ARTICLES

Clock synchronization and isotropy of the one-way speed of light

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(Received 30 May 1991)

Experimental tests of the isotropy of the speed of light using one-way propagation are analyzed using a test theory of special relativity. It is shown that, when properly expressed in terms of measurable quantities, the results of such experiments are independent of the method of global synchronization of clocks. Experiments analyzed include a Jet Propulsion Laboratory time-of-flight measurement, a resonant two-photon absorption experiment, the Smithsonian Astrophysical Observatory–NASA 1976 rocket gravitational redshift experiment, and Mössbauer rotor experiments. If the characteristic anisotropy is proportional to αw, where w is the velocity of the Earth relative to the cosmic background radiation, the best bound on α from these experiments is $|\alpha| < 9\times 10^{-8}$.

PACS number(s): 03.30.+p, 04.80.+z

I. INTRODUCTION AND SUMMARY

Advances in technology have made possible two recent experimental tests of a fundamental postulate of special relativity theory (SRT), the constancy or isotropy of the velocity of light in inertial reference frames. These measurements differ from many of the classic tests of special relativity, such as the Michelson-Morley experiment, in that they test the isotropy of the one-way velocity of light, without propagating the light on a closed path. The experiment of Riis *et al.* [1] monitored the frequency of light emitted by atoms excited resonantly via two-photon absorption (TPA) in an atomic beam as a function of the rotation of the Earth, thereby testing the isotropy of the first-order Doppler shift (henceforth called the TPA experiment). The experiment of Krisher *et al.* [2] monitored the time of flight of light signals along a fiber-optic link between two hydrogen maser clocks at the NASA–Jet Propulsion Laboratory (JPL) Deep Space Network, again as a function of the rotation of the Earth (henceforth called the JPL experiment). These experiments established limits of $\Delta c/c < 10^{-11}$ and $\Delta c/c < 3.5\times 10^{-7}$, respectively on an anisotropy in the one-way speed of light.

The TPA and JPL experiments supplement earlier tests of the one-way isotropy of light: the Mössbauer-rotor experiments of Turner and Hill [3] and of Champeney *et al.* [4,5], which measured the isotropy of the Doppler shift of an emitter mounted on the rim of a rotating disk, as sensed by an absorber at the center, and the Smithsonian Astrophysical Observatory- (SAO-)NASA Gravity Probe A (GP-A) Rocket Redshift experiment of Vessot *et al.* [6] which compared the rates of hydrogen maser clocks on a Scout rocket and on the ground, as a function of height and speed (gravitational redshift and time dilation effects) and as a function of the direction of the rocket's velocity (isotropy of speed of light).

In order to study the significance of a given experimental test of relativity, it is useful to employ a general framework which permits violations of special relativity. A popular framework for this purpose is the "test theory" of Mansouri and Sexl [7–11] (MS). This is a kinematical approach to violations of special relativity. It assumes: (1) there exists a preferred inertial reference system Σ with coordinates $(T,\mathbf{X})$, in which the speed of light is isotropic, i.e., $T^2-\mathbf{X}^2=0$ (units are chosen so that the speed of light in Σ is unity), and (2) the transformation between Σ and a reference system S which moves with velocity $\mathbf{w}$ relative to Σ is given by

$$T=a^{-1}(t-\boldsymbol{\varepsilon}\cdot\mathbf{x}) , \tag{1.1a}$$

$$\mathbf{X}=d^{-1}\mathbf{x}-(d^{-1}-b^{-1})\mathbf{w}\cdot\mathbf{x}\mathbf{w}/w^2+\mathbf{w}T , \tag{1.1b}$$

where a, b, and d are functions of w^2, and $\boldsymbol{\varepsilon}$ is a vector determined by the procedure adopted for the global synchronization of clocks in S.

MS have emphasized the arbitrary nature of the vector $\boldsymbol{\varepsilon}$ [7] (see also Ref. [12]). If, for example, one adopts the convention that clocks are to be synchronized in Σ, and that each clock in S is to be adjusted to be synchronized with the Σ clock momentarily at its location ("external synchronization"), then $\boldsymbol{\varepsilon}=0$. If one adopts Einstein synchronization by round-trip light signals in S, then $\boldsymbol{\varepsilon}=-\mathbf{w}a/b(1-w^2)$, while if one synchronizes by slow transport of clocks, then $\boldsymbol{\varepsilon}=b^{-1}\nabla_{\mathbf{w}}a$. In special relativity, the functions a, b, and d have the specific forms $a=b^{-1}=(1-w^2)^{1/2}$, $d=1$, but $\boldsymbol{\varepsilon}$ can be arbitrary, depending upon the procedure for synchronization; with *either* Einstein or transport synchronization, $\boldsymbol{\varepsilon}=-\mathbf{w}$. (For a brief review of the history of discussions of synchronization, see Ref. [7].) Theories that violate SRT will predict different functional forms for a, b, and d from those listed above.

On the other hand, the results of physical experiments should not depend on the synchronization procedure, except those which depend on a direct, one-time comparison of separated clocks. Thus a measurement of the absolute value of the speed of light in S by a time-of-flight technique between two points will depend on the synchronization of the two clocks (a particularly perverse choice of synchronization can make the apparent speed between those points infinite, for example). However, a test of the *isotropy* of the speed between the same two clocks as the orientation of the propagation path varies relative to Σ should not depend on how they were synchronized, as long as they were synchronized by some procedure initially. Similarly, a measurement of the Doppler shift of an atomic spectral line using a single "clock" as a receiver of the signal should not depend on synchronization, provided that the velocity of the atom is expressed in terms of observables measured by a single clock.

This point has sometimes been misunderstood [13]. Chang *et al.* [14] have suggested that there is a physically detectable difference between external synchronization and Einstein synchronization, and that it would be possible to measure this by means of a Mössbauer rotor-type experiment. Bay and White [15] argued that the TPA experiment could not in principle test the isotropy of the one-way speed of light once the arbitrariness of synchronization was accounted for.

In this paper, we will show that these claims are not correct. Provided that one deals with observable quantities, the outcome of physical experiments of this type is unique in all cases and is independent of synchronization; thus the TPA and other such one-way experiments do provide valid tests of possible violations of SRT. It is important to emphasize that those violations are embodied in functional forms of a, b and d that could differ from those quoted above, *not* in the form of $\boldsymbol{\varepsilon}$, which is arbitrary and irrelevant.

To the lowest order in the velocity $\mathbf{w}$ of S relative to Σ, the one-way experiments discussed above measure a variation or anisotropy controlled by the amplitude

$$A = \alpha \mathbf{w} \cos\theta \ , \tag{1.2}$$

where α is the coefficient in the expansion of the function a in powers of w^2 (our conventions here differ from those of MS; see Sec. II),

$$a(w) = 1 - (\tfrac{1}{2} - \alpha) w^2 + \cdots \ , \tag{1.3}$$

and θ is the angle between $\mathbf{w}$ and a direction relevant to the experiment in question, such as the propagation direction of light or the velocity vector of a moving clock. In SRT, $\alpha = 0$, so that the anisotropy vanishes.

For the velocity $\mathbf{w}$, it is natural to choose the velocity of the Earth relative to the cosmic microwave background, as inferred from the observed dipole anisotropy in the temperature [16]: $w = 350$ km/s, right ascension RA$=11.2$ h, declination $\delta = -6.1°$. The resulting bounds on the parameter α from these four experiments are listed in Table I.

From the point of view of the MS formalism, the connection between the experimental measurements and the "isotropy of the speed of light" is not entirely clear-cut. In the JPL time-of-flight experiment, for example, there are two effects: the behavior of the speed of light expressed in the MS coordinates of the moving frame, and the behavior of the atomic clocks used to measure the time of flight as the Earth's rotation transports them from one place to another in the moving frame. The combination of these effects leads to a predicted anisotropy in the measured phase difference between the two clocks at the ends of the light path that depends on α. If $\alpha = 0$, as in special relativity, no anisotropy is predicted.

TABLE I. Limits on α.

Experiment	Limit on α
JPL time of flight	1.8×10^{-4}
Two-photon absorption	1.4×10^{-6}
Rocket redshift	10^{-6}
Mössbauer rotor	9×10^{-8}

In this paper, we shall study these four experiments in detail using the MS test theory, with particular attention paid to the issue of synchronization (for discussion of other experiments of this type, see Refs. [8,10]). We shall also look at higher-order effects, to see what new tests of relativity might be provided by improved accuracy. In Sec. II, we derive some basic consequences and formulas of the MS test theory, and in Secs. III–VI we apply them to the JPL, TPA, GP-A, and Mössbauer-rotor experiments. In Sec. VII we make concluding remarks.

II. LIGHT PROPAGATION IN A TEST THEORY OF SPECIAL RELATIVITY

The experiments described in the preceding section all involve propagation of light from a source to a receiver; source and receiver may or may not be in motion in the "laboratory" frame S. Thus it will first be useful to derive a generic formula in the coordinates of S for the time of flight for propagation of a signal between two points.

Let a signal be emitted at time t_e at $\mathbf{x}_e$ and be received at time t_r at $\mathbf{x}_r$, these coordinates being defined in S. The values of $(T,\mathbf{X})$ in Σ corresponding to these events can be obtained by substitution into Eqs. (1.1). Because light is assumed to propagate isotropically in Σ with unit speed, we have

$$\begin{aligned}(T_r - T_e)^2 &= |X_r - X_e|^2 \\ &= d^{-2} x_{re}^2 + (b^{-2} - d^{-2})(\hat{\mathbf{w}}\cdot\mathbf{x}_{re})^2 \\ &\quad + 2b^{-1}\mathbf{w}\cdot\mathbf{x}_{re}(T_r - T_e) + w^2 (T_r - T_e)^2 \ ,\end{aligned} \tag{2.1}$$

where

$$\mathbf{x}_{re} \equiv \mathbf{x}_r - \mathbf{x}_e, \quad \hat{\mathbf{w}} \equiv \mathbf{w}/w \ . \tag{2.2}$$

Solving for $T_r - T_e$, we obtain

$$T_r - T_e = b^{-1}\gamma^2 [f(\mathbf{x}_{re}) + \mathbf{w}\cdot\mathbf{x}_{re}] \ , \tag{2.3}$$

where $\gamma \equiv (1-w^2)^{-1/2}$, and

$$f(\mathbf{x}_{re}) \equiv (b/d\gamma)|\mathbf{x}_{re}|\{1 - [1-(d\gamma/b)^2](\hat{\mathbf{w}}\cdot\mathbf{n}_{re})^2\}^{1/2} \ , \tag{2.4}$$

where $\mathbf{n}_{re} \equiv \mathbf{x}_{re}/x_{re}$. In frame S, the time of reception is then given by

$$t_r = t_e + a(T_r - T_e) + \boldsymbol{\varepsilon}\cdot\mathbf{x}_{re} \ . \tag{2.5}$$

In calculating the change in frequency of a signal transmitted between two points, it will be useful to determine from Eq. (2.4) the interval Δt_r between two signals (or between two crests of a continuous wave) received compared to the interval Δt_e between the signals emitted. We assume that the emitter and receiver have velocities in S, given by $\mathbf{v}_e$ and $\mathbf{v}_r$ at the times of emission and reception respectively. If we denote by $(\tilde{t}_e, \tilde{\mathbf{x}}_e)$ and $(\tilde{t}_r, \tilde{\mathbf{x}}_r)$ the respective events of emission and reception of the second signals, then, assuming that $\Delta t_e \equiv \tilde{t}_e - t_e \ll t_r - t_e$ we can approximate

$$f(\tilde{\mathbf{x}}_{re}) \approx f(\mathbf{x}_{re}) + (\tilde{\mathbf{x}}_{re} - \mathbf{x}_{re})\cdot[\partial f(\mathbf{x}_{re})/\partial \mathbf{x}_{re}] + O(f\Delta t^2/x_{re}^2) \ . \tag{2.6}$$

Substituting $(\tilde{\mathbf{x}}_{re} - \mathbf{x}_{re}) = \mathbf{v}_r \Delta t_r - \mathbf{v}_e \Delta t_e$, we obtain a relationship between Δt_r and Δt_e:

$$\Delta t_r = \Delta t_e \frac{1 - \boldsymbol{\varepsilon}\cdot\mathbf{v}_e - R(\mathbf{v}_e)}{1 - \boldsymbol{\varepsilon}\cdot\mathbf{v}_r - R(\mathbf{v}_r)} \ , \tag{2.7}$$

where

$$R(\mathbf{v}) \equiv \frac{a\gamma^2}{b} \left\{ \frac{1}{f(\mathbf{n}_{re})} \left[\frac{b^2}{d^2\gamma^2}(\mathbf{v}\cdot\mathbf{n}_{re}) + \left[1 - \frac{b^2}{d^2\gamma^2} \right] (\mathbf{v}\cdot\hat{\mathbf{w}})(\hat{\mathbf{w}}\cdot\mathbf{n}_{re}) \right] + \mathbf{v}\cdot\mathbf{w} \right\} \ . \tag{2.8}$$

The time intervals connected in Eq. (2.7) are coordinate times in reference frame S. It will be necessary to relate these intervals to intervals of time measured by an atomic clock moving through S with a given velocity $\mathbf{v}$. Consider an inertial reference frame S':$(t', \mathbf{x}')$ which moves at the same velocity as the moving clock. Relative to Σ, let this velocity be $\mathbf{U}$. The transformation from Σ to the frame of the clock is given by

$$T = (a')^{-1}(t' - \varepsilon'\cdot\mathbf{x}') \ , \tag{2.9a}$$

$$\mathbf{X} = (d')^{-1}\mathbf{x}' - [(d')^{-1} - (b')^{-1}]\mathbf{U}\cdot\mathbf{x}'\mathbf{U}/U^2 + \mathbf{U}T \ , \tag{2.9b}$$

where $a' \equiv a(\mathbf{U})$, $b' \equiv b(\mathbf{U})$, $d' \equiv d(\mathbf{U})$, and $\boldsymbol{\varepsilon}' \equiv \boldsymbol{\varepsilon}(\mathbf{U})$. Since the clock is assumed to be at rest at the origin $\mathbf{x}' = 0$ of its frame S', we have, for infinitesimal time intervals,

$$dT = dt'/a' \ , \tag{2.10}$$

and thus, since

$$dT = a^{-1}(dt - \boldsymbol{\varepsilon}\cdot d\mathbf{x}) = a^{-1}(1 - \boldsymbol{\varepsilon}\cdot\mathbf{v})dt \ ,$$

we can relate proper time of a moving atomic clock to coordinate time of S by

$$dt' = (a'/a)(1 - \boldsymbol{\varepsilon}\cdot\mathbf{v})dt \ . \tag{2.11}$$

Applying Eq. (2.11) to Δt_r and Δt_e in Eq. (2.7), and defining the frequencies measured by atomic clocks in the respective receiver and emitter frames to be $\nu_r \equiv (\Delta t_r')^{-1}$ and $\nu_e \equiv (\Delta t_e')^{-1}$, we obtain a formula relating the measured frequencies:

$$\frac{\nu_r}{\nu_e} = \frac{a(\mathbf{U}_e)}{a(\mathbf{U}_r)} \frac{1 - R(\mathbf{v}_r)/(1 - \boldsymbol{\varepsilon}\cdot\mathbf{v}_r)}{1 - R(\mathbf{v}_e)/(1 - \boldsymbol{\varepsilon}\cdot\mathbf{v}_e)} \ . \tag{2.12}$$

The relationship between the velocities $\mathbf{v}$, $\mathbf{U}$, and $\mathbf{w}$ can be obtained by noting that the origin of $\mathbf{S}'$, $\mathbf{x}' = 0$, moves relative to S according to $\mathbf{x} = \mathbf{v}t$. Substituting these relations into Eqs. (1.1) and (2.9) and eliminating T and $\mathbf{X}$, we obtain the "addition of velocities" formula of the MS framework:

$$\mathbf{U} = \mathbf{w} + (a/d)[\mathbf{v} - (1 - d/b)\hat{\mathbf{w}}(\mathbf{v}\cdot\hat{\mathbf{w}})](1 - \boldsymbol{\varepsilon}\cdot\mathbf{v})^{-1} \ . \tag{2.13}$$

Equations (2.3)–(2.5), (2.7), (2.8), (2.11)–(2.13), are the basic ingredients for analyzing light speed isotropy experiments.

In SRT with standard (Einstein or clock transport) synchronization, these formulas take the form

$$f(\mathbf{x}_{re}) = x_{re} \ , \tag{2.14a}$$

$$R(\mathbf{v}) = \mathbf{v}\cdot\mathbf{n}_{re} + \mathbf{v}\cdot\mathbf{w} \ , \tag{2.14b}$$

$$\mathbf{U} = \mathbf{w} + \gamma^{-1}[\mathbf{v} - (1 - \gamma^{-1})\hat{\mathbf{w}}\cdot\mathbf{v}\hat{\mathbf{w}}]/(1 + \mathbf{w}\cdot\mathbf{v}) \ , \tag{2.14c}$$

$$a(\mathbf{U}) = (1 - w^2)^{1/2}(1 - v^2)^{1/2}(1 + \mathbf{w}\cdot\mathbf{v})^{-1} \ , \tag{2.14d}$$

$$\frac{\nu_r}{\nu_e} = \frac{1 - \mathbf{v}_r\cdot\mathbf{n}_{re}}{1 - \mathbf{v}_e\cdot\mathbf{n}_{re}} \left[\frac{1 - v_e^2}{1 - v_r^2} \right]^{1/2} \ . \tag{2.14e}$$

In the low-velocity limit, it will be useful to expand the functions a, b, d, and $\boldsymbol{\varepsilon}$ in powers of velocity using arbitrary parameters. We write

$$a(\mathbf{w}) = 1 + (\alpha - \tfrac{1}{2})w^2 + (\alpha_2 - \tfrac{1}{8})w^4 + \cdots \ , \tag{2.15a}$$

$$b(\mathbf{w}) = 1 + (\beta + \tfrac{1}{2})w^2 + (\beta_2 + \tfrac{3}{8})w^4 + \cdots \ , \tag{2.15b}$$

$$d(w) = 1 + \delta w^2 + \delta_2 w^4 + \cdots \ , \tag{2.15c}$$

$$\boldsymbol{\varepsilon} = (\varepsilon - 1)\mathbf{w}(1 + \varepsilon_2 w^2 + \cdots) \ . \tag{2.15d}$$

In SRT, α, α_2, β, β_2, δ, and δ_2 all vanish, and with standard synchronization, so do ε and ε_2. Our convention for these parameters differs from that of MS; the translation between the two conventions is given in Table II.

Throughout this paper we will deal with low velocities; to keep track of the order of approximation, we will treat w^n and v^n as $O(n)$ (although in many cases, $v \ll w$). With this assumption, for example, we have

$$f(\mathbf{x}_{re}) = |\mathbf{x}_{re}|\{1 + (\beta - \delta)[w^2 - (\mathbf{w}\cdot\mathbf{n}_{re})^2]\} + O(4) \ , \tag{2.16a}$$

$$R(\mathbf{v}) = \mathbf{v}\cdot\mathbf{n}_{re}[1 + (\alpha - \delta)w^2 + (\beta - \delta)(\mathbf{w}\cdot\mathbf{n}_{re})^2] - 2(\beta - \delta)\mathbf{v}\cdot\mathbf{w}\mathbf{w}\cdot\mathbf{n}_{re} + \mathbf{v}\cdot\mathbf{w}[1 + (\alpha - \beta)w^2] + O(5) \ , \tag{2.16b}$$

$$\mathbf{U} = \mathbf{w} + \mathbf{v} + (\alpha - \delta - \tfrac{1}{2})w^2\mathbf{v} + (\delta - \beta - \tfrac{1}{2})\mathbf{w}\cdot\mathbf{v}\mathbf{w} + (\varepsilon - 1)\mathbf{w}\cdot\mathbf{v}\mathbf{v} + O(5) \ . \tag{2.16c}$$

We now turn to analysis of specific experiments.

TABLE II. Comparison of conventions for low-velocity expansions in Mansouri-Sexl framework.

This paper	Mansouri-Sexl
α	$\frac{1}{2}+\alpha$
β	$-\frac{1}{2}+\beta$
δ	δ
ε	$\varepsilon+1$

III. THE JPL EXPERIMENT

In the experiment performed at the NASA Deep Space Network [2], the phase of two hydrogen maser oscillators separated by a base line of 21 km were compared by propagating a laser carrier signal along a fiberoptic link connecting them. The phase comparisons could be performed simultaneously at each end using signals propagated in both directions along the fiber. The phase differences were monitored over a five-day period as the base line rotated relative to the Earth's velocity $\mathbf{w}$ relative to the cosmic microwave background. With an intrinsic clock stability of parts in 10^{15}, the expected accuracy in measuring a diurnal variation in phase difference can be estimated to be

$$\Delta\phi/\phi \approx \frac{\text{speed of light}}{\text{base line}} \times (\text{clock stability}) \times (\text{averaging time}) \approx 10^{-6}. \tag{3.1}$$

Before analyzing this experiment using the MS framework, it is useful to estimate the effects which could occur in order to establish the terms to be kept in an approximation. There are two relevant velocities in the problem, the velocity of the Earth itself, $w \approx 10^{-3}$, and the rotational velocity of the apparatus around the Earth's axis, $v \approx 10^{-6}$. If $\mathbf{n}$ denotes a unit vector parallel to the base line joining the clocks, then the fractional phase difference could vary by terms of order $\mathbf{n}\cdot\mathbf{w} \approx 10^{-3}$ and $(\mathbf{n}\cdot\mathbf{w})^2 \approx 10^{-6}$, which could be detectable. Terms of order $\mathbf{n}\cdot\mathbf{v} \approx 10^{-6}$ would be large enough, but are constant because of the rigid rotation of the Earth, and thus cannot be distinguished from constant phase offsets of instrumental and environmental origin. Other terms, such as $\mathbf{v}\cdot\mathbf{w}$, $(\mathbf{n}\cdot\mathbf{v})(\mathbf{n}\cdot\mathbf{w})$, and so on, are too small to be detected in the current version of the experiment, but might be detectable in future improved versions. Therefore, we shall consider only effects of order $\mathbf{n}\cdot\mathbf{w}$ and $(\mathbf{n}\cdot\mathbf{w})^2$. Because the actual rotational velocity of the Earth does not play a measurable role, we can ignore it, and can study the variation in phase difference by comparing phase differences at two different but fixed orientations of the base line [17].

We therefore consider the following idealized picture of the experiment. In a moving frame S, clock A is located at the origin $\mathbf{x}=0$; a traveling clock T moves slowly through S, passing points B and C, which are equidistant from the origin. In the JPL experiment, this motion is effected by the rotation of the Earth [18]. As the traveling clock passes each of these two points, it receives a light signal from the clock A, and compares its own phase with that of the received signal, the latter being given by $2\pi\nu\times$ the time of emission, where ν is the intrinsic oscillator frequency [19]. Consider the event of reception at B: $(t_B, \mathbf{x}_B)$. Let the phase of the traveling clock at this event be ϕ_0. The phase of the received signal is $2\pi\nu t_e$, where t_e is the time of emission of the signal. From Eq. (2.5) this is given by

$$t_e = t_B - a(T_r - T_e) - \varepsilon\cdot\mathbf{x}_B\ , \tag{3.2}$$

where

$$T_r - T_e = b^{-1}\gamma^2[f(\mathbf{x}_B) + \mathbf{w}\cdot\mathbf{x}_B]\ . \tag{3.3}$$

The measured phase difference at B is thus $\phi_0 - 2\pi\nu t_e$.

The proper time elapsed on the moving clock in traveling from B to the event of arrival at C: $(t_C, \mathbf{x}_C)$ is obtained from Eq. (2.11):

$$\Delta t' = \int_B^C (a'/a)(1-\varepsilon\cdot\mathbf{v})dt\ . \tag{3.4}$$

Assuming that $v \ll w$, we expand a' to first order in v, using the addition of velocities formula Eq. (2.13). The result is

$$a' \approx a(w)[1+(2\tilde{\alpha}-\gamma^2)(a/b)\mathbf{w}\cdot\mathbf{v}+O(v^2)], \tag{3.5}$$

where

$$\tilde{\alpha}\mathbf{w} \equiv \tfrac{1}{2}(a^{-1}\partial a/\partial\mathbf{w}+\gamma^2\mathbf{w})\ . \tag{3.6}$$

Substituting Eq. (3.5) into Eq. (3.4) and integrating, we obtain

$$\Delta t' = \Delta t + [(2\tilde{\alpha}-\gamma^2)(a/b)\mathbf{w}-\varepsilon]\cdot(\mathbf{x}_C - \mathbf{x}_B)\ , \tag{3.7}$$

where Δt is the coordinate time of travel from B to C. The coordinate time of arrival at C is thus $t_C = t_B + \Delta t$, and the phase of the traveling clock on arriving at C is thus

$$\phi_C = \phi_0 + 2\pi\nu\Delta t'\ . \tag{3.8}$$

When the clock reaches C, a second signal is received from A. The phase of this signal is $2\pi\nu t_e^{(2)}$, where $t_e^{(2)}$ is given from Eq. (2.5) by

$$t_e^{(2)} = t_B + \Delta t - a(T_r^{(2)} - T_e^{(2)}) - \varepsilon\cdot\mathbf{x}_C\ , \tag{3.9}$$

where

$$T_r^{(2)} - T_e^{(2)} = b^{-1}\gamma^2[f(\mathbf{x}_C) + \mathbf{w}\cdot\mathbf{x}_C]\ . \tag{3.10}$$

Thus the phase difference at C is $\phi_C - 2\pi\nu t_e^{(2)}$. However, the initial phase of the traveling clock at B was completely unknown; therefore, we must choose an initial phase arbitrarily, then measure subsequent phase differences relative to that initial choice. The most convenient choice is $\phi_0 = 2\pi\nu t_e$, so that the initial phase difference is zero. Since this is a constant phase, it will not affect a possible variation of phase with orientation. Consequently, at C, the measured phase difference is

$$\Delta\phi = \phi_C - 2\pi\nu t_e^{(2)} = \phi_0 + 2\pi\nu\Delta t' - 2\pi\nu t_e^{(2)} = 2\pi\nu(t_e - t_e^{(2)} + \Delta t')\ . \tag{3.11}$$

Substituting Eqs. (3.2), (3.7), and (3.9), we obtain

$$\Delta\phi = 2\pi\nu[a(T_r^{(2)} - T_e^{(2)}) - a(T_r - T_e) + (2\tilde{\alpha} - \gamma^2)(a/b)\mathbf{w}\cdot(\mathbf{x}_C - \mathbf{x}_B)] . \quad (3.12)$$

Then substituting Eqs. (3.3) and (3.10), we obtain

$$\Delta\phi = 2\pi\nu(a\gamma^2/b)\{[f(\mathbf{x}_C) - f(\mathbf{x}_B)] + 2\tilde{\alpha}\gamma^{-2}\mathbf{w}\cdot(\mathbf{x}_C - \mathbf{x}_B)\} . \quad (3.13)$$

Notice that the result is *independent* of the synchronization procedure embodied in the vector $\boldsymbol{\varepsilon}$. This is because the *initial* relative phase of the two oscillators must be chosen arbitrarily; this is tantamount to choosing a convention for synchronization. In SRT, $\tilde{\alpha} = 0$, and $f(\mathbf{x}) = |\mathbf{x}|$, and thus $\Delta\phi \equiv 0$ if B and C are equidistant from the origin, as they were in the JPL experiment (the base line was fixed in length).

Now, in the MS test theory, we must be careful about the interpretation of the term "equidistant." Since Lorentz invariance is assumed to be violated, we do not have the same concept of invariant length as in SRT. In SRT, the relation between the physically measured length and the coordinates in a given frame is $L = |\mathbf{x}|$. Since the change of the baseline from AB to AC is caused by the rigid rotation of the Earth, so that the physical distance is the same (the atoms along the base line are unchanged), we would conclude that $|\mathbf{x}_B| = |\mathbf{x}_C|$. But in the MS test theory, we do not know *a priori* the relationship between physical length and $|\mathbf{x}|$. Thus we must make an assumption.

One assumption is that $L \equiv |\mathbf{x}|$ as in SRT. The result is that $\mathbf{x}_C = L\mathbf{n}_C$ and $\mathbf{x}_B = L\mathbf{n}_B$, where $\mathbf{n}_C$ and $\mathbf{n}_B$ are unit vectors. In the limit $w^2 \ll 1$, using the expressions of Eqs. (2.15) and (2.16), the result is

$$\Delta\phi/\tilde{\phi} = 2\alpha w(\cos\theta - \cos\theta_0) + (\delta - \beta)w^2(\cos^2\theta - \cos^2\theta_0) + O(w^3) , \quad (3.14)$$

where we have defined $\tilde{\phi} \equiv 2\pi\nu L$, and where $\cos\theta \equiv \hat{\mathbf{w}}\cdot n_C$ and $\cos\theta_0 \equiv \hat{\mathbf{w}}\cdot n_B$.

In the JPL experiment, light signals were propagated simultaneously in both directions and phase comparisons were made at both ends of the fiberoptic link, yielding both $\Delta\phi(\theta)$ and $\Delta\phi(\theta+\pi)$. Thus by summing and differencing the phase differences at both ends, it was possible to separate the w term in Eq. (3.14) from the w^2 term and from effects that do not change sign when the propagation direction is reversed, such as diurnal temperature effects in the fiberoptic link. The resulting limits are [2]

$$|\alpha| < 1.8\times 10^{-4}, \quad |\delta - \beta| < 2\times 10^{-2} , \quad (3.15)$$

where the projection of the propagation base line on the direction of $\mathbf{w}$ has been taken into account.

However, there is an alternative assumption about the definition of physical length. Since the MS formalism specifies only the propagation of light, then we could define length only in those terms, specifically as one half the *round-trip* propagation time along each base line. From Eqs. (3.2) and (3.3), it is straightforward to show that this gives

$$L \equiv \frac{1}{2} t_{\text{round trip}} = (a\gamma^2/b) f(\mathbf{x}) . \quad (3.16)$$

Thus, if B and C are equidistant from the origin according to this definition of length, then $f(\mathbf{x}_B) = f(\mathbf{x}_C)$, and

$$\Delta\phi/\tilde{\phi} = 2f(\mathbf{x}_B)^{-2}\tilde{\alpha}\gamma^{-1}\mathbf{w}\cdot(\mathbf{x}_C - \mathbf{x}_B) . \quad (3.17)$$

In the limit $w^2 \ll 1$, using the expansions of Eqs. (2.15) and (2.16) we have

$$\Delta\phi/\tilde{\phi} = 2\alpha w(\cos\theta - \cos\theta_0) + O(w^3) , \quad (3.18)$$

with no w^2 dependence, and thus no bound on $\delta - \beta$.

IV. THE TPA EXPERIMENT

In this experiment [1], a fast beam of ^{20}Ne atoms was excited from an initial state l to an excited state n by two-photon absorption via an intermediate state m. The two photons were provided by counterpropagating laser beams of identical frequencies. The energy differences between the l and m and the m and n states are not equal, however, so the velocity of the atomic beam parallel to the laser beams was chosen so that, as seen in the atom's rest frame, the blueshifted photons from the forward direction are resonant with the transition with the larger energy difference, while the redshifted photons from the backward direction are resonant with the transition with the smaller energy difference. The laser frequency must be chosen and controlled so that the two photons can generate the required double transition, while the velocity of the beam is controlled so that the intermediate transition occurs. An analysis within SRT (see below) shows that this will occur if the velocity V of the atom and the laser frequency ν_L are related to the intrinsic transition frequencies ν_1 and ν_2 by

$$V = \frac{\nu_2 - \nu_1}{\nu_2 + \nu_1} , \quad (4.1a)$$

$$\nu_L = \sqrt{\nu_1 \nu_2} . \quad (4.1b)$$

In the experiment, the velocity and laser frequency were controlled using feedback loops which adjust them to maintain maximum resonant absorption. In practice, the measured quantities V and ν_L may differ from these exact values because of such systematic effects as ac Stark shifts, and deviations from SRT.

From the point of view of the atom, the important quantities are the forward and backward frequencies of laser photons as measured in the atom's rest frame, since those frequencies will determine the strength of any TPA resonance. If we define the frequency of the photon from the backward direction (i.e., moving in the same direction as the atom) as ν_+ and that of the photon from the forward direction as ν_-, then the actual measured quantities are

$$V \equiv \frac{\nu_- - \nu_+}{\nu_- + \nu_+} , \quad (4.2a)$$

$$\nu_L \equiv \sqrt{\nu_+ \nu_-} . \quad (4.2b)$$

We consider the following idealized model for this experiment. A laboratory moves with velocity $\mathbf{w}$ relative to the preferred frame Σ. A laser signal is emitted continuously from one end of a cavity and reflected from the other hand. The frequency of the laser is $\nu_L \equiv 1/t_0$, where t_0 is the interval between wave crests emitted, measured in the laboratory frame S. From stationarity or from direct calculation, the frequency of the reflected beam is also ν_L. An atom moves with coordinate velocity $\mathbf{v}=v\mathbf{n}$, where $\mathbf{n}$ is a unit vector parallel to the axis of the cavity, pointing in the same direction as the emitted laser signal. The atom receives adjacent wave crests from the emitted laser beam and from the reflected laser beam. The time interval between reception of adjacent crests Δt_r can be obtained from Eq. (2.7) by setting $\mathbf{v}_e=0$, and by noting that $\mathbf{x}_{re}=\pm x_{re}\mathbf{n}$, where the positive sign corresponds to the direct laser beam. But, since $\mathbf{v}_r=\mathbf{v}=v\mathbf{n}$, we find from Eq. (2.8) that

$$R(\mathbf{v})=\pm v(a\gamma^2/b)[f(\mathbf{n})\pm\mathbf{n}\cdot\mathbf{w}]$$

and, thus, from Eq. (2.12) we have [19]

$$\frac{\nu_\pm}{\nu_L}=\frac{a(\mathbf{w})}{a(\mathbf{U}_r)}\left[1\mp\frac{va\gamma^2}{b}\frac{f(\mathbf{n})\pm\mathbf{n}\cdot\mathbf{w}}{1-\boldsymbol{\varepsilon}\cdot\mathbf{v}}\right]. \tag{4.3}$$

In SRT, using $a=b^{-1}=\gamma^{-1}$, $d=1$, $\boldsymbol{\varepsilon}=-\mathbf{w}$, along with Eqs. (2.14a) and (2.14d), we find

$$\nu_\pm=\nu_L\frac{1\mp v}{\sqrt{1-v^2}}=\nu_L\left[\frac{1\mp v}{1\pm v}\right]^{1/2}, \tag{4.4}$$

which is the standard special-relativistic Doppler-shift formula. Notice that $(\nu_+\nu_-)^{1/2}=\nu_L$, independently of v, and $(\nu_- - \nu_+)/(\nu_- + \nu_+)=v$.

Now substituting the low-velocity expansions, Eqs. (2.15) and (2.16), and assuming that $v\approx w$, we obtain

$$(\nu_+\nu_-)^{1/2}\nu_L^{-1}=1-\alpha v^2-2\alpha v w\cos\theta+(\alpha^2-\tfrac{1}{2}\alpha-\alpha_2)v^4+(4\alpha^2-2\alpha\varepsilon-4\alpha_2)v^3w\cos\theta+(2\alpha\delta-\alpha^2-2\alpha_2)v^2w^2$$
$$+[2\alpha(\beta-\delta-\varepsilon)+4\alpha^2+\alpha-4\alpha_2]v^2w^2\cos^2\theta+(2\alpha\beta-4\alpha_2)vw^3\cos\theta+O(6), \tag{4.5a}$$

$$V=v[1+\varepsilon v w\cos\theta+(\alpha-\delta)w^2-(\beta-\delta)w^2\cos^2\theta]+O(4), \tag{4.5b}$$

where $\cos\theta\equiv\hat{\mathbf{w}}\cdot\mathbf{n}$. Notice that the expression for ν_L in Eq. (4.5a) depends on the arbitrary synchronization parameter ε. This dependence is only apparent, however, because the coordinate velocity v is not directly measurable. Instead, V is the measured quantity through the accelerating voltage on the beam required to maintain resonant absorption. Using Eq. (4.5b) to express v in terms of V and substituting into Eq. (4.5a), and noting that in the atom's rest frame the resonance condition corresponds to $\nu_+\nu_-=\nu_1\nu_2$, we obtain, finally

$$\nu_L=(\nu_1\nu_2)^{1/2}\{1+\alpha V^2+2\alpha Vw\cos\theta+(\tfrac{1}{2}\alpha+\alpha_2)V^4+4\alpha_2V^3w\cos\theta-(\alpha^2-2\alpha_2)V^2w^2$$
$$-(\alpha-4\alpha_2)V^2w^2\cos^2\theta-2[\alpha(\alpha+\beta-\delta)-2\alpha_2]Vw^3\cos\theta+2\alpha(\beta-\delta)Vw^3\cos^3\theta+O(6)\}. \tag{4.6}$$

Thus, in accord with the argument of Bay and White [15], there is no dependence on ε, but contrary to their argument, there *is* residual dependence on the SRT-violating parameters α, β, and so on. In SRT, all the V- and w-dependent terms vanish, as expected.

The TPA experiment used a beam of neon atoms with kinetic energy per atom of about 120 keV, corresponding to a velocity of 3.5×10^{-3} of the speed of light. The laser frequency and the beam voltage (which was related to the velocity V) were monitored to look for a diurnal variation generated by the $\cos\theta$ term as the Earth rotated. A limit of 10^{-11} was put on such a variation, leading to the bound

$$|\alpha+2\alpha_2V^2-[\alpha(\alpha+\beta-\delta)-2\alpha_2]w^2|<1.4\times10^{-6}. \tag{4.7}$$

However, since $V^2\approx w^2\approx10^{-6}$ the only useful bound is

$$|\alpha|<1.4\times10^{-6}. \tag{4.8}$$

An improved experiment, or one using higher beam velocities V could begin to constrain the parameter α_2. Such improvements could also lead to a bound on the semidiurnal term proportional to $V^2w^2\cos^2\theta$, which also depends on α_2.

V. THE ROCKET REDSHIFT EXPERIMENT (GRAVITY PROBE A)

The 1976 GP-A experiment [6] was intended primarily to be a test of the gravitational redshift, but it also was sensitive to special-relativistic effects. A Scout-D rocket carrying a hydrogen maser clock was launched to an altitude of 10 000 km; an identical hydrogen maser was located on the ground. A round-trip signal was sent from ground to rocket and back for the purpose of tracking the payload; at the same time, the rocket clock generated a signal which was sent to the ground. If ν_0 is the intrinsic frequency of a hydrogen maser in its own frame, and if ν' and ν'' are the received frequencies of the one-way and two-way signals, respectively, then the experiment was designed to measure two fractional frequency shift signals: the two-way Doppler signal D, defined by

$$D\equiv(\nu''-\nu_0)/\nu_0, \tag{5.1}$$

and the "redshift signal" R, defined by combining the one-way frequency shift with half the Doppler signal:

$$R\equiv(\nu'-\nu_0)/\nu_0-D/2=\nu'/\nu_0-\tfrac{1}{2}\nu''/\nu_0-\tfrac{1}{2}. \tag{5.2}$$

The Doppler signal D contains twice the first-order Doppler shift of the emitted frequency caused by the rocket's motion relative to the ground, but it does not contain the gravitational redshift, because the Earth-bound emitter and receiver are at the same gravitational potential. The first term in the redshift signal $(\nu'-\nu_0)/\nu_0$ contains the gravitational redshift of the signal produced by the potential difference between rocket and ground, and the first-order Doppler shift, once. The second term in R thus cancels the first-order Doppler shift, so that R contains only the gravitational redshift effect, plus second-order SRT effects. This "Doppler cancellation" scheme was built into the data-acquisition system. The gravitational redshift effects have been analyzed elsewhere [6]. For the purpose of this paper, we shall focus on the special-relativistic effects.

We consider the following idealization of the experiment. In an inertial frame S, which moves at velocity $\mathbf{w}$ with respect to Σ, a clock is at rest at the origin, and a second identical clock moves with velocity v. A signal of frequency ν_0 is sent from the rest clock to the moving clock and back, and a signal of the same frequency (as measured in the clock's rest frame) is emitted by the moving clock and received by the rest clock.

Consider first the round trip signal. Since $\mathbf{v}_e=0$, the frequency received at the rocket is given, from Eq. (2.12), by

$$\nu_r/\nu_0=[a/a(\mathbf{U})][1-R(\mathbf{v})/(1-\boldsymbol{\varepsilon}\cdot\mathbf{v})] , \tag{5.3}$$

where $\mathbf{v}$ and $\mathbf{U}$ denote the velocity of the receiving rocket clock (as measured in S and Σ, respectively), and where R is given by Eq. (2.8) with $\mathbf{x}_{re}=x_{re}\mathbf{n}$. This signal is returned with the same frequency as received (as measured in the rocket frame). Thus the frequency ν'' received on the ground is given by

$$\nu''/\nu_r=[a(\mathbf{U})/a][1-R'(\mathbf{v})/(1-\boldsymbol{\varepsilon}\cdot\mathbf{v})]^{-1} , \tag{5.4}$$

where R' is given by Eq. (2.8) with $\mathbf{x}_{re}=-x_{re}\mathbf{n}$. Combining Eqs. (5.3) and (5.4) we obtain

$$\nu''=\nu_0\frac{1-\boldsymbol{\varepsilon}\cdot\mathbf{v}-R(\mathbf{v})}{1-\boldsymbol{\varepsilon}\cdot\mathbf{v}-R'(\mathbf{v})} . \tag{5.5}$$

From Eq. (2.12), we find that the received frequency of the one-way signal from rocket to ground is given by

$$\nu'=\nu_0[a(\mathbf{U})/a](1-\boldsymbol{\varepsilon}\cdot\mathbf{v})[1-\boldsymbol{\varepsilon}\cdot\mathbf{v}-R'(\mathbf{v})]^{-1} . \tag{5.6}$$

If we now define

$$Q\equiv\frac{ab}{d^2f(\mathbf{n})}\left[\mathbf{v}\cdot\mathbf{n}+\left[\frac{d^2\gamma^2}{b^2}-1\right](\mathbf{v}\cdot\hat{\mathbf{w}})(\hat{\mathbf{w}}\cdot\mathbf{n})\right] , \tag{5.7}$$

then $R(\mathbf{v})=Q+(a\gamma^2/b)\mathbf{v}\cdot\mathbf{w}$ and $R'(\mathbf{v})=-Q+(a\gamma^2/b)\mathbf{v}\cdot\mathbf{w}$. The Doppler and redshift signals are then given by

$$D=\frac{-2Q}{1-\boldsymbol{\varepsilon}\cdot\mathbf{v}+Q-(a\gamma^2/b)\mathbf{v}\cdot\mathbf{w}} , \tag{5.8a}$$

$$R=\frac{[a(\mathbf{U})/a-1](1-\boldsymbol{\varepsilon}\cdot\mathbf{v})+(a\gamma^2/b)\mathbf{v}\cdot\mathbf{w}}{1-\boldsymbol{\varepsilon}\cdot\mathbf{v}+Q-(a\gamma^2/b)\mathbf{v}\cdot\mathbf{w}} . \tag{5.8b}$$

In SRT, $Q=\mathbf{v}\cdot\mathbf{n}$, and we recover the exact expressions

$$D=-2\mathbf{v}\cdot\mathbf{n}/(1+\mathbf{v}\cdot\mathbf{n}) ,$$
$$R=[(1-v^2)^{1/2}-1]/(1+\mathbf{v}\cdot\mathbf{n}) . \tag{5.9}$$

Expanding Eqs. (5.8) for low velocities as in the preceding section, we obtain

$$D=-2\mathbf{v}\cdot\mathbf{n}(1+\mathbf{v}\cdot\mathbf{n})^{-1}[1+(\alpha-\delta)w^2+(\beta-\delta)(\mathbf{w}\cdot\mathbf{n})^2+\boldsymbol{\varepsilon}\mathbf{w}\cdot\mathbf{v}]+4(\beta-\delta)(\mathbf{v}\cdot\mathbf{w})(\mathbf{w}\cdot\mathbf{n})+O(4) , \tag{5.10a}$$

$$R=[(-\tfrac{1}{2}+\alpha)v^2+(-\tfrac{1}{8}+\alpha_2)v^4+2\alpha\mathbf{v}\cdot\mathbf{w}](1+\mathbf{v}\cdot\mathbf{n})^{-1}-(2\beta\alpha-4\alpha_2)w^2\mathbf{v}\cdot\mathbf{w}-[(2\alpha-1)\delta-\alpha^2+\alpha-2\alpha_2]w^2v^2$$
$$+[(2\alpha-1)(\delta-\beta)+2\alpha\varepsilon-\alpha+4\alpha_2](\mathbf{v}\cdot\mathbf{w})^2+[(2\alpha-1)\varepsilon-\alpha+4\alpha_2]v^2\mathbf{v}\cdot\mathbf{w}+O(5) . \tag{5.10b}$$

Again, we see the presence of $\boldsymbol{\varepsilon}$-dependent terms in the higher-order corrections in R and D. As before, these effects are illusory, since the coordinate velocity v is not directly measured; instead it is inferred from the Doppler signal. By tracking the rocket from several tracking stations, thereby using different propagation directions $\mathbf{n}$, one can determine the vector velocity. If we define the *inferred* velocity component in a given direction $\mathbf{n}$ by $D_n\equiv\mathbf{V}\cdot\mathbf{n}/(1+\mathbf{V}\cdot\mathbf{n})$, then we have

$$\mathbf{V}\cdot\mathbf{n}=D_n/(1-D_n) \tag{5.11}$$

where D_n is given by Eq. (5.10a) with a chosen $\mathbf{n}$. Suppose that D has been measured using a propagation direction parallel to $\mathbf{w}$, and using two orthogonal directions perpendicular to $\mathbf{w}$. Combining Eqs. (5.11) and (5.10a), with $\mathbf{n}\cdot\mathbf{w}=|\mathbf{w}|$ and $\mathbf{n}\cdot\mathbf{w}=0$, respectively, we find the components of $\mathbf{V}$ parallel and perpendicular to $\mathbf{w}$ to be

$$V_\parallel=v_\parallel[1+(\alpha-\beta)w^2+\boldsymbol{\varepsilon}\mathbf{v}\cdot\mathbf{w}] , \tag{5.12a}$$

$$V_\perp=v_\perp[1+(\alpha-\delta)w^2+\boldsymbol{\varepsilon}\mathbf{v}\cdot\mathbf{w}] . \tag{5.12b}$$

Thus

$$\mathbf{V}\cdot\mathbf{w}=\mathbf{v}\cdot\mathbf{w}[1+(\alpha-\beta)w^2+\boldsymbol{\varepsilon}\mathbf{v}\cdot\mathbf{w}] , \tag{5.13a}$$

$$V^2\equiv V_\parallel^2+V_\perp^2=v^2[1+2(\alpha-\delta)w^2+2\boldsymbol{\varepsilon}\mathbf{w}\cdot\mathbf{v}]$$
$$-2(\beta-\delta)(\mathbf{w}\cdot\mathbf{v})^2 . \tag{5.13b}$$

Rewriting v in terms of V in Eq. (5.10b), we get

$$R=[(-\tfrac{1}{2}+\alpha)V^2+(-\tfrac{1}{8}+\alpha_2)V^4+2\alpha\mathbf{w}\cdot\mathbf{V}][1+\mathbf{V}\cdot\mathbf{n}]^{-1}$$
$$+(2\alpha_2-\alpha^2)w^2(V^2+2\mathbf{w}\cdot\mathbf{V})$$
$$+(4\alpha_2-\alpha)\mathbf{w}\cdot\mathbf{V}(V^2+\mathbf{w}\cdot\mathbf{V})+O(5) . \tag{5.14}$$

When written in terms of the observable velocity $\mathbf{V}$, the result is independent of $\boldsymbol{\varepsilon}$.

Focusing on the $O(2)$ terms in Eq. (5.14), we see the analogue of the second-order Doppler shift and a term dependent on the angle between $\mathbf{w}$ and the rocket's velocity. The absence of such an effect in the data from the 1976 experiment [6] led to a rough limit

$$|\alpha| < 10^{-6} . \tag{5.15}$$

VI. THE MÖSSBAUER ROTOR EXPERIMENT

In this experiment [3–5], a γ-ray emitter was placed on the rim of a rotating disk and an absorber was placed at the center. A detector was placed just behind the absorber. The disk was rotated, and the detector monitored changes in the transition of γ rays through the absorber as a function of the direction of propagation of the rays. Changes in transmission would occur if the frequency of γ rays, ν_r, received at the absorber differed from the intrinsic frequency corresponding to the peak of the γ-ray absorption cross section, ν_e. The combination of slight intrinsic differences between absorber and emitter and the ordinary time dilation of the emitter's frequency resulted in a small offset between ν_r and ν_e whose consequence was that any periodic changes in ν_r resulting from an anisotropy in the speed of light would lead to large changes in absorption because the incident frequencies would lie on the "wings" of the resonant absorption curve.

Consider a disk with its origin at rest in S. Light is emitted at $(\mathbf{x}_e, t_e)$ on the rim and received at $(\mathbf{x}_r = 0, t_r)$. Here $\mathbf{v}_r = 0$ and $\mathbf{v}_e \cdot \mathbf{x}_e = 0$, since the disk is assumed to rotate rigidly in S (we assume here that physical length is defined by $|\mathbf{x}|$). Thus from Eq. (2.12),

$$\frac{\nu_r}{\nu_e} = \frac{a(\mathbf{U}_e)}{a}\left[1 - \frac{R(\mathbf{v}_e)}{(1-\boldsymbol{\varepsilon}\cdot\mathbf{v}_e)}\right]^{-1} . \tag{6.1}$$

In SRT, using Eqs. (2.14), we get the usual time dilation

$$\nu_r/\nu_e = (1 - v_e^2)^{1/2} . \tag{6.2}$$

Substituting the slow-motion expansions Eqs. (2.15) and (2.16) we find

$$\begin{aligned}\nu_r/\nu_e = {} & 1 - (\tfrac{1}{2} - \alpha)v_e^2 + 2\alpha\mathbf{w}\cdot\mathbf{v}_e - 2(\beta-\delta)\mathbf{w}\cdot\mathbf{v}_e\mathbf{w}\cdot\mathbf{n}_{re} - (\tfrac{1}{8} - \alpha_2)v_e^4 + [2\alpha\boldsymbol{\varepsilon} + (2\alpha-1)(\delta-\beta) - \alpha + 4\alpha_2](\mathbf{w}\cdot\mathbf{v}_e)^2 \\ & + [2\alpha(\alpha-\beta) - 2\alpha^2 + 4\alpha_2]w^2\mathbf{w}\cdot\mathbf{v}_e + [\alpha^2 - \alpha + 2\alpha_2 - (2\alpha-1)\delta]w^2v_e^2 + [(2\alpha-1)\boldsymbol{\varepsilon} - \alpha + 4\alpha_2]v_e^2\mathbf{w}\cdot\mathbf{v}_e + O(5) .\end{aligned} \tag{6.3}$$

As in Sec. V, we must express the coordinate velocity $\mathbf{v}_e$ in terms of an appropriate physically measured velocity $\mathbf{V}$. In practice, $\mathbf{V}$ was presumably determined from the angular velocity, dimensions, and orientation of the disk. However, in the absence of a detailed accounting of how such measurements might have been made, and because of the difficulty of analyzing such details in a non-Lorentz-invariant framework, we shall adopt the Doppler-measured results of Sec. V, since in principle the velocity of the emitter *could* have been measured this way, using absorbers stationed at various locations around the laboratory. On substituting Eqs. (5.13), we find

$$\begin{aligned}\nu_r/\nu_e = {} & 1 - (\tfrac{1}{2} - \alpha)V^2 + 2\alpha\mathbf{w}\cdot\mathbf{V} - 2(\beta-\delta)\mathbf{w}\cdot\mathbf{V}\mathbf{w}\cdot\mathbf{n} \\ & - (\tfrac{1}{8} - \alpha_2)V^4 + (2\alpha_2 - \alpha^2)w^2(V^2 + 2\mathbf{w}\cdot\mathbf{V}) \\ & + (4\alpha_2 - \alpha)\mathbf{w}\cdot\mathbf{V}(V^2 + \mathbf{w}\cdot\mathbf{V}) + O(5) .\end{aligned} \tag{6.4}$$

Again, the final result is independent of synchronization parameter $\boldsymbol{\varepsilon}$. The most recent experiment, reported by Isaak [5], put a limit of 2×10^{-16} on variations of ν_r/ν_e with phase of the rotor, using emitter speeds of around 360 $\mathrm{m\,s^{-1}}$. Since $V \ll w$, only the terms linear in V in Eq. (6.4) are relevant. Taking into account the projection of $\mathbf{V}$ onto $\mathbf{w}$, we obtain the bound

$$|\alpha - (\alpha^2 - 2\alpha_2)w^2| < 9\times10^{-8} . \tag{6.5}$$

Barring fortuitous cancellations between terms, we can read off the separate bounds $|\alpha| < 9\times10^{-8}$ and $|\alpha_2| < 3\times10^{-2}$. The absence of a signal at twice the rotor frequency can also be used to limit the $O(3)$ term in Eq. (6.4) proportional to $\mathbf{w}\cdot\mathbf{V}\mathbf{w}\cdot\mathbf{n}$. The result is $|\beta-\delta| < 10^{-4}$.

VII. DISCUSSION

We have used a test theory of special relativity to analyze experiments in which the propagation of light along a one-way path was a central physical phenomenon. These experiments searched for an anisotropy associated with that one-way propagation, with null results. We found that the synchronization of clocks played no role in the interpretation of these experiments provided that one is careful to express the results in terms of physically measurable quantities.

In the JPL time-of-flight experiment, the measurable quantities were oscillator phase differences measured relative to an arbitrarily chosen initial clock phase. Here, the initial choice of phase difference amounted to a choice of synchronization, but once that choice was made, it was fixed once and for all, so that a resulting anisotropy in phase differences as the orientation of the propagation path changed became physically meaningful. In the three frequency-shift experiments, the TPA, GP-A, and Mössbauer rotor experiments, the measurable quantities were the frequency shifts and the velocities of the emitters. When the latter were properly interpreted in terms of Doppler-shift measurements, synchronization again dropped out of the problem.

One major limitation of the MS test theory is that it is purely kinematical: it deals only with relations between coordinates in different reference frames. The only physical assumption inherent in the formalism is the isotropy of the propagation of light in the preferred frame Σ. We have had to make additional *dynamical* assumptions that may or may not be justified. For example, we assumed that the proper time of a clock moving in the laboratory frame S was related to the coordinate time of S by Eq. (2.11). However, as Haugan and Will [20] have emphasized, violations of Lorentz invariance inevitably lead to altered dynamics for the rods and clocks on which the transformations between inertial frames are based. For example, two different clocks moving through S may measure different proper times. In other words, time dilation need no longer be universal. Because we focus on measurements made by a single clock (hydrogen maser, neon atom, Mössbauer absorber) in each experiment, and deal only with effects resulting from changing orientation, such dynamical effects on clocks should not play an important role.

On the other hand, we had to make an assumption about physical length in interpreting the JPL experiment and the Mössbauer rotor experiments. Again, if Lorentz invariance is violated, the behavior of moving rods may depend on their internal structure, so that a universal relation between coordinate length $|\mathbf{x}|$ and physical length L may not exist, or may not satisfy $L=|\mathbf{x}|$. Consequently there was an ambiguity in our interpretation of the JPL experiment at $O(w^2)$. The lowest-order effects were not affected by this problem.

Whether or not these ambiguities in the MS framework can be resolved remains to be seen. We are currently studying whether the dynamical framework for Lorentz noninvariance developed by Haugan and collaborators [20,21] could be combined with the MS approach to yield a formalism that avoids such ambiguities.

ACKNOWLEDGMENTS

This research was supported in part by National Science Foundation Grant No. PHY 89-22140.

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