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Renormalizable top-quark condensate models

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Recently it has been suggested that electroweak symmetry is broken by a top-quark vacuum condensate. We discuss the prospects for making a renormalizable version of this "top bootstrap" scenario using a strongly coupled spontaneously broken gauge interaction. Several specific models are considered. We argue that these models are somewhat similar to the extended technicolor models, but with a "technicolor" force which is broken, providing the mass-generation mechanism for the top quark. From this point of view, the top quark plays the role of a techniquark.

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I. INTRODUCTION

An unbroken electroweak symmetry would prohibit all of the particles in the standard model from obtaining masses. Indeed, the masses of the fermions in the standard model are directly proportional to the strengths of their Yukawa couplings to the Higgs boson. Therefore, the mass spectrum of quarks and leptons may contain important clues to the nature of the electroweak symmetry-breaking mechanism. From this point of view, the most striking feature of the fermion mass spectrum is that the top quark has a mass which is much larger than those of all of the other quarks and leptons, and is the only one comparable to the scale of electroweak symmetry breaking. Direct searches at the Collider Detector at Fermilab (CDF) indicate that the mass of the top quark is greater than 89 GeV, while all of the other quarks and leptons have masses at least 15 times smaller. Thus one might make a coarse summary of the fermion mass spectrum of the standard model by saying that the top quark, and only the top quark, has a significant coupling to the agent which breaks electroweak symmetry.

The relatively strong coupling of the top quark to the Higgs boson in the standard model suggests the possibility that the top quark plays an essential and unique role in a dynamical electroweak symmetry-breaking mechanism. In this spirit, several authors [1–4] have proposed that the electroweak symmetry is broken by a top-quark vacuum condensate rather than a fundamental Higgs boson. In this "top bootstrap" scenario, the top-quark condensate is supposed to be induced by a four-fermion interaction

$$\mathcal{L}_{\text{eff}} \sim \frac{g^2}{M^2}(\bar{Q}^i t)(\bar{t} Q_i) \tag{1.1}$$

introduced at a scale M which must be larger than the electroweak breaking scale. [Here $Q^i=(tb)_L$ is the left-handed third-generation quark doublet, the i is an $\mathrm{SU}(2)_L$ index, and t is the right-handed top quark. Color indices are suppressed.] If the coupling g is sufficiently large at the scale M, then it has been argued that a top-quark vacuum expectation value ("condensate")

$$\langle \bar{Q}^i t \rangle = \mu^3 \delta^{i1} \tag{1.2}$$

will form, and the Higgs scalar boson will be a composite top-quark–anti-top-quark state bound by the interaction (1.1). At scales far below M, the effective action in the top-quark condensate model should be the same as in the standard model. Several variations on the original top bootstrap idea have been discussed in [5–16].

The four-fermion interaction term (1.1) cannot be part of a renormalizable fundamental Lagrangian. Since renormalizability is one of the chief guiding principles in high-energy physics, one would like to understand how (1.1) can arise as an effective interaction in a renormalizable theory, and under what circumstances such a theory could lead to the standard model in the low-energy limit. Of course, there is a precedent for four-fermion interactions in elementary-particle physics. At low energies, the weak interactions are well described by an effective four-fermion interaction. It is now understood that these four-fermion interactions are the result of integrating out massive intermediate vector gauge bosons. Similarly, the effective interaction (1.1) might arise from integrating out some gauge bosons which obtain masses due to spontaneous symmetry breaking at a scale higher than the electroweak scale. Suppose that the top quark couples to a new gauge interaction which is spontaneously broken. In order for this interaction to produce a top-quark condensate, it is necessary that the new gauge interaction be strongly coupled. The most obvious way that this could happen is for the new interaction to be a non-Abelian gauge interaction with a negative β function. Then the running gauge coupling constant will increase as we go to lower-energy scales. If M is the scale at which the new interaction is spontaneously broken, and Λ is the scale at which the gauge coupling constant becomes large enough to drive the formation of a condensate, then $\Lambda > M$ so that the new gauge interaction has a chance to form condensates before it is broken.

In this scenario, the top quark is heavy because it has a special new gauge interaction not seen by other quarks and leptons. There is perhaps a natural prejudice against this sort of idea, because in the standard model the fermions come in three generations with identical gauge transformation properties. The most precise version of this prejudice follows from the constraint that the fermion representation be free of gauge anomalies. This constraint is already satisfied by the usual standard-model fermion generations; if the top quark has a special new non-Abelian gauge interaction, then some extra fermions will have to be included to cancel the new gauge anomalies. Obviously, these cannot be introduced in an arbitrary way. This will become particularly apparent when we consider specific models. It is even possible that the strongest experimental signature of the top-quark condensate scenario could come from direct or indirect evidence for the existence of the extra fermions necessary to cancel the gauge anomalies.

Our intentions in the present paper are not quite so ambitious as to present a completely realistic model which satisfies all known phenomenological constraints. In particular, we do not know of any mechanism in these models which can cause the ρ-parameter prediction to be sufficiently close to unity. Nor are we prepared to suggest here any mechanism which can give a realistic mass matrix to the lighter quarks and leptons without inducing unacceptable flavor-changing neutral currents. These are very difficult and important problems which certainly pose a serious threat to the whole top-quark condensate idea. Not surprisingly, these are the same phenomenological difficulties which have also plagued, e.g., the extended technicolor theories of dynamical electroweak symmetry breaking. Here we will simply ignore these important phenomenological problems. The point is that if these problems are ever to be successfully addressed, it should be done in the context of renormalizable theories, and this is not a trivial restriction.

It is often stated that the results of Ref. [4] show that the top-quark condensate models must be fine-tuned. However, the renormalization-group arguments used in [4] are really only reliable when the theory is already assumed to have been fine-tuned, so that the scale of new physics M is much larger than the electroweak scale. In the absence of such fine-tuning, we should simply refrain from drawing quantitative conclusions from such methods. This is particularly crucial since requiring the theory to be renormalizable typically forces us to introduce new particles with masses near the scale of new physics. Our prejudice is that fine-tuning should be avoided at all costs; this is a primary motivation for using dynamical-symmetry-breaking mechanisms in the first place. This implies a relatively low scale of new physics (very roughly in the TeV range). It cannot be overemphasized that the renormalization-group arguments of [4] are not to be trusted in this regime. Given the lack of reliable methods of calculation in the absence of fine-tuning, we will concentrate on the gross and mostly qualitative features of these models, in the (perhaps rather optimistic) hope that the more detailed phenomenological questions can be addressed later.

II. FOUR-FERMION INTERACTIONS AND HEAVY GAUGE BOSONS

By the use of a Fierz transformation and charge conjugation, it is always possible to rewrite every four-fermion interaction arising from heavy-vector-boson exchange as a sum of products of scalar fermion bilinears. Consider a notation in which all of the fermions are left-handed two-component Weyl fields assembled into a big column vector Ψ_I, with the generic index I ranging over all of the gauge and flavor degrees of freedom. Thus Ψ_I transforms as a direct sum of irreducible representations of the gauge group. Then the static quadratic part of the coupling of a heavy gauge boson H^a_μ can be represented by

$$\mathcal{L}_{\rm eff}=\frac{1}{2}M^2_{ab}H^a_\mu H^{b\mu}+gH^a_\mu(\overline{\Psi}\gamma^\mu S^a\Psi) , \tag{2.1}$$

where the S^a are the gauge generators (in the reducible representation carried by Ψ_I) corresponding to the heavy vector bosons with mass-squared matrix M^2_{ab}. Integrating out H^a_μ in (2.1) yields

$$\mathcal{L}_{\rm eff}=-\frac{g^2}{2}M_{ab}^{-2}(\overline{\Psi}\gamma_\mu S^a\Psi)(\overline{\Psi}\gamma^\mu S^b\Psi) . \tag{2.2}$$

Now, Fierz transforming (2.2) directly just yields another vector-vector representation of the interaction. However, the second term in (2.2) can always be rewritten in terms of the right-handed field Ψ^c (the charge conjugate of Ψ) according to $\overline{\Psi}\gamma^\mu S^b\Psi=-\overline{\Psi}^c\gamma^\mu(S^b)^T\Psi^c$. A subsequent Fierz transformation puts (2.2) into the form

$$\mathcal{L}_{\rm eff}=-g^2M_{ab}^{-2}S_I^{aL}S_J^{b^K}(\overline{\Psi}^I\Psi^{cJ})(\overline{\Psi}^c_K\Psi_L) . \tag{2.3}$$

Since, by Lorentz invariance, only a scalar fermion bilinear can acquire a vacuum expectation value, (2.3) is the relevant representation of the four-fermion interaction.

It is clear from (2.3) that, in general, there will be other four-fermion interactions in the effective action in addition to the one (1.1) which is our motivation. Let us consider under what circumstances these interactions can lead to scalar bound states and condensates. For now, we can use the relative strengths of the four-fermion interactions, together with some simple dynamical assumptions, to divine the condensation pattern; later we will deal directly with the renormalizable gauge theory, and a related set of dynamical assumptions, instead. Assume, for simplicity, that there is a gauge boson of mass M transforming as an irreducible representation of the standard model gauge group, so that the effective action is a sum of terms of the form

$$\mathcal{L}_{\rm eff}\supset\sum c_n\frac{g^2}{M^2}(\overline{\psi}\rho_n\chi)(\overline{\chi}\rho_n^\dagger\psi) , \tag{2.4}$$

where ψ and χ^c are left-handed two-component Weyl fermions which are each part of Ψ and transform as irreducible representations of the gauge group. (Thus χ is right handed.) The ρ_n are a set of matrices which act on gauge and flavor indices (but not on the spinor indices) in such a way that the fermion bilinears $\psi\rho_n\chi$ also transform as irreducible representations of the gauge group. They satisfy the normalization condition

$$\mathrm{Tr}[\rho_n\rho_m^\dagger]=\delta_{nm} \tag{2.5}$$

for $\psi\neq\chi^c$, and for $\psi=\chi^c$ satisfy

$$\mathrm{Tr}[\rho_n\rho_m^\dagger]=\tfrac{1}{2}\delta_{nm} . \tag{2.6}$$

The numerical constants c_n then characterize the attractiveness or repulsiveness and relative strengths of the interaction in each channel. For $c_n<0$, the interaction (2.4) is repulsive and leads to neither bound states nor condensates. For $c_n>0$, (2.4) is attractive, and scalar bound state(s) may form with the quantum numbers of $\bar{\psi}\rho_n\chi$. If we suppose further that g is sufficiently large at the scale M, then a condensate $\langle\bar{\psi}\rho_n\chi\rangle\neq 0$ may form in the particular channel for which c_n is the maximum. If several channels involving the same fermion have the same maximum value of c_n, then a plausible dynamical assumption is that the condensate will form in the one "most attractive channel" for which the corresponding bound state $\bar{\psi}\rho_n\chi$ has the lowest energy (cf. [17]).

To see how these considerations apply to the top-quark condensate idea, we perform a Fierz transformation on (1.1) to put it into the form that would arise from exchange of massive vector particles:

$$\mathcal{L}_{\rm eff}=\frac{g^2}{M^2}(\bar{Q}^i t)(\bar{t}Q_i)$$
$$=\frac{g^2}{M^2}[-(\bar{Q}^i\gamma^\mu T^a Q_i)(\bar{t}\gamma_\mu T^a t)-\tfrac{1}{6}(\bar{Q}^i\gamma^\mu Q_i)(\bar{t}\gamma_\mu t)]\ , \tag{2.7}$$

where the T^a are generators of color SU(3)c with normalization $\mathrm{Tr}[T^aT^b]=\frac{1}{2}\delta^{ab}$. The first term in (2.7) can arise in the low energy limit of the exchange of massive vector bosons which transform as an adjoint of color SU(3)c, while the second term could come from the exchange of massive vector bosons which do not carry color. We now consider the reverse Fierz transformations which follow from these two possibilities.

A color-octet vector boson of mass M which couples to the current $\bar{Q}^i\gamma_\mu T^a Q_i$ with strength g_Q and to the current $\bar{t}\gamma_\mu T^a t$ with strength g_t will produce an effective four-fermion interaction:

$$\mathcal{L}_{\rm eff}\supset-\frac{g_Qg_t}{M^2}(\bar{Q}^i\gamma^\mu T^a Q_i)(\bar{t}\gamma_\mu T^a t)$$
$$=\frac{g_Qg_t}{M^2}[\tfrac{8}{9}(\bar{Q}^i t)(\bar{t}Q_i)-\tfrac{2}{3}(\bar{Q}^iT^a t)(\bar{t}T^aQ_i)]$$
$$=\frac{g_Qg_t}{M^2}[\tfrac{8}{3}(\bar{Q}^i\rho_0 t)(\bar{t}\rho_0 Q_i)-\tfrac{1}{3}(\bar{Q}^i\rho^a t)(\bar{t}\rho^a Q_i)]\ , \tag{2.8}$$

where $\rho_0=1/\sqrt{3}$ and $\rho^a=\sqrt{2}T^a$ to agree with the normalization prescription (2.5). If $g_Qg_t>0$, this is just what we need to produce a top-quark–anti-top-quark bound state with the quantum numbers of the Higgs boson, because the first term in (2.8) represents an attractive force. If this binding force is sufficiently strong, a top-quark condensate $\langle\bar{Q}^i t\rangle\neq 0$ will occur. The second term in (2.8) represents a repulsive force which opposes the formation of a scalar bound state carrying color (and the corresponding condensate). However, there are necessarily other four-fermion interactions which may compete with the first term in (2.8). Consider

$$\mathcal{L}_{\rm eff}\supset-\frac{g_Q^2}{2M^2}(\bar{Q}^i\gamma^\mu T^a Q_i)(\bar{Q}^j\gamma_\mu T^a Q_j)$$
$$=\frac{g_Q^2}{M^2}[\tfrac{4}{3}(\bar{Q}^i\rho_\alpha Q^{cj})(\bar{Q}^c_j\rho^\alpha Q_i)$$
$$-\tfrac{2}{3}(\bar{Q}^i\rho_{\tilde{\alpha}}Q^{cj})(\bar{Q}^c_j\rho^{\tilde{\alpha}}Q_i)]\ , \tag{2.9}$$

where α and $\tilde{\alpha}$ are indices for the $\bar{\mathbf{3}}$ and $\mathbf{6}$ representations of SU(3)$_C$ respectively, and ρ_α and $\rho_{\tilde{\alpha}}$ are matrices in color space each satisfying the normalization (2.6). Now the first term in (2.9) is attractive and can produce a colored scalar bound state [which is an SU(2)$_L$ singlet because of Fermi statistics]. This is not a problem unless $g_Q^2>2g_Qg_t$, in which case the first term in (2.9) is a more attractive channel than the first term in (2.8), and the color-breaking condensate $\langle\bar{Q}^i\rho_\alpha Q^{cj}\rangle\neq 0$ will form instead of $\langle\bar{Q}^i t\rangle\neq 0$. To avoid this we must require $2g_Qg_t>g_Q^2$. There is also an interaction such as (2.9), but proportional to g_t^2 and with Q_i replaced by t. However, the term analogous to the attractive first term in (2.9) contains $\bar{t}\rho_\alpha t^c$ which vanishes identically by Fermi statistics, because ρ_α couples two 3's of SU(3)$_C$ antisymmetrically. So as long as g_t is large at M and $2g_Qg_t>g_Q^2$, we can indeed have the condensation pattern required by the top bootstrap scenario. (In the models we will consider in Sec. III, $g_Q=g_t$, so these requirements will be satisfied.)

Similarly, if instead we have a vector boson of mass M which is a singlet under SU(3)$_C$, then there is an effective four-fermion interaction

$$\mathcal{L}_{\rm eff}\supset-\frac{g_Qg_t}{M^2}(\bar{Q}^i\gamma^\mu Q_i)(\bar{t}\gamma_\mu t)$$
$$=\frac{g_Qg_t}{M^2}[\tfrac{2}{3}(\bar{Q}^i t)(\bar{t}Q_i)+4(\bar{Q}^iT^a t)(\bar{t}T^aQ_i)]$$
$$=\frac{g_Qg_t}{M^2}[2(\bar{Q}^i\rho_0 t)(\bar{t}\rho_0 Q_i)+2(\bar{Q}^i\rho^a t)(\bar{t}\rho^a Q_i)]\ . \tag{2.10}$$

Now if $g_Qg_t>0$, both terms in (2.10) produce attractive forces which favor scalar bound states. Taking into account the normalization of the fermion bilinears, one finds that the interactions for the two channels in (2.10) are actually equally attractive. However, the QCD interaction breaks this degeneracy in favor of the first term, since a $\mathbf{3}$ and a $\bar{\mathbf{3}}$ of SU(3)$_C$ feel an attractive force to combine into a singlet and a repulsive force to combine into an octet. Therefore the condensate $\langle\bar{Q}^i t\rangle\neq 0$ is favored and the condensate $\langle\bar{Q}^iT^a t\rangle\neq 0$, which would have broken SU(3)$_C$ and given the gluon a mass, should not occur. The four-fermion interaction terms which are proportional to g_t^2 and g_Q^2 are all repulsive. So again we can obtain the correct condensate pattern for the top bootstrap scenario, provided that g_t is large at M and $g_Qg_t>0$.

It should be emphasized that the requirements found in the previous two paragraphs are necessary but certainly not sufficient. We have so far only looked at the most attractive channels in the Q_i, t sector. In any particular model without gauge anomalies, there will be other fermions in addition to Q_i and t which couple to the strong guage group, and the possibility of condensates involving those other fermions must be considered.

The ascension to a renormalizable gauge theory brings with it the opportunity to analyze the condensate formation dynamics in a more fundamental and reliable way than the preceding comparison of the strengths of effective four-fermion interactions. It is important to note that the four-fermion interactions encode an unfortunate entanglement of two completely different effects. The first effect is the production of a condensate; this is really associated with the strength of the gauge coupling constant at the scale Λ. The second effect is the breaking of the strongly coupled gauge theory at the scale M, and is essentially extraneous to the condensation dynamics which occurs at the higher scale Λ. Of course, the second effect is necessary to explain why the top quark and the left-handed part of the bottom quark are not confined by the strongly coupled gauge interaction, and to explain the mass-generation mechanism for the top quark.

In a renormalizable version, the separate nature of these effects becomes manifest. There is a simple set of dynamical assumptions, based on the strength of gauge interactions, which can replace the cruder criteria based on the strength of four-fermion interactions. These assumptions are described beautifully in [17]. Consider a model which consists of a strong gauge theory which couples to some fermions but no scalars. The fermions may also have weakly coupled gauge interactions whose effects may be treated perturbatively. When the strong gauge coupling becomes sufficiently large, a scalar fermion bilinear condensate will form in an irreducible representation of the gauge group. Suppose that the fermions involved in the condensate transform under the strongly coupled gauge group in the irreducible representations R_1 and R_2, and the resulting condensate transforms as R_3. (We treat all the fermions here as left-handed two-component Weyl fermions.) Thus R_3 occurs in the direct sum decomposition of the direct product $R_1 \otimes R_2 = R_3 \oplus \cdots$. We need a way of deciding for which choices of R_1, R_2, and R_3 the condensate will occur. According to the single-gauge-boson-exchange approximation, the condensate appears in the "most attractive channel", for which $V = C_3 - C_1 - C_2$ is most negative. Here C_1, C_2, and C_3 are the quadratic Casimir invariants for the representations R_1, R_2, and R_3, respectively. For example, if the strongly coupled interaction is a U(1), and left-handed fermions have charges q_1 and q_2, then $V \propto (q_1+q_2)^2 - q_1^2 - q_2^2 = 2q_1q_2$, so that for a collection of charged fermions, the most attractive channel occurs when the product of charges is most negative. Thus in a general gauge theory the statement that V should be as negative as possible is the generalization of the familiar statement in electrodynamics that opposite charges attract.

In many simple cases, the criterion just mentioned gives exactly the same qualitative results that one would find from comparing the strengths of the four-fermion interaction. This is because both methods essentially apply a single-gauge-boson-exchange approximation. However, in more complicated models, the four-fermion interactions can be awkward and misleading in this regard, because strictly speaking they are only valid at scales below M, whereas the condensation dynamics is more properly associated with the higher scale Λ. [This happens for example if the heavy gauge boson transforms as a reducible representation of the standard model gauge group and the four-fermion interactions do not factor as in (2.4).] In this sense, the effective low-energy four-fermion interactions are inappropriate for the purposes of determining the qualitative pattern of condensation. Of course, once the pattern of condensation has been found, the low-energy four-fermion interactions are useful in determining, e.g., the fermion mass spectrum.

There are evidently three general ways to try to produce the top-quark condensate from a renormalizable gauge theory. The first way is to introduce a heavy gauge boson transforming as an adjoint of color, as in (2.8); this clearly must be associated with a strongly coupled non-Abelian gauge group. The second way is to use a heavy U(1) gauge boson transforming as a singlet of color, as in (2.10); and the third way is to introduce a non-Abelian color-singlet heavy gauge interaction, also as in (2.10). In each of these three cases, the strongly coupled gauge bosons must couple more strongly to Q^i and t than to the other standard model fermions. We have not yet found any models which employ the third way. Section III of this paper is devoted to the first way. In the remainder of this section we will briefly discuss the second way.

The difficulty of the second way is that we do not know how to explain the strength of the U(1) interaction, since it necessarily will have a positive β function. Nevertheless, let us ignore this problem and suppose that there is in addition to the standard model gauge group a strongly coupled $\mathrm{U}(1)_X$ interaction which is spontaneously broken at the scale M. For simplicity, let us make the gauge anomalies cancel without the introduction of extra fermions. The standard model fermion assignments for the gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_X$ are

$$\begin{aligned}
&(u_I d_I)_L \sim (\mathbf{3}, \mathbf{2}, \tfrac{1}{6}, x_{Q_I}) ,\\
&u^c_{IR} \sim (\overline{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}, x_{u_I}) ,\\
&d^c_{IR} \sim (\overline{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3}, x_{d_I}) ,\\
&(\nu_I e_I)_L \sim (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}, x_{L_I}) ,\\
&e^c_{IR} \sim (\mathbf{1}, \mathbf{1}, 1, x_{e_I}) ,
\end{aligned} \tag{2.11}$$

where $I = 1,2,3$ labels the three generations.[1] [The gauge transformation properties of fermions are always given in

[1]After the work described here was completed, we received [16] which also discusses the strongly coupled U(1) possibility. As pointed out there, it is not necessary for the $\mathrm{U}(1)_X$ interaction to be orthogonal to the standard model $\mathrm{U}(1)_Y$. However, the possible mixing between the U(1)'s does not affect anomaly cancellation considerations or the qualitative features of the condensate formation.

terms of left-handed two-component Weyl fields in (2.11) and throughout the rest of this paper.] Let us refer to this as model 1. There are 15 parameters associated with the x's. The condition that all of the gauge anomalies for the fermions in (2.11) must vanish implies five constraints on the $U(1)_X$ charges in model 1: namely,

$$\begin{aligned}
&\sum_I (3x_{Q_I}+x_{L_I})=0 \;, \\
&\sum_I (2x_{Q_I}+x_{u_I}+x_{d_I})=0 \;, \\
&\sum_I (x_{Q_I}+8x_{u_I}+2x_{d_I}+3x_{L_I}+6x_{e_I})=0 \;, \\
&\sum_I (x_{Q_I}^2-2x_{u_I}^2+x_{d_I}^2-x_{L_I}^2+x_{e_I}^2)=0 \;, \\
&\sum_I (6x_{Q_I}^3+3x_{u_I}^3+3x_{d_I}^3+2x_{L_I}^3+x_{e_i}^3)=0 \;,
\end{aligned} \tag{2.12}$$

which correspond respectively to the possible anomalies for $SU(2)_L^2U(1)_X$, $SU(3)_C^2U(1)_X$, $U(1)_Y^2U(1)_X$, $U(1)_YU(1)_X^2$, and $U(1)_X^3$.

Assume that $U(1)_X$ is strongly coupled at the scale Λ. A condensate will then form in the channel for which the product of $U(1)_X$ charges of the fermions (in their left-handed incarnation) is most negative. Since we want the top quark to condense first, we must have $x_{Q_3}x_{u_3}$ less than zero, and less than every other product of x's. If this is satisfied, then it is possible that only the condensate (1.2) will form [with an appropriate choice of $SU(2)_L$ orientation), because the other channels may not be sufficiently attractive to produce condensates. The $U(1)_X$ gauge symmetry can be broken, and the corresponding gauge boson given a mass, via the usual Higgs mechanism of introducing a complex scalar field ϕ which couples to $U(1)_X$ but not to the standard model gauge group and assuming that the self-coupling dynamics forces ϕ to obtain a vacuum expectation value. Alternatively, ϕ could itself be replaced by some unknown dynamical mechanism. In any case, once the $U(1)_X$ boson has a mass, it can be treated as static and integrated out for scales below M. A subsequent analysis of the resulting four-fermion interactions naturally yields the same condition for the top-quark condensate to be the most attractive channel in model 1.

There are many simultaneous solutions to the anomaly constraints (2.12) and the constraint that $x_{Q_3}x_{u_3}$ is most negative. Just to prove that it can be done, consider, e.g., a solution for which we arbitrarily choose to give the first two generations the same charges with respect to $U(1)_X$: $x_{Q_1}=x_{Q_2}=x_{L_I}=1$, $x_{d_1}=x_{d_2}=x_{e_I}=-2$, $x_{u_1}=x_{u_2}=0$, $x_{d_3}=2$, $x_{Q_3}=-3$, $x_{u_3}=4$. Clearly, $x_{Q_3}x_{u_3}$ is indeed more negative than any other possibility, and it is trivial to show that the anomaly constraints (2.12) are also satisfied. [We have also engineered this example to satisfy the mixed gravitational anomaly condition $\sum_I(6x_{Q_I}+3x_{u_I}+3x_{d_I}+2x_{L_I}+x_{e_I})=0$, which arises from considering triangle diagrams with one external $U(1)_X$ gauge boson and two external gravitons [18].]

Unfortunately, model 1 has very little predictive power for the following reasons. First, there is a large parameter space of solutions to the constraints on the $U(1)_X$ charges, which gets even larger if there are other fermions which couple to $U(1)_X$. Second, we do not yet have any dynamical picture with which to explain the behavior of the $U(1)_X$ coupling as a function of scale. It is possible that $U(1)_X$ gets strong because of some nontrivial renormalization-group behavior. Or it could be that the $U(1)_X$ is part of a non-Abelian gauge group which already has a large coupling when it gets broken down to $U(1)_X$. Rather than speculating further on this, let us move on to consider models in which the strength of the gauge interaction producing the condensate can be naturally explained by a negative β function.

III. TOP CONDENSATION WITH HEAVY COLORED GAUGE BOSONS

In the remainder of this paper we will concentrate on (2.8) rather than (2.10), so that the top-quark condensate scenario of dynamical electroweak symmetry breaking is to be produced by introducing a massive gauge boson which transforms as an adjoint of color and which couples strongly to Q^i and t. In fact, Hill [14] has recently presented a model which shows us how to accomplish this. His idea is that color $SU(3)_C$ is the diagonal subgroup of a larger symmetry $SU(3)_A\times SU(3)_B$. At the scale Λ, $SU(3)_A$ becomes strongly coupled, and at a lower scale M there is a spontaneous symmetry breaking $SU(3)_A\times SU(3)_B\to SU(3)_C$. The full unbroken gauge group is $SU(3)_A\times SU(3)_B\times SU(2)_L\times U(1)_Y$. The third-generation quarks and leptons are assigned to the representations

$$\begin{aligned}
&Q^i=(tb)_L\sim(3,1,2,\tfrac{1}{6}) \;, \\
&t^c\sim(\bar{3},1,1,-\tfrac{2}{3}) \;, \\
&b^c\sim(1,\bar{3},1,\tfrac{1}{3}) \;, \\
&L^i=(\tau\nu_\tau)_L\sim(1,1,2,-\tfrac{1}{2}) \;, \\
&\tau^c\sim(1,1,1,1) \;.
\end{aligned} \tag{3.1}$$

The first two generations are all just singlets under $SU(3)_A$, and transform under $SU(3)_B$ exactly as they do under the usual color $SU(3)_C$ of the standard model; i.e., they each transform as $(1,3,2,\frac{1}{6})\oplus(1,\bar{3},1,-\frac{2}{3})\oplus(1,\bar{3},1,\frac{1}{3})\oplus(1,1,2,-\frac{1}{2})\oplus(1,1,1,1)$.

The spontaneous symmetry breaking can be produced by introducing a scalar field Φ_β^α in the representation $(3,\bar{3},1,0)$ which is assumed to acquire a vacuum expectation value $\langle\Phi_\beta^\alpha\rangle=M\delta_\beta^\alpha$. (The field Φ_β^α might itself be replaced by some dynamical-symmetry-breaking mechanism, although we will not pursue that possibility here.) If the coupling constants at the scale M are g_A and g_B, then it is easy to show that the QCD coupling constant at M is given by $g_C=g_Ag_B/\sqrt{g_A^2+g_B^2}$. We are assuming that $SU(3)_A$ is strongly coupled at M and $SU(3)_B$ is not, so that $g_A\gg g_B$ and $g_C\approx g_B$. Of the sixteen gauge bosons associated with $SU(3)_A\times SU(3)_B$, eight remain massless after the symmetry is broken and are the gluons of QCD. The other eight gauge bosons H_μ^a obtain masses $M_H = M\sqrt{g_A^2+g_B^2}$ and also transform as an adjoint of $SU(3)_C$. If a fermion transforms under $SU(3)_A \times SU(3)_B$ in the representation $(\mathbf{R_A},\mathbf{R_B})$, then after spontaneous symmetry breaking it will transform under $SU(3)_C$ as the direct product representation $\mathbf{R_A}\otimes\mathbf{R_B}$.

The coupling of the heavy gauge bosons is given by [14]

$$\mathcal{L} \supset \frac{M_H^2}{2} H_\mu^a H^{a\mu} + \frac{g_A^2}{\sqrt{g_A^2+g_B^2}} H_\mu^a J_A^{a\mu} + \frac{g_B^2}{\sqrt{g_A^2+g_B^2}} H_\mu^a J_B^{a\mu} \quad (3.2)$$

with

$$J_A^{a\mu} = \bar{Q}^i \gamma^\mu T^a Q_i + \bar{t}\gamma^\mu T^a t \quad (3.3)$$

and

$$J_B^{a\mu} = \bar{b}\gamma^\mu T^a b + \sum \bar{q}\gamma^\mu T^a q \ , \quad (3.4)$$

where the last term represents the currents due to all of the quark fields of the first two generations. Integrating out the heavy gauge bosons H_μ^a produces the desired four-fermion interaction; comparing with (2.8) one finds [14]

$$\mathcal{L}_{\text{eff}} \supset \frac{8g_A^2}{9M_H^2} (\bar{Q}^i t)(\bar{t} Q_i) + \cdots \ . \quad (3.5)$$

If g_A gets sufficiently large at the scale M_H, the interaction term in (3.5) induces a top-quark condensate, gives a mass to the top quark, and binds the composite Higgs boson. [Of course there are other four-fermion terms involving the standard model quarks which arise from the exchange of H_μ^a, but only (3.5) is sufficiently attractive to lead to a condensate. We keep only the leading order in g_A/g_B in (3.5).]

As it stands, this theory is inconsistent because there are gauge anomalies involving both $SU(3)_A$ and $SU(3)_B$. Hill [14] proposes to eliminate this problem by introducing some extra fermions which transform according to

$$f_1 \sim (\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\tfrac{1}{3}) \ , \qquad f_2 \sim (\mathbf{1},\mathbf{3},\mathbf{1},-\tfrac{1}{3}) \ . \quad (3.6)$$

It is easy to see that (3.6) together with (3.1), which we shall call model 2, is free of all gauge anomalies. Unfortunately, this model seems a bit too simplistic. The problem is that f_2 and b^c together form a real representation of the gauge group even *before* spontaneous symmetry breaking. Therefore f_2 and b^c should pair up and decouple, obtaining a mass larger than any other scale of interest in the problem. (This might not occur if there were some ungauged unbroken chiral symmetry preventing such a mass term, but we consider such a possibility highly unnatural.) In the resulting low energy theory, f_1^c will play the role of the right-handed bottom quark, but since it also feels the strong $SU(3)_A$ interaction, t and f_1^c will form condensates with the two weak isospin components of Q^i with the same strength. Thus, in model 2, the physical bottom quark presumably gets an unacceptably large mass.

Fortunately, the preceding difficulty can be avoided by introducing a slightly more complicated set of "anomaly canceling" fermions. First let us make some general observations regarding the properties that the extra fermions must satisfy. The anomalies which need canceling in (3.1) are of the types $SU(3)_A^3$, $SU(3)_B^3$, $SU(3)_A^2U(1)_Y$, and $SU(3)_B^2U(1)_Y$. So there must be extra fermions with nontrivial quantum numbers for each of $SU(3)_A$ and $SU(3)_B$ and $U(1)_Y$. Those extra fermions that transform nontrivially under $SU(3)_A$ may participate in the condensation process when g_A gets large. These condensates can often produce potentially embarrassing results, such as breaking $SU(3)_C$ or $U(1)_{EM}$ and giving the gluon or the photon a mass. The absence of such unwanted condensates is an important and very restrictive constraint on these models as we will soon see. A no less embarrassing catastrophe would occur if the extra fermions made an unwanted appearance in the low energy limit of the theory. There are two ways the extra fermions can be banished from low energy physics. The first way, which we shall not pursue here, would be to introduce another non-Abelian gauge interaction which confines the extra fermions at some large scale. The second way is to arrange things so that the extra fermions are in a complex representation of $SU(3)_A\times SU(3)_B\times SU(2)_L\times U(1)_Y$ but in a real representation of $SU(3)_C\times SU(2)_L\times U(1)_Y$ [or at least a real representation of $SU(3)_C\times U(1)_{EM}$]. In this way the extra fermions can do their job of canceling anomalies and still get large enough masses to avoid present-day detection. Finally let us note that if there are too many fermions in nontrivial representations of $SU(3)_A$, then the β function which governs the running of g_A will be positive and we expect that g_A will not grow in the infrared. Since there is a key assumption that $SU(3)_A$ is strongly coupled at Λ, the fermion content should not be too large. This is a welcome restriction which relieves us of the responsibility of considering models which are too baroque.

Instead of (3.6), suppose we have fermions in the representations

$$\begin{aligned} &f_1 \sim (\bar{\mathbf{3}},\mathbf{1},\mathbf{1},q_1) \ , \quad f_4 \sim (\mathbf{1},\mathbf{3},\mathbf{1},-q_1) \ , \\ &f_2 \sim (\bar{\mathbf{3}},\mathbf{1},\mathbf{1},q_2) \ , \quad f_5 \sim (\mathbf{1},\mathbf{3},\mathbf{1},-q_2) \ , \\ &f_3 \sim (\mathbf{3},\mathbf{1},\mathbf{1},\tfrac{1}{3}-q_1-q_2) \ , \quad f_6 \sim (\mathbf{1},\bar{\mathbf{3}},\mathbf{1},-\tfrac{1}{3}+q_1+q_2) \ , \end{aligned} \quad (3.7)$$

with $q_1\neq\frac{1}{3}$, $q_2\neq\frac{1}{3}$, $q_1+q_2\neq-\frac{1}{3}$. (The restrictions on q_1 and q_2 are necessary to prevent any of the fermions from obtaining masses before spontaneous symmetry breaking.) It is easy to show that (3.7) together with (3.1), which we shall refer to as model 3, has no gauge anomalies. Since the Yukawa-type interactions $\Phi f_1 f_4$, $\Phi_2 f_5$, and $\Phi f_3 f_6$ respect all of the gauge symmetries, $f_1,\ldots,f_6$ will get masses which are dimensionless coupling constants times M. Note also that the β functions for $SU(3)_A$ and $SU(3)_B$ are given (to one-loop order) by

$$\beta_A=\mu\frac{dg_A}{d\mu}=-\frac{17}{32\pi^2}g_A^3\ , \tag{3.8}$$

$$\beta_B=\mu\frac{dg_B}{d\mu}=-\frac{13}{32\pi^2}g_B^3 \tag{3.9}$$

at scales above Λ. So it is quite plausible that $SU(3)_A$ could become strongly coupled above the electroweak scale while $SU(3)_B$ remains weak there. Unfortunately, the viability of model 3 is quite problematical. The difficulty is that there is a tendency for the extra fermions to participate in unwanted condensates. This is best understood after we make the following slight digression.

The types of models we are discussing here are actually close cousins of technicolor. In the original formulation using effective four-fermion interactions, this point is somewhat obscured. In the renormalizable version based on strongly coupled $SU(3)_A$, however, we can see the essential common ingredients: in both cases, a non-Abelian gauge theory gets strong, leading to spontaneous chiral breaking, exactly in the fashion familiar from QCD phenomenology. The electroweak symmetry is embedded in the chiral symmetry and so is broken down to $U(1)_{EM}$. The main differences here are that the top quark participates in the condensate (and thus plays the role of a techniquark) and that the strong interaction goes on to be spontaneously broken at a lower scale than the chiral-symmetry-breaking scale. It is useful, in the renormalizable version, to think about the chiral-symmetry breaking which takes place at the scale Λ as separate from the $SU(3)_A\times SU(3)_B$ breaking which occurs at the lower scale M. These play two distinct roles: the chiral symmetry breaking at Λ is responsible for the electroweak symmetry breaking condensate; the $SU(3)_A\times SU(3)_B$ breaking at M produces an extended-technicolor-like interaction contributing to the mass of the top quark, and ensures that Q^i and t will not be confined at scales above Λ_{QCD}. In our view, the enhanced understanding of the separate roles of the two scales Λ and M is one of several important advantages derived from thinking about top bootstrap models from the renormalizable point of view. Let us now return to model 3 with these ideas in mind.

Model 3 is very much like QCD with three quarks, with $SU(3)_A$ playing the role of color. For the purposes of analyzing the dynamics of the condensate formation, we can ignore all of the fermions except Q^i, t^c, f_1, f_2, and f_3, and neglect the weak couplings of $SU(3)_B\times SU(2)_L\times U(1)_Y$ and the scalar field Φ (since $M<\Lambda$). Then the $SU(3)_A$ interaction has an approximate global $SU(3)_L\times SU(3)_R$ chiral symmetry. The $SU(3)_L$ chiral transformations are unitary rotations of $Q^\uparrow$, $Q^\downarrow$, and f_3 into each other, and the $SU(3)_R$ transformations similarly rotate t^c, f_1, and f_2 into each other. Our experience with QCD tells us that the $SU(3)_A$ interactions will produce a fermion condensate which breaks $SU(3)_L\times SU(3)_R$ down to a vector $SU(3)_v$ symmetry. The condensate matrix takes the form

$$\begin{bmatrix} \langle Q^\uparrow t^c\rangle & \langle Q^\downarrow t^c\rangle & \langle f_3 t^c\rangle \\ \langle Q^\uparrow f_1\rangle & \langle Q^\downarrow f_1\rangle & \langle f_3 f_1\rangle \\ \langle Q^\uparrow f_2\rangle & \langle Q^\downarrow f_2\rangle & \langle f_3 f_2\rangle \end{bmatrix}=m_{\chi SB}^3 U\ , \tag{3.10}$$

where the $\uparrow$ and $\downarrow$ label $SU(2)_L$ eigenstates, $m_{\chi SB}$ is a parameter with dimensions of mass, and U is a unitary matrix. To find the precise form of U it is necessary to solve a vacuum alignment problem which depends on the values chosen for q_1 and q_2. Different possibilities for U correspond to different ways of orienting $SU(3)_v$ with respect to the weakly gauged $SU(2)_L\times U(1)_Y$ subgroup of $SU(3)_L\times SU(3)_R$. One component of the condensate matrix is the desired $\langle Q^\uparrow t^c\rangle\neq 0$, but there will necessarily be other nonzero condensates as well, since U is unitary. No matter what U is, the electroweak symmetry will be completely broken by the condensates and there cannot be a massless photon. It is amusing to note that the very restrictions on q_1 and q_2 which saved model 3 from suffering the same fate as model 2 are responsible for ensuring that $U(1)_{EM}$ cannot be respected by the condensate matrix. There are many variations on the theme of model 3 which have similar difficulties.

Bad condensates might be avoided in model 3 if f_1, f_2, and f_3 have sufficiently large masses to avoid forming condensates when g_A gets large. Then the only condensate will be $\langle Q^\uparrow t^c\rangle\neq 0$ [by a choice of orientation of $SU(2)_L$]. So after the $SU(3)_A\times SU(3)_B$ is broken, leading to an extended-technicolor-like-four-fermion interaction for the top quark, we will have the correct basic ingredients of the top-quark condensate idea. While this is of course a logical possibility, it seems to us unlikely that f_1, f_2, and f_3 can obtain such large masses in this model. This is because, as mentioned earlier, their masses are dimensionless Yukawa couplings times M, and there is no particular reason why those Yukawa couplings should be large. There is perhaps a useful comparison to be made here with the masses of the light fermions in the standard model; all of the quarks and leptons (except the top quark) have masses more than an order of magnitude less than the electroweak breaking scale. In the most extreme case, the mass of the electron is more than 5 orders of magnitude less than the scale at which the symmetry breaking allows it. The whole point of the top-quark condensate idea is that the top quark gets a Yukawa coupling of order one only because it actively participates in the dynamical electroweak symmetry-breaking mechanism. Since f_1, f_2, and f_3 do not play such a role in the breaking of $SU(3)_A\times SU(3)_B$, we might expect by analogy that the masses of f_1, f_2, and f_3 in model 3 will be somewhat smaller than M, which in turn must be smaller than the scale Λ at which $SU(3)_A$ gets strong enough to produce condensates.

Thus there are two possible outcomes for model 3, depending on what assumptions we make about the nonperturbative dynamics of the strongly coupled $SU(3)_A$. If $SU(3)_A$ produces condensates in a QCD-like manner, then model 3 has a massive photon and is an immediate failure. If $SU(3)_A$ somehow does not produce condensates involving f_1, f_2, and f_3, then model 3 at least will produce the correct electroweak symmetry breaking and a massive top quark. It should be emphasized that it lies beyond the scope of perturbative calculation to decide which of these assumptions is correct, and that in any case the most important parameters which would enter into such a calculation, namely the Yukawa couplings of Φ to the extra fermions, are not predictable. Still, for the reasons mentioned above, the former assumption seems to us more likely.

Let us next consider model 4 obtained by adding to (3.1) the extra fermions

$$\begin{aligned} f_1^i &\sim (\bar{3},1,2,q) , \\ f_2 &\sim (3,1,1,\tfrac{1}{3}-2q) , \\ f_3^i &\sim (1,3,2,-q) , \\ f_4 &\sim (1,\bar{3},1,-\tfrac{1}{3}+2q) , \end{aligned} \tag{3.11}$$

with $q \neq -\frac{1}{6}$ so that Q^i and f_1^i do not form a real representation of the unbroken-symmetry group. This model is in many ways very similar to model 3. The gauge anomalies are easily seen to cancel. The allowed Yukawa terms $\Phi f_1 f_3$ and $\Phi f_2 f_4$ mean that the extra fermions will end up getting masses which are dimensionless couplings times M. Above Λ, the β functions for $SU(3)_A$ and $SU(3)_B$ are given by the same formulas (3.8) and (3.9) as in model 3. There is again a chiral $SU(3)_L \times SU(3)_R$ symmetry; under $SU(3)_L$, $Q^\uparrow$, $Q^\downarrow$, and f_2 rotate into each other, while under $SU(3)_R$, t^c, $f_1^\uparrow$, and $f_1^\downarrow$ rotate into each other. Assuming QCD-like behavior of the strongly coupled $SU(3)_A$, $SU(3)_L \times SU(3)_R$ gets broken down to a vector $SU(3)_v$ by a condensate which is proportional to a 3×3 unitary matrix. The extent to which the electroweak symmetry is broken is determined by its alignment relative to the surviving $SU(3)_v$. Exactly this kind of vacuum alignment problem has been considered in the context of technicolor theories [19,20]. Using the methods of [19,20], one can show that the vacuum will align so that the matrix of condensates in model 4 breaks the electroweak symmetry in two different but equally bad ways depending on the relative strengths of the $SU(2)_L$ and $U(1)_Y$ interactions.

This can be understood heuristically as follows. The $SU(2)_L$ interaction favors the condensation of two 2's into the "most attractive channel" in the one-boson-exchange approximation, which is the singlet rather than the triplet of $SU(2)_L$. A singlet-$SU(2)_L$ condensate will not break $SU(2)_L$, so if $SU(2)_L$ dominates over $U(1)_Y$ in the vacuum-alignment problem, then the electroweak symmetry will break according to $SU(2)_L \times U(1)_Y \rightarrow SU(2)_L$. On the other hand, if q is sufficiently positive then the $U(1)_Y$ interaction dominates and favors condensation of fermions with opposite charges, so the t^c will condense with a component of Q^i [which we may choose to be $Q^\uparrow$ by a choice of orientation of $SU(2)_L$], and the f_2 will condense with a component of f_1^i. [How positive q has to be for this to happen is determined by the relative strengths of the $SU(2)_L$ and $U(1)_Y$ coupling constants at the scale Λ.] In fact f_2 is favored to condense with $f_1^\downarrow$ and $f_1^\uparrow$ condenses with $Q^\downarrow$, because the $SU(2)_L$ prefers that opposite weak isospins combine. The electroweak symmetry is completely broken by these condensates because we had to require $q \neq -\frac{1}{6}$. So model 4 is viable only if we once again make the slightly dubious assumption that the condensation is non-QCD-like and the extra fermions do not participate in the condensation.

We now present a model which does not have any bad condensates under the assumption that $SU(3)_A$ has QCD-like chiral-symmetry breaking. Consider fermions in the representations

$$\begin{aligned} f_1^i &\sim (3,1,2,\tfrac{7}{6}) , \\ f_2^I &\sim (\bar{3},1,3,-\tfrac{2}{3}) , \\ f_3^i &\sim (1,\bar{3},2,-\tfrac{7}{6}) , \\ f_4^I &\sim (1,3,3,\tfrac{2}{3}) . \end{aligned} \tag{3.12}$$

[Here $i=1,2$ and $I=1,2,3$ are $SU(2)_L$ indices.] Now (3.12) together with (3.1), which we will refer to as model 5, has no gauge anomalies and is again arranged so that (f_1,f_3) and (f_2,f_4) are each real representations of $SU(3)_c \times SU(2)_L \times U(1)_Y$, pairing up to get masses (by coupling to Φ) that are dimensionless couplings times M. The β functions for $SU(3)_a$ and $SU(3)_B$ are given to one-loop order by

$$\beta_A = \mu \frac{dg_A}{d\mu} = -\frac{47}{96\pi^2} g_A^3 , \tag{3.13}$$

$$\beta_B = \mu \frac{dg_B}{d\mu} = -\frac{35}{96\pi^2} g_B^3 \tag{3.14}$$

so that g_A can easily grow in the infrared more rapidly than g_B. Once again we can analyze the dynamics of the condensate formation by appealing to the comparison with QCD. The strong $SU(3)_A$ interaction couples to Q^i, t^c, f_1^i and f_2^I. There is an approximate $SU(4)_L \times SU(4)_R$ chiral symmetry; under $SU(4)_L$, $Q^\uparrow$, $Q^\downarrow$, $f_1^\uparrow$, and $f_1^\downarrow$ rotate into each other, while under $SU(4)_R$, f_2^+, f_2^0, f_2^-, and t^c rotate into each other. [The superscripts label $SU(2)_L$ quantum numbers in the obvious way.] Now one expects that the chiral symmetry will be broken down to a vector $SU(4)_v$ by fermion bilinear condensates at the scale Λ. The condensates will take the form $m_{\chi SB}^3 U$ where U is a unitary 4×4 matrix.

There is again a crucial question of vacuum alignment. The extent to which the electroweak symmetry is broken is determined by its alignment relative to the surviving $SU(4)_v$. Using the methods of [19,20], one can show that the electroweak interactions cause the condensate matrix to align so that

$$U = \begin{array}{c} \\ t^c \\ f_2^+ \\ f_2^0 \\ f_2^- \end{array} \begin{array}{c} \begin{array}{cccc} Q^\uparrow & Q^\downarrow & f_1^\uparrow & f_1^\downarrow \end{array} \\ \left[\begin{array}{cccc} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \end{array} \right] \end{array} . \tag{3.15}$$

[There is a simple heuristic explanation for (3.15). The $U(1)_Y$ interaction does not affect the vacuum alignment problem, because t^c and f_2^I have the same hypercharge. The $SU(2)_L$ interaction favors the condensation of a 3 and a 2 of $SU(2)_L$ into "the most attractive channel," which is the 2 rather than the **4**. So each of the condensates $\langle Q^i f_2^I \rangle$ and $\langle f_1^i f_2^I \rangle$ are favored to transform as 2's of $SU(2)_L$, as in (3.15). The t^c goes along with this, in such a way as to make U unitary, because it does not have $SU(2)_L$ interactions.] The matrix of condensates (3.15) breaks $SU(2)_L \times U(1)_Y$ down to $U(1)_{EM}$. Each of the nonzero entries in the condensate matrix (3.15) corresponds to a composite scalar field (with the quantum numbers of the standard model Higgs boson) which develops a vacuum expectation value. So model 5 at least exhibits the correct electroweak symmetry-breaking pattern. Let us now consider the masses obtained by fermions in this model.

In model 5, the top quark gets a mass from the combined effects of the condensate induced by the strength of $SU(3)_A$ and the effective four-fermion interaction produced by the symmetry breaking. Note that each of t^c and $Q^\uparrow$ has nonzero condensates with the extra fermions. There will also be corresponding four-fermion terms in the effective action, besides the term (3.5) which was our original motivation. Thus we have, instead of (3.5),

$$\mathcal{L}_{\text{eff}} \supset \frac{8g_A^2}{9M_H^2}[(\bar{Q}^i t)(\bar{t}Q_i)+(\bar{f}_1^i t)(\bar{t}f_{1i}) +(\bar{Q}^i f_2^I)(\bar{f}_{2I}Q_i)+(\bar{f}_1^i f_2^I)(\bar{f}_{2I}f_{1i})] \,. \quad (3.16)$$

From the point of view of the original four-fermion version of the top-quark condensate idea, (3.16) is responsible both for inducing the nonzero condensates of (3.15) and also for translating the condensates into masses for the participating fermions. Now, in general, four-fermion interactions of this type can be decomposed into products of scalar fermion bilinears which transform as irreducible representations of the gauge group. Thus each of the latter two terms in (3.16) can be decomposed into products of scalar fermion bilinears which transform as a 2 and a **4** of $SU(2)_L$. As we have already noted, the interaction which is a product of 2's is more attractive and so the condensate forms in those channels and does not form in the **4** channels. Furthermore, the interactions which transform as 2's couple to the condensates, producing effective mass terms for the participating fermions, while the **4** channels of the interactions do not couple to the condensates in lowest order and so do not produce such mass terms.

The physical-top-quark mass eigenstate in model 5 will be a mixture of what we have been calling the top quark and some of the extra fermions. In particular, the physical top quark at low energies will contain a mixture of t, f_2^{0c}, $f_3^{\uparrow c}$ as the right-handed part and of $Q^\uparrow, f_1^\downarrow, f_4^0$ as the left-handed part. The component of the physical top quark which consists of extra fermions increases as the Yukawa couplings between Φ and the extra fermions decrease. The mass of the physical top quark and the extent of its mixing with the extra fermions are not calculable because we do not know the Yukawa couplings of the extra fermions to Φ. But we can get an idea of the phenomenon by considering the form of the mass matrix for the fermions which have the same $SU(3)_c \times U(1)_{EM}$ quantum numbers as the top:

$$\mathcal{L}_{\text{eff}} \supset \begin{bmatrix} Q^\uparrow & f_1^\downarrow & f_4^0 \end{bmatrix} \begin{bmatrix} m_1 & 0 & m_1 \\ m_1 & m_2 & m_1 \\ 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} t^c \\ f_3^\uparrow \\ f_2^0 \end{bmatrix} \,. \quad (3.17)$$

Here m_1 schematically represents the masses which come from the condensates and the corresponding four-fermion terms, and m_2 represents the masses coming from the Yukawa interactions of Φ to the extra fermions when Φ gets an expectation value.

The left-handed part of the bottom quark, $Q^\downarrow$, also participates in a condensate with f_2^+. [Indeed, it is clear that if the chiral symmetry breaking produced by $SU(3)_A$ is QCD like, then $Q^\downarrow$ must participate in a condensate in all models of this type.] However, the right-handed part of the bottom quark does not participate in the condensation, and this is responsible for the lightness of the physical bottom quark. To appreciate this, we can write out the schematic mass matrix for the bottom quark and the extra fermions which have the correct $SU(3)_c \times U(1)_{EM}$ quantum numbers to mix with the bottom:

$$\mathcal{L}_{\text{eff}} \supset \begin{bmatrix} Q^\downarrow & f_4^- \end{bmatrix} \begin{bmatrix} 0 & m_1 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} b^c \\ f_2^+ \end{bmatrix} \,. \quad (3.18)$$

Here m_1 schematically represents the mass which comes from the condensate $\langle Q^\downarrow f_2^+ \rangle$ and m_2 represents the mass coming from the Yukawa interaction $\Phi f_2 f_4$. Now (3.18) corresponds to a massless bottom quark and a Dirac fermion with mass $\sim\sqrt{m_1^2+m_2^2}$. The left-handed part of the massless bottom quark is proportional to $Q^\downarrow-(m_1/m_2)f_4^-$ and the right-handed part is b. So we find that the left-handed part of the physical bottom quark should have an admixture of a weak-isospin-triplet state in model 5 instead of being entirely a weak isospin doublet state as in the standard model. That admixture can be small if m_1 is less than m_2. (The left-handed part of the bottom quark is known to be primarily a weak isospin $I^3=-\frac{1}{2}$ state from observations [21] of the forward-backward asymmetry in $e^+e^- \to \bar{b}b$.) The bottom quark presumably acquires a finite mass through radiative corrections.

It is also possible to try other choices for the strongly coupled gauge group. For example, consider the gauge group $SU(4)_A \times SU(3)_B \times SU(2)_L \times U(1)_{Y'}$. The standard-model third-generation fermions live in the representations

$$\begin{aligned} &Q^i, L^i \sim (\mathbf{4},1,2,0) \,, \\ &t^c, N^c \sim (\bar{\mathbf{4}},1,1,-\tfrac{1}{2}) \,, \\ &b^c \sim (1,\bar{3},1,\tfrac{1}{3}) \,, \\ &\tau^c \sim (1,1,1,1) \,, \end{aligned} \quad (3.19)$$

while the first- and second-generation fermions are all singlets under $SU(4)_A$ and so transform as

$$(1,3,2,\tfrac{1}{6}) \oplus (1,\bar{3},1,-\tfrac{2}{3}) \oplus (1,\bar{3},1,\tfrac{1}{3}) \oplus (1,1,2,-\tfrac{1}{2}) \oplus (1,1,1,1)$$

as before. Thus $SU(4)_A$ is a kind of Pati-Salam unification of color and lepton number for the third generation only. At the scale Λ, $SU(4)_A$ becomes strongly coupled enough to produce condensates, and at a lower scale M there is a spontaneous symmetry breaking $SU(4)_A \times SU(3)_B \times U(1)_{Y'} \rightarrow SU(3)_C \times U(1)_Y$. This can be accomplished by introducing a scalar field Φ_β^α in the representation $(\mathbf{4},\overline{\mathbf{3}},1,-\frac{1}{6})$, which is assumed to acquire a vacuum expectation value $\langle \Phi_\beta^\alpha \rangle = M\delta_\beta^\alpha$. [Here $\alpha=1,\ldots,4$ is an $SU(4)_A$ index and $\beta=1,2,3$ is an $SU(3)_B$ index.] N is a right-handed neutrino which is a gauge singlet after the symmetry breaking just mentioned. Of the 24 gauge bosons associated with $SU(4)_A \times SU(3)_B \times U(1)_{Y'}$, eight are the massless gluons; one is the hyperphoton associated with $U(1)_Y$, eight more transform as an adjoint of $SU(3)_C$ and get masses $M_H = M\sqrt{g_A^2+g_B^2}$, six more transform as a $\mathbf{3}$ and a $\overline{\mathbf{3}}$ of $SU(3)_C$ and get masses $Mg_A/\sqrt{2}$; and the last is an $SU(3)_C$ singlet with a mass $M\sqrt{g_A^2/4+g_{Y'}^2/6}$. Integrating out the heavy gauge bosons produces four-fermion interactions

$$\mathcal{L}_{\text{eff}} \supset -\frac{g_A^2}{M_H^2}[(\overline{Q}^i t)(\overline{t}Q_i)+2(\overline{Q}^i t)(\overline{N}L_i)+2(\overline{L}^i N)(\overline{t}Q_i)+3(\overline{L}^i N)(\overline{N}L_i) + (\overline{Q}^i L_j^c)(\overline{L}^{cj}Q_i)+(\overline{t}N^c)(\overline{N}^c t)+\tfrac{4}{3}(\overline{Q}^i \rho_\alpha Q^{cj})(\overline{Q}_j^c \rho^\alpha Q_i)] \tag{3.20}$$

where we have kept only the most strongly attractive terms for $g_A \gg g_B, g_{Y'}$. Now, it is inappropriate to try to understand the condensation pattern from (3.20), because there must be other fermions with $SU(4)_A$ interactions, and because (3.20) does not factor into the form (2.4) in a unique way, since the heavy gauge bosons do not transform as an irreducible representation of the standard model gauge group. Fortunately, we may simply appeal to the analogy between $SU(4)_A$ and QCD to find the condensation pattern.

As usual, extra fermions must be added to (3.19) to cancel the gauge anomalies. For each model based on $SU(3)_A$, one can construct a corresponding model based on $SU(4)_A$ by adding in extra fermions in an analogous way. For example, model 6 (the analogue of model 5) is given by adding to (3.19) the extra fermions

$$\begin{aligned} &f_1^i \sim (\mathbf{4},1,2,1)\ , \quad f_4^i \sim (1,1,2,-\tfrac{1}{2})\ , \\ &f_2^I \sim (\overline{\mathbf{4}},1,3,-\tfrac{1}{2})\ , \quad f_5^I \sim (1,3,3,\tfrac{2}{3})\ , \\ &f_3^i \sim (1,\overline{\mathbf{3}},2,-\tfrac{7}{6})\ , \quad f_6^I \sim (1,1,3,0)\ . \end{aligned} \tag{3.21}$$

The analysis of this model proceeds exactly as for model 5, with the same conclusions. Each of the extra fermions is eligible to receive a mass term after the spontaneous symmetry breaking $SU(4)_A \times SU(3)_B \times U(1)_{Y'} \rightarrow SU(3)_C \times U(1)_Y$ occurs, because they form a real representation of the standard model gauge group. There is again an approximate chiral symmetry $SU(4)_L \times SU(4)_R$ which is broken by the condensate down to a vector $SU(4)_v$ (assuming QCD-like behavior). The vacuum alignment problem is again solved by a condensate matrix proportional to (3.15) which breaks the electroweak symmetry to $U(1)_{EM}$. Among these condensates are $\langle \overline{Q}^i t \rangle = 3\langle \overline{L}^i N \rangle \neq 0$. The top quark and third-generation neutrino will both get large Dirac mass terms due to (3.20), as in [12]. Since N is a gauge singlet under the standard model gauge group, it also gets a Majorana mass of order M. So if M is much larger than the electroweak scale, the seesaw mechanism [22] dictates that there will be one light third-generation neutrino state. Alternatively, a smaller M can be used in a four-generation model, as in [10].

Similarly, if one is willing to make the assumption that the chiral symmetry breaking associated with $SU(4)_A$ is not QCD like, so that the extra fermions can avoid forming condensates, then models analogous to, e.g., model 3 and model 4 (which can be constructed in a fairly transparent way) may be viable.

IV. CONCLUSION

In this paper we have considered the possibility that a top-quark condensate can arise from a strongly coupled gauge theory which is spontaneously broken. Note that, qualitatively speaking, there are three possible things that can happen in the infrared to a non-Abelian gauge theory with a negative β function. First, it can be spontaneously broken before it has a chance to become strong. This is the fate of the electroweak $SU(2)_L$ in the standard model. Second, the gauge theory can become strong and not be spontaneously broken. This is what happens to color $SU(3)_C$ in the standard model. Our understanding of the low-energy behavior of this type of theory is due mostly to empirical observations of QCD. The third possibility is that the gauge theory can become strong but is then spontaneously broken at a lower scale. There is no example of this in the standard model, but there is also no good reason why this phenomenon could not occur one or more times between the electroweak scale and the Planck scale. Therefore, it seems worthwhile to try to understand this type of theory better, whether or not it is responsible for electroweak symmetry breaking as suggested here.

It is clear that none of the particular models discussed here is a candidate for a realistic theory. In this paper, we have not attempted to address such important phenomenological questions as the origin of the masses for the light standard model fermions, the strengths of flavor-changing neutral currents, and the value of the ρ parameter, which traditionally have posed the greatest challenges to technicolor and other dynamical electroweak symmetry-breaking models. One can imagine that there may be a host of more complicated renormalizable top-quark condensate models, for which the models we have described here are simple-minded prototypes, in which the more detailed phenomenological constraints of the standard model may or may not be satisfied. For example, one can imagine models based on strongly coupled gauge groups other than $SU(3)_A$ and $SU(4)_A$, or with additional broken or unbroken strongly coupled gauge groups. Then the technology of tumbling gauge theories [17] can perhaps be used to model the pattern of chiral symmetry breaking and mass generation, as a generalization of our understanding of the infrared properties of QCD-like theories. One particularly attractive possibility is that in a more sophisticated model the scalar field Φ can be replaced by some further dynamical mechanism, so that there are no fundamental scalars necessary at all in the theory.

Let us conclude by summarizing those qualitative features which are common to the models discussed in Sec. III, and which we presume will survive the passage to a more realistic and complete model. First, anomaly cancellation in the renormalizable theory generally implies that there must be extra fermions beyond those found in the standard model. These extra fermions are typically "exotic", i.e., their gauge quantum numbers differ from those of the standard model fermions. The extra fermions also participate in condensates and presumably obtain masses in the 100 GeV–few TeV range. Also, there should be pseudo Nambu-Goldstone bosons due to the spontaneous breakdown of the approximate chiral symmetry of the strongly coupled gauge interaction. In general, the existence of the extra fermions implies that this approximate chiral symmetry is larger than the electroweak symmetry. [For example, in model 5, the $SU(3)_A$ interaction has an approximate chiral symmetry $SU(4)_L \times SU(4)_R$ which is spontaneously broken by the condensates to $SU(4)_V$. This implies the existence of 15 pseudo Nambu-Goldstone bosons. Three of these are absorbed by the electroweak gauge bosons, giving masses to $W_\mu^\pm$ and Z_μ^0. The remaining 12 pseudo Nambu-Goldstone bosons will acquire masses due to the explicit violation of $SU(4)_L \times SU(4)_R$, including the Yukawa couplings of Φ to the extra fermions.]

As a corrollary of the preceding model-independent features, we may presume that in a renormalizable theory the "economical" four-fermion interaction (1.1) does not capture all of the important physics at the scale M, because the extra fermions must also couple to the strongly coupled gauge theory. This, along with our prejudice that the scale of new physics should not be too much higher than the electroweak scale, means that the renormalization-group methods used in [4] are not reliable and should not be used to infer quantitative constraints on the top-quark and Higgs-scalar-boson masses. Rather, the best general experimental signature of the top-quark condensate idea may simply be the existence of the heavy particles implied by the renormalizability of the underlying theory.

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