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Coupling first-order phase transitions to curvature-squared inflation

Luca Amendola, Salvatore Capozziello, Marco Litterio, and Franco Occhionero
Osservatorio Astronomico di Roma, Viale del Parco Mellini 84, I-00136 Rome, Italy
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A new way to couple first-order phase transitions to inflation is proposed. The mechanism, dubbed getaway inflation, bypasses the problem of the "graceful exit" by letting the inflationary phase of the background have a classical end. At the same time, a stage of bubble production via semiclassical tunneling occurs, allowing speculations on the role of bubbles on the early structure formation. A realization of the proposed process is found in a nonminimally coupled higher-order gravity theory which by itself deserves further investigation.

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I. INTRODUCTION

The decennial history of inflation appears as a sequence of rising, improving, and quitting of a small but fruitful set of ideas. The very beginning of inflation rested on the key words "phase transition," and all the play was based on the energetics of a supercooling universe and subsequent latent heat release [1] (old inflation). Later, it appeared that in fact a first-order phase transition caused more trouble than was able to be solved, and new mechanisms were proposed where essentially classical evolution took place: new inflation [2] and chaotic inflation [3]. Bubbles and related issues were suddenly discarded (or at least pushed outside of the present horizon), even because the observed Universe did not seem to keep any trace of such bulky objects. Recently, however, the binomial inflation and phase transition have gained new attention (for an "extended" review, see Ref. [4]). The extended inflation [5] (EI) proposed by La and Steinhardt suggests overcoming the "graceful exit" problem by slowing the inflationary expansion of that part of the Universe which remains trapped in the false-vacuum phase, and that we simply refer to as "background." While in the old models the background was supposed to undergo de Sitter inflation, i.e., an exponential growth of its dimensions, in EI it is found that nonminimal coupling (NMC) of matter to gravity allows a power-law inflationary expansion of the cosmic scale factor, $a \sim t^p$, $p>1$. In this case, the bubbles of a true vacuum, which semiclassically nucleate out of the false vacuum and expand with a Friedmann-like behavior rolling down toward the true minimum of the matter potential, are able to fill the greatest part of the disposable space. It is then possible to derive the fraction of space occupied by the bubbles at any time, and find the conditions to get values around unity. Several models have already been put forward which make use of this mechanism [6]. In Ref. [7] density and gravitational quantum fluctuations in EI are analyzed, while Ref. [8] deals with the impact of EI on baryogenesis. It is worth noting that an important feature is common to both old inflationary schemes and EI: in both cases the background is forever inflating, even if asymptotically almost all the space gets filled by nucleated bubbles. Very recently, another model has been proposed in Ref. [9] which exploits two coupled fields in pure Einstein gravity to modulate the bubble nucleation rate and to complete the phase transition. This model bears some resemblance to the one to be exposed here. However, contrary to our scenario, there again the background does not have a classical way out of its false-vacuum state: rather, the inflation is completed solely by semiclassical bubble production.

Here we propose a different way to gracefully exit from eternal inflation which at the same time keeps alive the idea of first-order phase transitions. Suppose we have two coupled fields ϕ and ψ, the former being the usual inflaton, and that we are on the false-vacuum minimum at $\phi=\phi_F$. During this phase, if the dynamics of the Universe along the ψ direction is slow enough, the description of bubbles nucleating from the false vacuum toward the true one at ϕ_T is still valid. This phase is very similar to old inflation. The difference arises when one considers the motion along the ψ direction: after some amount of time, the background, i.e., that part of the Universe which has not yet performed the phase transition, reaches a point where the potential barrier between ϕ_F and ϕ_T no longer exists. Then, the background may roll down classically, back along the ψ direction, toward the true-vacuum state. Thus, the background shares a common fate with the bubbles, becoming dynamically indistinguishable from them. In a word, we can say that the bubbles evolve semiclassically, while the background escapes purely classically from the false vacuum state. A pictorial representation of a ϕ,ψ potential which does the job is shown in Fig. 1. The false vacuum clearly disappears at some value of ψ, where a classical gateway connects it to the true vacuum. Then the "graceful exit" problem cannot even be posed, since there is no longer a forever inflating background. However, the presence of the bubble walls nucleated in the first phase can still be exploited as initial seeds for large-scale structure formation [6]. We call this scenario "getaway inflation."

The question now is the following: is there a simple model which can account for the features described above? The essential ingredient is a special coupling between the two fields able to shrink the barrier for ϕ. It is

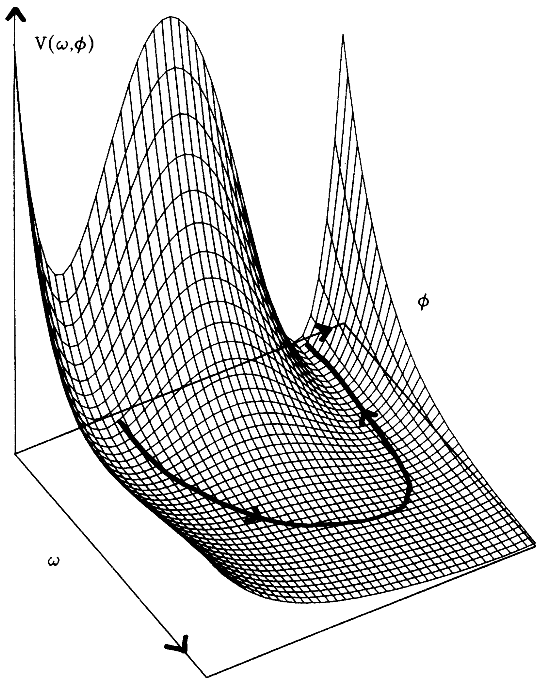

FIG. 1. Potential for ϕ and ψ. A classical path links the (unstable) false vacuum (on the left) with the true vacuum (on the right). This is the same potential (21): one axis is ϕ, the other one is $\psi \equiv \omega = \frac{1}{2}\ln|1-S/3|$.

clear that a simple decoupled model $V(\phi,\psi) = V_1(\phi)+V_2(\psi)$ does not work. Instead of looking for some special coupling among the fields involved, we propose in the next section a model which naturally holds the required features and which makes a step forward in the investigation of unconventional gravity theories, such as quadratic gravity Lagrangians and nonminimal couplings. The proposed model suffers from the same fine-tuning problem [10] of all viable inflationary theories put forward so far; a numerical estimate of the potential flatness required is given in Sec. II.

II. A HIGHER-ORDER NONMINIMAL THEORY

Higher-order gravity Lagrangians have been intensively studied in recent years (see, e.g., Ref. [11]). They are required as a low-energy limit of many theories attempting to quantize gravity and, as an interesting cosmological fallout, it has been recognized that higher-order corrections to the Hilbert Lagrangian behave like a kind of scalar field, thus producing inflation [12], removing the initial singularity [13], creating dark matter [14], modifying Newton's law [15], and so on. Another simple example of the generalization of Einstein's gravity theory is the nonminimal coupling (NMC) of matter to gravity (see, e.g., Ref. [16]), in the various forms of Jordan-Brans-Dicke (JBD) theory [17], of induced gravity [18], of Kaluza-Klein four-dimensionally reduced theory [19] and so on. The NMC generates new classes of inflation [20] (power-law, exactly de Sitter, chaotic) which only depend on the coupling constant, and it also has been proposed recently to account for the apparent periodicity in deep redshift surveys [21], and to reconcile cosmic-string production with inflation [22] (notice that the last paper in Ref. [6] makes extended inflation in a R^2 plus JBD theory). Here we take a logical step forward and let the quadratic term in the Ricci scalar in the gravity Lagrangian be nonminimally coupled to matter. Consider the Lagrangian (in Planck units, $c=\hbar=G=1$)

$$
\begin{aligned}
L(R,\phi) &= -R + e^{\tau\phi}\frac{R^2}{6M^2} + 16\pi L_m(\phi) , \\
L_m(\phi) &= \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi) ,
\end{aligned}
\tag{1}
$$

where τ and M are positive constants with physical dimensions mass^{-1} and mass, respectively. The form imposed to the coupling is a convenient one, but it is equivalent to a $\phi^2 R^2$ term, which closely reminds the usual NMC. As we will see later the relevant features of the model do not depend on this specific choice. We recall that the most general minimally coupled Lagrangian quadratic in the curvature and with fourth-order (in the metric) field equations is a linear combination of four terms: (a) R^2, (b) $R_{\alpha\beta}R^{\alpha\beta}$, (c) $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$, (d) $\epsilon_{\alpha\beta\gamma\delta}R^{\alpha\beta\mu\nu}R^{\gamma\delta}{}_{\mu\nu}$ (note that other terms such as $R\Box R$ which are quadratic in the curvature produce sixth-order equations in the metric, see Ref. [23]). Fortunately, things can be simplified. The term (d) (and all similar combinations of the totally antisymmetric tensor ϵ and of the Riemann tensor) is a total divergence in four dimensions [24] and does not contribute to the field equations; the terms (a), (b), and (c) can be combined in four dimensions in the Gauss-Bonnet term, $L_{\rm GB}=R^2-4R_{\alpha\beta}R^{\alpha\beta}+R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$, which is a total divergence; and, finally, in conformally flat metrics, where the Weyl conformal tensor $C_{\alpha\beta\gamma\delta}$ vanishes, another algebraic relation holds: $R^2-6R_{\alpha\beta}R^{\alpha\beta}+3R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}=3C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}=0$. Thus, in a conformally flat four-dimensional space-time the R^2 term is the only independent quadratic term. The R field is sometimes called a scalaron [12] because, as will be shown later, it behaves as a scalar field with mass M. However, it is clear that the Gauss-Bonnet term is no longer a total divergence when nonminimally coupled to an external field ϕ, and therefore it cannot be used to simplify the general Lagrangian. It then turns out that the most general quadratic Lagrangian with NMC is of the form[1]

$$
\begin{aligned}
L(R,\phi) = -R + \alpha\phi^2R^2 + \beta\phi^2R_{\alpha\beta}R^{\alpha\beta} \\
+\gamma\epsilon_{\alpha\beta\gamma\delta}R^{\alpha\beta\mu\nu}R^{\gamma\delta}{}_{\mu\nu} ,
\end{aligned}
\tag{2}
$$

plus the usual JBD term. This theory can be recast by means of a conformal transformation on the metric in a pure Einsteinian form plus "extra fields," but only if

[1] As usual it is tacitly assumed that there are no derivative couplings; furthermore, the functional form of the couplings could be generalized.

$\beta=\gamma=0$ can this "extra matter" be put in the form of scalar fields [25]; for $\beta\neq 0$ (but $\gamma=0$), for example, one needs to introduce a tensor field of spin 2. Here we confine ourselves to the simplest possibility $\beta=\gamma=0$ which, nevertheless, is shown to contain interesting features. Inclusion of a JBD term will not change the main conclusions.

Let us stress that the scalar-scalaron coupling in Eq. (1) naturally arises in a multidimensional quadratic gravity, where the dilation field gets coupled with R^2 (see, e.g., Ref. [25]). Suppose we start with the $(4+D)$-dimensional action

$$A_{4+D}(R)=\int\sqrt{-g}\,d^{4+D}x\left[R-\frac{R^2}{6M^2}\right], \tag{3}$$

where $R=R_A^{\,A}$ is the $(4+D)$-dimensional curvature scalar and $g=\det(g_{AB})$. Here upper case letters run in the range $(0,1,\ldots,3+D)$, lower case latin letters in the range $(4,5,\ldots,3+D)$, and greek letters in $(0,\ldots,3)$. We assume a $M_4\times S_D$ manifold with metric $g_{AB}=\mathrm{diag}(g_{\mu\nu},h_{ij})$, where h_{ij} is the D-dimensional metric of the compact homogeneous and isotropic space S_D (internal space) with scale factor e^{ϕ}. The curvature scalar can be split into two parts: $R=R_4+R_D$, where, in the homogeneous case,

$$\begin{aligned}R_4&=-6(\dot H+2H^2)\,,\\ R_D&=-D[2\ddot\phi+(D-1)\dot\phi^2+6H\dot\phi+(D-1)e^{-2D\phi}]\,,\end{aligned} \tag{4}$$

and $H=\dot a/a$ is the Hubble parameter. The last term in R_D takes into account the spatial curvature of the internal space. We may then perform the integration over the internal space dimensions (dimensional reduction):

$$\begin{aligned}A_4=\int\sqrt{-g_4}\Bigg[&e^{D\phi}R-e^{D\phi}\frac{R^2}{6M^2}+De^{D\phi}\Bigg[(D+1)\phi_{,\mu}\phi^{,\mu}-(D-1)e^{-2\phi}\\ &+\frac{1}{3M^2}[-(D-1)R\phi_{,\mu}\phi^{,\mu}-2R_{,\mu}R^{,\mu}+(D-1)Re^{-2\phi}]\Bigg]-\frac{R_D^2}{6M^2}\Bigg]\,,\end{aligned} \tag{5}$$

where among various kinds of dilaton-scalaron couplings a sector analogous to (1) is put in evidence. However, we will not follow here the multidimensional approach because we are going to interpret ϕ as an inflaton field with its own quartic bistable potential.[2]

As we said above, the constant M in Eq. (1) is the Starobinsky scalaron mass [12] and when $\tau=0$ it acts as a mass for the scalar field R. The effective scalaron mass in our model will be $M_{\rm eff}=Me^{-\tau\phi/2}$. Notice that it is not required to have $\phi\to-\infty$ to recover usual Einstein gravity: it is sufficient in fact to let $R\to 0$, as is commonly the case in cosmological models.

The gravitational field equations derived from the Lagrangian (1) are

$$3G_{\mu\nu}-S(R_{\mu\nu}-\tfrac{1}{4}Rg_{\mu\nu})+(S_{;\mu\nu}-g_{\mu\nu}\Box S)=24\pi T_{\mu\nu}\,, \tag{6}$$

where $S\equiv Re^{\tau\phi}/M^2$ and $G_{\mu\nu}\equiv R_{\mu\nu}-g_{\mu\nu}R/2$. The trace of the Einstein equations is a second-order equation (fourth order in the metric) where the S field plays as a Klein-Gordon scalar field. We have

$$\Box S+M^2e^{-\tau\phi}S+8\pi T(\phi)=0\,, \tag{7}$$

where $T=-\phi_{,\alpha}\phi^{,\alpha}+4V(\phi)$ is the trace of the energy-momentum tensor for ϕ. The field equation for ϕ is

$$\Box\phi+V'(\phi)-\frac{M^2\tau}{96\pi}S^2e^{-\tau\phi}=0\,, \tag{8}$$

where $V'\equiv\partial V/\partial\phi$. The last two equations are valid in any given metric and constitute our dynamical system. They are sufficient to completely determine the problem if we work in a homogeneous and isotropic flat metric $g_{\mu\nu}=\mathrm{diag}(1,-a^2,-a^2,-a^2)$ with the scale factor $a(t)$. The (0,0) component of Einstein equations is a Hamiltonian constraint useful as a check during numerical integration. In the above metric it reads

$$H\dot S=3H^2-8\pi\rho(\phi)-S\left[\frac{M^2e^{-\tau\phi}S}{12}+H^2\right], \tag{9}$$

where $\rho(\phi)\equiv T_{00}=\frac{1}{2}\dot\phi^2+V(\phi)$. It is easy to generalize the coupling to a generic term $f(\phi)R^2/6M^2$ in the Lagrangian (1). Then the scalaron field is to be defined as $S\equiv f(\phi)R/M^2$. While Eq. (6) remains untouched, the last term on the right-hand side of Eq. (8) becomes $-M^2S^2(df/d\phi)/(96\pi f^2)$.

The typical potential for the inflaton must show two minima: a local one (false vacuum) at $\phi=\phi_F$ and a global one (true vacuum) at $\phi=\phi_T$. Putting, for simplicity, $\phi_F=0$ we may write

$$V(\phi)=V_F\left[1+\frac{\lambda\phi^2}{\phi_T^4}(\phi^2+2b\phi_T\phi+c\phi_T^2)\right], \tag{10}$$

where $V_F=V(\phi_F=0)$ and λ is a positive dimensionless constant. The further condition that $V(\phi_T)=V'(\phi_T)=0$ implies $b=1/\lambda-1$ and $c=-3/\lambda+1$, with $\lambda>3$ for mathematical consistency. The height of the barrier (located at $\phi_M=c\phi_T/2$) between ϕ_T and ϕ_F grows with λ.

[2] The word "inflaton" is somewhat ambiguous in the context of two-field inflations where both fields cooperate at the same time; we leave here the denomination "inflaton" to the field which performs the first-order phase transiton, even if it is along the second field that the slow-rolling occurs. Eventually, we should speak of a *transiton* and an inflaton.

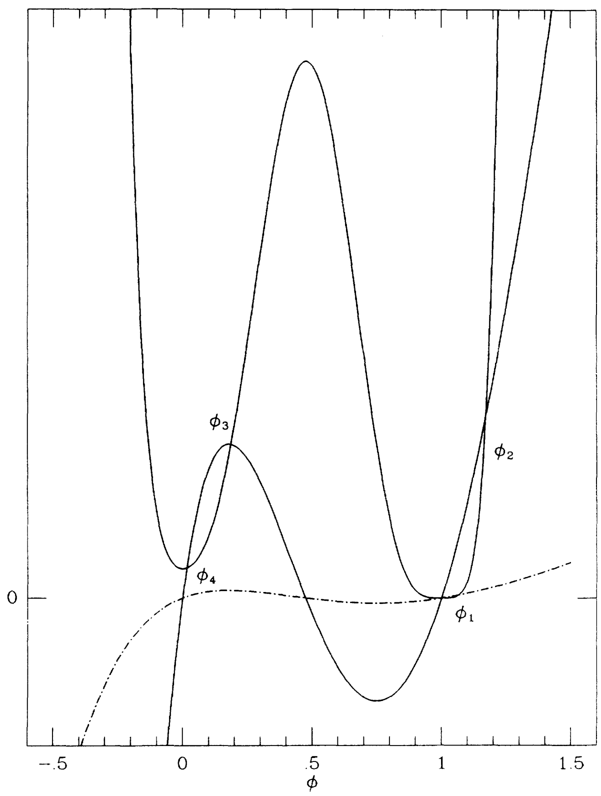

FIG. 2. Graphic solutions of Eq. (12). The solid lines are the left- and right-hand sides of (12). ϕ_4 is a (meta)stable false vacuum while ϕ_1 is the true vacuum. The broken line is the left-hand side of (12) for a lower value of M^2 which verifies (15): now there is only the stable true vacuum at ϕ_1 and the getaway scenario can be implemented.

Let us now search for the equilibrium points of the dynamical system (7),(8). Equilibrium points are found solving the algebraic system

$$S=-\frac{32\pi}{M^2}V(\phi)e^{\tau\phi}\ , \qquad V'(\phi)=\frac{M^2\tau}{96\pi}S^2e^{-\tau\phi}\ , \tag{11}$$

from which we get the equation

$$\beta V'e^{-\tau\phi}=V^2(\phi)\ , \tag{12}$$

where $\beta\equiv 3M^2/(32\pi\tau)$. A graphic solution is shown in Fig. 2. Notice that, taking Eq. (9) into account, the first equation in (11) is equivalent to the inflationary de Sitter condition $R=-12H^2$. This means that all the equilibrium points of the model (except the one with zero energy) are exact inflationary points, contrary to the (anomaly-free) Starobinsky model where no exact stationary solutions are allowed. We get for (12) two solutions (labeled ϕ_1,ϕ_2) for any value of the parameters, while two further solutions (ϕ_3,ϕ_4) are occasionally present depending on the specific choice of M,λ,V_F,ϕ_T. The true-vacuum state $\phi=\phi_T, S=0$ coincides with ϕ_1. Along a trajectory reaching this point the R^2 term becomes negligible and a Friedmann behavior is realized. Approximating $V(\phi)$ around ϕ_T and $\phi_F=0$ with two parabolas, we can give approximate values for the three other solutions ϕ_2,ϕ_3,ϕ_4 of Eq. (12). We have

$$\phi_2=\phi_T+\delta e^{-\tau\phi_T/3}\ , \tag{13}$$

where (for $\lambda\gg 1$)

$$\delta^3=\frac{3M^2\phi_T^2}{16\pi V_F\lambda\tau}>0\ , \tag{14}$$

while $\phi_{3,4}$ are solutions of an uninteresting second-order equation and are not real if

$$\frac{M^2}{\tau V_F\phi_T}\left[\frac{\lambda}{2}\right]^{1/2}<\frac{32\pi G}{3} \tag{15}$$

(we explicitly inserted G to check for dimensionality and for later use). As we show later, Eq. (15) is our "getaway" condition: a classical path between the false and the true vacua is found only when the condition (15) is satisfied (broken line in Fig. 2), i.e., when ϕ_3 and ϕ_4 do not exist. The values of S (and R) corresponding to the equilibrium points are to be found using one of Eqs. (11).

Before studying the nature of these equilibrium points, we will derive a potential $U(\phi,S)$ for our dynamical system. In order to do this we must perform a conformal transformation on the metric $g_{\mu\nu}$. As has been demonstrated in several papers [27], a conformal rescaling

$$\hat{g}_{\mu\nu}=e^{2\omega}g_{\mu\nu}\ , \tag{16}$$

with

$$\omega=\tfrac{1}{2}\ln\left|1-\frac{S}{3}\right|\ , \tag{18}$$

recasts a theory with Lagrangian $L=L(R,\phi)$ into an ordinary Einsteinian theory with two scalar fields ϕ,ω (this new metric is sometimes called the Einstein frame). In other words, the dynamical degrees of freedom of a fourth-order gravity theory are incorporated into the new field ω. With the rescaling specified by

$$e^{2\omega}=|\ln(1-S/3)|\ , \tag{18}$$

we get the Einstein equations (in the new metric)

$$\hat{G}_{\mu\nu}=\frac{1}{|L'|}\left[\hat{g}_{\mu\nu}\frac{L-L'R}{2L'}+8\pi(\phi_{,\mu}\phi_{,\nu}-\tfrac{1}{2}\hat{g}_{\mu\nu}\hat{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta})\right] +6(\omega_{,\mu}\omega_{,\nu}-\tfrac{1}{2}\hat{g}_{\mu\nu}\hat{g}^{\alpha\beta}\omega_{,\alpha}\omega_{,\beta})\ , \tag{19}$$

where in $L(\phi,R)$ are not included the kinetic terms of ϕ. A further rescaling $\tilde{\omega}=\sqrt{3/(4\pi G)}\omega$ would make the kinetic sector of the right-hand side of (19) completely canonical; $\tilde{\omega}$ also has the correct dimension of a mass and will be later employed. The field equations for $\tilde{\omega}$ and ϕ are in the canonical Klein-Gordon form. The conformal potential is now (notice that in our definition $L'<0$)

$$U(\phi,\omega)=\frac{L'R-L}{16\pi L'^2}\ . \tag{20}$$

Inserting ω as derived from (17) we obtain

$$U(\phi,\omega)=e^{-4\omega}\left[\frac{3}{32\pi}M^2e^{-\tau\phi}(1-e^{2\omega})^2+V(\phi)\right]. \tag{21}$$

It is not difficult to check that the equilibrium points $\partial U/\partial\phi=\partial U/\partial\omega=0$ are the same as those previously found. However, the expression (21) reveals us a new equilibrium point at infinity: for $\phi,\omega\to\infty$, in fact, $U(\phi,\omega)\to 0$. This "ground state" at infinity also arises in gravity theories containing curvature terms of order higher than two, in simple nonminimally coupled Lagrangians and in Kaluza-Klein theories. This new minimum is separated from ϕ_1 by the saddle ϕ_2; clearly it is not a Friedmannian limit and we will check numerically that our trajectories are not attracted by it. It would be also possible to remove completely this unwanted minimum of $U(\phi,\omega)$ by generalizing the effective scalaron mass [28] $M_{\text{eff}}\equiv Me^{-\tau\phi/2}$.

Let us describe now in some detail the evolutionary dynamics of the model. The behavior is radically different depending on whether or not the two paired points ϕ_3,ϕ_4 exist. When they do exist, which happens when (15) is not satisfied, ϕ_3 is a maximum and ϕ_4 a minimum with respect to the ϕ direction. This implies that the local minimum ϕ_4 is separated out from the true vacuum by a barrier at ϕ_3 (see Fig. 3), and that the model is completely equivalent to an old inflationary theory, where the bubbles of true vacuum never catch up the de Sitter background (notice that a pure R^2 model does not allow a mechanism similar to extended inflation). But if ϕ_3,ϕ_4 no longer exist, then the false vacuum $\phi=\phi_F$ is not an equilibrium point for the whole potential, although it is a minimum for $V(\phi)$. Figure 4 displays ϕ-constant sections of $U(\phi,\omega)$ for growing values of ω: at a certain value ω_t the barrier disappears; these are indeed sections of the same potential plotted in Fig. 1. Here, after an initial period of quasi-de Sitter inflation, the Universe rolls out purely classically from the false vacuum, turns around the barrier and finally reaches the true vacuum $\phi=\phi_T, R=0$, after the usual sequence of Friedmannian oscillations. A suitable coupling with a radiationlike field may then reheat the Universe [29], generating the huge amount of entropy observed today. During the first stage, however, bubbles of true vacuum nucleate on the other side of the barrier via semiclassical tunneling, leading to the formation of walls of unthermalized energy, possible seeds for galaxy formation. The completion of the inflationary phase is thus achieved both via the classical getaway and via the bubble filling of the space; contrary to other models, even if the true-vacuum bubbles are not sufficient to fill the Universe the end of the false-vacuum phase is guaranteed by the slow-rolling path down to the global minimum. The bubble nucleation rate is clearly time dependent, as in the double field model of Ref. [9] and in some extended inflationary models. Notice that the existence of a point where the barrier disappears is not a unique feature of the coupling $e^{\tau\phi}$: the same property holds for any $f(\phi)$ coupling such that $df/d\phi>0$, like, e.g., $f(\phi)=\xi\phi^n$, $n>0$.

Examples of numerical integrations in two relevant cases are displayed in Fig. 5. During the first and last phases the trajectories are well approximated by the

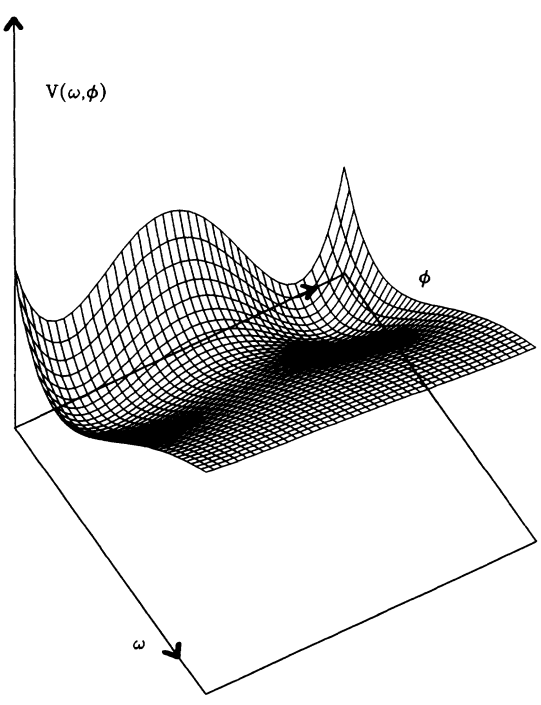

FIG. 3. Potential (21) with (meta)stable false vacuum. The model does not implement getaway inflation. The parameters do not verify Eq. (15).

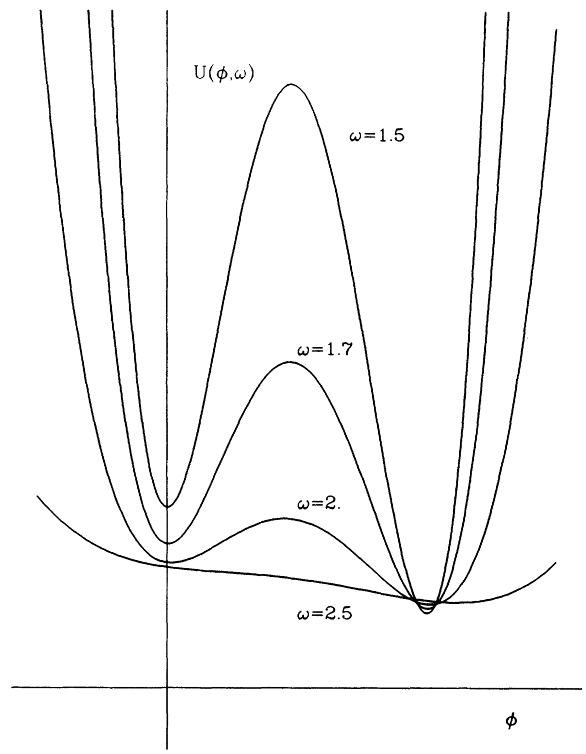

FIG. 4. Sections for $\phi=\text{const}$ of $U(\phi,\omega)$ in (21) for different values of ω, growing from top to bottom ($-S$ also grows). At the last section the barrier shrinks away and the fields may turn around and reach the true vacuum.

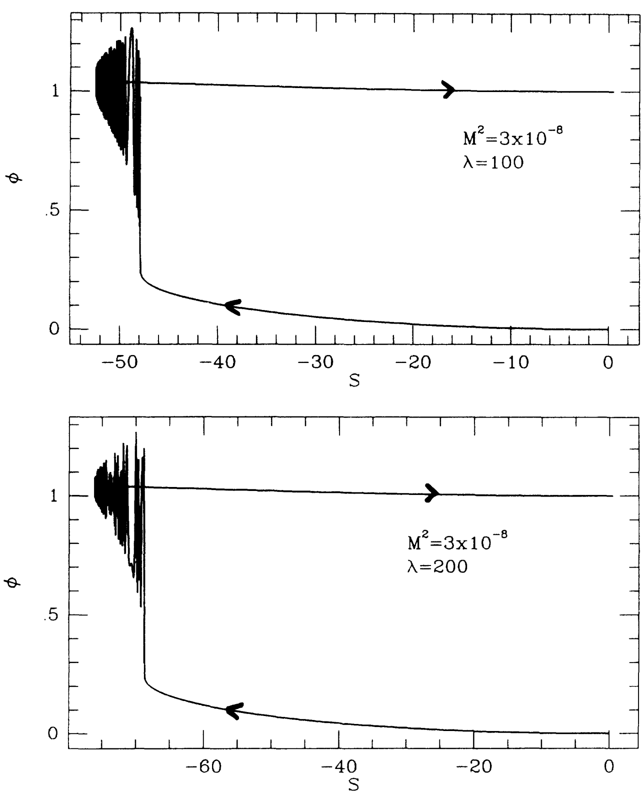

FIG. 5. Evolution of the model on the (ϕ,S) plane for two values of λ. The trajectories start at the origin, follow the false-vacuum valley, and reach the turnaround point. After a series of ϕ oscillations (damped through a phenomenological radiative coupling), the model rolls down to the true vacuum state at $(\phi=\phi_T, S=0)$, with a final epoch of S oscillations. In both cases the condition (15) is verified.

equations

$$\phi_B(S)=aS^2, \quad \text{before turnaround} , \tag{22}$$

$$\phi_A(S)=\phi_T+ae^{-\tau\phi_T}S^2 \quad \text{after turnaround} , \tag{23}$$

where $a=\tau M^2\phi_T^2/(192\pi V_F\lambda)$. These two parametric trajectories track the false-vacuum and true-vacuum "valleys" clearly visible in Fig. 1. The turnaround is located at

$$S_t^2=\frac{\phi T}{a(1-e^{-\tau\phi_T})} . \tag{24}$$

The number of e-foldings is given by $3H^2/M^2$ (where H is the Hubble function during inflation) in the uncoupled quadratic model; in our case this becomes (after the turnaround) $3H^2/M_{\rm eff}^2=S_t/4$. Too low a value for $M_{\rm eff}$ would imply a very long inflationary phase, even *after* the bubble nucleation, while the Universe is rolling back toward the true vacuum. This would dilute the bubbles to an unobservable level, making the whole model a useless replica of a slow rolling inflation. We must then check that the following constraints are compatible: the number of e-foldings after S_t must not exceed, say, 60, and the inequality (15) must hold. It is convenient to express quantities in units of the relevant mass scale, e.g., the grand-unified-theory (GUT) scale 10^{15} GeV$\approx 10^{-4}M_{\rm Pl}$. Then we may put $V_F=\phi_T=1$ and $G=10^{-8}$. The two conditions above read

$$M^2<50\tau G\lambda^{-1/2}, \quad M^2>\frac{G\lambda}{95\tau(1-e^{-\tau})} . \tag{25}$$

The two conditions are satisfied for a wide set of effective masses: $M_{\rm eff}\equiv Me^{-\tau\phi_T/2}$ lies in the range from 10^{11} GeV down to the lowest mass not in conflict with the current tests of gravity (the scalaron mass introduces in fact a Yukawa-type additive term $\exp(-M_{\rm eff}r)/r$ in the Newton law [15], with $M_{\rm eff}$ constrained to be larger than, say, 1 GeV). Note that our value of $M_{\rm eff}$ should be in any case much smaller than the preferred values in the typical fourth-order models (around 10^{15} GeV), where there is the need for having enough scalaron fluctuations to induce galaxy formation.

In a new inflation approach the initial values for the fields S,ϕ are to be assumed near the location of the high-temperature symmetric minimum: here this location can be put around the value of $\phi_3=\phi_4$ when the getaway condition (15) is marginally not satisfied. Then $\phi_{\rm in}\approx 3M^2/(64\pi G\tau V_F)$ and $S_{\rm in}\approx -3\sqrt{\lambda}/(\tau\phi_T)$. For the range of parameters we are interested in, they are not far from the origin (in particular, $|S_{\rm in}|\ll|S_t|$).

After the completion of the inflation the Universe reheats to a temperature suitable for baryogenesis and standard cosmology. The reheating should be the result of three distinct engines: bubble-wall collisions, damping of S-field oscillations, and damping of ϕ-field oscillations. The latter phase occurs just after the turnaround (see Fig. 5), i.e., during the inflationary era, and is thus expected to be much less influent than the other two. The fact that the reheating can be produced by the bubbles alone allows us in principle to choose the scalaron mass M, responsible for the amplitude of the scalaron field quantum fluctuation $\delta\rho/\rho$, as low as we like. In chaotic inflation, on the contrary, the mass m of the driving field is sandwiched between the conflicting requests of saving microwave isotropy on one side, of inducing large-scale structure via zero-point fluctuations and of reheating the Universe on the other. In this sense, getaway inflation, like extended inflation, does not suffer of the "lower end" of the fine-tuning problem. We now try to quantify the upper bounds on the model parameters.

A low value of M as required by (25) (in particular, much lower than $10^{-4}M_{\rm Pl}=10^{15}$ GeV) implies a very flat potential and a suppressed fluctuation production. The flatness of a potential $V(\psi)$ during an inflationary stage driven by ψ can be expressed following Ref. [10] by the dimensionless parameter $\lambda_{FT}=\Delta V/(\Delta\psi)^4$, where the differences Δ are to be taken between the initial and the final moment of the last sixty e-foldings of inflation (the "observable" part of inflation). A small λ_{FT} is a basic requirement for any inflation. In reality, the condition not to overproduce fluctuations leads to even smaller values of λ_{FT}, in fact so small that the whole matter has come to earn the name of the "fine-tuning" problem. The fluctuation amplitude during inflation is usually calculated by the expression $\delta\rho/\rho=0.1H^2/\dot{\psi}$ (the numerical factor is somewhat model and author dependent and has been chosen in agreement with Ref. [10]) but this expression can be used only in the Einstein frame where gravity is of general-relativity type and our inflaton field S behaves as a scalar field: the conformal transformation (16) is then required, the scalaron field being subject to the canonical equation

$$\Box\tilde{\omega}+U_{,\tilde{\omega}}=0\ , \tag{26}$$

where $U(\omega,\phi)$ is given in Eq. (21). We can then define $\lambda_{FT}=\Delta U/(\Delta\tilde{\omega})^4$. For the fluctuation amplitude not to be in conflict with the reported absence of anisotropies in the cosmic microwave background the requirement is

$$\frac{\delta\rho}{\rho}=0.1\frac{\hat{H}^2}{d\tilde{\omega}/d\hat{t}}=0.1e^{-\omega}\frac{H^2}{\dot{\tilde{\omega}}}<2\times10^{-5}\ , \tag{27}$$

where entries with carets make use of $\hat{g}_{\mu\nu}$: $d\hat{t}=e^{\omega}dt$, $\hat{a}=e^{\omega}a$, and so on. Numerical calculations show that the two conditions (25) are stronger than (27): parameters which fulfill (25) give $\delta\rho/\rho$ of order $10^{-6}-10^{-8}$ and $\lambda_{FT}\sim10^{-18}-10^{-30}$. We did not span, however, the whole parameter space. In other words, because of the presence of G in Eq. (15), the getaway condition itself constrains the model much more strongly than does the amount of fluctuations on the last scattering surface. In models where both fields are ordinary matter, G will not show up in the getaway condition (15) and this constraint should be consequently relaxed. Notice also that the above expression for $\delta\rho/\rho$ does not hold near the turnaround, when $\dot{\tilde{\omega}}=0$: this in fact signals that the other field ϕ is to be mainly responsible for the fluctuations at that moment.

Before closing the section, let us comment again on the condition (15). We identify in our model three interesting alternatives: (a) when (15) is not satisfied, the Universe can end the false-vacuum phase only via old-inflationary-like bubble nucleation toward ϕ_T; (b) when (15) is marginally not verified, the model is more similar to a new inflation: our observable Universe prefers to escape from the false vacuum phase with a single "thick" bubble nucleated along the S direction, which then evolves as already described; (c) when (15) is well satisfied the getaway mechanism is fully implemented and a purely classical rolling down takes place, along with bubble production up to the turnaround. Finally, it may be worth remarking that in any case there is the distinct possibility that the trajectory is able to climb over the saddle ϕ_2 and to roll toward infinity, i.e., toward the second ground state of $U(\phi,\omega)$: as the background always remains inflating this case is physically uninteresting.

III. BUBBLE NUCLEATION RATE

The domain-wall solution of Eq. (8) can be readily found in the thin-wall limit [19,30]. When the vacua are nearly degenerate ($\lambda\to\infty$) we can approximate our potential with $V_0=V_F\lambda\phi^2(\phi-\phi_T)^2/\phi_T^4$, and a domain-wall solution for Eq. (8) is

$$\phi^{\mathrm{TW}}(r)=\frac{1}{2}\phi_T\left[1-\tanh\left[\frac{r-R}{\Delta}\right]\right]\ , \tag{28}$$

where $R=\phi_T\sqrt{\lambda/V_F}$ is the bubble radius and $\Delta=\phi_T\sqrt{2/(V_F\lambda)}$ is the bubble-wall thickness: thin wall here means in fact $R/\Delta=\lambda/\sqrt{2}\to\infty$. (The derivation of the above solution requires some nontrivial approximations; for details, see e.g., Ref. [19].) The geometry of the nucleated bubbles changes as inflation proceeds, since the true potential $U(\phi,\omega)$ depends on the values of both fields. Actually, the above derivation is valid only for $S\sim0$, when the first term in the large parentheses of Eq. (21) is negligible, and it has been presented only to get a feeling of what parameters are involved. Near the end of inflation, when the residual barrier height is going to vanish, a thick-wall limit is more appropriate. Apart from the geometrical properties of bubbles, we are interested also in the probability of nucleation for unit time for unit volume $\Gamma=A\exp(-B)$, where B is the Euclidean action evaluated along the "bounce" path of the field, and in the fraction of space filled by the bubbles at any given time [31], which is controlled by the expression Γ/H^4. In the thin-wall limit $B=27\pi^2B_1^4/(2\epsilon^3)$, with

$$B_1=\int_{\Delta\phi}d\phi\sqrt{2U(\phi,S)}\ , \tag{29}$$

where the integration is performed between the minima (with respect to ϕ) $\phi_B(S)$, $\phi_A(S)$ for any slice S and where $\epsilon(S)$ (the thickness parameter) is the energy difference between $\phi_B(S)$ and $\phi_A(S)$. Since ϵ,ϕ_B,ϕ_A and the integrand of B_1 are functions of the second field S we may calculate a function $\Gamma=\Gamma(S)$. Actually, we are working here in the conformally rescaled (hatted) frame: the nucleation rate in the original frame is [32] $\Gamma=e^{4\omega}\hat{\Gamma}$, where $e^{4\omega}=(1-S/3)^2$. In Fig. 6 we plot the logarithm of $\Gamma(S)$, resting upon the following approximations: thin-wall limit (e.g., $\lambda=100$); tunneling along ϕ direction; no gravitational effects; approximation of the quartic potential $V(\phi)$ with a set of parabolas centered on the minima $\phi_B(S),\phi_A(S)$ and the maximum $\phi_M(S)$, for any value of S; slow rolling along S. Each of these conditions is inevitably broken near the turnaround, where the real dynamics of nucleation (as long as one can speak of nucleation) are unclear; the dominant effect for producing inhomogeneity during the latest stages is very likely the quantum stochastic motion of the fields (see, e.g., Ref. [33]). However, during the earliest phase, when the geometry of the barrier does not change dramatically, we expect that all the approximations are indeed reliable: the global meaning of Fig. 6 should not be spoiled by a refinement of the employed technique. The tunneling will occur along the ϕ direction provided the curvature (the second derivative) of the potential along ϕ is larger than along S, which on the other hand is what we need for having a slow rollover driven by S. As for the gravitational effects, they should enhance the bubble production [34] and thus are not expected to qualitatively modify the result of Fig. 6. As it is intuitive, the bubble nucleation rate grows during the model evolution ($|S|$ grows before the turnaround at S_t), leading to exponentially more small bubbles than big ones. In our model, however, the

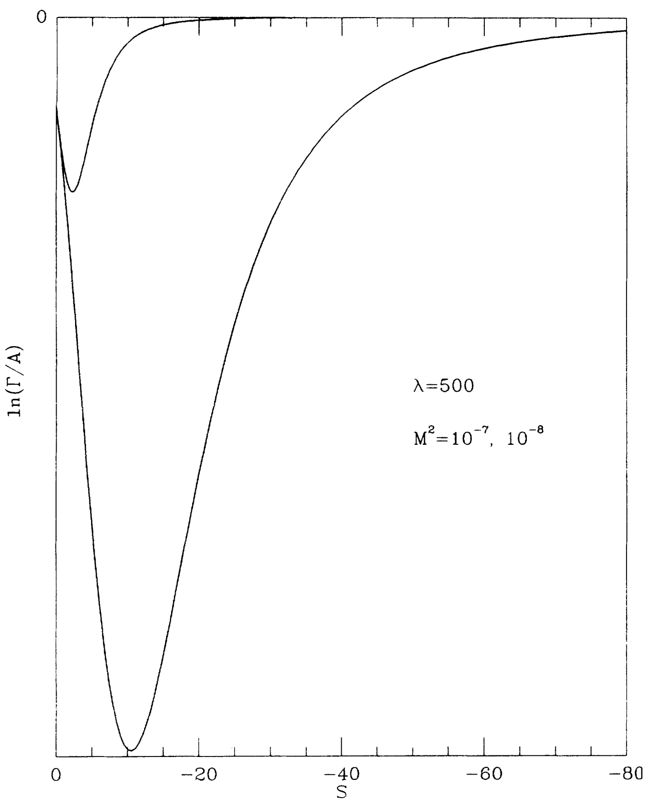

FIG. 6. Plot of the exponential part of the tunneling rate vs the S field in arbitrary units. After a first stage, the nucleation rate grows while the Universe reaches the gateway. The ratio of the vacuum energy difference to the barrier energy (dimensionless thickness parameter) remains as low as a few$\times 10^{-2}$ well after the minimum. The lower mass value refers to the lower plot.

"smallest" bubbles can be made astrophysically interesting simply by tuning the epoch of the turnaround (after which the nucleation rate is obviously suppressed).

IV. CONCLUSIONS

This paper does not intend to elaborate a complete model of what we call getaway inflation but rather to highlight a new class of models where the Universe exits from the false-vacuum phase due to both classical slow-rolling and bubble production. A first realization of this process is found in a nonminimally coupled higher-order theory of gravity where the new degrees of freedom allowed by the scalaron R are exploited to drive a slow-rolling inflation, while the coupled matter field (the "transiton") undergoes a first-order phase transition. Note that in this model the gravity is coupled to all the matter content of the Universe, unlike extended inflation where it couples only to one of the fields (the subdominant one). A further reason of interest in this kind of higher-order theory is that the effective potential for the scalaron field is directly built-in in the Lagrangian, aside from the coupling constants. We checked that inclusion of a JBD term of the form $\xi e^{\tau\phi}R$ with $\xi<0$ (which ensures $G_{\rm eff}=(G^{-1}-16\pi\xi e^{\tau\phi})^{-1}>0$) does not qualitatively alter the structure of the model; however, we do not know at the moment if the addition of different quadratic ϕ-coupled terms will substantially change the main features reported here. An advantage of this model is that the problem of the graceful exit is radically bypassed and no constraints are drawn from the condition of bubble space filling. The amount of space actually filled by the bubbles, when the background finally stops inflating, is an almost free parameter which should be eventually determined according to the observed large scale structure of the Universe. The numerically calculated degree of flatness of the adopted potential is admittedly very high.

Let us remark that the getaway mechanism can be easily implemented also in a simple two-field Einsteinian model where, however, the potential must be designed *ad hoc*. On the other hand, the proposed nonminimal quadratic coupling presented here appears worthy of deeper consideration by itself, and it will be of more detailed concern in further studies.

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