Intermittency in high-energy collisions and a phase transition in the Feynman-Wilson fluid

N. G. Antoniou*
CERN, Geneva, Switzerland

A. P. Contogouris†
Department of Physics, University of Athens, Athens, Greece

C. G. Papadopoulos
Institute of Nuclear Physics, National Research Centre for Physical Sciences Democritos, Athens, Greece

S. D. P. Vlassopulos
Department of Physics, National Technical University, Athens, Greece

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We study the intermittency effect in multihadron production processes as a critical phenomenon reflecting a higher-order quark-hadron phase transition in the hadronization process. We show that the production of a critical Feynman-Wilson fluid, representing the hadronized system at the critical temperature, has a fractal structure in a wide range of scales in the rapidity space. The intermittency pattern is specified by the critical exponent alone and, for each factorial moment, a minimal scale in rapidity emerges below which the power-law behavior breaks down. The relevance of the model for quark-gluon-plasma physics in present and future experiments is also discussed.

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I. INTRODUCTION

The existence of a nonconventional component of multiparticle processes, reflecting the development of collective phenomena in the strongly interacting system, is now considered as a possibility. Such a component may lead to a power-law behavior of the scaled factorial moments [1] so that intermittency effects in rapidity are very significant in high-energy collisions.

In this work we conjecture that a higher-order quark-hadron phase transition, which takes place during the hadronization process, gives rise to strong fluctuations in a wide range of rapidity scales and leads to intermittency patterns. More specifically, we attempt to study the phenomenon of intermittency in the context of a critical Feynman-Wilson (FW) fluid model, inspired by the following picture: In a high-energy collision there is a finite probability of creating a thermal quark-gluon system in the central region as a result of energy-density fluctuations followed by vacuum excitation. The space-time evolution of this parton fluid is assumed to follow the hyperbolas $Z=\tau\sinh y$, $t=\tau\cosh y$, where the proper time τ, a decreasing function of the temperature T, specifies the time scale in the evolution process and y is the rapidity along the collision axis Z (Fig. 1). In this picture, the longitudinal growth of the system is followed by a gradual decrease of the average transverse momentum $\langle p_T\rangle \sim T$, and when the temperature reaches the critical values $T_c \simeq 100-200$ MeV, a quark-hadron phase transition takes place, producing strongly correlated hadrons along the one-dimensional rapidity space. This hadronization process cannot be characterized by a single time scale τ_c since the generated critical system of correlated hadrons cannot be localized on a single hyperbola, $t^2-Z^2=\tau^2$, the space-time points of which can only accommodate totally uncorrelated events. Therefore, the time scale in the hadronization process $\tau(y)$ becomes, in

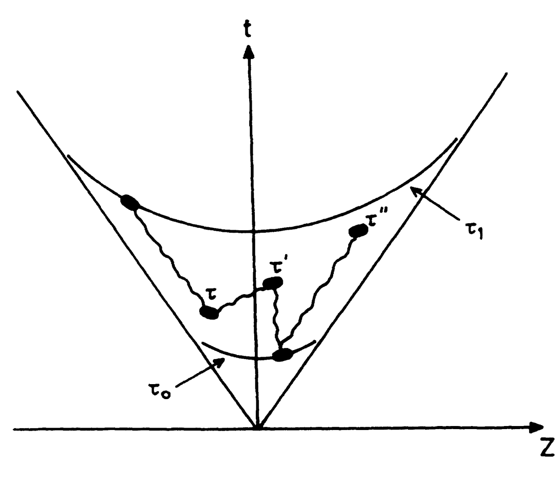

FIG. 1. The space-time evolution of the hadronization process.

*Also at Department of Physics, University of Athens, Athens, Greece.

†On leave from Department of Physics, McGill University, Montreal, Canada.

general, a random, rapidity-dependent quantity, directly related to the rapidity density of hadrons $\rho(y)$. In fact, a hadronization process along the worldline, $Z_h=\tau(y)\sinh y$, $t_h=\tau(y)\cosh y$, generates hadronic droplets with size $(\delta_y)_h$ and density $\rho(y)$ in rapidity, according to the equations [2]

$$\rho(y)=(\delta_y)_h^{-1}=(\delta Z)_h^{-1}\tau(y)\cosh y \ , \tag{1.1}$$

where $(\delta Z)_h$ is the spatial extension of the droplet along the Z axis. In the rest frame of the hadron we have $(\delta Z)_h=2R_h$, R_h being a typical hadronic radius and therefore $\tau(y)=2R_h\rho(y)$. This relation implies that a variation of $\tau(y)$ in a whole range of time scales, $\tau_0\leq\tau(y)\leq\tau_1$, as required by the underlying critical phenomenon, if $\tau_0\ll\tau_1$, may generate strong density fluctuations in a wide range of rapidity scales, which manifest themselves as intermittency patterns in high-energy collisions. It is of interest to note that, in a standard inclusive measurement of the rapidity distribution, after averaging over many events and smoothing out the irregularities of the time scale $\tau(y)$, one is likely to obtain a smooth inclusive density $\langle\rho(y)\rangle=\tau_h(2R_h)^{-1}$, where $\tau_h=\langle\tau(y)\rangle$ is a global measure of the hadronization time scale [3]. The above qualitative picture supports our proposal that the existence of a higher-order quark-hadron phase transition naturally leads to intermittency effects, suggesting at the same time that self-similarity in rapidity space may be the underlying simple and universal property which unifies the characteristic features of these complex and collective phenomena. Therefore, in order to formulate our proposal we consider the model of a FW fluid defined along the one-dimensional rapidity space and undergoing a higher-order phase transition. For this purpose, we impose Kadanoff scaling near the critical point and, in this context, the FW fluid becoming critical, simulates a newly hadronized system in a state of thermal equilibrium at the critical temperature $T=T_c$, produced in a high-energy collision. The cross section of this process is, in general, a very small fraction of the total cross section but it may become appreciable especially in relativistic heavy-ion collisions, where a quark-gluon plasma is expected to be easily produced. Phenomenologically, this production mechanism competes with conventional hadronic processes and, therefore, any new, cooperative effects due to the above phase transition coexist with conventional hadronic phenomena. This means that any attempt to compare the nonconventional features of this new mechanism with experimental measurements must seriously take into account the non-negligible background of the conventional hadronic physics at low transverse momenta.

The plan of this paper is as follows. In Sec. II, the statistical mechanics of the critical FW fluid is discussed and the properties of the corresponding critical hadronic system are investigated. In Sec. III, the intermittent behavior of the scaled factorial moments is established, and the existence and the origin of a minimal scale in intermittency patterns is discussed. In Sec. IV, the phenomenological implications of the model for present and future experiments are considered, especially in connection with the phenomenon of intermittency. In particular a two-component mechanism for intermittency is adopted, taking into account the effect of conventional, finite-range correlations. In this context, a qualitative study of recent experimental patterns of factorial moments is attempted and the connection of the intermittency phenomenon with the critical FW fluid production in high-energy collisions is discussed. Finally, in Sec. V, our results are summarized and our concluding remarks are presented.

II. THE STATISTICAL MECHANICS OF THE CRITICAL FW FLUID

We consider the system of hadrons generated by a quark-hadron phase transition in a high-energy collision and study the distribution in rapidity y, integrating over the transverse-momentum spectrum corresponding to the critical temperature $T=T_c$. In the FW fluid picture, the grand-canonical partition function near the critical point $(T=T_c, z\to1)$ is written as

$$Q_c(z,\Delta)=\sum_N z^N Z_c(N,\Delta) \ , \tag{2.1}$$

where $Z_c(N,\Delta)$ is the canonical partition function for N particles, at the critical temperature, z is the analogue fugacity, and Δ the total rapidity interval. Recent studies in QCD on a lattice appear to support a weak first-order confinement-deconfinement phase transition but, in order to reach a firm conclusion on this important issue, further investigation is needed [4]. In this work we assume that the quark-hadron phase transition is sufficiently close to a higher-order critical phenomenon [5] and therefore we impose Kadanoff scaling near the critical point of the corresponding FW fluid. The statistical mechanics of the system at $T=T_c$ is now specified by the boundary conditions

$$\ln Q_c(z,\Delta)=p(z)\Delta \quad (\Delta\to\infty, z>1) \ , \tag{2.2}$$

$$\ln Q_c(z,\Delta)=f[\Delta(z-1)^{1/(1-\eta)}] \quad (\Delta\to\infty, z\to1) \ , \tag{2.3}$$

where η is a critical exponent $(0<\eta<1)$ and $p(z)$ is the analogue pressure of the system. Equation (2.2) gives the normal behavior of the FW fluid in the thermodynamic limit, which is equivalent to the ordinary Regge behavior of the hadronic system in the high-energy limit. Equation (2.3), on the other hand, expresses Kadanoff scaling as a characteristic property of the newly hadronized system $(T=T_c)$ in a process of quark-hadron phase transition. The critical exponent η specifies the relation of the order parameter ρ (density in rapidity) with the ordering field p (pressure) near the critical point $\rho_c=p_c=0$. From Eqs. (2.2) and (2.3) we get

$$\begin{aligned}\bar{\rho}&=-\frac{1}{\Delta Q_c}\frac{\partial Q_c}{\partial \ln z}=f'(0)\frac{1}{1-\eta}(z-1)^{\eta/(1-\eta)} \quad (z\to1) \ ,\\ p&\sim(z-1)^{1/(1-\eta)} \quad (z\to1) \ ,\\ \bar{\rho}&\sim p^{\eta} \quad (p\to0) \ .\end{aligned} \tag{2.4}$$

The power-law behavior (2.4) corresponds to a typical liquid-gas phase transition and in the mean-field approximation the value of the critical exponent is $\eta=\frac{1}{3}$ [6]. Although this approximation is very crude for the critical FW fluid, in which strong density fluctuations are expected, this estimate of the critical exponent may be a useful guide in phenomenological studies.

In the limit $\Delta\to\infty$ we transform the series (2.1) to an integral representation, as follows:

$$\begin{aligned} Q_c(z,\Delta)&=\int_0^\infty R_c(x,\Delta)\exp(xw)dx \quad (w>0)\ , \\ R_c(x,\Delta)&\equiv\langle N\rangle Z_c(N,\Delta)\ . \end{aligned} \tag{2.5}$$

In Eq. (2.5) we have introduced the variables $x\equiv N/\langle N\rangle$, $w\equiv\langle N\rangle \ln z$, where $\langle N\rangle$ is the average multiplicity of hadrons at the critical point $z=1$, $T=T_c$. Combining now Eqs. (2.3) and (2.5) we find, in the limit $\Delta\to\infty$,

$$\langle N\rangle Z_c(N,\Delta)=h(x)\ , \tag{2.6}$$

$$\langle N\rangle=c\Delta^{1-\eta}\quad(z=1)\ , \tag{2.7}$$

$$p(z)=\left[\frac{1}{b}\ln z\right]^{1/(1-\eta)}\quad(z>1)\ , \tag{2.8}$$

where $h(x)$ is a scaling function and b,c are constants. Equation (2.6) indicates that Kadanoff scaling in the FW field is equivalent to Koba-Nielsen-Olesen (KNO) scaling in the hadronic system. Equation (2.7) suggests that the critical FW fluid is a fractal system [7] in rapidity space, but, in order to establish this important property, one must study the behavior of the correlation function $\langle\rho_c(y_1)\rho_c(y_2)\rangle$. Finally, Eq. (2.8) gives the ordering field $p(z)$ in the noncritical region $z>1$, where the FW fluid is expected to behave like a conventional hadronic system with short-range ordering in rapidity and a linear growth of multiplicity with the size of the system ($\langle N\rangle\sim\Delta$ for $z>1$). In what follows we elaborate on these issues by studying the implications of Eqs. (2.6)–(2.8) for the statistical mechanics of the critical hadronic system. For this purpose, using Eqs. (2.5)–(2.8) we write an integral equation for the scaling function $h(x)$ as

$$\int_0^\infty h(x)\exp(xw)dx=\exp(\gamma w^{1/(1-\eta)})\quad(w>0)\ , \tag{2.9}$$

where $\gamma=(bc)^{1/(\eta-1)}$. The solution of Eq. (2.9) in the limit $x\gg1$, obtained by the steepest-descent method, leads to the following form for the canonical partition function:

$$Z_c(N,\Delta)=\Delta^{(\eta-1)/2\eta}N^{(1-2\eta)/2\eta}\exp(-gN^{1/\eta}\Delta^{(\eta-1)/\eta})\quad(N\gg\langle N\rangle)\ , \tag{2.10}$$

where $g=\eta(1-\eta)^{(1-\eta)/\eta}b^{1/\eta}$. Taking the Laplace transform $\tilde{Z}_c(N,\xi)$ of the partition function $Z_c(N,\Delta)$, observe that there is a saddle point $\Delta_0\sim N$, which for $N\to\infty$ lies in the region of validity of Eq. (2.10) and gives the dominant contribution to $\tilde{Z}_c(N,\xi)$. This contribution represents the significant effect of large multiplicity fluctuations, expected at the critical point, and has the asymptotic form

$$\tilde{Z}_c(N,\xi)=\xi^{-\eta}\exp(-bN\xi^{1-\eta})\quad(N\gg1)\ . \tag{2.11}$$

In the thermodynamic limit, however, the properties of the system and especially its critical behavior are specified by the exponential factor in $\tilde{Z}_c(N,\xi)$ alone, and therefore we may consider the simplest solution, valid also for low multiplicites ($N=2,3,\ldots$), by iterating a factorizable kernel $\tilde{K}_f(\xi)$ according to the equations [2]

$$\tilde{K}_f(\xi)=\exp(-b\xi^{1-\eta})\ , \tag{2.12}$$

$$\tilde{Z}_f(N,\xi)=\exp[-b(N-1)\xi^{1-\eta}]\ , \tag{2.13}$$

$$\tilde{Q}_f(z,\xi)=z^2[\exp(b\xi^{1-\eta})-z]^{-1}\ . \tag{2.14}$$

Employing now the factorizability of the model, we obtain, at the critical point, the following expressions for the inclusive density $\langle\rho_c(y)\rangle$ and the correlation function $\langle\rho_c(y_1)\rho_c(y_2)\rangle$:

$$\langle\rho_c(y)\rangle=[Q_f(\Delta)]^{-1}Q_f\left[y+\frac{\Delta}{2}\right]Q_f\left[\frac{\Delta}{2}-y\right]\ , \tag{2.15}$$

$$\langle\rho_c(y_1)\rho_c(y_2)\rangle=[Q_f(\Delta)]^{-1}Q_f\left[y_1+\frac{\Delta}{2}\right]\times Q_f(y_2-y_1)Q_f\left[\frac{\Delta}{2}-y_2\right]\ , \tag{2.16}$$

where $y_1<y_2$ and, for any rapidity interval δ,

$$Q_f(\delta)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}[\exp(b\xi^{1-\eta})-1]^{-1}e^{\xi\delta}d\xi\ . \tag{2.17}$$

Equation (2.17) reveals, for the critical hadronic system, a natural scale δ_0 in the rapidity space, related to the parameter b. In fact, Eq. (2.17) leads to the scaling property

$$Q_f(\delta)=\delta_0^{-1}G_f\left[\frac{\delta}{\delta_0}\right]\ , \tag{2.18}$$

where $\delta_0=b^{1/(1-\eta)}$ and

$$G_f(\omega)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}[\exp(\lambda^{1-\eta})-1]^{-1}e^{\lambda\omega}d\lambda\ . \tag{2.19}$$

The contribution of the leading branch-point singularity at $\lambda=0$, to $G_f(\omega)$, leads to the following power-law behavior of $Q_f(\delta)$ for $\delta\gg\delta_0$:

$$Q_f(\delta)=[\delta_0\Gamma(1-\eta)]^{-1}\left[\frac{\delta}{\delta_0}\right]^{-\eta}\quad(\delta\gg\delta_0)\ . \tag{2.20}$$

From Eqs. (2.16) and (2.20), assuming $|y_1|\ll\Delta/2$, $|y_2|\ll\Delta/2$, in order to avoid the end effects, and $\delta_0\ll\Delta$, we obtain a power-law behavior for the correlation function

$$\langle\rho_c(y_1)\rho_c(y_2)\rangle=[\delta_0^{1-\eta}\Gamma(1-\eta)]^{-2}\left[\frac{\Delta}{4}\right]^{-\eta}(y_2-y_1)^{-\eta} \tag{2.21}$$

valid in the region $\delta_0\ll|y_1-y_2|\ll\Delta$. On the other hand, if $|y\pm\Delta/2|\gg\delta_0$, the density in the central region ($|y|\ll\Delta/2$) takes the form

$$\langle\rho_c(y)\rangle=[\delta_0^{1-\eta}\Gamma(1-\eta)]^{-1}\Delta^{\eta}\left[\frac{\Delta^2}{4}-y^2\right]^{-\eta}\ . \tag{2.22}$$

Integrating Eqs. (2.21) and (2.22) in a rapidity interval $\delta \ll \Delta$, around the central point $y=0$, we obtain the following behavior for the second-order factorial moment, normalized according to the Bialas and Peschanski proposal [1]:

$$F_2(\delta;\Delta)=\frac{\langle N(N-1)\rangle_\delta}{\langle N\rangle_\delta^2}=\frac{2}{(1-\eta)(2-\eta)}\left[\frac{4\delta}{\Delta}\right]^{-\eta} \quad (\delta_0 \ll \delta \ll \Delta)\ . \tag{2.23}$$

Equations (2.21) and (2.23) indicted that the critical hadronic system described by Eqs. (2.12)–(2.14) for $z=1$ has a random fractal structure in rapidity, for a wide range of scale δ ($\delta_0 \ll \delta \ll \Delta$). The two parameters of the model (η, b) are connected with the characteristic elements of the corresponding fractal geometry. The critical exponent η determines the fractal dimension $d_F=1-\eta$, and the parameter b introduces a minimal scale δ_m in rapidity ($\delta_m \sim b^{1/(1-\eta)}$) below which fractality breaks down. We also notice that, if $\delta_0 \to 0$, intermittency effects become important [Eq. (2.23)] whereas, for a sufficiently extended system ($\Delta \to \infty$), the fractal structure persists even at large scales [8].

In Sec. III, we study in detail the structure of the intermittency patterns resulting from our model by generalizing the power-law behavior (2.23) for the higher-order moments. For this purpose, one needs the inclusive densities $\langle \rho_c(y_1)\rho_c(y_2)\cdots\rho_c(y_p)\rangle$ for $p \ge 2$ which, owing to the factorization property of the model [Eqs. (2.12)–(2.14)], are written as

$$\langle \rho_c(y_1)\rho_c(y_2)\cdots\rho_c(y_p)\rangle = Q_f^{-1}(\Delta)Q_f\left[\frac{\Delta}{2}+y_1\right] \times Q_f\left[\frac{\Delta}{2}-y_p\right] \times \prod_{i=2}^{p} Q_f(y_i-y_{i-1})\ , \tag{2.24}$$

where $-\Delta/2<y_1<y_2\cdots<y_p<\Delta/2$. In the region $\Delta/2+y_1 \gg \delta_0$, $\Delta/2-y_p \gg \delta_0$, and $y_i-y_{i-1}\gg\delta_0$, the densities (2.24) take the form

$$\langle \rho_c(y_1)\rho_c(y_2)\cdots\rho_c(y_p)\rangle = \frac{\Delta^\eta}{[\delta_0^{1-\eta}\Gamma(1-\eta)]^p} \times \left[\frac{\Delta}{2}+y_1\right]^{-\eta} \times \left[\frac{\Delta}{2}-y_p\right]^{-\eta} \times \prod_{i=2}^{p}(y_i-y_{i-1})^{-\eta}\ . \tag{2.25}$$

It is now straightforward to show that the factorizable model defined by Eqs. (2.12)–(2.14) also satisfies asymptotic KNO scaling, the corresponding scaling function $\psi(x)$ being the unique solution of a well-defined moment problem. In fact, integrating Eqs. (2.22) and (2.25) in the domain of the rapidity space specified by the size Δ of the system, we obtain

$$\langle N(N-1)\cdots(N-p+1)\rangle = p!\frac{\Delta^\eta}{[\delta_0^{1-\eta}\Gamma(1-\eta)]^p} \times \int_{-\Delta/2}^{\Delta/2} dy_p \left[\frac{\Delta}{2}-y_p\right]^{-\eta} \times \int_{-\Delta/2}^{y_p}(y_p-y_{p-1})^{-\eta}dy_{p-1}\cdots\int_{-\Delta/2}^{y_2}\left[y_1+\frac{\Delta}{2}\right]^{-\eta}(y_2-y_1)^{-\eta}dy_1\ , \tag{2.26}$$

$$\langle N\rangle = \frac{\Delta^\eta}{\delta_0^{1-\eta}\Gamma(1-\eta)}\int_{-\Delta/2}^{\Delta/2}\left[\frac{\Delta^2}{4}-y^2\right]^{-\eta}dy\ . \tag{2.27}$$

Performing the integrations in Eqs. (2.26) and (2.27) we find, in the limit $\Delta \to \infty$,

$$C_p=\frac{\langle N^p\rangle}{\langle N\rangle^p}=\frac{p![\Gamma(1-\eta)]^{1-p}[\Gamma(2-2\eta)]^p}{\Gamma[(p+1)(1-\eta)]} \quad (p=2,3,\dots)\ . \tag{2.28}$$

Equation (2.28) shows the validity of asymptotic KNO scaling in the factorizable model, whereas the large-p behavior of the multiplicity moments, $\ln C_p \sim \eta p \ln p$, guarantees the uniqueness of the scaling function $\psi(x)$ as a solution of the corresponding moment problem [9]:

$$\int_0^\infty x^p\psi(x)dx = C_p \quad (p=2,3,\dots)\ . \tag{2.29}$$

In particular, the solution of Eq. (2.29) for $x \gg 1$ is obtained by the steepest-descent method, using the asymptotic expression C_p [Eq. (2.28)] in the limit $p\to\infty$. We find [10]

$$\psi(x)=\frac{\Gamma(1-\eta)}{\sqrt{2\pi\eta}}\left[\frac{\Gamma(2-\eta)}{\Gamma(2-2\eta)}\right]^{1/2\eta}x^{1/2\eta}\exp(-\alpha x^{1/\eta}) \quad (x\gg 1)\ , \tag{2.30}$$

where $\alpha\equiv\eta(1-\eta)^{(1-\eta)/\eta}[\Gamma(1-\eta)/\Gamma(2-2\eta)]^{1/\eta}$.

We close this section by studying the analytic properties of the grand-partition function $\tilde{Q}_f(z,\xi)$ in the ξ plane [Eq. (2.14)] in order to reveal the nature of the phase transition in the FW fluid when we approach the critical value $z=1$. In the thermodynamic limit, $\Delta\to\infty$, the grand-partition function $Q_f(z,\Delta)$ is dominated by the leading singularities in the ξ plane which, for $z>1$, are located at the points $\xi_c(z)=(b^{-1}\ln z)^{1/1-\eta}$ and $\xi_0=0$. The dominant singularity at $\xi=\xi_c(z)$ is a moving pole, as a function of z, whereas the secondary singularity is a fixed branch point at $\xi=0$. At the critical value $z=1$ these leading singularities collide, and for $0<z<1$ the fixed branch point at $\xi=0$ becomes dominant, whereas the moving pole disappears in the unphysical sheet. This mechanism in the ξ plane represents, formally, a critical phenomenon in the FW fluid, since the trajectory of the rightmost singularity, which dominates the thermodynamic behavior of the system, is not analytic at $z=1$.

Physically, the nature of this phase transition becomes transparent if one considers the behavior of the correlation function $C(y_1,y_2;z)$ near the critical value $z=1$ and for large rapidity differences y_2-y_1. The factorizability of the model enables us to determine $C(y_1,y_2;z)$ for $z>1$ in terms of the leading singularities at $\xi=\xi_c(z)$ and $\xi=0$ as follows. We have

$$C(y_1,y_2;z)=\langle\rho(y_1;z)\rho(y_2;z)\rangle-\langle\rho(y_1;z)\rangle\langle\rho(y_2;z)\rangle\ , \tag{2.31}$$

where $\langle\rho(y;z)\rangle$ and $\langle\rho(y_1;z)\rho(y_2;z)\rangle$ are inclusive densities for $z>1$. The contribution of the leading singularities to the grand partition function $Q_f(z,\delta)$ leads to the following approximation for large rapidity differences $(\delta\gg\delta_0)$ and $z>1$:

$$Q_f(z,\delta)=\frac{z[\xi_c(z)]^\eta}{b(1-\eta)}\exp[\delta\xi_c(z)]+\frac{z^2b(1-\eta)}{(1-z)^2\Gamma(\eta)}\delta^{\eta-2}\quad(z>1,\ \delta\gg\delta_0)\ . \tag{2.32}$$

On the other hand, the factorization property, which is also valid for $z>1$, leads to the inclusive densities

$$\langle\rho(y;z)\rangle=z^{-1}\frac{Q_f(z,y+\Delta/2)Q_f(z,\Delta/2-y)}{Q_f(z,\Delta)}\ , \tag{2.33}$$

$$\langle\rho(y_1;z)\rho(y_2;z)\rangle=z^{-2}\frac{Q_f(z,y_1+\Delta/2)Q_f(z,y_2-y_1)Q_f(z,\Delta/2-y_2)}{Q_f(z,\Delta)}\ , \tag{2.34}$$

Using now Eqs. (2.32)–(2.34), we obtain the leading terms for $\Delta\to\infty$,

$$\langle\rho(y;z)\rangle=\frac{[\xi_c(z)]^\eta}{b(1-\eta)}\ , \tag{2.35}$$

$$\langle\rho(y_1;z)\rho(y_2;z)\rangle=\frac{[\xi_c(z)]^{2\eta}}{[b(1-\eta)]^2}+\frac{z[\xi_c(z)]^\eta}{(1-z)^2\Gamma(\eta)}\times\frac{\exp(-\xi_c|y_1-y_2|)}{|y_1-y_2|^{2-\eta}}\ , \tag{2.36}$$

which, when combined with Eq. (2.31), lead to the following behavior of the correlation function [10]:

$$C(y_1,y_2;z)\sim\frac{\exp(-\xi_c|y_1-y_2|)}{|y_1-y_2|^{2-\eta}}\quad(z>1,|y_1-y_2|\gg\delta_0)\ . \tag{2.37}$$

At the critical value $z=1$, the previous analysis breaks down since the two leading singularities collide at the point $\xi=0$. In this case the correlation function is determined by the critical densities [Eqs. (2.15), (2.16), (2.21), and (2.22)] and in the limit $|y_1|\sim\Delta$, $|y_2|\sim\Delta$, $|y_1-y_2|\sim\Delta(\Delta\to\infty)$, which is suitable for studying the critical behavior of the correlation function at large distances, we find

$$C(y_1,y_2;z=1)\sim|y_1-y_2|^{-2\eta}\quad(|y_1|\sim\Delta,\ |y_2|\sim\Delta,\ |y_1-y_2|\sim\Delta)\ . \tag{2.38}$$

We observe that the grand-partition function (2.14) for $z>1$ describes a conventional hadronic system with low transverse momenta and short-range ordering in rapidity. The correlation length $\delta_c=\xi_c^{-1}(z)$ is finite [Eq. (2.37)] and the rapidity density is constant [Eq. (2.35)], leading to an average multiplicity which grows linearly with the size of the system. When we approach the critical point $(z=1)$, however, the system undergoes a higher-order phase transition. The correlation length becomes infinitely large, $\delta_c\sim(z-1)^{1/\eta-1}$, and asymptotic KNO scaling is established as a critical property, equivalent to Kadanoff scaling. Moreover, the critical system develops a fractal structure in rapidity, in a wide range of scales $(\delta_0\ll\delta\ll\Delta)$ and, as we shall discuss in the next section, an intermittency pattern for the scaled factorial moments is fully developed in the limit $\delta_0\to 0$.

III. INTERMITTENCY IN THE CRITICAL FW FLUID

Using Eqs. (2.22) and (2.25) for the critical inclusive densities and integrating in a rapidity interval $|y|\le\delta/2$, we obtain, for $\delta_0\ll\delta$, the forms

$$\langle N(N-1)\cdots(N-p+1)\rangle_\delta=\frac{p!\Delta^\eta}{[\delta_0^{1-\eta}\Gamma(1-\eta)]^p}\int_{-\delta/2}^{\delta/2}dy_p\left[\frac{\Delta}{2}-y_p\right]^{-\eta}\int_{-\delta/2}^{y_p}dy_{p-1}(y_p-y_{p-1})^{-\eta}\cdots\int_{-\delta/2}^{y_2}dy_1\left[y_1+\frac{\Delta}{2}\right]^{-\eta}(y_2-y_1)^{-\eta}\ , \tag{3.1}$$

$$\langle N\rangle_\delta=\frac{\Delta^\eta}{\delta_0^{1-\eta}\Gamma(2-\eta)}\int_{-\delta/2}^{\delta/2}\left[\frac{\Delta^2}{4}-y^2\right]^{-\eta}dy\ . \tag{3.2}$$

Changing the integration variables in Eq. (3.1) and performing the integration in (3.2), we obtain

$$\langle N(N-1)\cdots(N-p+1)\rangle_\delta=\frac{p!\Delta^{\eta}\delta^{p-\eta(p+1)}}{[\delta_0^{1-\eta}\Gamma(1-\eta)]^p}I\left[\frac{\delta}{\Delta};p,\eta\right], \tag{3.3}$$

$$\langle N\rangle_\delta=\frac{\Delta^{1-\eta}}{\delta_0^{1-\eta}\Gamma(1-\eta)}[B_{(\Delta+\delta)/2\Delta}(1-\eta,1-\eta)-B_{(\Delta-\delta)/2\Delta}(1-\eta,1-\eta)], \tag{3.4}$$

where $B_r(a,b)$ is the incomplete beta function and

$$I\left[\frac{\delta}{\Delta};p,\eta\right]=p!\int_0^1 d\tau_p(D_+-\tau_p)^{-\eta}\int_0^{\tau_p}d\tau_{p-1}(\tau_p-\tau_{p-1})^{-\eta}\cdots\int_0^{\tau_2}d\tau_1(D_-+\tau_1)^{-\eta}(\tau_2-\tau_1)^{-\eta}, \tag{3.5}$$

where $D_\pm=(\Delta\pm\delta)/2\delta$. In Eq. (3.5), a correction of order $(\delta_0/\delta)^{1-\eta}$, due to the integration along the forbidden intervals $|y_i-y_{i-1}|\lesssim\delta_0$ in Eq. (3.1), has been neglected. In the limit $\delta_0\to 0$, however, where strong intermittency effects are expected to be present, the integral representation (3.1) is exact. On the other hand, the effect of the scale δ_0 on the intermittency patterns is discussed in detail at the end of this section [Eqs. (3.17)–(3.21)]. Using now the Laplace convolution theorem in Eq. (3.5), we obtain the following expression for $I(\delta/\Delta;p,\eta)$:

$$I\left[\frac{\delta}{\Delta};p,\eta\right]=\frac{p![\Gamma(1-\eta)]^{p-1}}{\Gamma[(1-\eta)(p-1)]}\int_0^1 du(D_+-u)^{-\eta}\int_0^u dv(D_-+v)^{-\eta}(u-v)^{(1-\eta)(p-1)-1}. \tag{3.6}$$

Introducing now appropriate integral representations, we have

$$I\left[\frac{\delta}{\Delta};p,\eta\right]=L(p,\eta)\int_0^1 du(D_+-u)^{-\eta}(D_-+u)^{-\eta}u^{(1-\eta)(p-1)}$$
$$\times F\left[\eta,(1-\eta)(p-1);(1-\eta)(p-1)+1;\frac{u}{D_-+u}\right], \tag{3.7}$$

where $L(p,\eta)=p![\Gamma(1-\eta)]^{p-1}/\Gamma[1+(1-\eta)(p-1)]$ and, finally,

$$I\left[\frac{\delta}{\Delta};p,\eta\right]=\frac{p![\Gamma(1-\eta)]^{p-1}}{\Gamma[1+\eta+p(1-\eta)]}D_-^{-2\eta}F_3\left[\eta,\eta,1,1,1+\eta+p(1-\eta),-\frac{1}{D_-},-\frac{1}{D_-}\right], \tag{3.8}$$

where F is the hypergeometric series and F_3 a hypergeometric function of two variables [11].

The scaled factorial moments for $p\geq 2$, in the rapidity interval δ,

$$F_p(\delta;\Delta)=\frac{\langle N(N-1)\cdots(N-p+1)\rangle_\delta}{\langle N\rangle_\delta^p} \tag{3.9}$$

introduced by Bialas and Peschanski [1] in order to study intermittency effects, take, by virtue of Eqs. (3.3), (3.4), and (3.8), the form

$$F_p^{(c)}(\delta;\Delta)=\left[\frac{\delta}{\Delta}\right]^{\eta+(1-\eta)p}[B_{(\Delta+\delta)/2\Delta}(1-\eta,1-\eta)-B_{(\Delta-\delta)/2\Delta}(1-\eta,1-\eta)]$$
$$\times\frac{p![\Gamma(1-\eta)]^{p-1}}{\Gamma[1+\eta+p(1-\eta)]}\left[\frac{\Delta-\delta}{2\Delta}\right]^{-2\eta}F_3\left[\eta,\eta,1,1,1+\eta+(1-\eta)p,\frac{2\delta}{\delta-\Delta},\frac{2\delta}{\delta-\Delta}\right], \tag{3.10}$$

which corresponds to the critical densities (2.5) and is valid for $\delta_0\ll\delta\leq\Delta$. Equation (3.10) shows that the pattern of factorial moments is invariant under scale transformations in the rapidity space, owing to the Kadanoff or the KNO scaling property, provided that $\delta\gg\delta_0$. The fractal structure of the system is revealed by studying the behavior of $F_p^{(c)}(\delta;\Delta)$ in the appropriate range of scales, $\delta_0\ll\delta\ll\Delta$, as discussed in the previous section. In fact, in this limit, Eq. (3.10) leads to the power-law behavior [2]

$$F_p^{(c)}(\delta;\Delta)=\frac{p![\Gamma(1-\eta)]^{p-1}}{\Gamma[1+\eta+(1-\eta)p]}\left[\frac{4\delta}{\Delta}\right]^{-\eta(p-1)}$$
$$(p\geq 2,\delta_0\ll\delta\ll\Delta). \tag{3.11}$$

Equation (3.11) generalizes, at the level of higher-order moments ($p>2$), our previous result [Eq. (2.23)] on the second moment $F_2(\delta;\Delta)$ and establishes the fractality of the production process in the range of scales $\delta_0\ll\delta\ll\Delta$, with a single fractal dimension $d_F=1-\eta$. In the limit

$\delta_0 \to 0$, a clear intermittent behavior emerges with a linear spectrum of intermittency indices in accord with other studies on the connection of a higher-order phase transition with the phenomenon of intermittency [12].

Another important aspect of this model is the emergence of a minimal rapidity interval $\delta_m(p) \sim \delta_0$, below which the power-law behavior (3.11) breaks down. In order to study this intermittency-breaking effect and get an accurate estimate of $\delta_m(p)$, we consider the range of scales $0<\delta \ll \Delta$ which contains the nonfractality region $0<\delta \lesssim \delta_0$ and write for the moments $F_p^{(c)}(\delta;\Delta)$ the general form

$$F_p^{(c)}(\delta;\Delta)=\frac{p!}{Q_f(\Delta)\langle N\rangle_\delta^p}\int_{-\delta/2}^{\delta/2}dy_p\, Q_f\left[\frac{\Delta}{2}-y_p\right] \times \int_{-\delta/2}^{y_p}dy_{p-1}\, Q_f(y_p-y_{p-1})\cdots \int_{-\delta/2}^{y_2}dy_1\, Q_f(y_2-y_1)Q_f\left[\frac{\Delta}{2}+y_1\right], \tag{3.12}$$

where

$$\langle N\rangle_\delta=\frac{1}{Q_f(\Delta)}\int_{-\delta/2}^{\delta/2}Q_f\left[\frac{\Delta}{2}+y\right]Q_f\left[\frac{\Delta}{2}-y\right]dy\ . \tag{3.13}$$

In the integration region $|y_i|\le\delta/2$, we use the approximation $Q_f(\Delta/2\pm y_i)\approx Q_f(\Delta/2)$ and Eq. (3.12) is written as

$$F_p^{(c)}(\delta;\Delta)=p!\left[\frac{Q_f(\Delta)}{Q_f^2(\Delta/2)}\right]^{p-1}\delta^{-p}\mathcal{L}^{-1}[[\tilde{Q}_f(\xi)]^{p-1}\xi^{-2};\delta]\ , \tag{3.14}$$

where $\mathcal{L}^{-1}$ denotes the inverse Laplace transform. From Eq. (2.12) we have

$$[\tilde{Q}_f(\xi)]^{p-1}=\sum_{N=p-1}^{\infty}\frac{(N-1)!}{(p-2)!(N-p+1)!}\exp(-bN\xi^{1-\eta})\ , \tag{3.15}$$

so that Eq. (3.14) becomes

$$F_p^{(c)}(\delta;\Delta)=p!\left[\frac{Q_f(\Delta)}{Q_f^2(\Delta/2)}\right]^{p-1}\delta^{-p}\sum_{N=p-1}^{\infty}\frac{(N-1)!}{(p-2)!(N-p+1)!}\mathcal{L}^{-1}[\xi^{-2}\exp(-bN\xi^{1-\eta});\delta]\ . \tag{3.16}$$

Finally, using the Laplace convolution theorem, the saddle-point method, and the integral representation of the incomplete gamma function $\Gamma(a,u)$, we get, for $0<\delta\ll\Delta$,

$$F_p^{(c)}(\delta;\Delta)=p!\left[\frac{Q_f(\Delta)}{Q_f^2(\Delta/2)}\right]^{p-1}\frac{\delta^{1-p}}{\sqrt{2\pi(1-\eta)}} \times \sum_{N=p-1}^{\infty}g_p(N)\left[\Gamma\left[\tfrac{1}{2},u_N\right]-u_N^{\eta/(1-\eta)}\Gamma\left[\frac{1-3\eta}{2-2\eta},u_N\right]\right], \tag{3.17}$$

where

$$u_N=\eta(1-\eta)^{(1-\eta)/\eta}N^{1/\eta}\left[\frac{\delta_0}{\delta}\right]^{(1-\eta)/\eta},\quad g_p(N)=\frac{(N-1)!}{(p-2)!(N-p+1)!}\ . \tag{3.18}$$

In the limit $\delta\to 0$, Eq. (3.17) becomes

$$F_p^{(c)}(\delta;\Delta)=p!\left[\frac{Q_f(\Delta)}{Q_f^2(\Delta/2)}\right]^{p-1}\frac{\eta\delta^{1-p}}{\sqrt{2\pi(1-\eta)^3}}\sum_{N=p-1}^{\infty}g_p(N)u_N^{-3/2}\exp(-u_N)\ . \tag{3.19}$$

The dominant term $(N=p-1)$ in the series (3.19) gives the nonfractal behavior of $F_p^{(c)}(\delta;\Delta)$ in the region $\delta\ll\delta_0$,

$$F_p^{(c)}(\delta;\Delta)\sim\delta^{1-p}u_{p-1}^{-3/2}\exp(-u_{p-1})\quad(\delta\ll\delta_0)\ , \tag{3.20}$$

characterized by an exponential drop in the limit $\delta\to 0$. An estimate of $\delta_m(p)$, the minimal scale below which intermittency breaks down, is obtained by the value of δ for which the argument of the exponential in Eq. (3.20) becomes of order unity, $u_{p-1}\approx 1$. Using the definition of u_N [Eq. (3.18)], we find

$$\delta_m(p)=\eta^{\eta/(1-\eta)}(1-\eta)(p-1)^{1/(1-\eta)}\delta_0\ . \tag{3.21}$$

It is of interest to note that the existence of a minimal intermittency scale in this model reflects the very-short-distance behavior in rapidity space of the effective potential $V(y_i-y_j)$ corresponding to the kernel $\tilde{K}_f(\xi)$. In fact, using the steepest-descent method one finds [13]

$$V(y_i-y_j)=\frac{1+\eta}{2\eta}\ln\left|\frac{y_i-y_j}{\delta_0}\right| +\eta(1-\eta)^{(1-\eta)/\eta}\left|\frac{y_i-y_j}{\delta_0}\right|^{(\eta-1)/\eta} \tag{3.22}$$

The exponential behavior (3.20) in the nonfractality region $\delta \ll \delta_0$, leading to the estimate (3.21) of the minimal scale, is directly related to the presence of the second term in the potential (3.22), which dominates the correlation of hadrons for $|y_i-y_j| \ll \delta_0$ [2].

For completeness, one may show that the moments $F_p^{(c)}(\delta,\Delta)$ given by Eq. (3.17) have, in the region $\delta_m(p) \ll \delta \ll \Delta$, a fractal behavior of the form (3.11), as expected. For this purpose, we transform the series in Eq. (3.17) to an integral and, keeping the dominant term in the limit $\delta_m(p) \ll \delta$, we find the power law

$$F_p^{(c)}(\delta;\Delta)=G(p,\eta)\left[\frac{4\delta}{\Delta}\right]^{-\eta(p-1)} \quad [\delta_m(p) \ll \delta \ll \Delta]\ , \tag{3.23}$$

where

$$G(p,\eta)=[\Gamma(1-\eta)]^{p-1}\frac{p(p-1)\eta^{1-\eta(p-1)}}{(1-\eta)^{(p-1)(1-\eta)+1/2}} \times\int_0^{\infty}u^{\eta(p-1)-1}\left[\Gamma(\tfrac{1}{2},u)-u^{\eta/(1-\eta)}\Gamma\left[\frac{1-3\eta}{2-2\eta},u\right]\right]\frac{du}{\sqrt{2\pi}}\ . \tag{3.24}$$

Performing the integration in Eq. (3.24), we finally obtain

$$G(p,\eta)=\left[\frac{\Gamma(1-\eta)}{\eta^{\eta}(1-\eta)}\right]^{p-1}\frac{p\Gamma(\frac{1}{2}+\eta p-\eta)}{(1-\eta)^{1/2-\eta(p-1)}[1+(p-1)(1-\eta)]\sqrt{2\pi}}\ . \tag{3.25}$$

Comparing now Eqs. (3.11) and (3.23), which correspond to different approximations, we realize that the prefactors in the power law are identical for $\eta=\frac{1}{2}$ since, in this case, the model is soluble and the steepest-descend method gives the exact results [13]. For $\eta\neq\frac{1}{2}$, however, there is a difference, due to our approximations, but a numerical check has shown that this discrepancy is not significant.

In Fig. 2, the pattern of the factorial moments is shown for different values of the critical exponent η and the effect of the minimal scale $\delta_m(p)$ is illustrated. The nonuniformity of the fractality region with respect to the order p of the moment $[\delta_m(p)\sim(p-1)^{1/(1-\eta)}]$ is a characteristic property of the model which can be tested experimentally. Present experiments do not indicate any strong violation of the power-law behavior of $F_p(\delta;\Delta)$ for $0.1<\delta<1$, suggesting a very small value for the parameter δ_0. Therefore, systematic measurements of the higher moments ($p\geq 5$) in very small domains of the rapidity space ($\delta \ll 0.1$) are needed [14] in order to reveal the characteristics of this important effect and probe the scale δ_0 which, together with the critical exponent η, completely specifies, within our model, the dynamics of the intermittency phenomenon and its breaking.

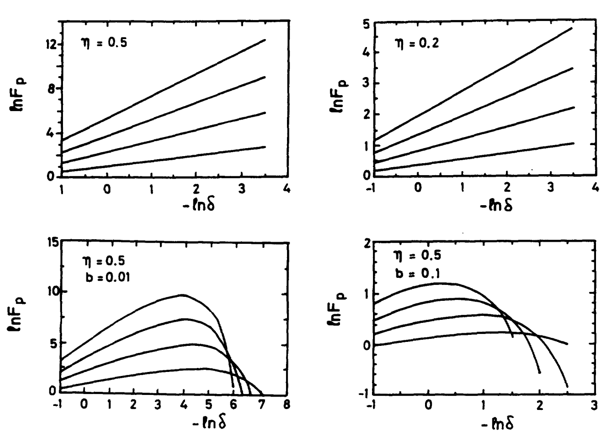

FIG. 2. The pattern of the moments $F_p^{(c)}$ for $p=2,3,4,5$ and for different values of the critical exponent η. Two upper figures: Eq. (3.10). Two lower figures: Eq. (3.17) with $\delta_0=b^{1/(1-\eta)}$.

IV. INTERMITTENCY IN PRESENT EXPERIMENTS

Summarizing the main features of our model, which may be relevant for present and future experiments on multiparticle production, one may distinguish between the large-scale properties of the critical system in rapidity space, including the validity of KNO scaling and the fractal growth of the average multiplicity ($\langle N\rangle\sim\Delta^{1-\eta}$), and the intermittent behavior at very small rapidity scales δ, indicating the onset of a nonconventional component in the production process, characterized by large density fluctuations. In the present experiments, however, it is very difficult to verify the critical behavior of the system at large scales since the production processes are dominated by conventional events, which violate KNO scaling and give rise to rapidly increasing average multiplicites in the limit $\Delta\to\infty$ ($\langle n\rangle/\Delta\to\infty$ for $\Delta\to\infty$). On the contrary, the very-small-scale aspects ($\delta\to 0$) of the quark-hadron phase-transition process, discussed in this work, may be tested experimentally, even in the presence of a strong conventional component, by studying in detail the intermittency effects, especially at the level of higher moments [15]. Recently, a good deal of moment measurements with increasing precision have become available in nucleus-nucleus, hadron-nucleus, hadron-hadron, as well as e^+e^- collisions [16–23]. In this work we adopt the point of view that the pattern of moments, observed experimentally, is due to the incoherent superposition of the inclusive densities corresponding (a) to a critical FW fluid [Eq. (2.24)] and (b) to a conventional hadronic system with finite-range correlations in rapidity [15]. This two-component model is further specified by introducing a mixing parameter λ_c, which gives the probability for producing a critical system at a given energy, $\lambda_c=\sigma_c/\sigma$, σ_c being the corresponding cross section and σ the total cross section of the collision $(0\le\lambda_c\le 1)$. Hence, the inclusive densities in the production process are written as [15]

$$\langle\rho(y_1)\rho(y_2)\cdots\rho(y_p)\rangle=\lambda_c\langle\rho_c(y_1)\rho_c(y_2)\cdots\rho_c(y_p)\rangle+(1-\lambda_c)\times\langle\rho_s(y_1)\rho_s(y_2)\cdots\rho_s(y_p)\rangle\ , \tag{4.1}$$

where $\langle\rho_c(y_1)\rho_c(y_2)\cdots\rho_c(y_p)\rangle$ is given by Eq. (2.24) and $\langle\rho_s(y_1)\rho_s(y_2)\cdots\rho_s(y_p)\rangle$ corresponds to a conventional hadronic system. Integrating Eq. (4.1) in the rapidity interval δ, in the central region $|y|\le\delta/2$, we find

$$F_p(\delta;\Delta)=\lambda_c\left[\frac{\langle\rho_c(0)\rangle}{\langle\rho(0)\rangle}\right]^p F_p^{(c)}(\delta;\Delta)+(1-\lambda_c)\left[\frac{\langle\rho_s(0)\rangle}{\langle\rho(0)\rangle}\right]^p F_p^{(s)}(\delta;\Delta)\ , \tag{4.2}$$

where $F_p^{(c)}$ and $F_p^{(s)}$ are the two components of the scaled factorial moment F_p, corresponding to the densities $\langle\rho_c(y_1)\rho_c(y_2)\cdots\rho_c(y_p)\rangle$ and $\langle\rho_s(y_1)\rho_s(y_2)\cdots\rho_s(y_p)\rangle$, respectively. In the present experiments, we expect that $\langle\rho_c(0)\rangle\gg\langle\rho(0)\rangle$, and, therefore, the critical component may contribute significantly in Eq. (4.2) for large p, even if the cross section σ_c is very small $(\lambda_c\ll 1)$. On the contrary, the lowest moments $(p=2,3)$ are dominated, in the same limit $\lambda_c\ll 1$, by the conventional component $F_p^{(s)}$. Therefore, independently of the detailed structure of the components $F_p^{(c)}$ and $F_p^{(s)}$, a genuine intermittency effect in the present experiments is expected to appear at the level of higher moments, whereas in the lowest moments $(p\le 3)$ this new phenomenon is likely to be masked by the conventional correlation mechanism.

In order to be able to compare the two-component model [Eq. (4.2)] with experiments and previous studies of the conventional component alone, we adopt, for the second conventional moment $F_2^{(s)}(\delta;\Delta)$, the form used in Refs. [24,25]; namely,

$$F_2^{(s)}(\delta;\Delta)=1+\frac{\gamma\xi_s}{\delta}(1-e^{-\delta/\xi_s})\ , \tag{4.3}$$

where ξ_s is the correlation length in rapidity and γ the strength of the two-particle correlation. For the higher moments $(p>2)$ we use the recursion formula

$$F_p^{(s)}(\delta;\Delta)=F_{p-1}^{(s)}(\delta;\Delta)[1+(p-1)(F_2^{(s)}-1)]\ , \tag{4.4}$$

which is based on the negative-binomial distribution [26]. It is of interest to note that, in the limit $(F_2^{(s)}-1)\ll 1$, a simplified expression of $F_p^{(s)}(p>2)$ in terms of $F_2^{(s)}$ is valid:

$$F_p^{(s)}(\delta;\Delta)=1+\frac{p(p-1)}{2}[F_2^{(s)}\delta;\Delta)-1](p>2)\ , \tag{4.5}$$

where terms of order $(F_2^{(s)}-1)^2$ have been neglected.

In comparing the two-component model with experiments, we have used the detailed forms (3.10), (3.17), and (4.4), but in order to clarify the complementary effects of $F_p^{(c)}$ and $F_p^{(s)}$ in Eq. (4.2) one may write, using Eqs. (3.11) and (4.5), a simplified expression for $F_p(\delta;\Delta)$ which remains a good approximation in a wide range of δ, as follows:

$$F_p(\delta;\Delta)=\lambda_c\left[\frac{\rho_c}{\rho}\right]^p\frac{p![\Gamma(1-\eta)]^{p-1}}{\Gamma[1+\eta+(1-\eta)p]}\left[\frac{4\delta}{\Delta}\right]^{-\eta(p-1)}+(1-\lambda_c)^{1-p}\left[1-\lambda_c\frac{\rho_c}{\rho}\right]^p\left[1+\frac{\gamma p(p-1)}{2}G_s\left[\frac{\delta}{\xi_s}\right]\right]\ , \tag{4.6}$$

where $\rho_c\equiv\langle\rho_c(0)\rangle$, $\rho\equiv\rho(0)\rangle$, and $G_s(z)=z^{-1}(1-e^{-z})$. In the limit $\delta_0\to 0$, an intermittency pattern emerges for $\delta\ll 1$ and from Eq. (4.6) it is clear that the intermittency effect may become very strong for sufficiently high moments, even in the limit $\lambda_c\ll 1$.

In Fig. 3 we compare the two-component model with the NA22 measurements of the factorial moments in hadron-hadron collisions [18]. It is of interest to note

TABLE I. Values of parameters used in our calculations.

	ξ_s	γ	λ_c	η	δ_0
NA22, π^+p, K^+p (250 GeV)	1.8	0.40	10^{-5}	0.30	2.5×10^{-2}
EMC, μp (280 GeV)	1	0.70	10^{-4}	0.20	4.6×10^{-2}
EMU01, S+Au (200 GeV/nucleon)	2	0.10	10^{-2}	0.20	6.2×10^{-3}

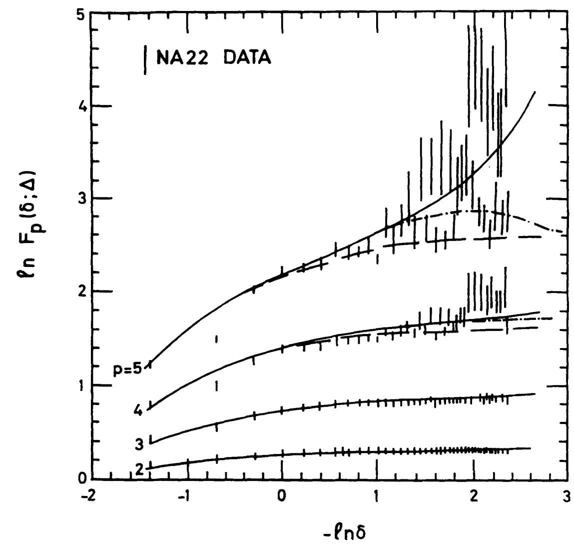

FIG. 3. Comparison of our two-component model [Eq. (4.2)] with the NA22 data (from Ref. [18]). Solid lines: In the critical component, the effect of the scale δ_0 is ignored [Eq. (3.10)]. Dashed-dotted lines: In the critical component, the effect of δ_0 is included [Eq. (3.17)]. The conventional component is given by the dashed lines.

that, in this experiment, at least one unusual event has been observed corresponding to a production cross section 0.2 μb and characterized by local density of 100 particles per unit of rapidity [17]. If we associate these exceptional events with the production of a critical hadronic system in a quark-hadron phase transition process, we may get, for this experiment, a phenomenological, order-of-magnitude estimate of the mixing parameter λ_c in our two-component model: $\lambda_c \geq 10^{-5}$. For the rest of the parameters, we fix $\rho_c/\rho = 5$ and restrict the critical exponent η in the region 0.2–0.4, close to its mean-field approximation value, $\eta = \frac{1}{3}$. In the conventional component, the parameters γ, ξ_s are fixed close to the values used in Refs. [25,26]. In the same figure, employing the expression (3.17) for the moments $F_p^{(c)}(\delta;\Delta)$, the effect of the scale δ_0 is also shown. One observes that, for the rather low value of the critical density $\rho_c = 10$, corresponding to a scale $\delta_0 = 2.5\times 10^{-2}$ (see Table I), intermittency-breaking effects become important for $p \geq 5$, but in order to study quantitatively this phenomenon, accurate measurements of the factorial moments are needed, especially for $\delta \leq 0.1$.

In Fig. 4, our two-component model is compared with European Muon Collaboration (EMC) measurements in muon-proton collisions at 280 GeV/c [19]. Again, for a very small value of the mixing parameter λ_c (see Table I) and neglecting the effect of the scale δ_0, it is seen that the intermittency effect is non-negligible for $p \geq 5$, which is consistent with the trend of data. If is of interest to note that, given the statistics of the experiment and the order of magnitude of the mixing parameter ($\lambda_c = 10^{-4}$), suggested by our preliminary study, a small number of exceptional events, with characteristic fluctuations in rapidity, is expected in an event-by-event analysis of the data. These events, according to the interpretation proposed in our model, belong to the critical component and are responsible for the intermittency effects of this process.

Finally, in Fig. 5, a comparison of the model with the EMU01 measurements of the factorial moments in nucleus-nucleus collisions is presented. In relativistic heavy-ion collisions the density ρ is sufficiently high so that the choice $\rho_c/\rho = 1$ is a suitable assumption. In this experiment a strong intermittency-breaking effect is observed for $\delta \leq 0.1$, at the level of higher moments ($p = 6$). This phenomenon can be easily accommodated within our two-component model [Eqs. (4.2) and (3.17)] suggest-

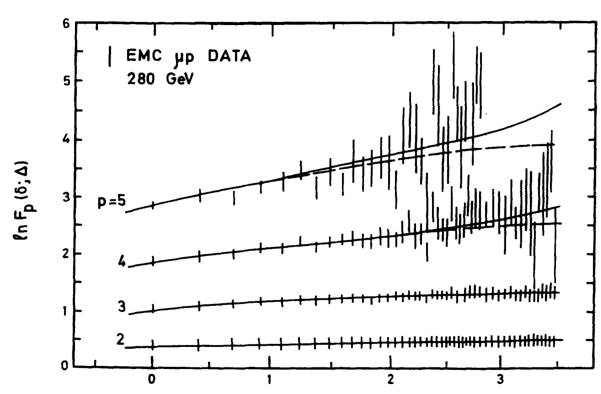

FIG. 4. Comparison of our two-component model [Eq. (4.2)] with the EMC data on μp collisions (from Ref. [19]). In the critical component the effect of the scale δ_0 is ignored [Eq. (3.10)]. The conventional component is given by the dashed lines.

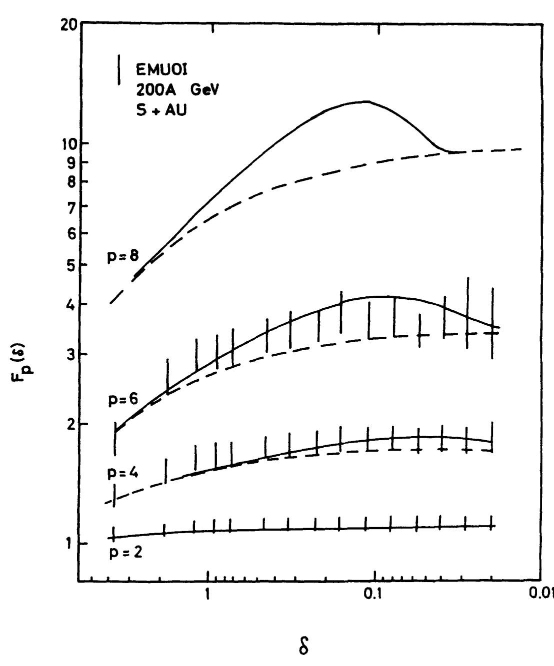

FIG. 5. Comparison of the two-component model [Eq. (4.2)] with the EMU01 data (from Ref. [14]). In the critical component, the effect of the scale δ_0 is included [Eq. (3.17)]. The conventional component is given by the dashed lines.

ing a very small value for the scale parameter $\delta_0 \le 10^{-2}$. The rest of the parameters are given in Table I. With this choice, the prediction of our model for the moment of order $p=8$ is also shown in the same figure, illustrating the effects (a) of the conventional mechanism, (b) of the fractal component, and (c) of the minimal scale δ_0.

V. CONCLUSIONS AND REMARKS

We have put forward a model for a critical FW fluid, involving a higher-order phase transition, and have shown that, in this model, at the critical point, the scaled factorial moments of the rapidity distribution exhibit a clear intermittency pattern. As has been stressed [5], such a phase transition provides one of the two natural scenarios leading to intermittency (the other scenario being a self-similar cascade).

In our model, the critical hadronic system is specified by two basic parameters: the critical exponent η (a universal index connected with the fractal dimension $d_F = 1-\eta$ of the production process) and a characteristic length δ_0 in rapidity space which is related to the maximal time scale in the hadronization process [2]. The scale δ_0 leads to an intermittency-breaking effect in the limit $\delta \to 0$, and an important feature of the model is the existence, for each factorial moment $F_p^{(c)}$, of a minimal rapidity interval $\delta_m(p) \sim \delta_0$ [Eq. (3.21)] below which the power-law behavior of $F_p^{(c)}$ breaks down (Fig. 2).

Furthermore, we carried out a qualitative study of recent experimental data by adopting a two-component model; in this, the inclusive densities arise as the (incoherent) superposition of one part due to FW fluid in a critical condition, and of another part due to conventional finite-range correlations. From our study, the main characteristics of the intermittency phenomenology may be summarized as follows:

(a) The scaled factorial moments of lowest order ($p=2,3$) are dominated by the conventional component.

(b) The intermittency pattern becomes visible at the level of higher factorial moments ($p \ge 5$).

(c) The intermittency effect due to a quark-hadron phase transition may be significant even if the production cross section for the critical system is very small ($\sigma_c/\sigma < 10^{-4}$).

(d) The intermittency indices are likely to be much larger ($\eta \ge 0.2$) than the effective ones extracted from the data without taking into account the conventional component [16].

(e) In the presence of the conventional component, a linear spectrum of the intermittency indices, indicating a single fractal dimension in the nonconventional process, may well be consistent with the experimental measurements.

(f) The intermittency effect in a multiparticle production process may be consistently associated with the presence of exceptional events in the sample, characterized by a strongly fluctuating rapidity density. One may conjecture that these events belong to a critical hadronic system which can be interpreted as a newly hadronized quark-gluon plasma.

(g) The breakdown of intermittency for $\delta \to 0$, at the level of higher moments, may be associated with the presence of a characteristic scale $\delta_0 \ll 0.1$ in the critical system, which introduces a lower limit $\delta_m(p)$ in the fractality region, increasing with the order p of the factorial moment.

Finally, our study of intermittency effects within a two-component model suggests that, in order to establish a firm connection between the intermittency phenomenon and the quark-hadron phase-transition process [5], one needs precision measurements of the factorial moments, in experiments with high statistics, and a very good knowledge of the conventional background. On the other hand, in future experiments, especially with relativistic heavy ions [at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC)] we expect that the probability for producing a quark-gluon plasma will increase significantly and the role of the conventional component will become less important. In the extreme case of a collision with $\lambda_c \approx 1$, our model predicts a genuine intermittency pattern with a linear spectrum of intermittency indices [Eq. (3.11)] as a consequence of a higher-order quark-hadron phase transition in a nuclear process of quark-gluon-plasma formation.

Having completed, in connection with the phenomenon of intermittency, a detailed study of the critical FW fluid and its fractal structure, a clarification regarding a number of conceptual questions is now in order.

(a) Despite the fact that the FW system is one dimensional, a phase transition of the nature discussed in this work is feasible because the two-particle (hadron-hadron) effective potential at the critical temperature [Eq. (3.22)] found in this model has a long-range tail of logarithmic behavior at large distances in rapidity space. This property makes our critical solution consistent with the well-known restriction according to which there is no phase transition in one-dimensional systems with short-range forces.

(b) In our S-matrix approach towards intermittency dynamics in multihadron production, the connection of the critical phenomenon in the FW fluid with the confinement-deconfinement transition cannot be easily established since the quark-gluon degrees of freedom are missing in the treatment of the correlation functions in rapidity space [Eq. (2.24)]. However, the fractality of the hadronic system in the central region ($y \approx 0$) of the rapidity space induces a similar structure in ordinary space ($Z \approx c\tau y$), where the QCD correlation functions operate (Fig. 1), and this remark may provide us with a link between the underlying quark-hadron phase transition and the phase transition of the FW fluid, at a geometrical level. In fact, our investigation suggests that, near the critical temperature ($T = T_c$), the configurations of the hadronic currents in a finite region of the three-dimensional space have a fractal structure with anomalous dimension $\tilde{D}_F = 3 - 3\eta$, where η is the critical exponent in the FW fluid. It is now plausible to assume that this geometrical property remains valid also for the quark-gluon field configurations near the critical temperature of the confinement-deconfinment transition. This remarkable geometrical constraint connects our effective theory with the underlying quark-hadron phase-transition mechanism and, furthermore, using the phenomenological values of the critical exponent $\eta \approx 0.2-0.3$ found in our study, one may estimate the fractal dimension of the quark-gluon system near the critical temperature, $\tilde{D}_F \approx 2.1-2.4$, as a consequence of a second-order phase transition in the FW fluid. In connection with this estimate, it is of interest to note that recent studies in lattice gauge theory have shown that the statistical mechanics of the SU(2) interaction, near the critical temperature builds in the deconfinement region a fractal structure with $2 < \tilde{D}_F < 3$ [27].

(c) The above remarks indicate that the intermittency phenomenon in rapidity space, as was first suggested by Bialas and Peschanski, has a particular significance due to the fact that rapidity may be viewed, either as a phase-space degree of freedom,

$$y = \tfrac{1}{2} \ln \left| \frac{E+P_L}{E-P_L} \right| ,$$

or as a space-time coordinate,

$$y \approx \tfrac{1}{2} \ln \left| \frac{Z+ct}{Z-ct} \right| ,$$

for the hadronized system (Fig. 1). We have also seen that, because of this characteristic property, any intermittency pattern of the hadronic distribution in rapidity space necessitates a similar fractal structure in the ordinary space of the related hadronic current distribution and the underlying QCD configurations. This link of rapidity space with ordinary space time is expected to differentiate its fractality from the corresponding structure in the remaining dimensions of the phase space. Recent phenomenological studies extending intermittency effects in transverse momentum p_T and azimuthal angle φ [28] may lead through further investigation to a quantification and clarification of this expectation.

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