QCD, relativistic flux tubes, and potential models

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Relativistic spin-independent corrections to the confinement of heavy quarks are examined. A comparison is made between QCD, a QCD-motivated model, and the predictions of scalar confinement. We find that a linearly confining Lorentz-scalar potential is inconsistent with QCD but that the relativistic flux-tube model is consistent with QCD. The relativistic corrections are identified with the rotational energy of the flux tube.

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Progress in understanding the nature of quark bound states has been somewhat uneven. From the beginning, the potential model [1] seemed to provide the clearest and most accurate picture of meson states, particularly those composed of heavy quarks. Going beyond the Schrödinger equation has proven a difficult task since relativity introduces new ambiguities. Some of the best theoretical guidance has been provided by the reduced Wilson loop reduction formalism [2–4] which, although it is valid only for slowly moving quarks, reflects the nonperturbative aspects of the QCD bound-state problem.

The relativistic corrections that have previously been studied most intensely (and successfully) involve spin dependence. The experimental signatures in spin dependence are unambiguous and much evidence now suggests that the short-range interaction is a Lorentz vector and the long-range (confining) interaction is effectively a Lorentz scalar. This latter identification rests upon the characteristic lack of any long-range spin interaction except the kinematic Thomas spin-orbit interaction. In a potential model the Lorentz-scalar potential fits this requirement but this does not necessarily mean that one should look no further. On the contrary, the lack of long-range spin interactions may be realized in other dynamical schemes. The chromoelectric flux-tube model illustrates this point perfectly. Buchmüller points out [5] that in the rest frame of a flux tube extending from the antiquark to the quark there is no possibility for long-range spin-spin correlations to occur.

Recently Brambilla and co-workers [4] extended the work of Eichten and Feinberg [2] and Gromes [3] to the spin-independent sector of the heavy $Q\bar{Q}$ interaction. As before, the relativistic corrections are valid only for slowly moving quarks but are correct even in the confining (nonperturbative) regime. Any specific model of the $Q\bar{Q}$ interaction must satisfy this general QCD framework. In particular, the scalar confinement potential model or the flux-tube model must yield spin-independent corrections consistent with QCD or these models must be discarded.

In Sec. I we slightly extend the results of Brambilla and co-workers to the unequal-mass case. It is important to consider the general case of unequal-mass quarks since the appearance of $1/m_1^2 + 1/m_2^2$ versus $1/m_1 m_2$ corrections terms can be significant. In Sec. II we examine the predictions of a Lorentz-scalar potential model. We find strong evidence indicating rejection of this type of potential model. We formulate and compute the relativistic corrections of the flux-tube model in Sec. III and determine that they are consistent with QCD.

I. QCD RELATIVISTIC REDUCTION

An expansion in v^2/c^2 of a Wilson loop formulation of QCD yields [4] a Hamiltonian of the form

$$H = m_1 + m_2 + \frac{p^2}{2\mu} - \frac{p^4}{8}\left(\frac{1}{m_1^3} + \frac{1}{m_2^3}\right) + V(r) + H_{\rm SD} + H_{\rm SI} \;, \tag{1}$$

where p is the c.m. momentum and $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass. The first four terms follow from the expansion of $\sqrt{p^2+m_1^2} + \sqrt{p^2+m_2^2}$. In this reduction formalism the static potential remains unspecified but when postulated the relativistic corrections to this static potential can be computed. In addition there are consistency relations which severely limit the nature of the static potential. A simple allowed static potential is

$$V(r) = -\frac{\kappa}{r} + ar \;. \tag{2}$$

In terms of this static potential the spin-dependent relativistic corrections are

$$H_{\rm SD} = \frac{\kappa}{m_1 m_2}\left[\frac{8\pi}{3}\delta(\mathbf{r})\mathbf{S}_1\cdot\mathbf{S}_2 + \frac{1}{r^3}(3\mathbf{S}_1\cdot\hat{\mathbf{r}}\mathbf{S}_2\cdot\hat{\mathbf{r}} - \mathbf{S}_1\cdot\mathbf{S}_2) + \frac{\mathbf{L}\cdot\mathbf{S}}{r^3}\right] + \frac{1}{2}\left(\frac{\mathbf{L}\cdot\mathbf{S}_1}{m_1^2} + \frac{\mathbf{L}\cdot\mathbf{S}_2}{m_2^2}\right)\left(\frac{\kappa}{r^3} - \frac{a}{r}\right) \tag{3a}$$

and the spin-independent corrections are [6]

$$H_{\rm SI} = \kappa \left[\frac{1}{2}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{4}{m_1 m_2}\right)\pi\delta(\mathbf{r}) + \frac{1}{m_1 m_2 r}\left(-p^2+\frac{L^2}{2r^2}\right)\right] + \frac{a}{r}\left[\frac{1}{36}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{8}{m_1 m_2}\right) - \frac{1}{6}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}-\frac{1}{m_1 m_2}\right)L^2\right] . \tag{3b}$$

In addition to explicitly demonstrating the general corrections for unequal masses, we have further simplified the results using integration-by-parts identities. The effective Hamiltonian can only be consistently evaluated perturbatively about the Schrödinger solution, so assuming that expectation values are always to be taken we have used

$$i\hat{\mathbf{r}}\cdot\mathbf{p} = -\frac{1}{r} , \tag{4a}$$

$$p_i \frac{1}{r} p_i = \frac{1}{r}p^2 - 2\pi\delta(\mathbf{r}) . \tag{4b}$$

In addition, expectation values of p^2 are calculable using the Schrödinger equation

$$p^2 = 2\mu\Big[M - m_1 - m_2 - V(r)\Big] , \tag{4c}$$

where M is the meson mass eigenstate.

We have grouped the corrections in Eqs. (3) according to the coefficients of κ and a. In the subsequent sections these coefficients will be interpreted in terms of either Lorentz properties in potential models or in terms of the QCD-inspired relativistic flux-tube model.

II. RELATIVISTIC REDUCTION OF POTENTIAL MODELS

From the reduction of the Bethe-Salpeter equation or by other methods, relativistic corrections [7, 8] can be developed corresponding to an interaction potential of a specified Lorentz nature. The resulting generalized Breit-Fermi Hamiltonian is generally assumed to provide a categorization of relativistic corrections. In this section we compile previously known results on these corrections. As in the preceding section we assume that the actual evaluation of these corrections is to be done perturbatively and we have used Eqs. (4) to simplify our final results. To compare with the QCD reduction given in Eqs. (1) to (3) we have again assumed the static potential of Eq. (2) of Eichten and Feinberg. Following conventional wisdom we shall assume the "short-range $-\kappa/r$" term of Eq. (2) is a Lorentz vector and the "long-range ar" term of Eq. (2) is a Lorentz scalar. As we will shortly see, this choice goes far, but not all the way, toward understanding the $Q\bar{Q}$ interaction in terms of potential interactions.

Using well-known results [7–9] for Lorentz-vector and -scalar potentials, the relativistic potential model corrections to the static potential of Eq. (2) are

$$H_{\rm SD} = \frac{\kappa}{m_1 m_2}\left[\frac{8\pi}{3}\delta(\mathbf{r})\mathbf{S}_1\cdot\mathbf{S}_2 + \frac{1}{r^3}(3\mathbf{S}_1\cdot\hat{\mathbf{r}}\mathbf{S}_2\cdot\hat{\mathbf{r}} - \mathbf{S}_1\cdot\mathbf{S}_2) + \frac{\mathbf{L}\cdot\mathbf{S}}{r^3}\right] + \frac{1}{2}\left(\frac{\mathbf{L}\cdot\mathbf{S}_1}{m_1^2}+\frac{\mathbf{L}\cdot\mathbf{S}_2}{m_2^2}\right)\left(\frac{\kappa}{r^3}-\frac{a}{r}\right) , \tag{5a}$$

$$H_{\rm SI} = \kappa\left[\frac{1}{2}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{4}{m_1 m_2}\right)\pi\delta(\mathbf{r}) + \frac{1}{m_1 m_2 r}\left(-p^2+\frac{L^2}{2r^2}\right)\right] - \frac{a}{2}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\left(\frac{1}{2r}+rp^2\right) + \left[\frac{a}{m_1 m_2}\left(\frac{L^2}{2r}-rp^2\right)\right]_{\rm ret} , \tag{5b}$$

where the final terms are the so-called confinement retardation corrections. In a popular method the $Q\bar{Q}$ scattering amplitude is evaluated assuming the quarks are on mass shell. As pointed out [8–10], this is incomplete in the bound-state problem and a retardation term should be added.

By comparing Eqs. (5) to the corresponding QCD relativistic corrections of Eqs. (3) we conclude that (1) the relativistic terms proportional to κ are correctly identified with the "short range" Lorentz-vector corrections in both $H_{\rm SD}$ and $H_{\rm SI}$, (2) the spin-dependent confining corrections [i.e., the coefficients of a in Eq. (3a)] are identical to those generated as if the linear confinement interaction were a scalar potential, and (3) the spin-independent confinement corrections of Eq. (3b) are not the same as those resulting if the linear confinement interaction were a Lorentz scalar. *This conclusion holds whether or not retardation terms are included.* The main result of this section is that the normal historical assumption that quark confinement acts like a Lorentz-scalar potential is inconsistent with QCD in the form of a low-quark velocity Wilson loop expansion.

From a purely phenomenological point of view, similar conclusions were previously made by Jacobs *et al.* [11]. Using a momentum-space formulation, Breit-Fermi-type corrections were included nonperturbatively. A comparison with $c\bar{c}$ and $b\bar{b}$ data showed that assuming scalar confinement (with or without retardation) spin-independent corrections had a negative effect toward improving the agreement of the potential model with data.

Another aspect of this inconsistency has been pointed out by Gara *et al.* [12]. These authors found that in the limit of light-quark masses the scalar confinement kernel in the Bethe-Salpeter equation partially cancels for zero momentum transfer. Since the confinement interaction is quite singular at this point, the result is the destruction of the linear Regge trajectory which ordinarily should result from linear confinement and relativistic kinematics. This result identifies a real difficulty for the potential model picture with scalar confinement.

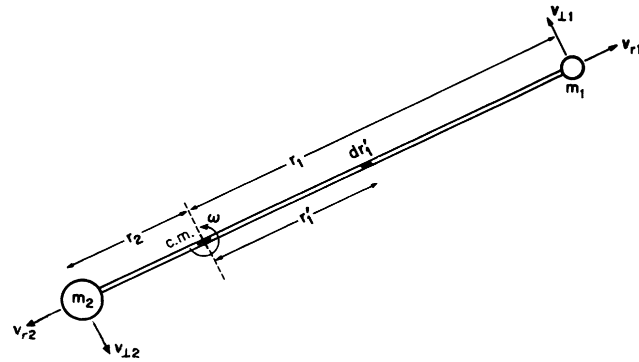

FIG. 1. Geometrical quantities used in the asymmetric flux-tube formulation in Sec. IV.

III. THE RELATIVISTIC FLUX-TUBE MODEL

One seems to have reached an impasse with the potential model including scalar confinement. The spin dependence requires pure scalar but the spin-independent part is not only phenomenologically wrong, it appears to disagree with QCD predictions. Fortunately for model building, there is a possible way out. The electric flux-tube model [5, 13–16] has many attractive features. In its simplest form a straight flux tube connects the quark and antiquark. A wide range of advantages over the potential model can be enumerated.

(1) It makes more physical sense than the potential model for light quarks since the field becomes a dynamical entity carrying momentum and angular momentum as well as energy [17].

(2) In the ultrarelativistic limit the tube carries all of the rotational energy and angular momentum. The potential model cancellation of Gara *et al.* [12] will therefore not change the Regge structure.

(3) The spin dependence is similar to scalar confinement [5].

(4) For heavy quarks the leading-order interaction is just the potential model with linear confinement. The flux-tube model augmented by short-distance $Q\bar{Q}$ interactions reproduces all of the successes of the potential model [15, 16].

(5) Unification with glueballs and hybrid mesons is natural [14].

(6) The flux-tube model is consistent with QCD. In the next section we will demonstrate that this is true for relativistic corrections to heavy-quark bound states.

IV. RELATIVISTIC CORRECTIONS IN THE ASYMMETRIC FLUX-TUBE MODEL

The asymmetric flux-tube model consists of a quark and antiquark of masses m_1 and m_2 connected by a thin flux of energy per unit length a as shown in Fig. 1. The origin of our coordinate system is the center-of-momentum point and the total tube length $r = r_1 + r_2$. For the part of the tube from the c.m. to m_1 the tube momentum is [18]

$$\mathbf{P}_1^{\text{tube}} = \int_0^{r_1} a\, dr_1' \frac{\omega r_1' \hat{\theta}}{\sqrt{1-\omega^2 r_1'^2}} = \frac{a r_1}{v_{\perp 1}}\left(1 - \frac{1}{\gamma_{\perp 1}}\right)\hat{\theta}\,, \tag{6a}$$

where $\gamma_{\perp 1} = (1 - v_{\perp 1}^2)^{-1/2}$. The angular momentum is

$$\begin{aligned} L_1^{\text{tube}} &= \int_0^{r_1} a\, dr_1' \frac{\omega r_1'^2}{\sqrt{1-\omega^2 r_1'^2}} \\ &= \frac{a r_1^2}{2 v_{\perp 1}}\left(\frac{\arcsin v_{\perp 1}}{v_{\perp 1}} - \frac{1}{\gamma_{\perp 1}}\right) \end{aligned} \tag{6b}$$

and the tube segment energy is

$$H_1^{\text{tube}} = \int_0^{r_1} \frac{a\, dr_1'}{\sqrt{1-\omega^2 r_1'^2}} = a r_1 \frac{\arcsin v_{\perp 1}}{v_{\perp 1}}\,. \tag{6c}$$

The quark m_1 carries momentum $m_1\gamma_1(\dot{r}_1\hat{\mathbf{r}}_1 + v_{\perp 1}\hat{\theta})$, angular momentum $L_1 = m_1\gamma_1 r_1 v_{\perp 1}$ and energy $m_1\gamma_1$. Since the flux tube carries no radial momentum, the total canonical momentum in the direction $\hat{\mathbf{r}}_1$ is the particle momentum

$$p_r = m_1\gamma_1\dot{r}_1\,, \tag{7}$$

from which it follows that

$$W_{r1}\gamma_{\perp 1} = m_1\gamma_1\,, \tag{8}$$

where $W_{r1} = \sqrt{p_r^2 + m_1^2}$. Radial momentum conservation is achieved by $p_{r1} = p_{r2} \equiv p_r$. Combining the tube and particle contributions and using Eq. (8) the three conserved quantities are

$$P_\perp = 0 = W_{r1}\gamma_{\perp 1} v_{\perp 1} + \frac{a r_1}{v_{\perp 1}}\left(1 - \frac{1}{\gamma_{\perp 1}}\right) - (1 \to 2)\,, \tag{9a}$$

$$\begin{aligned} L = {} & W_{r1}\gamma_{\perp 1} v_{\perp 1} r_1 + \frac{a r_1^2}{2 v_{\perp 1}}\left(\frac{\arcsin v_{\perp 1}}{v_{\perp 1}} - \frac{1}{\gamma_{\perp 1}}\right) \\ & + (1 \to 2)\,, \end{aligned} \tag{9b}$$

$$M = W_{r1}\gamma_{\perp 1} + a r_1 \frac{\arcsin^{-1} v_{\perp 1}}{v_{\perp 1}} + (1 \to 2)\,, \tag{9c}$$

where M is the total meson energy and $(1 \to 2)$ indicates terms due to the second-quark and second-tube segment. These equations constitute the complete classical description of this system. For equal quark masses the momentum-conservaton equation is trivially satisfied by $r_1 = r_2$ but in general Eq. (9a) is required to fix r_2 relative to r_1.

For our purposes we wish to compute the relativistic corrections to Eqs. (9). We expand in the small quantities

$$v_\perp^2 \,, \quad (p_r/m)^2 \,, \quad \frac{ar}{m} \ll 1 \,, \tag{10}$$

giving

$$0 = \sum_{i=1}^{2} (-)^i m_i v_{\perp i} \left\{ 1 + \frac{1}{2} v_{\perp i}^2 + \frac{1}{2}\left(\frac{p_r}{m_i}\right)^2 + \frac{a r_i}{2 m_i} \right\} , \tag{11a}$$

$$L = \sum_{i=1}^{2} m_i v_{\perp i} r_i \left\{ 1 + \frac{1}{2} v_{\perp i}^2 + \frac{1}{2}\left(\frac{p_r}{m_i}\right)^2 + \frac{1}{3}\frac{a r_i}{m_i} \right\} , \tag{11b}$$

$$H = \sum_{i=1}^{2} \left\{ m_i + \frac{p_r^2}{2m_i} - \frac{p_r^4}{8m_i^3} + a r_i + \frac{1}{2} m_i v_{\perp i}^2 \left[1 + \frac{3}{4} v_{\perp i}^2 + \frac{1}{2}\left(\frac{p_r}{m_i}\right)^2 + \frac{1}{3}\frac{a r_i}{m_i} \right] \right\} , \tag{11c}$$

where $r = r_1 + r_2$ is the interquark separation. Using Eq. (11a) we rewrite Eq. (11b) as

$$\frac{L}{r} = m_i v_{\perp i} \left\{ 1 + \frac{1}{2} v_{\perp i}^2 + \frac{1}{2}\left(\frac{p_r}{m_i}\right)^2 + \frac{a r_i}{3 m_i} - \frac{a(r_1^2 + r_2^2 - r_1 r_2)}{6 m_i r_i} \right\} ,$$

where $i = 1$ or 2. Inverting and using the approximations (10) and $v_{\perp i} \simeq L/m_i r$ in the $\frac{1}{2} v_{\perp i}^2$ term we obtain

$$v_{\perp i} = \frac{L}{r m_i} \left\{ 1 - \frac{p^2}{2m_i^2} - \frac{a r_i}{2 m_i} + \frac{a(r_1^2 + r_2^2 - r_1 r_2)}{6 m_i r_i} \right\} , \tag{12}$$

where we also have combined the classical radial and angular terms by using

$$p^2 = p_r^2 + \frac{L^2}{r^2} \,. \tag{13}$$

The Hamiltonian of Eq. (11c) can now be evaluated from Eqs. (11c) and (12)

$$H = m_1 + \frac{p_r^2}{2m_1} - \frac{p_r^2}{8m_1^3} + a r_1 + r \frac{L^2}{2 m_1 r^2} \left[1 - \frac{p^2}{m_1^2} - \frac{2 a r_1}{3 m_1} - \frac{a r_2}{3}\left(\frac{1}{m_1} - \frac{1}{m_2}\right) \right] \times \left[1 + \frac{p^2}{2m_1^2} + \left(\frac{L}{2 m_1 r}\right)^2 + \frac{a r_1}{3 m_1} \right] + (1 \leftrightarrow 2) \,,$$

where we have again used $m_1 r_1 \simeq m_2 r_2$ and $v_{\perp i} \simeq \frac{L}{m_i r}$ in the correction terms. Multiplying out and combining terms we obtain

$$H = m_1 + m_2 + \frac{p^2}{2\mu} - \frac{p^4}{8}\left(\frac{1}{m_1^3} + \frac{1}{m_2^3}\right) + ar - \frac{a}{6r}\left[\frac{1}{m_1^2} + \frac{1}{m_2^2} - \frac{1}{m_1 m_2}\right] L^2 \,. \tag{14}$$

The above result should be compared to the QCD result in Eqs. (1)–(3).

The quarks in the flux-tube model presented here have no spin and furthermore momenta and coordinates have been commuted classically. One can reasonably expect that only the semiclassical terms would be correct. In Eq. (3b) the coefficient of L^2 matches that in Eq. (14). The L^2-independent terms may well arise from spin effects or by commutation. We emphasize again that the spin dependence of a flux tube model is expected [5] to be that of an effective Lorentz scalar and hence to match that of Eq. (3a). Also, as mentioned previously [16], one can easily introduce a direct Lorentz-vector interaction between the quarks and hence account for the short-range QCD interaction.

Finally, it is interesting to provide a physical picture for the flux-tube relativistic corrections. To first order the system c.m. is the quark c.m. and the quarks carry most of the angular momentum and energy. Consider now only the rotational energy ($p_r \equiv 0$). The rotational energy of the tube alone is

$$E_{\rm rot}^{\rm tube} = \frac{1}{2} I^{\rm tube} \omega^2 \,, \tag{15}$$

where the tube moment of inertia is

$$I^{\rm tube} = ar\left[\frac{1}{12} r^2 + \frac{1}{4}\left(\frac{m_1 - m_1}{m_1 + m_2}\right)^2 r^2\right] = \frac{ar^3}{3}\frac{(m_1^2 + m_2^2 - m_1 m_2)}{(m_1 + m_2)^2} \,. \tag{16}$$

The angular velocity is fixed by the quark angular momentum to be

$$\omega = \frac{L^{\rm part}}{\mu r^2} \simeq \frac{L}{\mu r^2} \,. \tag{17}$$

Combining the last three equations gives

$$E_{\rm rot}^{\rm tube} = \frac{a}{6r}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} - \frac{1}{m_1 m_2}\right) L^2 \,, \tag{18}$$

which is the negative of the tube relativistic correction. Going back to the leading rotational energy term in Eq. (14),

$$\frac{p^2}{2\mu} = \frac{L^2}{2\mu r^2} = \frac{\left(L^{\rm part} + L^{\rm tube}\right)^2}{2\mu r^2} \simeq \frac{(L^{\rm part})^2}{2\mu r^2} + \frac{2 L^{\rm tube} L^{\rm part}}{2\mu r^2} \,. \tag{19}$$

Using $L^{\rm tube} = \frac{2}{\omega} E_{\rm rot}^{\rm tube}$ and $\omega \simeq \frac{L^{\rm part}}{\mu r^2}$ we have

$$\frac{p^2}{2\mu} \simeq E_{\rm rot}^{\rm part} + 2E_{\rm rot}^{\rm tube} , \tag{20}$$

and hence the spin-independent relativistic correction $-E_{\rm rot}^{\rm tube}$ is needed to make the total rotational energy the sum of the particle and tube contributions.

CONCLUSIONS

Starting from the nonperturbative relativistic QCD corrections to the heavy-quark bound-state interaction [4] we note first that the usual short-range-vector and/or long-range-scalar potential model is not entirely correct. The relativistic corrections generated by the short-range (Lorentz-vector) part of the potential (2) of Eichten and Feinberg are identical to those expected from the QCD reduction [4]. However, the spin-independent corrections implied by Lorentz-scalar linear confinement are inconsistent with QCD even though the spin dependence is correctly given. This discrepancy remains regardless of whether retardation terms are included. This observation reinforces the conclusion that scalar confinement relativistic corrections in heavy-quarkonia potential models made the agreement between experiment and potential-model predictions worse, and Gara *et al.* [12] demonstrated that for light quarks, a cancellation in the Bethe-Salpeter scalar interaction kernel causes deviation from Regge behavior.

The relativistic flux-tube model is a more viable model as at least its semiclassical behavior is consistent with QCD. By examining the flux-tube relativistic corrections for heavy quarkonia we show that the semiclassical (large angular momentum) correction corresponds exactly to the large angular momentum QCD correction. We further demonstrate that this correction is equivalent to adding the angular momentum and rotational energy of the rotating flux tube of effective mass ar. Because of this identification, the failure of the potential model is understandable. The qualitative difference between the usual potential model and the flux-tube model is that the gluon field's angular momentum is taken into account. This is necessary even at the level of the lowest relativistic corrections according to the QCD results of Eqs. (1)–(3).

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[1] For recent reviews, see D. B. Lichtenberg, Int. J. Mod. Phys. A **2**, 1669 (1987); A. Martin, Comments Nucl. Part. Phys. **16**, 249 (1986); W. Lucha, F. F. Schöberl, and D. Gromes, Phys. Rep. **200**, 127 (1990).

[2] E. Eichten and F. Feinberg, Phys. Rev. D **23**, 2724 (1981).

[3] D. Gromes, Z. Phys. C **22**, 265 (1984); **26**, 401 (1984).

[4] A. Barchielli, N. Brambilla, and G. M. Prosperi, Nuovo Cimento **103A**, 59 (1989); N. Brambilla and G. M. Prosperi, Phys. Lett. B **236**, 69 (1990); A. Barchielli, E. Montaldi, and G. M. Prosperi, Nucl. Phys. **B296**, 625 (1988); **B303**, 752(E) (1974).

[5] W. Buchmüller, Phys. Lett. **112B**, 479 (1982).

[6] In addition to carrying through the unequal-mass case calculation we also present the result in an alternative form using the perturbative identities (4).

[7] A. B. Henriques, B. H. Keller, and R. G. Moorhouse, Phys. Lett. **64B**, 85 (1976); Lai-Him Chan, *ibid.* **71B**, 422 (1977); C. E. Carlson and Franz Gross, *ibid.* **74B**, 404 (1978); N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1979); H. J. Schnitzer, Phys. Lett. **76B**, 461 (1978); Phys. Rev. D **18**, 3482 (1978).

[8] D. Gromes, Nucl. Phys. **B131**, 80 (1977); M. G. Olsson and K. J. Miller, Phys. Rev. D **28**, 674 (1983).

[9] T. Barnes and G. I. Ghandour, Phys. Lett. **118B**, 411 (1982).

[10] G. C. Bhatt, H. Grotch, and Xingguo Zhang, J. Phys. G **17**, 231 (1991).

[11] S. Jacobs, M. G. Olsson, and C. J. Suchyta III, Phys. Rev. D **33**, 3338 (1986).

[12] Alan Gara, Bernice Durand, and Loyal Durand, Phys. Rev. D **40**, 843 (1989).

[13] A. Chodos and C. B. Thomas, Nucl. Phys. **B72**, 509 (1974); I. Bars, *ibid.* **B111**, 413 (1976).

[14] N. Isgur and J. Paton, Phys. Rev. D **31**, 2910 (1985); Phys. Lett. **124B**, 247 (1983).

[15] M. Ida, Prog. Theor. Phys. **59**, 1661 (1978).

[16] D. LaCourse and M. G. Olsson, Phys. Rev. D **39**, 2751 (1989).

[17] For the development of the string model see, for example, the following review articles: C. Rebbi, Phys. Rep. **12C**, 1 (1974); S. Mandelstam, *ibid.* **13C**, 259 (1974); T. Gotō, Prog. Theor. Phys. **46**, 1560 (1971); X. Artru, Phys. Rep. **97**, 147 (1983).

[18] The heuristic arguments given here follow consistently from a Lagrangian formulation [15, 16].