$$V(\phi)=-\frac{\mu^2}{2}\mathrm{Tr}(\phi^2)+\frac{a}{4}[\mathrm{Tr}(\phi^2)]^2+\frac{b}{2}\mathrm{Tr}[\phi^4] \, , \qquad (3.21a)$$

$$G_{\mu\nu}=\sum_{i=1}^{24}G^i_{\mu\nu}\frac{\lambda^i}{\sqrt{2}} \, , \qquad (3.21b)$$

$$W_\mu=\sum_{i=1}^{24}W^i_\mu\frac{\lambda^i}{\sqrt{2}} \, , \qquad (3.21c)$$

$$\phi=\sum_{i=1}^{24}\phi^i\frac{\lambda^i}{\sqrt{2}} \, , \qquad (3.21d)$$

$$D_\mu\phi=\partial_\mu\phi-\frac{ig}{\sqrt{2}}\mathrm{Tr}[W_\mu,\phi] \qquad (3.21e)$$

and λ^i $(i=1,\ldots,24)$ are the generators of SU(5) in the fundamental representation (normalized so that $\mathrm{Tr}[\lambda^i\lambda^j]=2\delta^{ij}$). We also impose that $b>0$ and $a>-(7/15)b$.

This model exhibits two different topological defects: domain walls and magnetic monopoles. The background field describing a domain wall is the type of solution which one is interested in and is given by

$$\bar{\phi}_w=\frac{\mu}{\sqrt{\lambda}}\tanh\left[\frac{\mu}{\sqrt{2}}x\right]\frac{\lambda_{24}}{\sqrt{2}} \, , \qquad (3.22a)$$

$$\bar{W}^a_\mu=0 \qquad (3.22b)$$

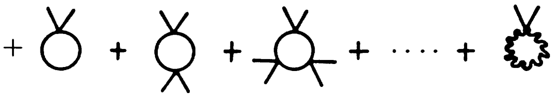
with $\lambda=a+\frac{7}{15}b$. Note that this solution depends only on one spatial coordinate, which we choose to be x.

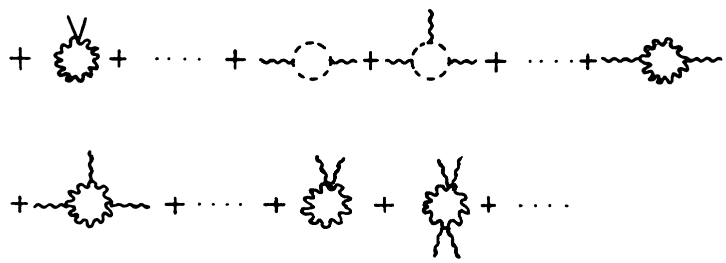
Let us exhibit the structure of the free energy of the system under this background field in the one-loop approximation. In the zero-loop approximation one has, from (3.16),

$$\sigma_w(0)\equiv\Delta\varepsilon^0_w=\frac{1}{L^2}[\Gamma_0(\bar{\phi}_w)-\Gamma_0(\phi_v)]$$
$$=\frac{T}{L^2}[S_{\mathrm{cl}}(\bar{\phi}_w)-S_{\mathrm{cl}}(\phi_v)] \, , \qquad (3.23)$$

where ϕ_v is the vacuum value of the classical potential $V(\phi_v)$, Eq. (3.21a), given by $\phi_v=\mu/\sqrt{\lambda}$, with $\lambda=a+\frac{7}{15}b$. Then from (3.23), in the zero-loop approximation, the free energy of the topological defect is just the difference between the classical action associated with the wall and the energy of the vacuum. $\Delta\varepsilon^0_w$ is the mass per unit area of the wall at $T=0$.

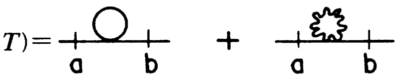
Within the one-loop approximation $\Gamma(\bar{\phi},\bar{W}_\mu)$ will have the structure predicted from (3.8) which, for the example that we are considering, has the structure

$\Gamma(\bar{\phi},\overline{W}_\mu)=S_{\rm cl}(\bar{\phi},\overline{W}_\mu)+$ 

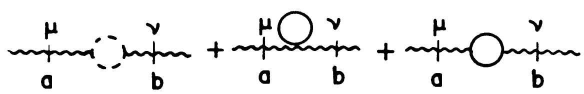


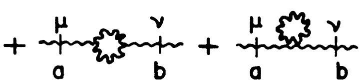
$$=S_{\rm cl}(\bar{\phi},\overline{W}_\mu)-\frac{1}{2!}\Sigma^{ab}(T)\int_0^\beta d\tau\int d^3\bar{x}\,\bar{\phi}^a\bar{\phi}^b=\frac{1}{2!}\Pi^{ab}_{\mu\nu}(T)\int_0^\beta d\tau\int d^3\bar{x}\,\overline{W}{}^a_\mu\overline{W}{}^b_\nu+\cdots\ , \qquad (3.24)$$

where $S_{\rm cl}$ is the classical action associated with the background field, $\Sigma^{ab}(T)$ can be represented graphically as

$\Sigma^{ab}(T)=$  (3.25)

whereas $\Pi^{ab}(T)$ can be represented as

$\Pi^{ab}_{\mu\nu}(T)=$ 

 (3.26)

The wavy, solid, and dashed lines stand, respectively, for the gauge boson, Higgs boson and ghost fields (for the fluctuations we are working in Landau gauge). $\Pi^{ab}(T)$ can be identified as the polarization tensor for zero external momenta [10]. Following our earlier prescription (3.12), we also split (3.25) and (3.26) into zero-temperature and temperature-dependent parts:

$$\Sigma^{ab}(T)=\Sigma^{ab}_0+\Sigma^{ab}_T(\{\mathbf{k}_i,\omega_i=0\}) \qquad (3.27a)$$

and

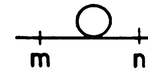
$$\Pi^{ab}_{\mu\nu}(T)=\Pi^{ab}_{\mu\nu_0}+\overline{\Pi}{}^{ab}_{\mu\nu}(T)\ . \qquad (3.27b)$$

First of all one notes, looking at (3.24), the appearance of ultraviolet divergences. These, however, can be treated, as usual, by adding appropriate renormalization counterterms, which are just the usual ones at zero temperature. This means that the zero-temperature renormalization scheme suffices for getting finite expressions for free energies of topological defects. Substituting (3.27) into (3.24), one can obtain the topological-defect free energy of the SU(5) model, which for a wall with $\bar{\phi}$ and $\overline{W}_\mu$ given by (3.22), one has

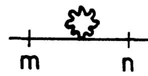
$$\sigma_w(T)=\Delta\varepsilon_w-\frac{1}{2!}\frac{\overline{\Sigma}{}^{24,24}(T)}{L^2}\times\int_0^\beta d\tau\int d^3\bar{x}[\bar{\phi}{}^w_{24}(x)\bar{\phi}{}^w_{24}(x)-\bar{\phi}{}^2_v]+\cdots, \qquad (3.28)$$

where $\Delta\varepsilon_w$ stands for the classical energy density of the wall, $\bar{\phi}{}^w_{24}(x)$ is given by (3.22), $\bar{\phi}_v=\mu/\sqrt{2\lambda\lambda_{24}}$, $\overline{\Sigma}{}^{24,24}(T)$ is given by (3.27a) and the dots represent one-loop contributions not included explicitly in (3.28). One could go further and write down similar expression for all the one-loop graphs for the topological-wall structure of the SU(5) model. However, instead of doing this explicitly, we will just analyze the high-temperature limit of the free energy. In this limit, the form (3.15) is particularly useful, since the leading power in T of series (3.15) is easily obtained. Property (3.13) permits us to identify these contributions, which are the ones with higher superficial degrees of divergence. These contributions are precisely the ones we have written explicitly.

In the high-temperature limit, the graphs appearing in (3.25) are the ones we need and yield

 $=-\left[26a+\frac{282}{15}b\right]\frac{T^2}{12}\delta^{mn}$ (3.29a)

and



$$= -\tfrac{5}{4}g^2T^2\delta^{mn} \ . \qquad (3.29b)$$

From (3.29) we have the asymptotic expression for $\bar{\Sigma}^{mn}(T)$

$$\bar{\Sigma}^{mn}(T) = -\frac{T^2}{4}[5g^2+\tfrac{1}{3}(26a+\tfrac{282}{15}b)]\delta^{mn} \qquad (3.30)$$

and from (3.28) one obtains the high-temperature behavior for $\sigma_w(T)$

$$\sigma_w(T) = \Delta\varepsilon_w + \frac{T^2}{4}[5g^2+\tfrac{1}{3}(26a+\tfrac{282}{15}b)] \times \int dx[\bar{\phi}_w^2(x)-\phi_v^2] \ . \qquad (3.31)$$

The substitution of (3.22) into (3.31) and use of (3.23) leads to

$$\sigma_w(T) = \frac{(2\mu^2)^{3/2}}{3\lambda} - \frac{T^2}{12}\frac{\mu\sqrt{2}}{\lambda}(26a+\tfrac{282}{15}b+5g^2) \qquad (3.32)$$

with $\lambda = a+\frac{7}{15}b$.

From the expression for $\sigma_w(T)$ and from the condition (2.10) we obtain the critical temperature T_c:

$$(T_c)^2 = \frac{60\mu^2}{\frac{225}{2}g^2+13(15a+7b)+50b} \ . \qquad (3.33)$$

From (3.33) and (3.32) one can also write $\sigma_w(T)$ as

$$\sigma_w(T) = \sigma(0)\left[1-\frac{T^2}{T_c^2}\right] , \qquad (3.34)$$

where $\sigma(0) = (2\mu^2)^{3/2}/3\lambda$. This is the result predicted in (2.12), where $\sigma(0)$ and T_c depend on the parameters (masses and coupling constants) of the SU(5) model.

C. Semiclassical approach

Admitting that the system under study is described by a scalar field ϕ one can write a path-integral representation for the partition function Z

$$Z = \int [D\phi]\exp[-S(\phi)] , \qquad (3.35)$$

where S is the effective action of the field ϕ,

$$S(\phi) = \int d^4x[\tfrac{1}{2}(\partial_\mu\phi)^2+V(\phi)] \ . \qquad (3.36)$$

In the semiclassical limit, the leading contributions to Z, given by (3.35) and (3.36), come from the field configurations, ϕ_c, which minimize the effective action,

$$\left.\frac{\delta\Gamma(\phi)}{\delta\phi}\right|_{\phi=\phi_c} = 0 \qquad (3.37)$$

and therefore obey the Euler-Lagrange equation

$$\Box\phi_c - V'(\phi_c) = 0 \ . \qquad (3.38)$$

If one makes a functional Taylor expansion of $S(\phi)$ around ϕ_c keeping only the quadratic terms in $\eta=\phi-\phi_c$, one obtains the partition function (3.35) up to the one-loop level,

$$Z^{(1)} = e^{-S(\phi_c)}\int [D\eta]\exp\left[-\int_0^\beta d\tau\int d^3\mathbf{x}[\tfrac{1}{2}(\partial_\mu\eta)^2+\tfrac{1}{2}\eta V''(\phi_c)\eta]\right] \ . \qquad (3.39)$$

The Gaussian integral in (3.39) is easy to evaluate and one gets, formally,

$$Z^{(1)} = e^{-S(\phi_c)}\det{}^{-1/2}[-\Box_{\rm Eucl}+V''(\phi_c)] \ . \qquad (3.40)$$

This expression gives the contribution of just the one-bounce solution.

Using the dilute-gas approximation one gets

$$Z = Z^{(0)}\exp\left[\frac{Z^{(1)}}{Z^{(0)}}\right] , \qquad (3.41)$$

where $Z^{(0)}$ is the partition function (3.40) computed at the vacuum field configuration, ϕ_v, of the theory, that is

$$Z^{(0)} = e^{-S(\phi_v)}\det{}^{-1/2}[-\Box_{\rm Eucl}+V''(\phi_v)] \ . \qquad (3.42)$$

Since the free energy F is given by

$$F = -\beta^{-1}\ln Z \qquad (3.43)$$

one gets, by treating the zero eigenvalues separately [11],

$$F = -T\left[\frac{S(\phi_c)}{2\pi}\right]^{\gamma/2} \times \left[\frac{\det'[-\Box_{\rm Eucl}+V''(\phi_c)]}{\det[-\Box_{\rm Eucl}+V''(\phi_v)]}\right]^{-1/2} e^{-S(\phi_c)} , \qquad (3.44)$$

where the prime indicates that the zero eigenvalues must be omitted from the determinant and γ is the number of these eigenvalues, which, in theories of three spatial dimensions is three.

Let Λ be the ratio of determinants which appear in (3.44). We shall develop a formal expansion for Λ that will be useful in order to extract its dependence on T at high temperatures. Λ can be written as

$$\Lambda = \exp(-\tfrac{1}{2}\{{\rm Tr}'\ln[-\Box_{\rm Eucl}+V''(\phi_c)] - {\rm Tr}\ln[-\Box_{\rm Eucl}+V''(\phi_v)]\}) \qquad (3.45)$$

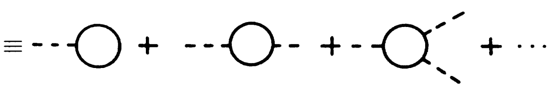
(3.45) can be written in the alternative form

$$\Lambda=\exp[-\tfrac{1}{2}(\mathrm{Tr}\ln\{1+G_\beta[V''(\phi_c)-V''(\phi_v)]\})] , \tag{3.46}$$

where $G_B=1/[-\Box_{\rm Eucl}+V''(\phi_v)]$ is the free propagator at finite temperature, with mass $\sqrt{V''(\phi_v)}$.

If we expand the natural log in (3.46) in powers of $G_\beta[V''(\phi_c)-V''(\phi_v)]$, we get formally

$$\mathrm{Tr}\ln\{1+G_\beta[V''(\phi_c)-V''(\phi_v)]\}\equiv \tag{3.47}$$



where the dashed lines correspond to the *background field* $[V''(\phi_c)-V''(\phi_v)]$, and the internal lines stand for the propagator G_β.

As in Sec. III B, one can isolate (in the high-T limit, $\beta\to0$) the terms which have higher superficial degree of divergence and then the contribution with leading power in T. This contribution is just the first term of the series (3.47). Then, we have, for $\beta\to0$ ($T\to\infty$),

$$\Lambda=\exp\left[-\frac{1}{2}\mathrm{Tr}\left[\frac{1}{-\Box_{\rm Eucl}+V''(\phi_v)}[V''(\phi_c)-V''(\phi_v)]\right]\right] . \tag{3.48}$$

We can compute explicitly the exponent in (3.49) so that one can write

$$\Lambda=\exp\left[-\frac{1}{2}\frac{1}{\beta}\sum_{n=-\infty}^{+\infty}\frac{d^3k}{(2\pi)^3}\frac{1}{(2\pi n/\beta)^2+k^2+V''(\phi_v)}\int_0^\beta d\tau\int d^3\mathbf{x}[V''(\phi_c)-V''(\phi_v)]\right] , \tag{3.49}$$

where we have used the usual representation for the trace and the Feynman rules at finite temperature for the first graph in (3.47). Performing the n summation and taking into account a static classical field ϕ_c one obtains, for a renormalizable theory [and making the argument of the exponent in (3.49) free of divergences, which is associated with the temperature-independent part of the first graph in (3.47)], Λ can be written as

$$\Lambda=\exp\left[-\frac{1}{2}\beta\int d^3\mathbf{x}[V''(\phi_c)-V''(\phi_v)]\int\frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{k^2+V''(\phi_v)}(e^{\beta\sqrt{k^2+V''(\phi_v)}}-1)}\right] . \tag{3.50}$$

In the high-temperature limit, Λ behaves as

$$\Lambda\sim\exp(-AT) , \tag{3.51}$$

where

$$A=\tfrac{1}{2}\int d^3\mathbf{x}[V''(\phi_c)-V''(\phi_v)]\int\frac{d^3k}{(2\pi)^3}\frac{1}{k(e^k-1)} . \tag{3.52}$$

Therefore (3.44) in the high-temperature limit becomes

$$F\simeq -T\left[\frac{S(\phi_c)}{2\pi}\right]^{\gamma/2}\left[\frac{1}{\beta}\right]^{\gamma}\exp[-S(\phi_c,T)] , \tag{3.53}$$

where $S(\phi_c,T)=S(\phi_c)+AT$, with AT being the result (3.51) for Λ. The factor $(1/\beta)^\gamma$ comes from a careful manipulation of (3.45) when one takes into account the zero eigenvalues of the determinant.

In the following sections we will use this expression to obtain the total free energy for a specific field configuration ϕ_c describing a spherical bubble.

IV. FIELD THEORETICAL DESCRIPTION OF CONDENSATION

A. The droplet picture of phase transitions and critical radius

In this section we will show how the evaluation of the partition function, for a collection of noninteracting droplets, may lead to the thermodynamic properties of a condensing system and the derivation of macroscopic features of a two-phase system. This is the so-called condensation problem.

For a nondegenerate system one can treat the condensation problem by using the droplet picture of phase transitions. The basic idea of the droplet picture, suggested as early as 1939 [12], is that a transition from a phase A to a phase B might be preceded by the formation of small nuclei of the phase B within A. The droplet model in field theory at zero temperature has been developed by Langer [4] as a statistical theory of the condensation phenomenon.

Under the hypothesis of noninteracting bubbles, the thermodynamics of the system can be derived from the knowledge of the partition function of a simple bubble. For a system of n particles, one can write the partition function, $Z_l(T)$, associated with an isolated cluster of l

particles moving in the volume V.

Within the droplet picture, and this is the basic assumption of the model, $Z_l(T)$ is written phenomenologically as [13]

$$\frac{Z_l(T)}{V}=q_0 l^{-\tau}\exp\{a_0 l^{\gamma}[(W-\omega t)/T]\}\exp\{l[F_V/K_B T]\} \tag{4.1}$$

where $q_0 l^{-\tau}$ is a geometric term, whereas the other terms represent the surface and bulk contribution to the free energy. The surface term has a contribution associated with the surface energy W and a surface entropy ω associated with the wiggles of the surface. $a_0 l^{\gamma}$ is the effective surface. q_0, τ, and γ are phenomenological parameters. $F_V/K_B T$ is the bulk contribution to the free energy.

An expression analogous to (4.1) has already been obtained, within the context of field theory, for a phase transition in which the system goes to a metastable phase (the vacuum is metastable). For cosmological phase transitions this metastability implies that there is a difference in energy density of the vacua of the theory (the true one and the false, the one in which the system is trapped). In this context it is possible, in semiclassical approximation, to identify all the elements present in (4.1). In fact, the classical action $S_{\rm cl}(R)$ associated with a bound solution, describing a bubble of radius R, can be cast (in three dimensions) in the form

$$S_{\rm cl}(R)=-\frac{4\pi}{3}R^3\Delta\Gamma+4\pi R^2\sigma \ , \tag{4.2}$$

where ΔT is just the difference in energy density between the vacuum states and σ is the surface tension. The first term thus represents the bulk contribution (volume energy) and the second one represents the surface energy.

A term analogous to the geometric one can be obtained only within the one-loop approximation [11]. Taking into account just the zero modes we have a preexponential term that goes like $S^{\gamma/2}$, where γ is the number of zero modes [11].

The droplet model pictures the system as a *dilute gas* of small droplets of radius R. The number of bubbles of size R might be approximated by a simple Boltzmann factor, that is

$$N(R)\sim\exp[-\beta\Delta F(R)] \ , \tag{4.3}$$

where $\Delta F(R)$ is the energy cost for introducing a bubble into the system.

As shown in the previous sections, the cost in energy for introducing an interface in the system can be defined by (3.7)

$$\Delta F=F(\phi_B)-F(\phi_v)=-\beta^{-1}\ln\left[\frac{Z(\phi_B)}{Z(\phi_v)}\right] . \tag{4.4}$$

For spherical bubbles of radius R, ΔF is a function of R, and one can write $\Delta F\equiv\Delta F(R)$.

Only bubbles whose size R is above a critical value $R_{\rm cr}$ are stable and they survive in the system. This critical value is given by the condition

$$\left.\frac{d\Delta F(R)}{dR}\right|_{R=R_{\rm cr}}=0 \ . \tag{4.5}$$

Bubbles with radius smaller than $R_{\rm cr}$ are unstable and disappear again. These bubbles are assumed to be macroscopic objects. The value $R=R_{\rm cr}$ determined by (4.5) corresponds to the limit beyond which large quantities of the new phase begin to be formed. Bubbles beyond the critical range (with $R>R_{\rm cr}$) will inevitably develop into a new phase.

For a nondegenerate system one can picture the condensation process as a two-stage process. In the first stage (metastable phase) the system is metastable. In this stage there is formation of critical bubbles. In the second stage (condensed phase) there is the growth of the critical droplets. Bubbles of sizes larger than the critical one become stable and grow.

Within the one-loop approximation and for temperature below, but close to, the critical one, one can write

$$\Delta F(R)=-\frac{4\pi}{3}R^3[\Gamma(\bar\phi_{\rm out},T)-\Gamma(\bar\phi_{\rm in},T)]+4\pi R^2\sigma(T) \ , \tag{4.6}$$

where $\bar\phi_{\rm out}$ is the local minimum, of the potential $V(\phi)$, which dominates the region outside the bubble and $\bar\phi_{\rm in}$ is the global minimum which dominates the inside region. In this case the solution which interpolates between these two minima is the kinklike solution $\phi_k(\mathbf{x})$ and it describes the surface of the bubble. $\sigma(T)$ in (4.6) is the surface tension.

The result (4.6) above can be seen when one uses (3.53) for a nondegenerate system, replacing ϕ_c by ϕ_B, which represents a spherical bubble of radius R. ϕ_B consists of: $\bar\phi_{\rm in}$, for $R<R_{\rm cr}-\Delta R$; ϕ_k, the kink field configuration, for $R\in(R_{\rm cr}-\Delta R,R_{\rm cr}+\Delta R)$ and $\bar\phi_{\rm out}$, for $R>R_{\rm cr}+\Delta r$. Then one can divide the integral of the classical action into three regions [11]: the inside of the bubble, the skin of the bubble, and the outside of the bubble. In the thin-wall approximation, that is, $\Delta R\ll R_{\rm cr}$ and using the result (3.52) (in the case of a nondegenerate system), the temperature corrections to the classical action, (4.6) give a good approximate description to the bubble action [14]. From (4.6) one obtains that $R_{\rm cr}$ is given by

$$R_{\rm cr}(T)=\frac{2\sigma(T)}{\Delta\Gamma(T)} \ . \tag{4.7}$$

As an example (of a nondegenerate system) one can consider the Hamiltonian density for a scalar field theory given by

$$\mathcal{H}=\tfrac{1}{2}\pi^2(\mathbf{x},t)+\tfrac{1}{2}[\nabla\phi(\mathbf{x},t)]^2+\tfrac{1}{2}m^2\phi^2(\mathbf{x},t) + \left[\frac{\lambda}{4!}\right]\phi^4(\mathbf{x},t)+j\phi(\mathbf{x},t)+\tfrac{3}{2}m^4/\lambda \ , \tag{4.8}$$

where j is an external current assumed to be time and position independent.

Following the ideas of Langer [4], metastability arises if we consider what happens as we vary the value of the external current j, for suitable low temperatures.

A simple analysis of the classical potential shows that at the vicinity of $j=0$, one can have two minima, one local and the other global. The semiclassical correction around each minimum brings the temperature into the problem and leads to a two-phase picture of the system: large regions dominated by the global minimum configuration where, due to thermal fluctuations, there occur bubbles (or droplets) dominated by the local minimum [14].

From (4.7) one can make contact with the phase transition (second order) that take place as $j\to 0$ and $T\to T_c$ by remarking that, at the transition temperature, the critical radius $R_{\rm cr}(T_c)$ should vanish. From (4.7) one can see that at $T=T_c$, $R_{\rm cr}(T_c)=0$, if $\sigma(T_c)=0$.

Let us assume that the distribution of bubbles is a dilute one. Under these circumstances one can write for the partition function Z Eq. (3.41)

$$\frac{Z}{Z^{(0)}}=\exp\left[\frac{Z^{(1)}}{Z^{(0)}}\right], \tag{4.9}$$

where $Z^{(0)}$ now stands for the partition function in the vacuum field configuration, ϕ_v, and $Z^{(1)}$ in the bubble field configuration ϕ_B.

From the results of Sec. III C, one can write the free energy of the bubble, $F=-\beta^{-1}\ln Z$, Eq. (3.53), with ϕ_c replaced by ϕ_B. In the high-temperature limit and considering spherical bubbles one can find a general form for F given by

$$F=-T\left[\frac{-(4\pi/3)R^3\Delta\Gamma+4\pi R^2\sigma(0)}{2\pi T}\right]^{3/2}\left[\frac{1}{\beta}\right]^3 \times\exp\left[\frac{(4\pi/3)R^3\Delta\Gamma(T)-4\pi R^2\sigma(T)}{T}\right], \tag{4.10}$$

where we have used (4.2) for $S_{\rm cl}(\phi_B)$ and (4.6) for $S(\phi_B,T)$, the classical action associated with the bubble appearing in (3.53).

In (4.10) we have taken the factor γ appearing in (3.53) as being three (for bubbles in three spatial dimensions there are three translational zero modes and therefore three zero eigenvalues).

For the situations in which the difference between the energies of the degenerate vacua is zero [$\Delta\Gamma$ in (4.2) is zero] there is no external source to make one of them energetically preferred with regard to the other. In this circumstance, the determination of the critical radius of bubbles, for instance, cannot be done by using (4.2). So that one has to adopt, as an improved version, the droplet picture in field theory.

Within the droplet-model picture, symmetry restoration occurs as a result of formation of droplets of the new phase within the old phase. Close to the critical temperature these bubbles are more numerous and larger. In fact, at the critical temperature bubbles with infinite radius are favored to appear in the system. That is why we could consider these bubbles, close to T_c, as infinite domain walls ("magic carpets" in Fisher's words [13]).

One can consider (4.10) in the two situations of vacuum, the nondegenerate and the degenerate. In the case of a nondegenerate vacuum, we have essentially the picture sketched in the beginning of this section and the radius of the critical bubbles is given by (4.7). When one takes the vacuum as a degenerate one, i.e., $\Delta\Gamma$ in (4.10) is equal to zero, one can again have critical bubbles in the system, with radius given by the minimization of (4.10) with $\Delta\Gamma=0$,

$$F=-T^4\left[\frac{4\pi R^2\sigma(0)}{2\pi T}\right]^{3/2}\exp\left[\frac{-4\pi R^2\sigma(T)}{T}\right]. \tag{4.11}$$

One obtains then the following expression for $R_{\rm cr}(T)$, in the degenerate case,

$$R^2_{\rm cr}(T)=\frac{3T}{8\pi\sigma(T)}. \tag{4.12}$$

From this expression for $R_{\rm cr}(T)$, one can see that for $T=T_c$ the bubble radius becomes infinite.

B. The total free energy of bubbles and their densities

Within the droplet picture of phase transitions, and admitting a dilute gas of droplets, the total free energy of a collection of bubbles, with radius R, can be written as [4,5]

$$\mathcal{F}\sim\int dR\,F(R), \tag{4.13}$$

where $F(R)$ is the free energy associated with a single bubble of radius R, Eq. (4.10), and it represents the cost in energy for introducing a single bubble in the system.

In the dilute-gas approximation one can also estimate the bubble density, defined as

$$\rho_{\rm bubble}=M_{\rm bubble}(R)N(R), \tag{4.14}$$

where $M_{\rm bubble}(R)$ is the bubble mass and $N(R)$ is the average number of bubbles, which in the dilute-gas approximation is given by

$$N(R)=\frac{Z^{(1)}}{Z^{(0)}}. \tag{4.15}$$

By using the capillarity approximation [5] one writes, using (4.6),

$$N(R)\sim e^{-\beta[-(4\pi/3)R^3\Delta\Gamma(T)+4\pi R^2\sigma(T)]}. \tag{4.16}$$

For the critical bubbles one predicts

$$N(R_{\rm cr})\sim e^{-16\pi\sigma^3(T)/3T\Delta\Gamma^2(T)}, \tag{4.17}$$

where we have used the expression (4.7) for $R_{\rm cr}$.

Finally, as a simple exercise, let us consider the degenerate case ($\Delta\Gamma=0$). In this case one obtains for (4.14) the expression

$$\rho_{\rm bubble}(R)=4\pi R^2\sigma(0)T^3\left[\frac{4\pi R^2\sigma(0)}{2\pi T}\right]^{3/2}\times\exp\left[\frac{-4\pi R^2\sigma(T)}{T}\right]. \quad (4.18)$$

For $R=R_{\rm cr}$ given by (4.12), one obtains

$$\rho_{\rm bubble}(R_{\rm cr})=\frac{3}{2}\left[\frac{3}{4\pi e}\right]^{3/2}\left[\frac{\sigma(0)}{\sigma(T)}\right]^{5/2}T^4 . \quad (4.19)$$

Note that all of these expressions are strongly dependent upon the surface tension σ.

V. CONCLUSIONS

Bubbles might appear in cosmological phase transitions for theories with nondegenerate or degenerate vacua. In both cases one can predict phase coexistence in the Universe and the appearance of bubbles as a result of thermal fluctuations. The basic ingredient for making predictions relevant to cosmology is the cost in energy to introduce such an object in the system.

In this paper we have proposed an extension of the droplet picture of phase transitions in field theory that allows us to get estimates of the critical radius, their dependence on temperature and the contrast density due to bubbles. The droplet picture has been applied, in field theory, to the description of phase transitions in which the system goes through a metastable phase [4]. These situations are characterize by the existence, at least for a certain range of temperatures, of nondegenerate vacua [16]. In the case of theories with nondegenerate vacua, expression (4.10) permits us to make better estimates of physical quantities than the usual "classical theory" [5] since this expression takes into account the translational modes as well as the temperature dependence of the surface tension.

When the theory exhibits degenerate vacua as a result of a discrete symmetry, as has been suggested in the literature [9,15,17] the phase transition is supposed to be Ising-like. In this paper we have shown how the droplet picture can be applied in these circumstances.

We have shown that the knowledge of $\sigma(T)$, the surface tension, is relevant in all of our expressions, such that $\sigma(T)$ is a fundamental quantity to be determined. Within the formalism of the present work we have shown how to compute this quantity in field theory at finite temperature.

Whereas in the nondegenerate case the radius of critical bubbles tends, formally, to zero at the critical temperature, in the degenerate case the critical bubbles tend to infinity. This, on the other hand, implies that only for temperatures above the critical one does condensation of domain walls take place and consequently above this temperature there will be formation of domain walls [18]. Below this temperature domain walls are not favorable.

Some phenomenological applications of this framework can be found in Ref. [19].

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