Light-cone quark-model axial-vector-meson wave function

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Within the framework of the light-cone formalism we construct a model wave function appropriate for axial-vector mesons. The spin bilinear covariant component of our model wave function is consistent with the Melosh transformation procedure. We have applied our model to the a_1 axial-vector meson and have compared the first six moments of the quark distribution amplitude with those obtained from the QCD sum-rule technique. In addition to our axial-vector-meson results, we further investigate the simple light-cone quark model by computing moments for pseudoscalar π and K and vector ρ mesons and compare our results with those from various QCD sum-rule techniques and lattice calculations.

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I. INTRODUCTION

Even though more fundamental nonperturbative QCD methods such as QCD sum-rule techniques [1,2] and lattice QCD calculations exist [3–6], there is still growing interest in using simple relativistic quark models [7–12] to describe hadron properties. Since currently no rigorous criteria exist for assessing how well such simple models approximate the actual QCD solution, it is important to document how well this approach, which accurately describes numerous hadron properties, can reproduce features characteristic of nonperturbative QCD methods. Our analysis [7] is a generalization and extension of the work by Dziembowski and Mankiewicz [9], who developed a relativistic description of the π and ρ-meson valence-quark structure using the constituent-quark model formulated in the light-cone Fock approach [13–15]. Perhaps their most significant finding was that such a model could indeed reproduce the important features for meson amplitudes obtained by Chernyak and Zhitnitsky [1], who used QCD sum-rule techniques and by others performing lattice QCD calculations [3].

A surprising, controversial aspect of meson structure which has emerged from nonperturbative QCD calculations is the substantial meson spin dependence associated with the quark momentum distribution amplitude for mesons containing light quarks. In particular, the QCD sum-rule approach of Refs. [1,2] and lattice QCD calculations of Refs. [3,4] find that the π distribution amplitude is double peaked or camel shaped as a function of the fraction of the meson momentum carried by the quarks, while the ρ meson only exhibits a single broad peak. Because this signature is generated by both QCD sum-rule and lattice gauge approaches it was initially thought to be a consequence of nonperturbative theory even though it was difficult to attribute which aspect of the QCD dynamics was directly responsible for the spin-dependent behavior. Now, however, since this same behavior is also reproduced in the simpler model calculations by Ref. [9] and, more recently, Ref. [12], there are physically more intuitive explanations available. Reference [9] argues that the only necessary ingredients to reproduce the striking distribution feature are a nonstatic relativistic spin wave function and a small radial transverse size for the valence configuration. Reference [12], which uses a different meson wave-function ansatz, claims the behavior arises from quark spin interactions and predicts that for heavier open-flavored mesons such as D and B the camel-shaped distribution will disappear because the spin interaction is suppressed due to decoupling between the spin of a heavy quark and the gluon field [16]. More recently, however, Ref. [6], using the unquenched approximation of Ref. [4], reported a lattice calculation which does not generate a camel-shaped distribution for the π and supports the result obtained by Mikhailov and Radyushkin [17], indicating the issue needs further clarifying study.

Previously, we extended the pion model for Ref. [9] and derived general, analytic formulas for the electromagnetic form factor and the decay constant which are valid for any pseudoscalar meson [7]. Our predictions for the kaon (K^+ and K^0) charge radii, the kaon form factor, and the decay constant f_K compared favorably with experimental data. In this paper we further extend that analysis to the a_1 axial-vector meson and perform a comparative meson study using the Lorentz-invariant light-cone wave functions for the a_1, π, K, and ρ mesons. In particular, we find that the spin bilinear covariant component (spin covariant) of our model wave function is consistent with the Melosh transformation procedure. We also generate various π quark distribution amplitudes including one which is not camel shaped and compare the first six moments of the a_1, π, K, and ρ quark distribution amplitudes with those obtained from various QCD sum-rule and lattice QCD calculations.

In Sec. II we discuss the essential aspects of our relativistic quark model and develop the spin covariant for the a_1 meson wave function. In this section we also demonstrate the equivalence of our spin-covariant result with that obtained by a Melosh transformation and provide a more formal derivation in the Appendix. In Sec. III we present our numerical results for the π, K, ρ, and a_1 mesons moments and compare with recent QCD sum-rule and lattice results. Conclusions are followed in Sec. IV.

II. MODEL DESCRIPTION AND a_1 MESON WAVE FUNCTION

The model used in this paper is based upon the light-cone quantization method [13–15] which provides an improved Fock-state expansion for hadron states since in the light-cone formalism the vacuum and hadron states are rigorously orthogonal. The key approximation in this approach is to truncate the expansion by retaining only the lowest Fock state. The meson state $|M\rangle$ considered in this paper is, therefore, represented by

$$|M\rangle=\Psi^M_{q,\bar{q}}|q\bar{q}\rangle , \quad (2.1)$$

where $|q\bar{q}\rangle$ is the two-body Fock state for a quark q and an antiquark $\bar{q}$. The model wave function $\Psi^M_{q,\bar{q}}$ is given by the product of the light-cone harmonic-oscillator wave function Φ_M, which is prescribed by Brodsky, Huang, and Lepage [18], and the light-cone spin-covariant component χ, which in the rest frame should be equivalent to the light-cone spin wave function obtained by a Melosh transformation [19,20] of the equal-time static spin wave function. These wave functions depend on the light-cone variables

$$x_i=p_i^+/P^+ , \quad (2.2)$$

$$\mathbf{k}_{\perp i}=\mathbf{p}_{\perp i}-x_i\mathbf{P}_\perp , \quad (2.3)$$

and λ_i, where

$$P=(P^+,P^-,\mathbf{P}_\perp)=(P^0+P^3,(m_M^2+\mathbf{P}_\perp^2)/P^+,\mathbf{P}_\perp)$$

is the four-momentum of the meson M, and p_i are the four-momenta of the constituent quarks. The particle masses and helicities are specified by m_M and λ for the meson and $m_i(i=1,2)$ and λ_i for the two quarks. For the $\pi(K)$ and ρ mesons [i.e., $M=\pi(K),\rho$], $\Psi^M_{q,\bar{q}}$ is given by

$$\Psi^M_{q,\bar{q}}=\Psi_{M,\lambda}(x_i,\mathbf{k}_{\perp i},\lambda_i)$$
$$=\Phi_M(x_i,\mathbf{k}_{\perp i})\chi_{M,\lambda}(x_i,\mathbf{k}_{\perp i},\lambda_i) , \quad (2.4)$$

where

$$\Phi_M(x_i,\mathbf{k}_{\perp i})=A\exp\left[-\sum_{i=1}^{2}\frac{\mathbf{k}_{\perp i}^2+m_i^2}{x_i}\Big/8\beta^2\right] , \quad (2.5)$$

and the spin covariants for $\pi(K)$ and ρ are given by

$$\chi_p(x_i,\mathbf{k}_{\perp i},\lambda_i)=\bar{u}_{\lambda_1}[m_p+\not{P}]\gamma_5 v_{\lambda_2} \quad \text{for } p=\pi(K) , \quad (2.6)$$

$$\chi_{\rho,\lambda}(x_i,\mathbf{k}_{\perp i},\lambda_i)=\bar{u}_{\lambda_1}\left[m_\rho\not{\epsilon}(\lambda)+\frac{[\not{P},\not{\epsilon}(\lambda)]}{2}\right]v_{\lambda_2} \quad \text{for } \rho . \quad (2.7)$$

Here the normalization of light-cone spinors is given by $\bar{u}_{\lambda_i}(p_i)u_{\lambda_i}(p_i)=2m_i/p_i^+$ and Eq. (2.7) involves the commutator between the Feynman slashed momentum $\not{P}$ and polarization four-vector $\not{\epsilon}(\lambda)$. In Eq. (2.5) A is specified by normalizing $\Psi^M_{q,\bar{q}}$ to unity and β, the oscillator parameter, is the only dynamical constant entering the model.

As shown in the Appendix the spin covariants $\chi_{M,\lambda}$ given by Eqs. (2.6) and (2.7) are equivalent in form to the light-cone spin wave functions obtained by a two-step process consisting of a Melosh transformation from the equal-time static spin wave functions $\bar{\chi}_{M,\lambda}(\mu_i)$ in the rest frame and then boosting to the arbitrary frame. Here the equal-time static spin wave functions are

$$\bar{\chi}_p(\mu_i)=\hat{\chi}^T_{\mu_1}[i\sigma_2/\sqrt{2}]\hat{\chi}_{\mu_2} , \quad (2.8)$$

and

$$\bar{\chi}_{\rho,\lambda}(\mu_i)=\hat{\chi}^T_{\mu_1}[i(\hat{\mathbf{e}}(\lambda)\cdot\boldsymbol{\sigma})\sigma_2/\sqrt{2}]\hat{\chi}_{\mu_2} , \quad (2.9)$$

for $p=\pi(K)$ and ρ, respectively, where $\hat{\chi}_{\mu_i}$ is the two-component Pauli spinor and the superscript T is the transpose operation. The canonical spin projection μ_i is related to the helicity λ_i by a Melosh rotation in the arbitrary frame and is equal to helicity in the rest frame. The polarization vectors $\hat{\mathbf{e}}(\lambda)$ are the space components of the polarization four-vectors $\epsilon(\lambda)$ in the rest frame and have the components $\hat{\mathbf{e}}(\pm)=\mp(1,\pm i,0)/\sqrt{2}$, $\hat{\mathbf{e}}(0)=(0,0,1)$.

Equivalence between Eqs. (2.6) and (2.8) as well as Eqs. (2.7) and (2.9) mentioned above can be shown by substituting for μ_1 and μ_2 at equal time (t) by λ_1 and λ_2 at equal light-cone (LC) time:

$$\begin{bmatrix}|p_i,\uparrow\rangle_t\\|p_i,\downarrow\rangle_t\end{bmatrix}=U_{\mathcal{M}}\begin{bmatrix}|p_i,\uparrow\rangle_{\rm LC}\\|p_i,\downarrow\rangle_{\rm LC}\end{bmatrix} , \quad (2.10)$$

where

$$U_{\mathcal{M}}=\frac{1}{[2p_i^+(p_i^0+m_i)]^{1/2}}\begin{bmatrix}p_i^++m_i & -p_i^1-ip_i^2\\ p_i^1-ip_i^2 & p_i^++m_i\end{bmatrix} . \quad (2.11)$$

Equation (2.11) is the unitary Melosh transformation.

Now, for the a_1 the issue is the spin covariant χ_{a_1}. Since the a_1 is an axial-vector meson it is tempting to use the simple ansatz of inserting a γ_5 in the vector-meson spin covariant given by Eq. (2.7): i.e.,

$$\chi_{a_1,\lambda}(x_i,\mathbf{k}_{\perp i},\lambda_i)=\bar{u}_{\lambda_1}\left[\left[m_{a_1}\not{\epsilon}(\lambda)+\frac{[\not{P},\not{\epsilon}(\lambda)]}{2}\right]\gamma_5\right]v_{\lambda_2} . \quad (2.12)$$

However, we find that Eq. (2.12) is not equivalent to the light-cone spin wave function obtained from a Melosh transformation of the equal-time static spin wave function having the standard quark-model assignment. This is because the a_1 has nonzero angular momentum while Eq. (2.12) is based upon Eq. (2.7) which describes a zero angular momentum vector meson. The correct form which describes nonzero orbital angular momentum and is consistent with the Melosh transformed spin wave function is

$$\chi_{a_1,\lambda}(x_i,\mathbf{k}_{\perp i},\lambda_i)$$
$$=\bar{u}_{\lambda_1}\left[(m_{a_1}+\not{P})\left[\frac{k\cdot P}{m_{a_1}}\not{\epsilon}(\lambda)+\frac{[\not{\epsilon}(\lambda),\not{k}]}{2}\right]\gamma_5\right]v_{\lambda_2} , \quad (2.13)$$

where the relative momentum is defined as

$$k=(p_2-p_1)/2 . \qquad (2.14)$$

As in the cases of the pseudoscalar and vector mesons, one can also verify Eq. (2.13) by a straightforward substitution using Eq. (2.11) involving the static equal-time spin wave function:

$$\overline{\chi}_{a_1,\lambda}(\mathbf{k},\mu_i)=\hat{\chi}^T_{\mu_1}\left[\left[\frac{\sqrt{3}}{2}\hat{\mathbf{e}}(\lambda)\cdot(\mathbf{k}\times\sigma)\right]\sigma_2\right]\hat{\chi}_{\mu_2} . \qquad (2.15)$$

In the same spirit of Refs. [9] and [10], we construct a simple model wave function for a_1 as a product of Eqs. (2.13) and (2.5). A more formal derivation of Eqs. (2.6), (2.7), and (2.13) is presented in the Appendix. Further, we note that our results are consistent with the vertex functions based on the time-ordered diagrams for conventional perturbation theory in the infinite-momentum frame [21].

III. NUMERICAL RESULTS AND COMPARISON WITH QCD SUM-RULE AND LATTICE QCD RESULTS

The meson's quark distribution amplitude is defined to be the probability amplitude for finding quarks in the $L_z=0$ (s-wave) projection of the wave function collinear up to the scale Q [13]:

$$\phi_{M,\lambda}(x_i,Q)=\int^Q[d^2k_\perp]\Psi_{M,\lambda}(x_i,\mathbf{k}_{\perp i},\lambda_i) . \qquad (3.1)$$

For the value $\beta\approx0.30-0.45$ GeV [7,9,10], the scale is $Q\approx1$ GeV. Because of the presence of the damping Gaussian factor in Eq. (2.5), we can extend the integral limit to infinity without loss of accuracy. Using the wave functions (2.4) and (2.5)–(2.7), the following results were obtained for π, K, and ρ mesons: for the π and K,

$$\phi_\pi(\xi)=\hat{\phi}_\pi(\xi)[\mu^2+\mu\hat{\mu}+(\hat{\mu}^2-2)(1-\xi^2)/4] , \qquad (3.2)$$

$$\phi_K(\xi)=\hat{\phi}_K(\xi)[\mu_1\mu_2+(x_1\mu_2+x_2\mu_1)\hat{\mu}+(\hat{\mu}^2-2)(1-\xi^2)/4] , \qquad (3.3)$$

and for the ρ meson with longitudinal and transverse polarizations,

$$\phi_{\rho_L}(\xi)=\hat{\phi}_\rho(\xi)[\mu^2+\mu\hat{\mu}+(\hat{\mu}^2+2)(1-\xi^2)/4] , \qquad (3.4)$$

$$\phi_{\rho_T}(\xi)=\hat{\phi}_\rho(\xi)[\mu^2+\mu\hat{\mu}+\hat{\mu}^2(1-\xi^2)/4] , \qquad (3.5)$$

where

$$\mu=\frac{m}{2\beta} \text{ (equal-mass constituents case) },$$

$$\mu_1=\frac{m_1}{2\beta},\quad \mu_2=\frac{m_2}{2\beta} \text{ (unequal-mass constituents case) }, \qquad (3.6)$$

$$\hat{\mu}=\frac{m_M}{2\beta},\quad \xi=x_1-x_2 ,$$

and

$$\hat{\phi}_M(\xi)=N_M\exp[-2\mu^2/(1-\xi^2)] . \qquad (3.7)$$

For the a_1 meson, using the wave functions (2.4), (2.5), and (2.13), we obtain

TABLE I. First three nonzero moments of the quark distribution amplitude for the pion.

β	$\langle\xi^2\rangle^\pi$	$\langle\xi^4\rangle^\pi$	$\langle\xi^6\rangle^\pi$
0.30	0.20	0.08	0.04
0.36	0.25	0.11	0.06
0.43	0.37	0.20	0.12
0.45	0.46	0.26	0.16
0.47	0.64	0.39	0.25
Chernyak and Zhitnitsky (CZ)[a]	0.43	0.24	0.15
Mikhailov and Radyushkin[b]	0.26	0.13	0.08
Mikhailov and Radyushkin[c]	$\leq0.32\pm0.03$	$\leq0.20\pm0.03$	$\leq0.15\pm0.02$
Narison[d]	0.38–0.60	0.22–0.35	0.17–0.22
Guo and Huang[e]	0.36	0.17	...
Martinelli and Sachrajda[f]	0.26 ± 0.13	...	...
DeGrand and Loft[g]	0.30 ± 0.13	...	...
Daniel *et al.*[h]	0.10 ± 0.01	...	...

[a]Reference [1].
[b]Equation 45 in Ref. [17].
[c]Equation 47 in Ref. [17].
[d]Reference [2].
[e]Reference [12].
[f]Reference [4].
[g]Reference [5].
[h]Reference [6].

TABLE II. First six moments of the quark distribution amplitude for the kaon.

β	$\langle \xi \rangle^K$	$\langle \xi^2 \rangle^K$	$\langle \xi^3 \rangle^K$	$\langle \xi^4 \rangle^K$	$\langle \xi^5 \rangle^K$	$\langle \xi^6 \rangle^K$
0.30	0.058	0.17	0.026	0.06	0.014	0.03
0.36	0.046	0.20	0.025	0.08	0.015	0.04
0.43	0.034	0.25	0.023	0.11	0.016	0.06
0.45	0.030	0.26	0.023	0.12	0.017	0.07
0.47	0.026	0.28	0.022	0.14	0.017	0.08
CZ[a]	0.107	0.34	0.060	0.18	0.038	0.11
Guo and Huang[b]	0.15	0.34	0.075	0.30	· · ·	· · ·

[a]Reference [1].
[b]Reference [12].

$$\phi_{a_{1L}}(\xi)=\hat{\phi}_{a_1}(\xi)[(\hat{\mu}+2\mu)(1-\xi^2)/4] , \quad (3.8)$$

$$\phi_{a_{1T}}(\xi)=\hat{\phi}_{a_1}(\xi)\hat{\mu}[(\hat{\mu}^2-2)(1-\xi^2)/4+\hat{\mu}\mu+\mu^2]\xi/2 , \quad (3.9)$$

where $\hat{\phi}_{a_1}(\xi)$ is also given by Eq. (3.7). In comparing these quark distribution amplitudes with the QCD sum-rule and lattice QCD results, we used constituent-quark masses $m_u=m_d=0.33$ GeV and $m_s=0.45$ GeV and spin-averaged meson masses $m_{\pi(\rho)}=0.612$ GeV, $m_K=0.793$ GeV, and $m_{a_1}=1.120$ GeV. Tables I–IV summarize the first six moments of the quark distribution amplitudes of π, K, ρ_L, ρ_T, a_{1L}, and a_{1T} and list for comparison the various QCD sum-rule and lattice QCD results. The nth moment is defined by

$$\langle \xi^n \rangle_\lambda^M=\int_{-1}^{+1} d\xi\, \xi^n \phi_{M,\lambda}(\xi) . \quad (3.10)$$

Here the normalization of $\phi_{M,\lambda}(\xi)$, N_M, is fixed by the zeroth moment $\langle \xi^0 \rangle_\lambda^M \equiv 1$ if $\phi_{M,\lambda}(\xi)$ is an even function of ξ. Because Eq. (3.9) is an odd function of ξ, the lowest $n=0$ moment vanishes for the transverse a_1 and thus we present the moment ratios $\langle \xi^n \rangle_T^{a_1}/\langle \xi^1 \rangle_T^{a_1}$ in Table IV.

As shown in Table I, the present model can reproduce the pion result of Chernyak and Zhitnitsky remarkably well with the value of $\beta=0.45$ GeV. Even though the present model does not give the same level of agreement with Chernyak-Zhitnitsky ρ-meson result (see Table III) the important qualitative features of the ρ-meson quark distribution amplitude are reproduced (i.e., no camel shape). Such agreement supports the conclusion [9,10] that this model can reproduce both QCD sum-rule and lattice QCD calculation results. However, if the recent claim [6,17] is correct that the second moment for the pion is sufficiently small to remove the camel shape in the quark distribution amplitude, then the value of β should be further reduced (see Fig. 1). Such reduction might be related to nonlocal condensate contributions in the QCD sum-rule result or the unquenched approximation in the lattice QCD result. This issue needs further study to resolve. In any case, the present model qualitatively can reproduce the moments from QCD sum-rule and lattice QCD approaches by adjusting only one parameter β. For the kaon, the odd power moments do not vanish because of unequal-mass constituents (see Table II). Nevertheless, the shape of quark distribution amplitude is still predominantly governed by even power moments and the basic features of the Chernyak-Zhitnitsky kaon quark distribution amplitude can also be reproduced by this model using $\beta=0.45$ GeV (see Fig. 2). However, for larger β values the computed meson decay constants are far below the observed values (e.g., for $\beta=0.45$ GeV, $f_\pi=36$ MeV,

TABLE III. First three nonzero moments of the transverse-quark distribution amplitude for the transverse ρ meson and the ratio of the transverse to the longitudinal moments.

β	$\langle \xi^2 \rangle_T^\rho$	$\langle \xi^4 \rangle_T^\rho$	$\langle \xi^6 \rangle_T^\rho$	$\langle \xi^2 \rangle_{T/L}^\rho$	$\langle \xi^4 \rangle_{T/L}^\rho$	$\langle \xi^6 \rangle_{T/L}^\rho$
0.30	0.180	0.067	0.033	1.05	1.10	1.14
0.36	0.200	0.082	0.042	1.08	1.14	1.17
0.43	0.217	0.095	0.052	1.11	1.20	1.24
0.45	0.222	0.098	0.054	1.13	1.21	1.26
0.47	0.226	0.101	0.057	1.14	1.22	1.30
CZ[a]	0.143	0.048	0.022	0.53	0.36	0.28
Narison[b]	<0.14	<0.06	· · ·	<1.0–1.2	<1.0–1.2	· · ·

[a]Reference [1].
[b]Reference [2].

TABLE IV. First three nonzero moments of the quark distribution amplitude for the longitudinal and transverse a_1 meson.

β	$\langle \xi^2 \rangle_L^{a_1}$	$\langle \xi^4 \rangle_L^{a_1}$	$\langle \xi^6 \rangle_L^{a_1}$	$\frac{\langle \xi^3 \rangle_T^{a_1}}{\langle \xi^1 \rangle_T^{a_1}}$	$\frac{\langle \xi^5 \rangle_T^{a_1}}{\langle \xi^1 \rangle_T^{a_1}}$	$\frac{\langle \xi^7 \rangle_T^{a_1}}{\langle \xi^1 \rangle_T^{a_1}}$
0.30	0.143	0.046	0.020	0.376	0.181	0.100
0.36	0.155	0.053	0.024	0.414	0.216	0.127
0.43	0.165	0.059	0.028	0.454	0.255	0.159
0.45	0.167	0.061	0.029	0.465	0.266	0.169
0.47	0.169	0.062	0.030	0.477	0.278	0.179
CZ[a]	0.04–0.07	...	...	...	...	...
Narison[b]	0.07–0.08	0.03–0.04	0.02	...	...	...

[a]Reference [1].
[b]Reference [2].

and $f_K = 97$ MeV). Finally, the significant reduction of the second moment of the longitudinal a_1 meson is also reproduced by the present model (see Table IV).

IV. CONCLUSIONS

In this paper we constructed a Lorentz-invariant light-cone wave function for the axial-vector a_1 meson following the same procedure applied for the zero angular momentum pseudoscalar and vector mesons. We found that the proper spin covariant consistent with the Melosh transformed light-cone spin wave function for the a_1 is given by Eq. (2.13) instead of Eq. (2.12). While more complete fundamental results are not yet available to determine whether our light-cone model wave functions are good approximations to the actual QCD solutions, a comparison has been made for the first six moments of the π, K, ρ_L, ρ_T, a_{1L}, and a_{1T} quark distribution amplitudes obtained from the light-cone model wave functions and the recent QCD sum-rule and lattice QCD results. It is both interesting and significant to note that the basic features of the quark distribution amplitudes obtained from nonperturbative QCD methods can be reproduced by the present model which uses only one parameter β. Finally, we note that in Ref. [21], the a_1 form factors and transition form factor in the decay process $a_1 \to \pi\gamma$ using the vertex function consistent with Eq. (2.13) were calculated and found to quantitatively agree with QCD sum-rule predications. In summary, this model provides a remarkably good description of static properties for the π and K mesons and reproduces the basic features of the QCD sum-rule results for four different mesons. Because of its conceptual and computational simplicity it merits

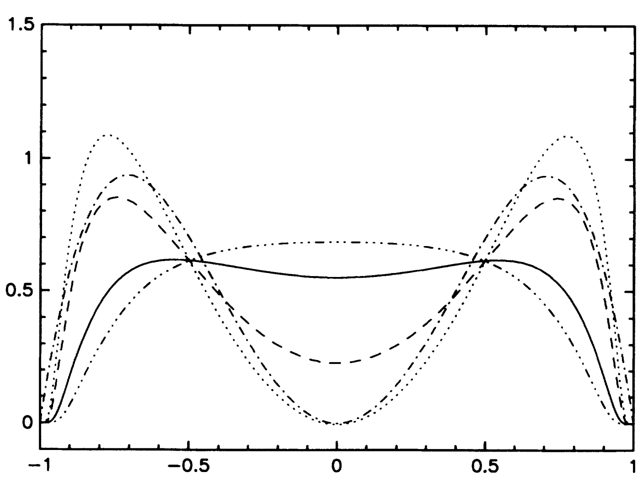

FIG. 1. The quark distribution amplitude for the pion. The following line codes are used: dash-dot-dot-dotted, $\beta=0.30$; solid, $\beta=0.36$; dashed, $\beta=0.43$; dotted, $\beta=0.45$; dash-dotted, for the QCD sum-rule results of Chernyak and Zhitnitsky (Ref. [1]).

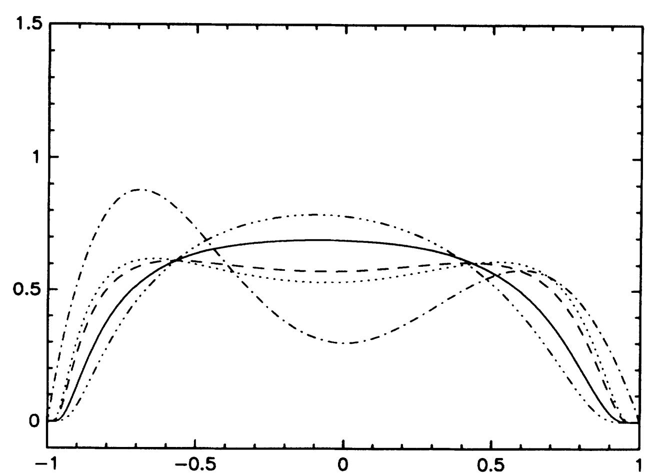

FIG. 2. The quark distribution amplitude for the kaon. The line codes are the same as Fig. 1.

further consideration and should be comprehensively tested by applications to other hadron systems.

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APPENDIX: COVARIANT FORM OF MESON WAVE FUNCTIONS

In this appendix we derive the explicit covariant wave function for pseudoscalar, vector and axial-vector mesons based on the standard quark-model assignments [22]. The meson wave function in the rest frame $\hat{P}=(M,0,0,0)$ can be written in terms of the standard Clebsch-Gordan coefficient $(j_1,j_2,\mu_1,\mu_2|j,\mu)$ as

$$\begin{aligned}\Psi_{M,\mu}(x_i,\mathbf{k}_{\perp i},\mu_1,\mu_2)&=\sum_{\mu_s,\mu_l}\sqrt{4\pi}Y_{l,m_l}(\hat{\mathbf{k}})(\tfrac{1}{2},\tfrac{1}{2},\mu_1,\mu_2|s,\mu_s)(s,l,\mu_s,\mu_l|j,\mu)\Phi_M(x_i,\mathbf{k}_{\perp i})|k|^l\\&=\hat{\chi}^T_{\mu_1}(\Gamma_{M,\mu})\hat{\chi}_{\mu_2}\Phi_M(x_i,\mathbf{k}_{\perp i})\\&=(\bar{\chi}_{M,\mu})_{\mu_1,\mu_2}\Phi_M(x_i,\mathbf{k}_{\perp i})\ ,\end{aligned} \tag{A1}$$

where $\Gamma_{M,\mu}$ are the operators and

$$\begin{aligned}\Gamma_p&=\frac{1}{\sqrt{2}}i\sigma_2\ ,\\\Gamma_{v,\mu}&=\frac{i}{\sqrt{2}}[\hat{\mathbf{e}}(\mu)\cdot\boldsymbol{\sigma}]\sigma_1\ ,\\\Gamma_{a,\mu}&=\tfrac{1}{2}[\sqrt{3}\hat{\mathbf{e}}(\mu)\cdot(\mathbf{k}\times\boldsymbol{\sigma})]\sigma_2,\end{aligned} \tag{A2}$$

for pseudoscalar, vector, and axial-vector mesons respectively.

First, we specify the spin-state vector of meson M at equal time as $|\bar{\chi}_{M,\mu}\rangle_t$:

$$\begin{aligned}|\bar{\chi}_{M,\mu}\rangle_t&=(\bar{\chi}_{M,\mu})_{\mu_1,\mu_2}|\mu_1\rangle_t|\mu_2\rangle_t\\&=(|p_1,\uparrow\rangle_t,|p_1,\downarrow\rangle_t)\Gamma_{M,\mu}\begin{bmatrix}|p_2,\uparrow\rangle_t\\|p_2,\downarrow\rangle_t\end{bmatrix}\ .\end{aligned} \tag{A3}$$

Next, we use the Melosh transformation $U_{\mathcal{M}}$ given by Eq. (2.11) to relate equal-time states $|\mu_i\rangle_t$ and the light-cone states $|\lambda_i\rangle_{LC}$. Thus, we have the matrix

$$|\bar{\chi}_{M,\mu}\rangle_t=(|p_1,\uparrow\rangle_{LC},|p_1,\downarrow\rangle_{LC})\Omega_{M,\mu}\begin{bmatrix}|p_2,\uparrow\rangle_{LC}\\|p_2,\downarrow\rangle_{LC}\end{bmatrix}\ , \tag{A4}$$

where $\Omega_{M,\mu}=(U^T_{\mathcal{M}})_{\lambda_1,\mu_1}(\Gamma_{M,\mu})(U_{\mathcal{M}})_{\mu_2,\lambda_2}$. Now, we use the fact that the Melosh transformation is simply the product of the light-cone spinors and the equal-time spinors:

$$(U^T_{\mathcal{M}})_{\lambda_1,\mu_1}=\left[\frac{p_1^+}{2m_1}\right]^{1/2}\bar{u}_{\lambda_1}(p_1)u^c_{\mu_1}(p_1)\ , \tag{A5}$$

and

$$(U_{\mathcal{M}})_{\mu_2,\lambda_2}=\left[\frac{p_2^+}{2m_2}\right]^{1/2}\bar{v}^c_{\mu_2}(p_2)v_{\lambda_2}(p_2)\ , \tag{A6}$$

where u_{λ_1} and v_{λ_2} are the light-cone spinors used in Sec. II and $u^c_{\mu_1}$ and $v^c_{\mu_2}$ are the equal-time spinors such as

$$(u^c_{1/2},u^c_{-1/2},v^c_{1/2},v^c_{-1/2})=\frac{1}{\sqrt{2m(E+m)}}\begin{bmatrix}E+m&0&p_x-ip_y&-p_z\\0&E+m&-p_z&-(p_x+ip_y)\\p_z&p_x-ip_y&0&-(E+m)\\p_x+ip_y&-p_z&E+m&0\end{bmatrix}\ . \tag{A7}$$

However, the crucial observation of the present, more elegant derivation shown in this appendix is

$$\begin{aligned}(U^T_{\mathcal{M}})_{\lambda_1,\mu_1}&=\left[\frac{p_1^+}{2m_1}\right]^{1/2}\bar{u}_{\lambda_1}(p_1)\\&\quad\times\left[\frac{2m_1}{E_1+m_1}\right]^{1/2}u^c_{\mu_1}(\mathbf{p}_1=0)\\&=\left[\frac{p_1^+}{E_1+m_1}\right]^{1/2}\bar{u}_{\lambda_1}(p_1)u^c_{\mu_1}(\mathbf{p}_1=0)\ ,\end{aligned} \tag{A8}$$

and

$$(U_{\mathcal{M}})_{\mu_2,\lambda_2}=\left[\frac{p_2^+}{E_2+m_2}\right]^{1/2}\bar{v}^c_{\mu_2}(\mathbf{p}_2=0)v_{\lambda_2}(p_2)\ . \tag{A9}$$

Thus, we find that

$$(\Omega_{M,\mu})_{\lambda_1,\lambda_2}=\bar{u}_{\lambda_1}(p_1)\mathbf{X}_{M,\mu}(\hat{P})v_{\lambda_2}(p_2)\ , \tag{A10}$$

where

$$\begin{aligned}\mathbf{X}_{M,\mu}(\hat{P})&=\left[\frac{p_1^+}{E_1+m_1}\right]^{1/2}\left[\frac{p_2^+}{E_2+m_2}\right]^{1/2}u^c_{\mu_1}(\mathbf{p}_1=0)\\&\quad\times(\Gamma_{M,\mu})_{\mu_1,\mu_2}\bar{v}^c_{\mu_2}(\mathbf{p}_2=0)\ .\end{aligned} \tag{A11}$$

From this, it is not very difficult to show that $\mathbf{X}_{M,\mu}(\hat{P})$ is given by

$$\mathbf{X}_{M,\mu}(\hat{P})=N^{-1}\mathbf{\Gamma}_{M,\mu}(\hat{P}) , \tag{A12}$$

where N is given by

$$N=[8(p_1^0+m_1)(p_2^0+m_2)/x_1x_2]^{1/2} , \tag{A13}$$

and

$$\mathbf{\Gamma}_p(\hat{P})=m_p(1+\beta)\gamma_5 , \tag{A14a}$$

$$\mathbf{\Gamma}_{v,\mu}(\hat{P})=m_v(1+\beta)\not\epsilon(\mu) , \tag{A14b}$$

$$\mathbf{\Gamma}_{a,\mu}(\hat{P})=m_a(1+\beta)\left[\frac{k\cdot\hat{P}}{m_a}\not\epsilon(\mu)+\frac{[\not\epsilon(\mu),\not k]}{2}\right]\gamma_5 . \tag{A14c}$$

To obtain Eq. (A14c) we have used the identity $\not a\not b=a\cdot b-i\sigma_{\mu\nu}a^\mu b^\nu$ and the subsidiary condition $\epsilon\cdot P=0$.

Hence the light-cone bound-state wave function in an arbitrary frame becomes

$$\Psi_{M,\lambda}(x_i,\mathbf{k}_\perp,\lambda_i)=N^{-1}\bar{u}_{\lambda_1}(p_1)\mathbf{\Gamma}_{M,\lambda}(P)\times v_{\lambda_2}(p_2)\Phi(x_i,\mathbf{k}_{\perp i}) , \tag{A15}$$

where $\mathbf{\Gamma}_{M,\lambda}(P)$ in the arbitrary frame are given by

$$\mathbf{\Gamma}_p(P)=[m_p+\not P]\gamma_5 ,$$

$$\mathbf{\Gamma}_{v,\lambda}(P)=[m_v+\not P]\not\epsilon(\lambda)=m_v\not\epsilon(\lambda)+\frac{[\not P,\not\epsilon(\lambda)]}{2} , \tag{A16}$$

$$\mathbf{\Gamma}_{a,\lambda}(P)=(m_a+\not P)\left[\frac{k\cdot P}{m_a}\not\epsilon(\lambda)+\frac{[\not\epsilon(\lambda),\not k]}{2}\right]\gamma_5 .$$

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