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Predictions for proton-antiproton decay rates of $^3D_2$ charmonium states in exclusive QCD

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We use the methods of exclusive QCD to derive the amplitude for the coupling of $J=2$ quarkonium triplet D states to proton-antiproton pairs. Beginning with this amplitude, we find that $\Gamma(^3D_2\to p\bar{p})/\Gamma(\psi\to p\bar{p})$ is in the range 0.08–0.19 depending on the assumed nucleon wave function.

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In the last decade, much progress has been made in the development of calculations for exclusive processes in QCD [1]–[3]. Specifically, quarkonium two-body decays have been considered in detail, beginning with the decay width for $\psi(^3S_1)\to p\bar{p}$ [4] and continuing with calculations for $^3P_1,{}^3P_2\to p\bar{p}$ [5]–[7]. Confrontation of QCD predictions for quarkonium decay widths with the data has led to somewhat mixed results [7]–[11], but more detailed information should soon be available since the first set of data on exclusive charmonium production from $p\bar{p}$ collisions in the next-generation experiment (Fermilab E760 [12] has recently begun to appear [13, 14]. It may eventually be possible to explore charmonium D states whose masses have been predicted in potential models [15]. Effects from D-state processes are expected to be small since D states couple to the second derivative of the wave function at the origin rather than the wave function itself as S states do or its first derivative as P states do. However, the presence of large numerical factors found for several processes [16]–[18] can counteract the previous argument and produce larger than anticipated D-state effects.

Familiar helicity-conservation arguments from exclusive QCD lead to the conclusion that the couplings of $^3D_{1,2,3}$ to $p\bar{p}$ pairs are allowed at full strength, but the coupling to the $^1D_2$ state will be suppressed (just as predicted for the $^1S_0$ and $^1P_1$, $^3P_0$). Furthermore, the $^3D_2$ and $^1D_2$ states are both predicted to be relatively narrow [15] because parity and angular momentum conservation forbid their decays into $D\bar{D}$ pairs and they lie below the $D\bar{D}^*$ threshold. The $^3D_1$ state is expected to mix with the $^3S_1$ state [15], and a recent calculation derived decay rates for $\psi',\psi''\to p\bar{p}$ which included this mixing [19]. This paper extends that calculation to a second D state, $^3D_2$, which is expected to be a pure D state.

The calculation of the $^3D_2\to p\bar{p}$ decay rate follows the same pattern as the previous calculation for the $^3D_1$ [19]. The only difference between the calculations for the $^3D_1$ and the $^3D_2$ decay rates is the projection onto the polarization states of the quarkonium. However, the matrix algebra involved in the $J=2$ triplet case makes this calculation more complex. We use the methods described in Refs. [7,20] to extract the nucleon and antinucleon spinor from the basic quark diagram, which is the same as that required for the $^3D_1$ case. These methods are then combined with the covariant formalism developed to describe D-state annihilation decays [21]. Here we find the amplitude for the $S=1$ D state is given by

$$A=\left[\frac{15}{8\pi M}\right]^{1/2}\left[\frac{\phi_D''(0)}{2M^2}\right]\times\left[\mathrm{Tr}[\gamma^\alpha\mathcal{O}_0\gamma^\beta(\not{P}+M)\gamma^\rho]+M\,\mathrm{Tr}[\{\mathcal{O}_1^\alpha,\gamma^\beta\}_+(\not{P}+M)\gamma^\rho]+\frac{M^2}{2}\mathrm{Tr}[\mathcal{O}_2^{\alpha\beta}(\not{P}+M)\gamma^\rho]\right]\Pi_{\alpha\beta\rho}\ , \tag{1}$$

where $\mathcal{O}_0$, $\mathcal{O}_1^\alpha$, and $\mathcal{O}_2^{\alpha\beta}$ are the zeroth, first, and second derivatives, respectively, of $\mathcal{O}$ (the relevant Dirac operator) with respect to k^α (with $k=0$), M is the quarkonium mass, P is the quarkonium momentum, and $\phi_D''(0)$ is the second derivative of the radial wave function evaluated at the origin.

For the $J=2$ state, the projection operator is

$$\Pi_{\alpha\beta\gamma}(J=2)=\frac{i}{M\sqrt{6}}(e_{\alpha\sigma}\epsilon_{\tau\beta\rho\sigma'}P^\tau g^{\sigma\sigma'}+e_{\beta\sigma}\epsilon_{\tau\alpha\rho\sigma'}P^\tau g^{\sigma\sigma'})\ , \tag{2}$$

where $e_{\alpha\beta}$ is the symmetric spin-2 polarization tensor which satisfies

$$g^{\alpha\beta}e_{\alpha\beta}=0,\quad P^\alpha e_{\alpha\beta}=0\ , \tag{3}$$

and where the sum over polarizations is given by the familiar expression

$$\sum_{m=-2}^{2} e_{ab}(m)e^*_{xy}(m)=\tfrac{1}{2}(\mathcal{P}_{ax}\mathcal{P}_{by}+\mathcal{P}_{ay}\mathcal{P}_{bx})-\tfrac{1}{3}\mathcal{P}_{ab}\mathcal{P}_{xy} , \tag{4}$$

where

$$\mathcal{P}_{ab}=-g_{ab}+\frac{P_aP_b}{M^2} . \tag{5}$$

For comparison, the corresponding expression for the $^3S_1$ state is given by

$$A=-\frac{\phi_S(0)}{2\sqrt{4\pi M}}\mathrm{Tr}[\mathcal{O}_0(\not P+M)\not\epsilon] . \tag{6}$$

Equations (1) and (2) thus generalize the results of the appendix of Ref. [7] to D states. The use of an explicitly covariant formalism enabled us to work directly with popular algebraic manipulation packages and we have made extensive use of FORM in deriving the results below.

The amplitude for $^3S_1\to p\bar{p}$ has been derived in Ref. [7] and in our language the result reads

$$A(^3S_1\to p(q_1)\bar{p}(q_2))=(4\pi\alpha_s)^3\left[\frac{3}{\pi M}\right]^{1/2}\left[\frac{10\phi_S(0)f_N^2}{81M^5}\right]\times\bar{N}(q_1)\not\epsilon N(q_2)\mathcal{M}_S , \tag{7}$$

where

$$\mathcal{M}_S=\mathcal{M}_S^{(0)}=\int[\widetilde{dx}]\int[\widetilde{dy}]\mathcal{J}_S^{(0)} \tag{8}$$

and

$$\mathcal{J}_S^{(0)}=\frac{\phi(x)\phi(y)x_1y_3}{D_1D_3}+\frac{2T(x)T(y)x_2y_3}{D_1D_2} . \tag{9}$$

In the above we have defined $D_i\equiv(2x_i-1)(2y_i-1)-1$ and

$$\int[\widetilde{dx}]\equiv\int_0^1dx_1\int_0^1dx_2\int_0^1dx_3\frac{\delta(1-x_1-x_2-x_3)}{x_1x_2x_3} . \tag{10}$$

Also, $T(z)$ is a function related to a leading-twist nucleon wave function $\phi(z)=\phi(z_1,z_2,z_3)$ via

$$2T(1,2,3)=\phi(1,3,2)+\phi(2,3,1) . \tag{11}$$

The corresponding decay rate is then

$$\Gamma(^3S_1\to p\bar{p})=(\pi\alpha_s)^6\left[\frac{2^{12}\times5^2}{3^8\pi^2}\right]\frac{|\phi_s(0)|^2f_N^4}{M^{10}}\mathcal{M}_S^2 . \tag{12}$$

Using our D-wave formalism, we find (after extensive algebra) that the corresponding amplitude for $^3D_2$ states is given by

$$A(^3D_2\to p(q_1)\bar{p}(q_2))=(4\pi\alpha_s)^3\left[\frac{15}{\pi M}\right]^{1/2}\left[\frac{5\phi_D''(0)f_N^2}{27M^7}\right]\times\bar{N}(q_1)\gamma_\alpha N(q_2)e_{\alpha\beta}\frac{\Delta_\beta}{M}\mathcal{M}_D , \tag{13}$$

where $\Delta\equiv q_2-q_1$ and

$$\mathcal{M}_D\equiv\mathcal{M}_D^{(0)}+\mathcal{M}_D^{(1)}+\mathcal{M}_D^{(2)}=\int[\widetilde{dx}]\int[\widetilde{dy}](\mathcal{J}_D^{(0)}+\mathcal{J}_D^{(1)}+\mathcal{J}_D^{(2)}) , \tag{14}$$

corresponding to the contributions from the zeroth-, first-, and second-derivative terms in Eq. (1), respectively. The integrands are given by

$$\mathcal{J}_D^{(0)}=0 , \tag{15}$$

$$\mathcal{J}_D^{(1)}=\phi(x)\phi(y)\left[\frac{4}{D_1D_3}\left[\frac{E_1}{D_1}+\frac{E_2}{D_3}\right]+\frac{E_3}{D_1D_2}\right]+4T(x)T(y)\left[\frac{F_1}{D_1D_3}+\frac{2}{D_1D_2}\left[\frac{F_2}{D_1}+\frac{F_3}{D_2}\right]\right] , \tag{16}$$

where

$$E_1=2x_1^2y_3-2x_1y_3-x_1^2+x_1 , \tag{17}$$

$$E_2=x_1x_3-2x_1x_3y_3+2x_1y_3^2-x_1y_3-x_3^2+x_3y_3 , \tag{18}$$

$$E_3=x_3 , \tag{19}$$

$$F_1=1-2x_3 , \tag{20}$$

$$F_2=2x_1^2y_2-2x_1y_2-x_1^2+x_1 , \tag{21}$$

$$F_3=2x_1y_2^2-2x_1y_2-x_2^2+x_2 , \tag{22}$$

and

$$\mathcal{J}_D^{(2)}=\frac{8\phi(x)\phi(y)}{3D_1D_3}\left[\frac{2G_1}{D_1^2}+\frac{G_2}{D_1D_3}+\frac{G_3}{D_3^2}\right]+\frac{2\phi(x)\phi(y)}{3D_1D_2}\left[\frac{H_1}{D_1}+\frac{H_1}{D_2}+\frac{2H_2}{D_1D_2}\right]+\frac{16T(x)T(y)}{3D_1D_3}\left[\frac{I_1}{D_1}-\frac{I_2}{D_3}\right]+\frac{32T(x)T(y)}{3D_1D_2}\left[\frac{J_1}{D_1^2}+\frac{J_2}{D_1D_2}+\frac{J_3}{D_2^2}\right] , \tag{23}$$

where

$$G_1=2x_1^3y_3-2x_1x_3y_1^2+x_1^2x_3-x_1^2y_3 , \tag{24}$$

$$G_2=x_1x_3-x_1^2x_3-2x_1^2x_3y_3+4x_1^2y_3^2-x_1^2y_3-2x_1x_3^2+2x_1x_3y_3-2x_1y_3^2+x_1y_3 , \tag{25}$$

$$G_3=4x_1y_3^3-x_1y_3^2-6x_1x_3y_3^2+2x_1x_3^2y_3+x_1x_3^2 , \tag{26}$$

$$H_1=x_1(x_2-y_2) , \tag{27}$$

$$H_2=x_1^2x_2-2x_1^2x_2y_2+x_1^2y_2-2x_1x_2^2y_1+2x_1x_2y_1+x_1x_2^2+2x_1x_2y_2-2x_1x_2+x_1y_2^2-2x_1y_2 , \tag{28}$$

$$I_1=x_1(x_3-y_3) , \tag{29}$$

$$I_2=x_3(1-x_3) , \tag{30}$$

$$J_1=2x_1^3y_2-2x_1x_2y_1^2+x_1^2x_2-x_1^2y_2\ , \tag{31}$$

$$J_2=2x_1^2y_2^2-x_1^2x_2-x_1^2y_2-x_1x_2^2+x_1x_2+x_1y_2-x_1y_2^2\ , \tag{32}$$

$$J_3=2x_1y_2^3-2x_1x_2y_2^2-x_1y_2^2+x_1x_2^2\ . \tag{33}$$

The corresponding decay rate is then

$$\Gamma(^3D_2\to p\bar{p})=(\pi\alpha_s)^6\left[\frac{2^9\times5^2}{3^5\pi^2}\right]\frac{|\phi_D''(0)|^2f_N^4}{M^{14}}\mathcal{M}_D^2\ . \tag{34}$$

To evaluate the hard-scattering amplitudes, we use several parametrizations of the nucleon wave function which satisfy sum-rule constraints and lead to reasonable predictions for nucleon form factors and $\psi,\chi_{1,2}$ decays. Specifically, we consider, for $\phi(x)=\phi(x_1,x_2,x_3)$,

$$\phi_{\rm CZ}=\phi_{\rm as}(x)(18.06x_1^2+4.62x_2^2+8.82x_3^2-1.68x_3-2.94)\ [3]\ , \tag{35}$$

$$\phi_{\rm KS}=\phi_{\rm as}(x)[20.16x_1^2+15.12x_2^2+22.68x_3^2-6.72x_3+1.68(x_1-x_2)-5.04]\ [22]\ , \tag{36}$$

and

$$\phi_{\rm ZOC}=\phi_{\rm as}(x)(23.814x_1^2+12.978x_2^2+6.174x_3^2+5.88x_3-7.098)\ [23]\ . \tag{37}$$

In these expressions, one defines $\phi_{\rm as}(x)\equiv120x_1x_2x_3$ which is the asymptotic wave function that should be reached at infinitely large values of Q^2. Using these wave functions, we can evaluate the various overlap integrals in Eqs. (8) and (14) and we give the results in Table I.

In order to minimize the strong dependence of the decay widths on factors such as α_s^6, we examine ratios of $p\bar{p}$ decay widths. Thus, with our results for the $^3D_2\to p\bar{p}$ amplitudes, we find that

$$\frac{\Gamma(^3D_2\to p\bar{p})}{\Gamma(\psi\to p\bar{p})}=\left[\frac{\alpha_s(M^2(^3D_2))}{\alpha_s(M_\psi^2)}\right]^6\left[\frac{M(\psi)}{M(^3D_2)}\right]^{10}\times\left\{\left[\frac{\sqrt{27/8}\phi_D''(0)}{\phi_{1S}(0)M^2}\right]\left[\frac{\mathcal{M}_D}{\mathcal{M}_S}\right]\right\}^2\ . \tag{38}$$

The final ingredients are values of the potential-model wave functions which we take from previous fits [15, 24] to charmonium and bottomium data. Specifically we use

$$\phi_{1S}(0)=1.01\ {\rm GeV}^{3/2}\ ,$$
$$\phi_{1D}''(0)=0.121\ {\rm GeV}^{7/2}\ .$$

TABLE I. Values of the overlap integrals $\mathcal{M}_S$ and $\mathcal{M}_D$ for various proposed nucleon wave functions, $\phi_{\rm CZ}$ [3], $\phi_{\rm ZOC}(x)$ [23], and $\phi_{\rm KS}(x)$ [22].

	$\mathcal{M}_S$	$\mathcal{M}_D^{(1)}$	$\mathcal{M}_D^{(2)}$	$\mathcal{M}_D$
$\phi_{\rm CZ}$	0.73×10^4	2.28×10^5	2.58×10^5	4.86×10^5
$\phi_{\rm ZOC}$	0.86×10^4	2.83×10^5	4.03×10^5	6.86×10^5
$\phi_{\rm KS}$	1.11×10^4	4.09×10^5	6.84×10^5	1.09×10^6

For the three model wave functions we find the results

$$\Gamma(^3D_2)/\Gamma(\psi)=(0.081-0.085),\ (0.11-0.12)\ ,\ (0.176-0.185) \tag{39}$$

for CZ, ZOC, and KS wave functions, respectively.

In conclusion, we have calculated the exclusive QCD amplitude for the coupling of $^3D_2$ quarkonium states to $p\bar{p}$ pairs. Previous studies using this formulation of exclusive QCD have focused on examining different processes to prove the extent to which these methods can offer a coherent picture of exclusive hadronic processes and shed light on the nucleon wave function. As a further test of these methods, this paper extends a previous calculation for the coupling of $^3D_1$ quarkonium states to $p\bar{p}$ pairs. Unlike the $^3D_1$ case which involves mixing with the S state, the $^3D_2$ case does not exhibit interference effects since it only has contributions from the D state. Consequently, the variation of the ratios of decay widths with wave function is smaller than in the $^3S_1$-$^3D_1$ case. Here the ratios vary by a factor of 2.2 while the relevant factor involved in the decay rate of ψ'' is about 4.4. The sensitivity to variations in nucleon wave function is also reduced by calculating ratios of decay rates making the use of exclusive QCD methods to extract physics from other systems more believable than relying on absolute predictions.

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