Energy dependence of parameters of negative-binomial distributions in NN collisions

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We have discussed the energy dependence of the parameters $\langle n \rangle$ and K of the negative-binomial distributions for the full phase-space charged-particle multiplicity distributions in $pp/p\bar{p}$ collisions. It was shown that, under the assumption that multiparticle production is a stationary branching process, there exists a relation between the two parameters $\langle n \rangle$ and K. A similar relation can also be derived empirically, from information-theoretic entropy considerations. The energy dependence of one parameter then automatically determines the energy dependence of the other. It was then argued from entropy considerations that there should be an upper bound for $\langle n \rangle$ or conversely a lower bound for K as a function of energy. New parametrizations for the energy dependence of K were given, taking into account that K should have a lower bound, which together with the relation between the parameters $\langle n \rangle$ and K predicts that the average number of particles cannot be increased indefinitely with increasing c.m. system energy.

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I. INTRODUCTION

In recent years the negative-binomial distributions (NBD's)

$$P(n)=\frac{\Gamma(n+K)}{\Gamma(n+1)\Gamma(K)}[\langle n \rangle/(\langle n \rangle+K)]^n \times[K/(\langle n \rangle+K)]^K , \quad (1)$$

with two parameters $\langle n \rangle$ (average of the distribution) and K (width of the distribution), are being widely used to describe the multiplicity distributions (MD's) of produced charged particles in full phase space as well as in limited phase space, from a variety of collisions such as hadronic [1], hadron-nucleus [2], nucleus-nucleus [3,4], e^+e^- [5], etc. The energy dependence of the parameters $\langle n \rangle$ and K for hadronic collisions has also been studied by the UA5 Collaboration [1] in the energy range 10–900 GeV. It was found that, while the parameter $\langle n \rangle$ increases, the parameter K decreases with the c.m. energy. While the energy dependence of $\langle n \rangle$ can be understood from general physical considerations, the same cannot be said about the energy dependence of the parameter K.

The true theory of the multiparticle production process should be QCD. At these high energies (several hundreds of GeV), scattering of two nucleons proceeds via the scattering between the constituents, namely, the quarks and gluons. However, most of the particles emitted in the process are soft particles, rendering the perturbative approach inapplicable. Thus, presently, we cannot describe the multiparticle production process in terms of its true theory, i.e., QCD. We can at best make some models to describe it. Various theoretical attempts have been made to obtain a NBD law for hadronic collisions from general principles, such as the stochastic model [6,7], quantum-statistics model [8], cluster model [9–11], stochastic branching model [12,13], and string model [14]. But we have not understood why the NBD works so well for such a wide variety of reactions. More importantly, none of the models proposed until now can explain satisfactorily what K represents or why it decreases with energy. However, it will be unrealistic to attach too much importance to the fact that the NBD can describe multiplicity distributions from a variety of collisions. Being a two-parameter ($\langle n \rangle$ and K) distribution, the NBD can go from a wide distribution (geometric or Bose-Einstein) for $K=1$ to a narrow distribution (Poisson) for $K=\infty$. It is thus flexible enough to accommodate a wide variety of distributions with changing $\langle n \rangle$ and K.

In the present paper, we study the energy dependence of the parameters of the NBD. It will be shown that the energy dependence of K can be understood from a simple model of stochastic branching process. We assume that the multiparticle production process is a "stationary" branching process. We also assume that the charged-particle multiplicity distributions obey, strictly, the NBD law, which is the result of the stationary branching process. Then the two parameters $\langle n \rangle$ and K of the NBD are not independent; rather, they are connected by a simple relation. The energy dependence of one parameter automatically determines the energy dependence of the other. It will also be shown that empirically a similar relation can be obtained from (information) entropy considerations.

In Sec. II we shall describe briefly the stationary branching process and obtain the relation between the parameters $\langle n \rangle$ and K. In Sec. III an empirical relation between $\langle n \rangle$ and K will be obtained from entropy considerations. It will be argued that the maximum-entropy consideration constrains the average number $\langle n \rangle$ to have an upper bound, corresponding to maximum entropy. In Sec. IV new parametric forms for the parameter K will be considered and its consequences will be discussed. A summary will be given in Sec. V.

II. STOCHASTIC BRANCHING PROCESS AND RELATION BETWEEN $\langle n \rangle$ AND K

If the random variable is discrete, like the multiplicity of electrons in a cosmic-ray shower, and if the conditional probability $P(n_2t_2/n_1t_1)$ is zero for n_2 less than n_1 at any time t_2 greater than t_1, then we say that the process involves only branching and no recombination. A branching process need not be Markovian, but it must have the property that the n particles at any given time do not interact with each other; they can only create more particles separately and independently. We define the generating function for a branching process as

$$F_l(s,t)=\sum s^n P(n,t|l,0) , \tag{2}$$

where $P(n,t|l,0)$ is the conditional probability that, given l particles at time $t=0$, there will be n particles at time $t=t$. The central property of the branching process can be expressed by the equation

$$F_l(s,t)=|F(s,t)|^l , \tag{3}$$

where

$$F(s,t)=F_1(s,t)=\sum s^n P(n,t|1,0) .$$

Now, for a branching process with only one type of particle produced and a continuous evolution parameter t, the generating function (F) for the process satisfies the reverse Kolmogorov differential equation

$$\frac{dF}{dt}=-G(F,t) . \tag{4}$$

The specific nature of the evolution parameter (t) will not be needed for our purpose. A stochastic evolution generally involves real time, which has no counterpart in particle physics. Hwa [12] has discussed this aspect in detail. We only say that it is possible to connect the evolution parameter t with energy in an indirect way.

For a stationary branching process, the reverse Kolmogorov differential equation factorizes into an F-dependent and a time-dependent part:

$$\frac{dF}{dt}=-f(F)g(t) . \tag{5}$$

With this brief introduction of the branching process, we now proceed to obtain a relation between the parameters $\langle n \rangle$ and K of the NBD for a charged-particle multiplicity distribution for $pp/p\bar{p}$ collisions. We assume that the negative-binomial distribution law for multiplicity distributions is the result of a stationary branching process.

The generating function of the NBD is given by

$$\begin{aligned} F(x)&=[1+(1-x)\langle n \rangle /K]^{-K} \\ &=[1+(1-x)m]^{-K} , \end{aligned} \tag{6}$$

where we have introduced a new parameter $m=\langle n \rangle /K$.

From Eq. (6) we obtain

$$\frac{dF}{dt}=F\ln F(1/K)\frac{dK}{dt}-F(1-F^{1/K})(K/m)\frac{dm}{dt} . \tag{7}$$

Equation (7) satisfies the boundary conditions as required by a branching process, namely, $G(F=1,t)=0$ and $G(F,t)\to 0$ as $F\to 0$. Chliapnikov and Tchikilev [13] also obtained the same relation. They then demanded that the right-hand side (RHS) of Eq. (7) factorizes into F and t if

$$(1/K)\frac{dK}{dt}=\text{const}\times(K/m)\frac{dm}{dt} , \tag{8}$$

and solving (8) obtained the relation

$$1/K=a+b\ln m , \tag{9}$$

where a and b are two constants to be determined. However, the factorization property of the stationary branching process is not satisfied until the term $F^{1/K}$ is present in Eq. (7). Indeed, putting Eq. (8) into (7), we obtain

$$\begin{aligned} \frac{dF}{dt}&=[\text{const}\times F\ln F-F(1-F^{1/k})](k/m)\frac{dm}{dt} \\ &=f(F,k(t))g(t)\neq f(F)g(t) . \end{aligned} \tag{10}$$

This clearly shows that relation (9) has no theoretical basis. We also note that Chliapnikov and Tchikilev [13], while fitting data, noticed that a quadratic term $\ln^2 m$ is required by the data. The relation is also not consistent with the energy dependence of K as obtained by the UA5 Collaboration [1].

It is imperative that the term $F^{1/K}$ should be eliminated from Eq. (7) before the condition (5) of a stationary branching process can be applied. One possible way to eliminate the term $F^{1/K}$ is to consider Eq. (7) near $F\approx 1$. Then

$$(1-F^{1/K})\approx(1-F)/K , \tag{11}$$

and Eq. (7) becomes

$$\frac{dF}{dt}=F\ln F(1/K)\frac{dK}{dt}-F(1-F)(1/m)\frac{dm}{dt} . \tag{12}$$

Equation (12) clearly factorizes into F- and t-dependent terms when

$$(1/K)\frac{dK}{dt}=\text{const}\times(1/m)\frac{dm}{dt} . \tag{13}$$

Solving (13), the desired relation between the parameters K and m of the NBD can be obtained:

$$K=Am^B , \tag{14}$$

where A and B are the two constants to be determined.

We would like to add a few words about approximation (11), which is valid for $F\to 1$. Approximation (11) is exact at $F=1$ and also for $K=1$. We note that as such the generating function $[F(x)]$ of a probability distribution gives no information about the measurable quantities; rather, its various-order differentiations in the limit $x\to 1$ give measurable quantities. This in turn means that the generating function near 1 determines the physically measurable quantities. Approximation (11) is very accurate in the neighborhood of $x=1$ or $F=1$. Then, since we are interested in obtaining a relation between the parameters $\langle n \rangle$ and K of the NBD, which are determined in terms of the first and second derivatives of the generating function, relation (14) will necessarily be a good one. As will be evident later, experimental data also suggest a relation such as (14).

In Fig. 1 we present the experimental K and m as obtained by the UA5 Collaboration [1] by fitting $pp/p\bar{p}$

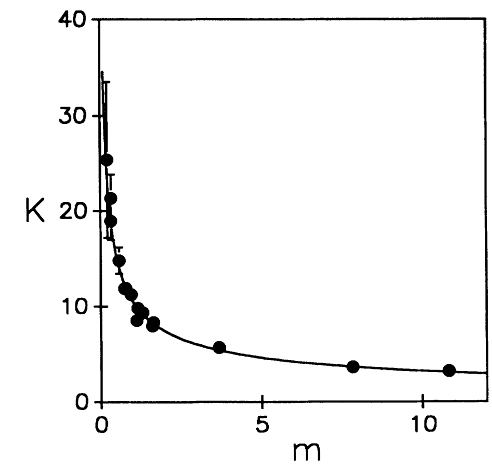

FIG. 1. NBD parameters K and $m=\langle n\rangle/K$ in the energy range 10–900 GeV for $pp/p\bar{p}$ charged-particle multiplicity distribution as obtained by the UA5 Collaboration. The solid line is a fit with Eq. (14).

multiplicity data over the energy range 10–900 GeV. The experimental values of $\langle n\rangle$ and K were taken from a compilation by Gupta and Sarma [15]. The data are well fitted with Eq. (14) with $A=10.52\pm0.15$ and $B=-0.516\pm0.009$. The experimental data then confirm the existence of such a relation. Equation (14) can be rewritten to bring out in an absolutely transparent way the relation between $\langle n\rangle$ and K:

$$\langle n\rangle=95.37/K^{0.936}\ . \tag{15}$$

Unlike the relation obtained by Chliapnikov and Tchikilev [13], this relation is consistent with the observed energy dependence of $\langle n\rangle$ and K. As the energy increases, $\langle n\rangle$ increases (more particles are produced with more energy), and the parameter K then decreases.

III. ENTROPY AND RELATION BETWEEN $\langle n\rangle$ AND K

It has been recently observed that the experimental multiplicity distributions in a hadronic interaction obey a new kind of scaling law [16,17]. The scaling variable involved is the entropy or more exactly the information-theoretic entropy defined as

$$W=-\sum P_n \ln P_n\ . \tag{16}$$

In Fig. 2 we show the calculated entropy for the charged-particle multiplicity in $pp/p\bar{p}$ collisions in the energy range 10–900 GeV. They were calculated by taking the NBD law for the multiplicity distributions with the parameters $\langle n\rangle$ and K as obtained by the UA5 Collaboration [1,15]. As observed earlier [16], the entropy is a smooth function of the c.m. energy ($\sqrt{s}$), the functional form of which can be conveniently parametrized as

$$W=\alpha+\beta\ln\sqrt{s}\ , \tag{17}$$

with $\alpha=1.378\pm0.031$ and $\beta=0.433\pm0.008$. In Fig. 2 we show the parametrization (17) for the entropy against the experimental values. The parametrization (17) agrees very well with experiment. We note that the observed energy dependence of (information) entropy is expected from physical considerations also. Entropy, as defined in Eq. (16), is the average of the self-information. With increasing energy we expect to generate more information; consequently, entropy will also increase with energy.

For hadronic collisions the parameters $\langle n\rangle$ and K of the NBD are a smooth function of the c.m. energy. It is then expected that the entropy will also be a smooth function of the parameters $\langle n\rangle$ and K. In Figs. 3 and 4, we show the calculated entropy as a function of $\langle n\rangle$ and K, respectively. It is observed that while entropy is a smoothly increasing function of $\langle n\rangle$, it decreases smoothly with K. The $\langle n\rangle$ dependence of entropy can be parametrized as

$$W=a\langle n\rangle^b\ , \tag{18}$$

with $a=1.30\pm0.01$ and $b=0.0340\pm0.004$. This behavior of entropy with the average of the multiplicity is also expected. Again, the information content of the system

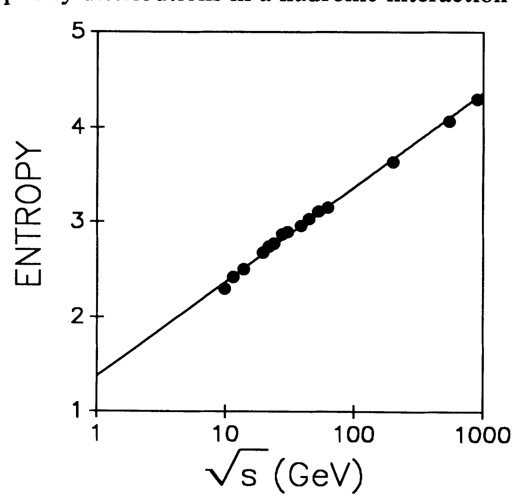

FIG. 2. Entropy of $pp/p\bar{p}$ charged-particle multiplicity distributions as a function of c.m.s. energy ($\sqrt{s}$). The solid line is a fit with Eq. (17).

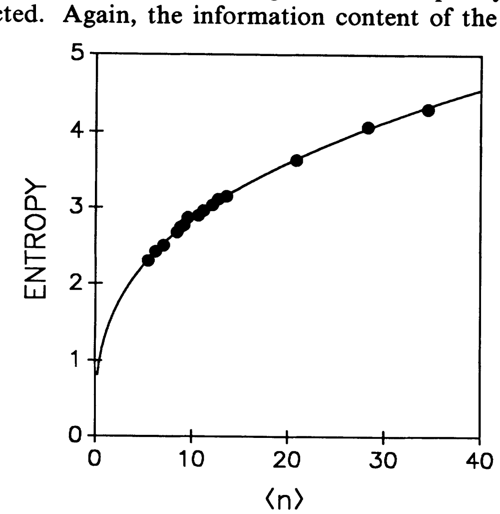

FIG. 3. Entropy as a function of the average of charged-particle multiplicity distributions. The solid line is a fit with Eq. (18).

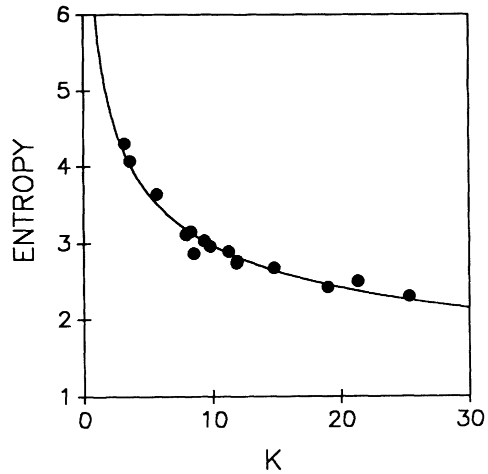

FIG. 4. Entropy as a function of the NBD parameter K. The solid line is a fit with Eq. (19).

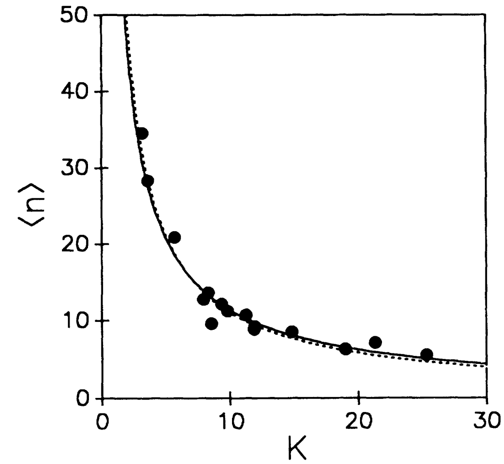

FIG. 5. Experimental average charged-particle multiplicity $\langle n \rangle$ as a function of the NBD parameter K. The solid line is a fit to it with Eq. (15); the dotted line is a fit with Eq. (20).

with a greater number of particles is expected to be more. The dependence of the entropy on the parameter K can also be parametrized as

$$W = AK^{B} , \tag{19}$$

with $A = 5.89 \pm 0.22$ and $B = -0.297 \pm 0.016$. In Figs. 3 and 4, we show the parametrizations (18) and (19) against the experimental values. We observe that the simple forms (18) and (19) agree very well with the calculated values.

The relations (18) and (19) can be equated to obtain an empirical relation between $\langle n \rangle$ and K, which is given by

$$\langle n \rangle = 85.15/K^{0.876} . \tag{20}$$

Relation (20) is very similar to the relation between the parameters $\langle n \rangle$ and K (Eq. (15)] obtained from the consideration that multiparticle production is a stationary branching process. Thus, empirically also, a relation exists between the parameters $\langle n \rangle$ and K of the NBD for charged-particle multiplicity distributions in $pp/p\bar{p}$ collisions. Relations (15) and (20) give very much similar results, as can be seen from Fig. 5, where we show the experimental $\langle n \rangle$ as a function of experimental K, along with the prediction from (15) and (20). Both relations gave a good description of the data. We rewrite Eqs. (15) and (20) as a single equation showing the relationship between the parameters $\langle n \rangle$ and K of the NBD, for charged-particle multiplicity:

$$\langle n \rangle = \alpha/K^{\beta} , \tag{21}$$

with $\alpha = 90.26 \pm 5.11$ and $\beta = 0.905 \pm 0.029$.

The entropy defined in Eq. (16) can be written in a different fashion, introducing the Koba-Nielsen-Olesen (KNO) function $\psi(z) = \langle n \rangle P_n$, $z = n/\langle n \rangle$. The concept of the KNO function is useful in the regime where the KNO scaling law holds [18]. The KNO function then does not change with energy. However, we now know that the KNO scaling law does not hold; the approximate scaling observed in low-energy $pp/p\bar{p}$ collisions were accidental [1]. Using the KNO function $\psi(z)$, the entropy in Eq. (16) can be written as

$$W = \ln\langle n \rangle - \int \psi(z) \ln\psi(z) dz , \tag{22}$$

with

$$\int \psi(z) dz = \int z\psi(z) dz = 1$$

as the normalizing conditions.

Equation (22) can be maximized to obtain the KNO function for which entropy will be maximum. It can be seen that for the exponential KNO function $\psi(z) = \exp(-z)$, the entropy is maximized. Thus, as the c.m. system (c.m.s.) energy for $pp/p\bar{p}$ collisions increases, entropy increases, until the condition of maximum entropy is reached (exponential KNO function). A further increase in energy will not increase entropy; i.e., it will saturate. As evident from Fig. 2, presently available data are not sufficient to confirm this prediction. However, from general theoretical considerations, we can argue that entropy should saturate with energy. With increasing energy the system under consideration is probed with higher and higher resolution. Consequently, the system reveals more and more information as we probe it with higher and higher energy. However, once the limit where the system has revealed all its information content is reached, probing with finer resolution does not produce additional information. Then, from (22), under the condition of maximum entropy ($W^{\max}$), we obtain for the average charged-particle multiplicity an upper bound given by

$$\langle n \rangle^{\max} = \exp(W^{\max} - 1) . \tag{23}$$

Thus, with increasing energy, the average number of charged particles will increase until they reach a maximum value corresponding to the exponential KNO distribution. In other words, even by increasing the energy indefinitely, the charged-particle multiplicity cannot be increased indefinitely. As mentioned earlier, the presently available data cannot confirm this prediction. We also note that in view of the relation between $\langle n \rangle$ and K [Eq. (21)], if there is an upper bound on $\langle n \rangle$, there will also be a lower bound on K. Now the KNO function for the NBD for $\langle n \rangle \gg K$ is given by the gamma distribution

$$\psi(z)=K^K z^{K-1}\exp(-Kz)/\Gamma(K)\ , \tag{24}$$

which reduces to the exponential distribution for $K=1$. Thus the desired lower bound for K corresponding to maximum entropy is 1. We expect K to saturate at unity. Then, in view of Eq. (21), the maximum value for $\langle n \rangle \approx 90$.

IV. ENERGY DEPENDENCE OF $\langle n \rangle$ AND K

We have seen in the previous sections that the parameters $\langle n \rangle$ and K of the NBD for $pp/p\bar{p}$ collisions are not independent parameters; rather, they are connected by a simple relation given by Eq. (21). Thus the energy dependence of one of them automatically determines the energy dependence of the other. We have also argued that the parameter $\langle n \rangle$ should have an upper bound and the parameter K should have a lower bound, as a function of energy. We also expect that minimum value of K is unity.

The UA5 Collaboration [1] has studied the energy dependence of $\langle n \rangle$ and K. The following parametric form for $\langle n \rangle$ was given by them:

$$\langle n \rangle = a + b\ \ln s + c\ \ln^2 s\ , \tag{25}$$

where $a=2.7\pm0.7$, $b=-0.03\pm0.21$, and $c=0.167\pm0.016$. An alternate parametrization was also given for $\langle n \rangle$ [1]:

$$\langle n \rangle = \alpha + \beta s^{\gamma}\ , \tag{26}$$

where $\alpha=-7.0\pm1.3$, $\beta=7.2\pm1.0$, and $\gamma=0.127\pm0.009$.

Both the parametrizations, [Eqs. (25) and (26)] are in good agreement with the experimental data. The energy dependence of K was parametrized as [1]

$$1/K = a + b\ \ln\sqrt{s}\ , \tag{27}$$

with $a=-0.104\pm0.004$ and $b=0.058\pm0.001$.

The parametrizations of $\langle n \rangle$ or K as given by the UA5 Collaboration [1] do not include the constraint imposed on $\langle n \rangle$ or K from entropy considerations that $\langle n \rangle$ should have an upper bound and that K should have a lower bound. Furthermore, the UA5 parametrization for K has a serious defect, that for energies below 10 GeV it gives negative values for K, which is unphysical in any model of multiparticle production. Thus the parametrization is unsuitable for extrapolation to energies below 10 GeV. In the following we shall consider two simple parametrizations of K and use them to find the energy dependence of $\langle n \rangle$.

First, we consider the following simple form to represent the energy dependence of K:

$$\begin{aligned} 1/K &= A\sqrt{s}^{\,B} \quad \text{for } 0<\sqrt{s}<\sqrt{s_{\max}}\ , \\ &= 1 \quad \text{for } \sqrt{s}>\sqrt{s_{\max}}\ , \end{aligned} \tag{28}$$

where we have assumed that the parameter K is bounded between the two limiting values 1 and ∞ corresponding to the two limiting distributions of the NBD: namely, the geometric or Bose-Einstein distribution ($K=1$) and the Poisson distribution ($K=\infty$). The parameters A and B were obtained by fitting experimental K values in the energy range 10–900 GeV. The best fitted values are $A=0.024\pm0.001$ and $B=0.385\pm0.015$. Then, around 16 TeV, the asymptotic condition $K=1$ is reached. In Fig. 6 we show the parametrization (28) labeled as a, against the experimental points. Also shown are the parametrization of the UA5 Collaboration [Eq. (27)] (labeled as c). Both of them give similar fit to data.

In parametric form (28), the minimum value $K=1$ was imposed by hand. To see whether the available data give any indication of the minimum value of K, we now consider the form

$$K=\alpha(1+\beta/\sqrt{s}^{\,\gamma})\ , \tag{29}$$

The form (29) gives α as the minimum of K as the energy tends to infinity. In Fig. 6 the parametrization (29) is shown (labeled as b) for the best-fitted values $\alpha=1.89\pm0.70$, $\beta=32.36\pm3.83$, and $\gamma=0.565\pm0.115$. Parametrization (29) also describes the energy dependence of K very well. We note that the parametrization (27) of the UA5 Collaboration [1] and the present parametrization (29) yield very similar results when extrapolated to higher energies. Parametrization (28), on the other hand, yields a different result: $1/K$ increasing very fast. For the parametrization (29), around 16 TeV again, K starts to saturate. Thus both the parametrizations (28) and (29) indicate that around energies of the CERN Large Hadron Collider (LHC) (16 TeV) the asymptotic condition will be reached. However, in contrast with the parametrization (28), where unity was taken as the minimum value for K, parametrization (29) gives $K_{\min}\approx1.89$ as the asymptotic value. Thus, in this parametrization, the entropy will never be maximum. However, if we consider the fact that with increasing en-

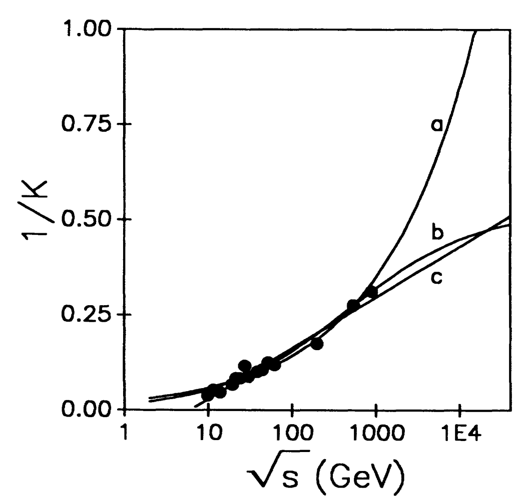

FIG. 6. Experimental NBD parameter K as a function of c.m. energy. The curves labeled as a, b, and c correspond to the parametric from Eqs. (28), (29), and (27), respectively.

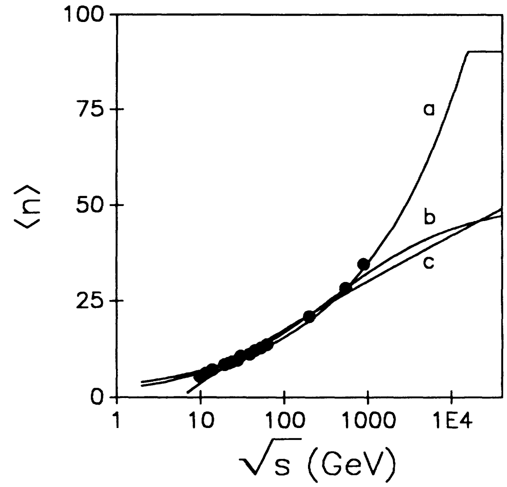

FIG. 7. Experimental NBD parameter $\langle n \rangle$ as a function of c.m. energy. The curves labeled as a, b, and c are obtained from Eq. (21) with the parametric form of K given by Eqs. (28), (29), and (27), respectively.

ergy leading particles produce more and more particles, then the minimum value $K_{\min} \approx 1.89$ has the added significance of representing two particle-producing sources. Indeed, with increasing energy, the large-x_F quarks will fly through the other hadron, carrying a significant fraction of momenta [19]. The small relative momenta among these quarks enable them to cocoon themselves with gluons to form the two leading systems (possibly) with quantum numbers the same as the incident hadron. The leading systems will be excited also (transverse momentum transfer increasing with energy) and can radiate pions sequentially. Then, in a simple model, the multiplicity distribution of produced particles from the two leading systems will be the NBD with the parameter $K=2$, corresponding to two sources. The value of $K=1.89$ obtained from fitting is very close to 2 [indeed the available data can be fitted well with Eq. (29) with $\alpha=2$].

Relation (21) between the parameters $\langle n \rangle$ and K can now be used to obtain the energy dependence of $\langle n \rangle$. In Fig. 7 we show the average number $\langle n \rangle$ as calculated from Eq. (21) using the parametric form of K as given by Eqs. (28), (29), and (27), labeled as a, b, and c, respectively. The energy dependence of $\langle n \rangle$ is correctly reproduced by all the three forms of K (except for the UA5 parametrization, the $\langle n \rangle$ value at 900 GeV is underestimated by few percent). While the parametric forms (28) and (29) indicate saturation at $\langle n \rangle \approx 90$ and 45, respectively, around LHC energies form (27) shows no such indication.

V. SUMMARY AND CONCLUSION

We have obtained a relation between the two parameters $\langle n \rangle$ and K of the negative-binomial distribution assuming that the multiparticle production process is a stochastic stationary branching process. The full phase-space multiplicity data for $\bar{p}p/pp$ collisions in the energy range 10–900 GeV confirm the relation. Unlike the earlier relation (which we have shown to have no theoretical basis), it is consistent with the energy dependence of $\langle n \rangle$ and K as observed by the UA5 Collaboration [1]. It was seen that empirically also a similar relation can be derived from the information-theoretic entropy considerations. The relation between $\langle n \rangle$ and K enables one to consider the energy dependence of either one of the two parameters, the energy dependence of the other being automatically fixed from the relation. It was shown that from maximum-entropy considerations one can obtain an upper bound on the average of the charged-particle multiplicity or, conversely, a lower bound on K. It thus seems that the number of particles cannot be increased indefinitely with energy. It will be interesting to see in future experiments at LHC energy whether or not data show such saturation, as it will indicate the limitation of the NBD model and its entropy interpretation. We have also given two simple parametrizations for the parameter K incorporating a lower bound on it. One is with the assumption that K should be bounded between the two limits 1 and ∞, the two limiting distributions, Bose-Einstein and Poisson, of the NBD and should be a decreasing function of the c.m.s. energy. In the other parametric form, we tried to obtain the minimum value of K from existing data. Both forms give a good description of the data and indicate that around 16 TeV (LHC energies) the asymptotic condition will be reached. An extrapolation of the different parametric forms for K at higher energies indicates that around LHC energies we shall be able to distinguish between different kinds of parametrizations.

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