PCAC consistency. II. Charmed meson two- and three-body nonleptonic weak decays

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(Received 10 June 1991; revised manuscript received 12 November 1991)

We extend the notion of PCAC (partial conservation of axial-vector current) consistency to calculate eight two-body weak decays $D \to \bar{K}\pi$, $\bar{K}K$, $\pi\pi$ and seven three-body weak decays $D \to \bar{K}\pi\pi$, $\bar{K}K\pi$, $\pi\pi\pi$. Most of the fifteen predicted amplitudes are in reasonable agreement with the experimental results.

PACS number(s): 13.25.+m, 11.30.Rd, 11.40.Ha

I. INTRODUCTION

We have shown in the preceding paper (I) [1] that partially conserved axial-vector current (PCAC) techniques can be quite successful in reproducing the experimental data for kaon nonleptonic weak decays. In that paper we demonstrated that PCAC consistency accurately predicts all seven $K \to 2\pi, 3\pi$ decay amplitudes.

In this paper (II), we generalize PCAC consistency to the charmed sector and show that the same prescription that worked so well for kaon decays in paper I will also give reasonable results for charmed decays, as indicated in Ref. [2]. Specifically, in Sec. II we predict eight decay amplitudes for the two-body $D \to \pi\pi$, $\bar{K}K$, $\bar{K}\pi$ decays. In Sec. III we extend our analysis to seven nonresonant three-body $D \to \bar{K}\pi\pi$, $\bar{K}K\pi$, 3π decays. We then summarize our findings in Sec. IV.

We emphasize that although we will be applying pion and kaon PCAC to charmed decays, where either final-state meson can be taken soft alone with the initial D meson always on mass shell, the resulting soft PCAC amplitude will *not* be the entire physical amplitude, just as it is not for kaon decays. This is because overall four-momentum conservation dictates that there must be energy dependence due the nonsoft final-state mesons. As explained in paper I and explicitly demonstrated in the Appendix, the amplitude at the soft point (M_{CC}) is part of the constant background that, when combined with the four-momentum variation of the pole term (M_P), leads to the physical amplitude [3]

$$M = M_{CC} + M_P - M_P(0) \ . \tag{1}$$

Even though the values of M_{CC} and $M_P(0)$ depend on which final-state particle is taken soft, the difference $M_{CC} - M_P(0)$ is independent of this choice. This is what is meant by "PCAC consistency."

When applying PCAC consistency in paper I to $K \to \pi\pi$ decays, we required M_{CC} for $q_1 \to 0$ to be the surface term $M_P - M_P(0)$ when $q_2 \to 0$, up to PCAC corrections $O(m_\pi^2/m_K^2) \approx 7\%$. This corresponded to the double PCAC consistency relation for $K \to 2\pi$:

$$M = M_{CC1} + M_{CC2} + O(m_\pi^2/m_K^2) \ , \tag{2a}$$

where the subscripts 1,2 refer to the final-state particle taken soft. For the D decays the analog of (2a) is

$$M = M_{CC1} + M_{CC2} + O(m_K^2/m_D^2) \ , \tag{2b}$$

where the corrections are again of order $O(m_K^2/m_D^2) \approx 7\%$. In the double PCAC consistency relation (2a), used in paper I, the decaying kaon was always kept on mass shell when the final-state pions were taken soft. Similarly, in this paper we study the double PCAC consistency relation (2b) with the decaying charmed D meson always kept on mass shell when the final-state pions or kaons are taken soft.

The prescriptions of Eqs. (2) call for calculating the equal-time charge commutator for each final-state particle and then adding the two commutator terms together. The advantage of using Eqs. (2) as opposed to Eq. (1) is that one need not calculate the detailed pole terms corresponding to Fig. 1. The charge commutator amplitude M_{CC}, is found from the traditional chiral-symmetry (CS) PCAC reduction (i.e., soft meson theorem). For pseudoscalar meson state P and general hadron states A (in this paper the states A will also be pseudoscalar mesons), this reduction is

$$\begin{aligned} \langle P_j A_f | H_w | A_i \rangle \\ \to M_{CC} &= (-i/f_P) \langle A_f | [Q_5^j, H_w] | A_i \rangle \\ &= (i/f_P)[i f_{fjk} \langle A_k | H_w | A_i \rangle \\ &\quad - i f_{jik} \langle A_f | H_w | A_k \rangle] \ , \end{aligned} \tag{3}$$

where f_P is the appropriate decay constant (f_π or f_K). The last equality follows from the chiral-symmetry statement that the usual weak Hamiltonian density H_w, generated by left-handed currents, is orthogonal to right-handed charges. This corresponds to the CS equal-time commutation relations

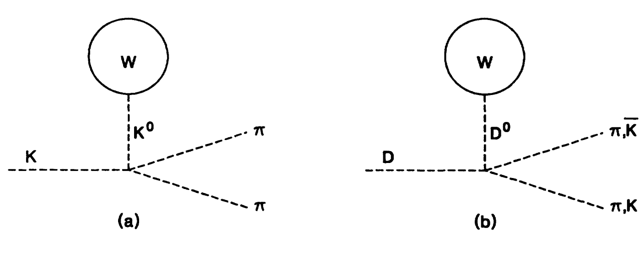

FIG. 1. Tadpole graphs for (a) $K \to \pi\pi$ decays and (b) $D \to \bar{K}K$ decays.

$$[Q+Q_5,H_w]=0 \quad \text{or} \quad [Q_5,H_w]=-[Q,H_w] \ . \tag{4}$$

In (3) we have also employed the strong-interaction transformation law for hadron states:

$$Q_j|A_i\rangle = if_{jik}|A_k\rangle \ . \tag{5}$$

Again, we stress that the initial hadron state A_i will not be taken off mass shell even though its flavor structure may change. That is, a charmed D meson could obtain the flavor structure of a charmed F meson via the transformation law in Eq. (5), but it will remain on the D mass shell.

II. TWO-BODY D DECAYS

In this section we apply the $M_{\text{CC1}}+M_{\text{CC2}}$ prescription (2b) to the nine $D\to PP$ transitions. We begin with the Cabibbo-suppressed $D\to\pi\pi$ decays that are quite similar in structure to the $K\to\pi\pi$ decays of paper I. If we apply Eq. (2b) together with the CS-PCAC reduction (3), we obtain the real amplitudes

$$a_{+-}=-i\langle\pi^+\pi^-|H_w|D^0\rangle = (1/\sqrt{2}f_\pi)\langle\pi^+|H_w|D^+\rangle \ , \tag{6a}$$

$$a_{00}=-i\langle\pi^0\pi^0|H_w|D^0\rangle=(1/f_\pi)\langle\pi^0|H_w|D^0\rangle \ , \tag{6b}$$

$$a_{+0}=-i\langle\pi^+\pi^0|H_w|D^+\rangle = (1/2f_\pi)[\sqrt{2}\langle\pi^0|H_w|D^0\rangle-\langle\pi^+|H_w|D^+\rangle] \ . \tag{6c}$$

The next task is to compute the reduced matrix elements appearing in (6). In Ref. [4] the reduced matrix element in (6b) was estimated to be

$$\langle\pi^0|H_w|D^0\rangle=-\langle\pi^0|H_w|K^0\rangle \approx +2.5\times10^{-8}\ \text{GeV}^2 \ , \tag{7}$$

where the numerical value in (7) is the reduced matrix element that fits the kaon decays so well in paper I. The meson loop model of Ref. [4] finds the same intermediate-state mesons for both Cabibbo-suppressed $K^0\to\pi^0$ and $D^0\to\pi^0$ transitions, suggesting the equality in (7). A quark-model calculation of (7) gives essentially the same numerical result.

The reduced matrix element in (6a) has two contributions, the first of which is a $\Delta I=\frac{1}{2}$ loop contribution given by [4]

$$\langle\pi^+|H_w|D^+\rangle_{\text{loop}}=\sqrt{2}\langle\pi^0|H_w|D^0\rangle \approx 3.5\times10^{-8}\ \text{GeV}^2 \ . \tag{8a}$$

The second contribution is from a W-pole graph depicted in Fig. 2, which is the exact analog of Fig. 1 and Eq. (8) for $K\to2\pi$ decays in paper I:

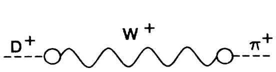

FIG. 2. W-pole graph for $\langle\pi^+|H_w|D^+\rangle$.

$$\langle\pi^+|H_w|D^+\rangle_{\text{pole}}=-(G_F/\sqrt{2})s_1c_1f_\pi f_D m_D^2 \approx -9.9\times10^{-8}\ \text{GeV}^2 \ , \tag{8b}$$

where $f_D\approx1.8f_\pi$ [5], $f_\pi\approx93$ MeV, and $s_1c_1\approx0.22$. In order to keep the reduced matrix element (8b) constant, we employ overall momentum conservation for $D\to\pi\pi$, which requires $2p_D\cdot p_\pi=m_D^2$. The minus sign in (8b) is due to the Glashow-Iliopoulos-Maiani (GIM) mechanism [6]. Adding (8a) and (8b) together yields the net reduced matrix element

$$\langle\pi^+|H_w|D^+\rangle\approx-6.4\times10^{-8}\ \text{GeV}^2 \ . \tag{8c}$$

The loop-plus-pole combination in (8c) is quite similar to that for $\langle\pi^+|H_w|K^+\rangle$ in paper I.

In order to compare (6)–(8) with experimental data, one must in general introduce final-state interactions (FSIs). We shall assume that FSIs only rotate the amplitudes in isospin space without introducing additional inelasticity parameters. The experimental amplitudes including FSIs for $D\to\pi\pi$ decays are basically the same [4] as for $K\to\pi\pi$ decays:

$$M_{+-}=a_0e^{i\delta_0}+\tfrac{2}{3}a_2e^{i\delta_2} \ , \tag{9a}$$

$$M_{00}=a_0e^{i\delta_0}-\tfrac{4}{3}a_2e^{i\delta_2} \ , \tag{9b}$$

$$M_{+0}=-\sqrt{2}a_2e^{i\delta_2} \ . \tag{9c}$$

To link the real theoretical $D\to\pi\pi$ predictions (6) to the experimental complex amplitudes (9), we decompose Eqs. (6) into their isospin components in analogy with (9):

$$a_{+-}=a_0+\tfrac{2}{3}a_2\approx-0.49\times10^{-6}\ \text{GeV} \ , \tag{10a}$$

$$a_{00}=a_0-\tfrac{4}{3}a_2\approx0.27\times10^{-6}\ \text{GeV} \ , \tag{10b}$$

$$a_{+0}=-\sqrt{2}a_2\approx0.53\times10^{-6}\ \text{GeV} \ , \tag{10c}$$

where the numerical evaluations in (10) result from substituting the reduced matrix elements of Eqs. (7) and (8c) into Eqs. (6). We can now solve the real Eqs. (10) for the real $a_{0,2}$ and insert the latter into Eqs. (9) to find the complex amplitudes M. In order to avoid the unknown phase shifts $\delta_{0,2}$ in (9), we can take a combination of $|M_{+-}|$ and $|M_{00}|$ that does not depend on phase shifts [7]. Specifically, our theoretical $D^0\to\pi\pi$ amplitudes obey the constraint

$$2|M_{+-}|^2+|M_{00}|^2=3a_0^2+\tfrac{8}{3}a_2^2 \approx0.56\times10^{-12}\ \text{GeV}^2 \ , \tag{11}$$

where we have inserted the values $a_0\approx-0.24\times10^{-6}$ GeV and $a_2\approx-0.38\times10^{-6}$ GeV, derived from (10), into (11). The experimental analog of (11) is proportional to the sum of the $D^0\to\pi^+\pi^-,\pi^0\pi^0$ branching ratios and is given by

$$2|M_{+-}|^2_{\text{expt}}+|M_{00}|^2_{\text{expt}}=(0.62\pm0.12)\times10^{-12}\ \text{GeV}^2 \ . \tag{12}$$

The matrix element $|M_{+-}|$ in (12) is found from the Particle Data Group (PDG) listings in Ref. [8], while

$|M_{00}|$ follows from Ref. [9]. The prediction (11) compares well with the experimental data in (12). There is no phase shift complication in the decay $D^+ \to \pi^+\pi^0$, which has a pure $I=2$ final state. We simply compare the prediction (10c) with the experimental bound [8]

$$|M_{+0}|_{\rm expt} < 0.56\times 10^{-6}\ {\rm GeV}\ . \tag{13}$$

Although (10c) is only slightly below the bound of (13), the W-pole estimate in (8b) means that the theoretical value of $a_{+0} = -i\langle \pi^+\pi^0|H_w|D^+\rangle$ in (6c) contains a mixture of $\Delta I = \frac{3}{2}$ and $\frac{1}{2}$ parts. Since in fact a_{+0} must be pure $\Delta I = \frac{3}{2}$, one can show that the reduced matrix element $\langle \pi^0|H_w|D^0\rangle$ in (6c) also has an additional (but small) $\Delta I = \frac{1}{2}$ component which reduces the resulting pure $\Delta I = \frac{3}{2}$ a_{+0} in (10c) to be (safely) below the experimental bound in (13). Our predictions as well as the experimental branching ratios for $D \to \pi\pi$ are tabulated in Table I.

In principle, one could use the experimental decay rates to obtain a best fit to the amplitudes in Eqs. (9), thus replacing M_{+-}, M_{00}, M_{+0} with $a_0, a_2, \delta_0 - \delta_2$. Then, by using this fitted $\delta_0 - \delta_2$ and the theoretical values for $a_{0,2}$ from (10), one could predict the $D^0 \to \pi^+\pi^-, \pi^0\pi^0$ decay amplitudes separately. However, because $B(D^+ \to \pi^+\pi^0)$ is not yet known, such a procedure is not yet possible for $D \to \pi\pi$, but we will employ this procedure for $D \to \bar{K}K$, $\bar{K}\pi$ later in this section.

In particular, we next study the Cabibbo-suppressed $D \to \bar{K}K$ decays, where FSIs are very important. The double PCAC consistency relation (2b) and the CS-PCAC reduction (3) predict the real amplitudes

$$a_{+-} = -i\langle K^+K^-|H_w|D^0\rangle = (1/\sqrt{2}f_K)\langle K^+|H_w|F^+\rangle\ , \tag{14a}$$

$$a_{00} = -i\langle \bar{K}^0K^0|H_w|D^0\rangle = 0\ , \tag{14b}$$

$$a_{+0} = -i\langle \bar{K}^0K^+|H_w|D^+\rangle = (1/\sqrt{2}f_K)\langle K^+|H_w|F^+\rangle\ . \tag{14c}$$

TABLE I. Two-body charmed-meson decay branching ratios.

	$\mathrm{BR_{th}}$ (%)	$\mathrm{BR_{expt}}$ (%)
$D^0 \to K^-\pi^+$	3.7[a]	3.71 ± 0.25[b]
$D^0 \to \bar{K}^0\pi^0$	1.9[a]	1.9 ± 0.5[c]
$D^0 \to K^-\pi^+ + \bar{K}^0\pi^0$	5.6	5.6 ± 0.6[b,c]
$D^+ \to \bar{K}^0\pi^+$	2.8	2.8 ± 0.4[b]
$D^0 \to K^+K^-$	0.38[a]	0.45 ± 0.07[b]
$D^0 \to \bar{K}^0K^0$	0.10[a]	$0.11^{+0.06}_{-0.04}$[d]
$D^0 \to K^+K^- + \bar{K}^0K^0$	0.48	0.56 ± 0.09[b,d]
$D^+ \to \bar{K}^0K^+$	1.2	0.84 ± 0.27[b]
$D^0 \to \pi^+\pi^- + \pi^0\pi^0$	0.20	0.21 ± 0.04[b,e]
$D^+ \to \pi^+\pi^0$	0.48	<0.53[b]

[a]These branching ratios are computed using phase shifts (20b) and (30) found from a fit to the experimental amplitudes.
[b]Reference [8].
[c]Reference [12].
[d]Reference [10].
[e]Reference [9].

The main contribution to the reduced matrix element in Eqs. (14) is from the W-pole graph of Fig. 3, which in analogy to (8b) is given by

$$\langle K^+|H_w|F^+\rangle_{\rm pole} = (G_F/\sqrt{2})s_1c_1f_Kf_Fm_D^2 \approx 0.12\times10^{-6}\ {\rm GeV}^2\ , \tag{15a}$$

where the F is on the D mass shell, $2p_D\cdot p_K = m_D^2$, and $f_F \approx 1.8f_\pi$ [5]. There is also a loop contribution to this reduced matrix element that, when calculated through the quark model, is simply related by SU(3) to Eq. (8a):

$$\langle K^+|H_w|F^+\rangle_{\rm loop} = (f_\pi/f_K)\langle \pi^+|H_w|D^+\rangle_{\rm loop} \approx 0.03\times10^{-6}\ {\rm GeV}^2\ , \tag{15b}$$

given that $f_F \approx f_D$ [5]. [In Refs. [2] and [4] we ignored the small contribution of (15b) relative to (15a)]. The net reduced matrix element is then found by adding (15a) and (15b) to arrive at

$$\langle K^+|H_w|F^+\rangle \approx 0.15\times10^{-6}\ {\rm GeV}^2\ . \tag{15c}$$

The experimental amplitudes for $D \to \bar{K}K$, which include FSIs, can be written as

$$M_{+-} = a_0e^{i\delta_0} + a_1e^{i\delta_1}\ , \tag{16a}$$

$$M_{00} = -a_0e^{i\delta_0} + a_1e^{i\delta_1}\ , \tag{16b}$$

$$M_{+0} = 2a_1e^{i\delta_1}\ . \tag{16c}$$

Decomposing Eqs. (14) into their isospin components as well as inserting the reduced matrix element (15c) yields

$$a_{+-} = a_0 + a_1 \approx 0.91\times10^{-6}\ {\rm GeV}\ , \tag{17a}$$

$$a_{00} = -a_0 + a_1 = 0\ {\rm GeV}\ , \tag{17b}$$

$$a_{+0} = 2a_1 \approx 0.91\times10^{-6}\ {\rm GeV}\ . \tag{17c}$$

Again, we eliminate the phase shift dependence by computing the combination of theoretical $D^0 \to \bar{K}K$ amplitudes:

$$|M_{+-}|^2 + |M_{00}|^2 = 2(a_0^2 + a_1^2) \approx 0.83\times10^{-12}\ {\rm GeV}^2\ . \tag{18}$$

The branching-ratio data for this $D^0 \to \bar{K}K$ combination gives

$$|M_{+-}|^2_{\rm expt} + |M_{00}|^2_{\rm expt} = (0.97\pm0.16)\times10^{-12}\ {\rm GeV}^2\ , \tag{19}$$

which is near the theoretical prediction in (18). We derive $|M_{+-}|$ in (19) from the PDG compilation [8], while $|M_{00}|$ is found from Ref. [10].

Alternatively, we could solve Eqs. (16) for $\delta_0 - \delta_1$, which, assuming $a_0 = a_1$ from (17b), results in

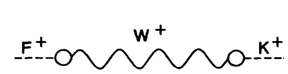

FIG. 3. W-pole graph for $\langle K^+|H_w|F^+\rangle$.

$$\tan[(\delta_0-\delta_1)/2]=|M_{00}|_{\text{expt}}/|M_{+-}|_{\text{expt}}\approx 0.51 , \quad (20a)$$

$$\delta_0-\delta_1\approx 54° . \quad (20b)$$

[Equation (20b) is not significantly changed when the assumption $a_0=a_1$ is not used.] The value (20b) then leads to the complex amplitudes for $D\to\bar{K}K$ decays,

$$|M_{+-}|=2a_1\cos[(\delta_0-\delta_1)/2]\approx 0.81\times 10^{-6}\text{ GeV} , \quad (21a)$$

$$|M_{00}|=2a_1\sin[(\delta_0-\delta_1)/2]\approx 0.41\times 10^{-6}\text{ GeV} , \quad (21b)$$

$$|M_{+0}|=2a_1\approx 0.91\times 10^{-6}\text{ GeV} , \quad (21c)$$

which compare well with the experimental $D\to\bar{K}K$ amplitudes

$$|M_{+-}|_{\text{expt}}=(0.87\pm 0.08)\times 10^{-6}\text{ GeV} , \quad (22a)$$

$$|M_{00}|_{\text{expt}}=(0.44\pm 0.10)\times 10^{-6}\text{ GeV} , \quad (22b)$$

$$|M_{+0}|_{\text{expt}}=(0.77\pm 0.13)\times 10^{-6}\text{ GeV} , \quad (22c)$$

where (22a) and (22c) are found from Ref. [8] and (22b) follows from Ref. [10]. The theoretical and experimental branching ratios for $D\to\bar{K}K$ are listed in Table I. One additional point to note is that Eqs. (21a) and (21c) predict $|M_{+-}|\le|M_{+0}|$ irrespective of the reduced matrix element (15c) or the phase shift difference (20b).

Finally, we apply the double PCAC prescription (2b) to the Cabibbo-enhanced $D\to\bar{K}\pi$ decays [11], which leads to

$$a_{+-}=-i\langle K^-\pi^+|H_w|D^0\rangle$$
$$=(1/\sqrt{2}f_K)\langle\pi^+|H_w|F^+\rangle$$
$$+(1/\sqrt{2}f_K)(1-f_K/f_\pi)\langle\bar{K}^0|H_w|D^0\rangle , \quad (23a)$$

$$a_{00}=-i\langle\bar{K}^0\pi^0|H_w|D^0\rangle$$
$$=(1/2f_K)(2f_K/f_\pi-1)\langle\bar{K}^0|H_w|D^0\rangle , \quad (23b)$$

$$a_{+0}=-i\langle\bar{K}^0\pi^+|H_w|D^+\rangle$$
$$=(1/\sqrt{2}f_K)\langle\pi^+|H_w|F^+\rangle$$
$$+(1/\sqrt{2}f_\pi)\langle\bar{K}^0|H_w|D^0\rangle . \quad (23c)$$

The experimental $D\to\bar{K}\pi$ amplitudes, with FSIs included, can be expressed as

$$M_{+-}=\sqrt{2}a_{1/2}e^{i\delta_{1/2}}+a_{3/2}e^{i\delta_{3/2}} , \quad (24a)$$

$$M_{00}=-a_{1/2}e^{i\delta_{1/2}}+\sqrt{2}a_{3/2}e^{i\delta_{3/2}} , \quad (24b)$$

$$M_{+0}=3a_{3/2}e^{i\delta_{3/2}} . \quad (24c)$$

The reduced matrix element $\langle\pi^+|H_w|F^+\rangle$ in (23), having no loop contributions, is found solely from the W-pole graph of Fig. 4:

$$\langle\pi^+|H_w|F^+\rangle\approx(G_F/\sqrt{2})c_1^2 f_\pi f_F m_D^2$$
$$\approx 0.42\times 10^{-6}\text{ GeV}^2 , \quad (25)$$

where once again the F is on the D mass shell and $2p_D\cdot p_\pi\approx m_D^2$. The Cabibbo-enhanced $D^0\to\bar{K}^0$ transition in (23) was found in Ref. [4] to have the same intermediate meson states as the Cabibbo-suppressed $K^0\to\pi^0$ transition, thereby yielding

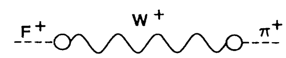

FIG. 4. W-pole graph for $\langle\pi^+|H_w|F^+\rangle$.

$$\langle\bar{K}^0|H_w|D^0\rangle=\sqrt{2}(c_1/s_1)\langle\pi^0|H_w|K^0\rangle$$
$$\approx -0.16\times 10^{-6}\text{ GeV}^2 . \quad (26)$$

If we decompose Eqs. (23) into their isospin components as well as insert the reduced matrix elements (25) and (26) therein, we find, for $D\to\bar{K}\pi$ decays,

$$a_{+-}=\sqrt{2}a_{1/2}+a_{3/2}\approx 2.80\times 10^{-6}\text{ GeV} , \quad (27a)$$

$$a_{00}=-a_{1/2}+\sqrt{2}a_{3/2}\approx -1.03\times 10^{-6}\text{ GeV} , \quad (27b)$$

$$a_{+0}=3a_{3/2}\approx 1.34\times 10^{-6}\text{ GeV} . \quad (27c)$$

We can again choose the following $D^0\to\bar{K}\pi$ amplitude combination where the phase-shift difference does not enter:

$$|M_{+-}|^2+|M_{00}|^2=3(a_{1/2}^2+a_{3/2}^2)$$
$$\approx 8.9\times 10^{-12}\text{ GeV}^2 . \quad (28)$$

The corresponding experimental branching ratio sum for $D^0\to\bar{K}\pi$ decays is seen to be

$$|M_{+-}|^2_{\text{expt}}+|M_{00}|^2_{\text{expt}}=(8.9\pm 0.9)\times 10^{-12}\text{ GeV}^2 , \quad (29)$$

where $|M_{+-}|$ is taken from Ref. [8] and $|M_{00}|$ from Ref. [12]. The data sum in (29) is in excellent agreement with the prediction (28).

Alternatively, we can solve Eqs. (24) self-consistently for the phase-shift difference, then giving [13]

$$\delta_{1/2}-\delta_{3/2}\approx 87° . \quad (30)$$

Utilizing this phase-shift difference in (30) together with $a_{1/2}\approx 1.66\times 10^{-6}$ GeV and $a_{3/2}\approx 0.45\times 10^{-6}$ GeV, deduced from (27), we find that the absolute magnitudes of Eqs. (24) for $D\to\bar{K}\pi$ decays become

$$|M_{+-}|=2.41\times 10^{-6}\text{ GeV} , \quad (31a)$$

$$|M_{00}|=1.67\times 10^{-6}\text{ GeV} , \quad (31b)$$

$$|M_{+0}|=1.34\times 10^{-6}\text{ GeV} . \quad (31c)$$

The theoretical $D\to\bar{K}\pi$ predictions (31) are in very good agreement with the experimental data

$$|M_{+-}|_{\text{expt}}=(2.43\pm 0.12)\times 10^{-6}\text{ GeV} , \quad (32a)$$

$$|M_{00}|_{\text{expt}}=(1.74\pm 0.23)\times 10^{-6}\text{ GeV} , \quad (32b)$$

$$|M_{+0}|_{\text{expt}}=(1.32\pm 0.10)\times 10^{-6}\text{ GeV} . \quad (32c)$$

The numerical evaluations in (32a) and (32c) follow from the PDG [8], while (32b), as stated before, is derived from Ref. [12]. The close agreement between (31c) and (32c) (also noted in Refs. [2,4]) is a check on the theoretical link in (23c) between the apparently unrelated dynamical reduced matrix elements (25) and (26). All the two-body D decay branching ratios, both theoretical and experimental, are listed in Table I.

III. THREE-BODY D DECAYS

We now extend PCAC consistency to the nonresonant three-body decays of D mesons. We generalize the amplitude (2b) for two-body decays to the three-body decay amplitude (up to 10% PCAC corrections) as the triple PCAC consistency relation

$$M = M_{CC1} + M_{CC2} + M_{CC3} , \tag{33}$$

where we first apply the PCAC and charge-algebra reduction (3) to each of the three final-state mesons and add together the resulting charge commutator matrix elements. Underlying Eq. (33) are pole terms, such as the K and π poles, as needed in paper I for $K \to 3\pi$ decays, and D and $\bar{K}$ poles for $D \to \bar{K}\pi\pi$, etc. But the simpler triple PCAC consistency prescription (33) proceeds as in the two-body decay case (2b) without explicitly having to calculate pole terms. However, in contrast with the two-body decays, for three-body decays we have charge operators acting on two-particle states. To analyze this configuration we allow the charge operator to act on each final-state particle and then add the results together, generating an extension of the strong-interaction transformation law (5):

$$\langle A_i A_j | Q_k = i f_{ikl} \langle A_l A_j | + i f_{jkl} \langle A_i A_l | . \tag{34}$$

We now use (3), (33), and (34) to compute the five nonresonant $D \to \bar{K}\pi\pi$ decay amplitudes A in terms of the three $D \to \bar{K}\pi$ amplitudes:

$$\begin{aligned} A^{-++} &= -i \langle K^- \pi^+ \pi^+ | H_w | D^+ \rangle \\ &= (\sqrt{2}/f_\pi)[\langle \pi^+ K^- | H_w | D^0 \rangle \\ &\quad - (1 - f_\pi/f_K) \langle \pi^+ \bar{K}^0 | H_w | D^+ \rangle] , \end{aligned} \tag{35a}$$

$$\begin{aligned} A^{0+0} &= -i \langle \bar{K}^0 \pi^+ \pi^0 | H_w | D^+ \rangle \\ &= (1/\sqrt{2} f_\pi) \langle \pi^0 \bar{K}^0 | H_w | D^0 \rangle \\ &\quad - (1/2 f_K) \langle \pi^+ \bar{K}^0 | H_w | D^+ \rangle , \end{aligned} \tag{35b}$$

$$\begin{aligned} A^{-+0} &= -i \langle K^- \pi^+ \pi^0 | H_w | D^0 \rangle \\ &= (1/2 f_K)[\langle \pi^+ K^- | H_w | D^0 \rangle \\ &\quad + \sqrt{2}(1 - f_K/f_\pi) \langle \pi^0 \bar{K}^0 | H_w | D^0 \rangle] , \end{aligned} \tag{35c}$$

$$\begin{aligned} A^{0+-} &= -i \langle \bar{K}^0 \pi^+ \pi^- | H_w | D^0 \rangle \\ &= (1/\sqrt{2} f_\pi)[\langle \pi^+ \bar{K}^0 | H_w | D^+ \rangle \\ &\quad - (1 - f_\pi/f_K) \langle \pi^+ K^- | H_w | D^0 \rangle] , \end{aligned} \tag{35d}$$

$$\begin{aligned} A^{000} &= -i \langle \bar{K}^0 \pi^0 \pi^0 | H_w | D^0 \rangle \\ &= (1/f_\pi)(2 - f_\pi/f_K) \langle \pi^0 \bar{K}^0 | H_w | D^0 \rangle . \end{aligned} \tag{35e}$$

If we use the usual $\bar{K}\pi$ sum rule $\sqrt{2} M_{00} = M_{+0} - M_{+-}$ [obeyed by the complex $D \to \bar{K}\pi$ amplitudes in Eqs. (24)], to eliminate $\langle \pi^0 \bar{K}^0 | H_w | D^0 \rangle$ from the above Eqs. (35b) and (35c), we find the $D \to \bar{K}\pi\pi$ relationships

$$A^{-+0} = -A^{0+0} = A^{-++}/2\sqrt{2} , \tag{36}$$

which are approximately satisfied by the data.

Because of the complex nature of the $D \to \bar{K}\pi$ amplitudes (24) and the sizable effects of the FSI, we cannot simply insert the experimental $D \to \bar{K}\pi$ amplitudes into Eqs. (35) when calculating the $D \to \bar{K}\pi\pi$ amplitudes. Instead, we insert the complex forms (24) along with the predicted $D \to \bar{K}\pi$ values of $a_{1/2} \approx 1.66 \times 10^{-6}$ GeV, $a_{3/2} \approx 0.45 \times 10^{-6}$ GeV, and, from (30), $\delta_{1/2} - \delta_{3/2} \approx 87°$ and finally take the absolute magnitudes to find the theoretical nonresonant amplitudes:

$$\begin{aligned} |A^{-++}| &= (\sqrt{2}/f_\pi)[2a_{1/2}^2 + (2 - 3f_\pi/f_K)^2 a_{3/2}^2 - 2\sqrt{2}(2 - 3f_\pi/f_K) a_{1/2} a_{3/2} \cos(\delta_{1/2} - \delta_{3/2})]^{1/2} \\ &\approx 36 \times 10^{-6} , \end{aligned} \tag{37a}$$

$$|A^{0+0}| = |A^{-++}|/2\sqrt{2} \approx 13 \times 10^{-6} , \tag{37b}$$

$$|A^{-+0}| = |A^{-++}|/2\sqrt{2} \approx 13 \times 10^{-6} , \tag{37c}$$

$$\begin{aligned} |A^{0+-}| &= (1/\sqrt{2} f_\pi)[2(1 - f_\pi/f_K)^2 a_{1/2}^2 + (2 + f_\pi/f_K)^2 a_{3/2}^2 \\ &\quad - 2\sqrt{2}(2 + f_\pi/f_K)(1 - f_\pi/f_K) a_{1/2} a_{3/2} \cos(\delta_{1/2} - \delta_{3/2})]^{1/2} \\ &\approx 10 \times 10^{-6} , \end{aligned} \tag{37d}$$

$$|A^{000}| = (1/f_\pi)(2 - f_\pi/f_K) |\langle \pi^0 \bar{K}^0 | H_w | D^0 \rangle| \approx 23 \times 10^{-6} . \tag{37e}$$

To extract the experimental amplitudes from the measured decay rates, we must compute the standard three-body phase-space integral [14]. Assuming that the amplitude is constant results in the decay rate

$$\begin{aligned} \Gamma &= [2/N(8\pi M)^3] |A|^2 \int_{4\mu^2}^{(M-m)^2} ds \left\{ \frac{[s - 4\mu^2][s - (M+m)^2][s - (M-m)^2]}{s} \right\}^{1/2} \\ &= I |A|^2 , \end{aligned} \tag{38}$$

where N is a statistical factor for identical particles and, in this case, $\mu=\pi$ mass, $m=K$ mass, and $M=D$ mass. The values for I in (38) for the five cases of interest are $I(-++)=3.09$, $I(0+0)=6.16$, $I(-+0)=6.17$, $I(0+-)=6.09$, and $I(000)=3.08$ in units of 10^{-5} GeV. Using the experimental nonresonant branching ratios [8] along with (38) then leads to

$$|A_{\rm expt}^{-++}|=(36.4\pm3.0)\times10^{-6}\ , \qquad (39a)$$

$$|A_{\rm expt}^{0+0}|=(11.0\pm3.9)\times10^{-6}\ , \qquad (39b)$$

$$|A_{\rm expt}^{-+0}|=(17.4\pm4.4)\times10^{-6}\ , \qquad (39c)$$

$$|A_{\rm expt}^{0+-}|=(21.5\pm3.0)\times10^{-6}\ . \qquad (39d)$$

The first three experimental $D\to\bar{K}\pi\pi$ amplitudes in (39) agree fairly well with the first three theoretical predictions in (37).

If we now apply our triple PCAC consistency relation (33) to the two measured Cabibbo-suppressed decays $D^+\to K^+K^-\pi^+,\pi^+\pi^+\pi^-$, we find

$$-i\langle K^+K^-\pi^+|H_w|D^+\rangle =(1/\sqrt{2}f_\pi)[\langle K^+K^-|H_w|D^0\rangle -(1-f_\pi/f_K)\langle K^+\bar{K}^0|H_w|D^+\rangle]\ , \qquad (40a)$$

$$-i\langle\pi^+\pi^+\pi^-|H_w|D^+\rangle=(\sqrt{2}/f_\pi)\langle\pi^+\pi^-|H_w|D^0\rangle\ . \qquad (40b)$$

To estimate (40a) numerically, we must insert the complex amplitude structure (16) together with the predicted values $a_0=a_1\approx0.455\times10^{-6}$ GeV and $\delta_0-\delta_1\approx54°$ from (20b), which results in

$$|\langle K^+K^-\pi^+|H_w|D^+\rangle| =(a_1/\sqrt{2}f_\pi)[1+2(f_\pi/f_K)\cos(\delta_0-\delta_1) +(f_\pi/f_K)^2]^{1/2} \approx5.6\times10^{-6}\ . \qquad (41a)$$

Since there is no interference in (40b), we can simply use the experimental magnitude of $\langle\pi^+\pi^-|H_w|D^0\rangle$ to predict [15]

$$|\langle\pi^+\pi^+\pi^-|H_w|D^+\rangle|\approx3.1\times10^{-6}\ . \qquad (41b)$$

In order to compute the experimental amplitudes, we employ Eq. (38) with $\mu=K$ mass and $m=\pi$ mass for $D^+\to K^+K^-\pi^+$ and, for $D^+\to\pi^+\pi^+\pi^-$, $\mu=m=\pi$ mass. The phase-space integrals I, in these two cases, are $I(\bar{K}K\pi)=3.25$ and $I(3\pi)=4.88$ in units of 10^{-5} GeV. Substituting these integrals I into (38) and employing the experimental nonresonant decay widths [8], we then find the two Cabibbo-suppressed transition amplitudes to be

$$|\langle K^+K^-\pi^+|H_w|D^+\rangle|_{\rm expt}=(8.6\pm1.0)\times10^{-6}\ , \qquad (42a)$$

$$|\langle\pi^+\pi^+\pi^-|H_w|D^+\rangle|_{\rm expt}=(5.2\pm0.7)\times10^{-6}\ , \qquad (42b)$$

in rough agreement with theory (41). Although both (41a) and (41b) are somewhat short of the experimental results in (42), the ratio between the two does agree with the data. We have compiled the theoretical and experimental branching ratios for the seven nonresonant three-body decays, examined in this section, in Table II.

TABLE II. Three-body nonresonant charmed-meson decay branching ratios.

	$\mathrm{BR_{th}}$ (%)	$\mathrm{BR_{expt}}$ (%)[a]
$D^+\to K^-\pi^+\pi^+$	6.5	6.6 ± 1.1
$D^+\to\bar{K}^0\pi^+\pi^0$	1.8	$1.2^{+1.0}_{-0.7}$
$D^0\to K^-\pi^+\pi^0$	0.72	1.2 ± 0.6
$D^0\to\bar{K}^0\pi^+\pi^-$	0.39	1.8 ± 0.5
$D^0\to\bar{K}^0\pi^0\pi^0$	1.0	
$D^+\to K^+K^-\pi^+$	0.16	0.39 ± 0.09
$D^+\to\pi^+\pi^+\pi^-$	0.08	0.21 ± 0.06

[a]See Ref. [8].

IV. CONCLUSION

In this paper we have computed decay amplitudes for the nine $D\to\pi\pi,\bar{K}K,\bar{K}\pi$ decays as well as for seven $D\to\bar{K}\pi\pi,\bar{K}K\pi,\pi\pi\pi$ decays, the majority of which compare rather well with the experimental data. In particular, we gave a brief summary of our theoretical technique in Sec. I. We explained that our PCAC consistency predictions are not simply the soft-meson amplitudes, but also include (rapidly varying) surface-term corrections. Then, by requiring PCAC consistency (giving the same overall amplitude no matter which pion or kaon is reduced in), the resulting surface term (i) was recast as a second charge commutator, leading to $M=M_{\rm CC1}+M_{\rm CC2}$, and (ii) accounted for the necessary momentum and energy variation of the amplitude.

We then proceeded to calculate PCAC consistency amplitudes for the two-body D meson decays in Sec. II. We used two techniques to handle the phase-shift dependence of the D^0 matrix elements. The first was to take combinations, proportional to the sum of the two D^0 branching ratios, that do not depend on phase shifts. The second method was to fit the phase shifts to the experimental complex amplitudes (M) and then incorporate the theoretical real amplitudes (a) to make predictions for the individual D^0 decays. The latter procedure was only possible for the $D\to\bar{K}K,\bar{K}\pi$ processes because of lack of information in $D\to\pi\pi$ decays. Most of the predictions were in very good agreement with the experimental two-body decay data as listed in Table I, and in fact all predictions were within or near one standard deviation from data.

Finally, in Sec. III we generalized our PCAC consistency analysis to include the three-body $D\to\bar{K}\pi\pi,\bar{K}K\pi,\pi\pi\pi$ decays and tabulated the results in Table II. We introduced the logical extension of the double PCAC consistency amplitude (2) to triplet PCAC consistency in Eq. (33): $M=M_{\rm CC1}+M_{\rm CC2}+M_{\rm CC3}$. This prescription allowed us to relate the three-body amplitudes of Sec. III to the two-body amplitudes from Sec. II, which, with the inclusion of the two-body final-stateinteraction relations, again for the most part reproduced the experimental results.

When applying PCAC consistency in these two papers (I and II), we have always kept the decaying K or D meson on mass shell. The final-state (Nambu-Goldstone) pions and/or kaons are taken soft separately, while simultaneously accounting for surface-term momentum variation in the double and triple PCAC consistency relations (2) and (33). In our opinion the key unifying idea for PCAC consistency applied to both K and D decays is this momentum variation. As long as such momentum variation, as represented by the addition of two and three equal-time charge commutators, is taken into account, one almost always finds approximate agreement with low-energy physics.

The underlying issue is, what is the upper bound for momentum variation in low-energy chiral-symmetry-breaking physics? Most people believe that 1 GeV sets this scale. We suggest that, even though the typical decay momenta for D decays of 700–900 MeV are only slightly below this 1-GeV scale, we will not violate the latter if we always keep the D meson on mass shell when consistently folding in the momentum variation in D decay channels.

For example, the momentum dependence in the reduced matrix element $\langle \pi^+|H_w|D^+\rangle$ in Eq. (8b) is $p_\pi \cdot p_D$. Always keeping the D meson on mass shell, we account for this two-body momentum variation by conserving the overall $D\to\pi\pi$ four-momenta $p_D=p_\pi+p'_\pi$ so that $2p_\pi\cdot p_D=m_D^2$ when both pions are on mass shell. If instead $p'_\pi\to 0$, then $p_\pi\cdot p_D\to m_D^2$. But if $p_\pi\to 0$, then $p_\pi\cdot p_D\to 0$. The average of the latter two limits then corresponds to our overall momentum variation prescription $p_\pi\cdot p_D=(m_D^2+0)/2=m_D^2/2$ used in Eq. (8b) and in Eqs. (7), (10), and (11) in Ref. [2]. Three-body momentum variation corresponds to the double PCAC consistency relations (2), while four-body momentum variation corresponds to the triple PCAC consistency relation (33). A similar pattern holds for $K\to 2\pi, 3\pi$ decays in paper I, and in this sense a 1-GeV chiral-symmetry-breaking scale appears to be continued slightly higher by our PCAC consistency approach. The fact that some of the D meson three-body decay amplitudes do not agree well with the data might indicate that we have reached the limit of applicability of PCAC.

As always, PCAC results must be compared with data in order to know the size of the PCAC corrections. We suggest that the compatibility of data with our PCAC consistency analysis for 7 K and 16 D decays almost uniformly shows that the PCAC errors are within 10%, as anticipated. One expects the PCAC consistency corrections for $K\to 2\pi, 3\pi$ decays to be $O(m_\pi^2/m_K^2)\sim 7\%$, as stated in (2a) and shown in paper I, and likewise the corrections to (2b) and (33) for charmed meson decays to be $O(m_K^2/m_D^2)\sim 7\%$. We suggest the almost exact parallel between the PCAC consistency for K decays (K on shell) and D decays (D on shell) is more than the small 7% PCAC error, but includes the overall compatibility with experimental data of our 7 $K\to 2\pi, 3\pi$ amplitudes in paper I and our 15 $D\to\pi\pi, \bar{K}K, \bar{K}\pi, \bar{K}\pi\pi, \bar{K}K\pi, \pi\pi\pi\pi$ amplitudes in this paper II.

In effect, in this paper we have predicted 15 charmed-D-meson weak-decay amplitudes based only on the Cabibbo-enhanced scale $\langle \bar{K}^0|H_w|D^0\rangle$ and the Cabibbo-suppressed scale $\langle \pi^0|H_w|D^0\rangle$. In fact, these two scales are related to the $K\to\pi\pi$ reduced matrix element $\langle \pi^0|H_w|K^0\rangle$, and the latter was the basis for the seven kaon decays in paper I. This $K\to\pi$ transition, in turn, is determined by the charmed D mass or c-quark mass due to dynamical loop graphs [3,4,16]. Thus, not surprisingly, the charmed mass sets the dynamical scale for charmed weak decays and also for the $\Delta I=\frac{1}{2}$ enhanced $K^0\to 2\pi$ decays.

ACKNOWLEDGMENTS

The authors appreciate the partial support of the U.S. Department of Energy and discussions with A. N. Kamal, N. Paver, and Riazuddin.

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