Dissipative cosmology with decaying vacuum energy

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A closed cosmological model that begins with a phase of generalized inflation followed by a Friedmann-like period is proposed.

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I. INTRODUCTION

The idea of a decreasing vacuum energy with cosmic expansion has been invoked by different authors [1–4] in order to understand phenomenologically the incredibly small value of the cosmological "constant." This is in the same spirit as the idea of the slow-rolling potentials utilized in the literature, more or less *ad hoc*, to give rise to inflation [5].

In a recent paper [4], Chen and Wu proposed a homogeneous and isotropic cosmological model which incorporates an effective cosmological term Λ that varies with the scale factor of the Universe as

$$\Lambda=\frac{\gamma}{R^2}\ , \tag{1}$$

where γ is a constant of order unity. Considerations based on the second law of thermodynamics lead to the restriction $\gamma\geq 0$ [1]. Chen and Wu heuristically argue that the law given by Eq. (1) reflects the simplest dependence of Λ on R compatible with the nonappearance of $\hbar$ on Einstein's equations, which would be rather disturbing. Furthermore, at variance with some other laws so far proposed for the decay of Λ [2,3], the thermodynamic fluctuations of energy associated with this one behave correctly [6] and do not conflict with the high degree of isotropy of the cosmic microwave background radiation (CMBR).

This model, aside from being compatible with astrophysical data, alters the predictions of the standard model for the matter-dominated epoch and has two interesting consequences for inflation. These are (i) the current value of the deceleration parameter no longer has to be strictly 0.5, as the standard model demands, but it can have any value satisfying $0<q_0<0.5$ (if $\gamma>0$) or else $q_0>0.5$ (if $\gamma<0$), and (ii) likewise, the parameter Ω_0 ($\equiv\rho_0/\rho_{\text{cr},0}$) can take any value in the range $\frac{2}{3}<\Omega_0<1$ instead of the strict value $\Omega_0=2q_0=1$ required by the standard model.

Both consequences can be traced, on the one hand, to the fact that the scalar curvature k gets replaced by $k-(\gamma/3)$ and $k-\gamma$ for the radiation- and matter-dominated epochs, respectively, and, on the other hand, to the conversion of the vacuum energy into matter and radiation.

However, the model of Chen and Wu does not incorporate particle production or dissipation. As is well known, both effects play an important role on cosmic evolution (see, for instance, Barrow [7] and Murphy [8]) and may, in principle, have notable observational consequences.

Our target is to explore the consequences of generalizing this model by means of the inclusion of the mentioned effects, but retaining homogeneity and isotropy, however. This will be done by supplementing the hydrostatic (equilibrium) pressure with a dissipative term

$$\sigma=-\zeta\theta\quad(\theta\equiv 3\dot{R}/R)\ , \tag{2}$$

which represents a bulk viscous pressure. ζ stands for the coefficient of bulk viscosity (which is positive semidefinite), and θ denotes the expansion of the fluid. It is worth stressing that this term, when incorporated into Einstein's equations, can be interpreted as the effect of particle production instead [7,9]. So we will look at expression (2) in both ways simultaneously, with regard to the usual bulk-viscous pressure and the effect of particle production.

We find that cosmologies with either $k=0$ or -1 turn out to be incompatible with observation. By contrast, our model, which exhibits an initial inflationary period, predicts $q_0 \simeq 0.5$ and $\Omega_0 \simeq 1$, in good agreement with astrophysical data.

In Sec. II we propose our model, show their solution curves, and compare them with those arising from the model of Chen and Wu. Section III is devoted to briefly studying its observational consequences. Finally, in Sec. IV, we summarize our conclusions and point out some possible generalizations of our work.

II. THE MODEL

We adopt a Friedmann-Robertson-Walker universe filled with an imperfect fluid whose stress-energy tensor reads

$$T^{\mu\nu}=(\rho+P-\zeta\theta)u^{\mu}u^{\nu}+(P-\zeta\theta)g^{\mu\nu} , \tag{3}$$

where ρ denotes the energy density of matter and radiation and P the hydrostatic pressure. Accordingly, the Einstein equations

$$R^{\mu\nu}-\tfrac{1}{2}Rg^{\mu\nu}-\Lambda g^{\mu\nu}=-8\pi G T^{\mu\nu} \tag{4}$$

reduce to

$$H^2+\left[k-\frac{\gamma}{3}\right]\frac{1}{R^2}=\frac{8\pi G}{3}\rho , \tag{5}$$

$$2\frac{\ddot{R}}{R}+H^2+\left[\frac{k-\gamma}{R^2}\right]=-8\pi G(P-3\zeta H) , \tag{6}$$

with $H \equiv \dot{R}/R$, and the energy-conservation equation

$$8\pi G u_{\mu}T^{\mu\nu}{}_{;\nu}=u^{\mu}\Lambda_{,\mu} \tag{7}$$

takes the form

$$\dot{\rho}+3(\rho+P)H=\frac{1}{8\pi G}\left[\frac{2\gamma}{R^2}+3\zeta\right]H . \tag{8}$$

The right-hand side of (8) does not vanish in general since, on the one hand, as a result of the varying character of Λ, particles are "emerging" from the vacuum and, on the other hand, because of the presence of the bulk-viscous pressure and/or particle production by gravitational and quantum fields.

By combining Eqs. (5) and (6) with the equation of state

$$P=(\lambda-1)\rho \tag{9}$$

and the customary expression for ζ in terms of the energy density [7,8,10,11],

$$\zeta=\alpha\rho^{m} , \tag{10}$$

where λ is a constant lying in the range $1<\lambda<2$, and α and m are constants of order unity too, we obtain

$$2\frac{\ddot{R}}{R}=(2-3\lambda)\left[H^2+\frac{\kappa}{R^2}\right]+\frac{3^{m+1}}{(8\pi G)^{m-1}}H\left[H^2+\frac{\kappa}{R^2}\right]^{m}+\frac{2}{3}\frac{\gamma}{R^2} , \tag{11}$$

where κ is shorthand for $k-(\gamma/3)$.

For a radiative fluid m picks up the value 1, whereas for a fluid of fundamental strings one has $m=\frac{3}{2}$ [7]. In what follows we will take $m=1$ for mathematical simplicity.

There is a variety of mechanisms capable of generating a non-negligible bulk-viscosity pressure [12], notably the one connected with the matter-radiation interaction. In addition to this, the existence of dark matter suggests a new one, not considered up to now. Nonbaryonic dark matter seems to exist abundantly, pervading the whole Universe, and it is thought to interact with normal matter only gravitationally. This interaction may conceivably give rise, on the average, to a "drag effect" of one kind of matter on the other; this, in its turn, may be viewed phenomenologically as viscosity. This is the usual outcome of mixing up two different fluids. Of course, in the case we are considering, the viscous effect so generated should appear as shear viscosity as well. However, the latter has no room in our analysis because of the high degree of symmetry imposed by the metric we have adopted.

Equation (11) has no analytical solution in general, and so we resort to the method of qualitative analysis of differential equations [13], of ample use in the gravitational context, to gain some insight about the behavior of its solutions. As has been stressed, it gives a very good picture of the general behavior of the different solutions and helps us in pointing in the direction in which the search for specific solutions should be undertaken [14].

We skip the details of the analysis, which are valid for any λ in the range $1<\lambda<2$ and any positive α, and show the main results in the phase plane $(R,\dot{R})$ of Fig. 1 ($\kappa>0$) and Fig. 2 ($\kappa<0$). Singular points are indicated by capital letters. Before discussing these figures it is worth mentioning that, because of the successive coordinate transformations involved in bringing the points of infinity near the origin, the scale is not uniform throughout the phase planes, but varies continuously.

The first thing to note is precisely the sharp difference between the solution curves belonging to the case $\kappa>0$ and those to $\kappa<0$ (the latter includes $k=0$ and -1, for they are qualitatively identical). For $\kappa>0$ all physically relevant solutions start from the origin (point O), whereas for $\kappa<0$ only the one ending in P originates in O. Point P in either figure corresponds to a de Sitter expansion, and the curves joining it from O correspond to unstable solutions. The hatched region in Fig. 2 is of negative-energy density [see Eq. (5) for $k<\gamma/3$]; hence, it is reasonable to dismiss as unphysical the solution curves with points in that region.

Those solution curves originated in O ($\kappa>0$) or in P_+ ($\kappa<0$) and with an end in S diverge and are incompatible with observation. The closed solution curves, of Fig. 2, passing through point Q (a Minkowski space), aside from crossing the mentioned region of negative-energy density, seem to be incompatible with observation too since they do not go through any period of small scale factor, something required if one wishes to have a very hot epoch able to generate the CMBR.

Likewise, the solution curves with an origin in Q and an end either in P_- or in S' cannot be reconciled with observation since, aside from the CMBR problem, they

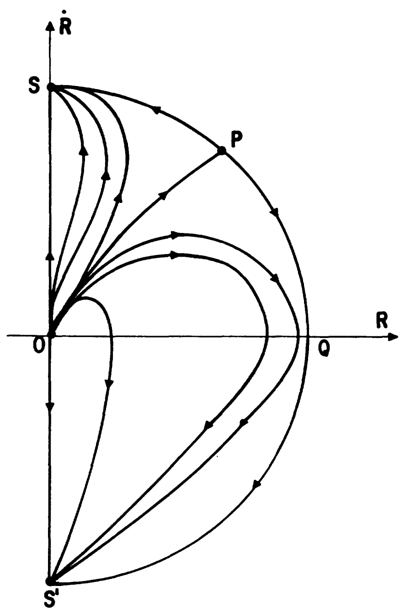

FIG. 1. Phase plane $(R,\dot{R})$ for $\gamma=1$, $\kappa=\frac{2}{3}$, and $\alpha>0$.

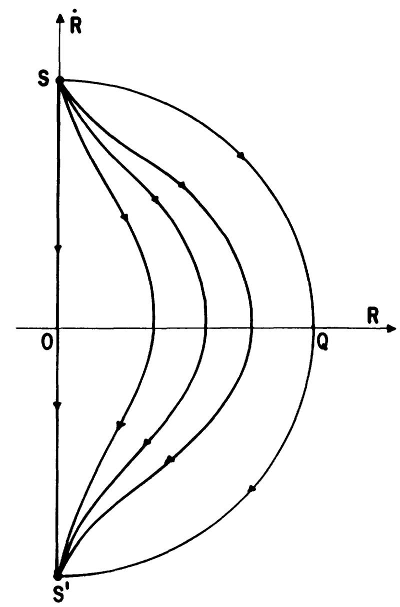

FIG. 3. Phase plane $(R,\dot{R})$ for $\gamma=1$, $\kappa=\frac{2}{3}$, and $\alpha=0$ [4].

have $H<0$ throughout.

Solution curves compatible with observation can only be found among those of Fig. 1 ($\kappa>0$) with an origin in O and an end in S'. These are stable. Any of them represents the evolution of a closed universe that begins with an epoch of generalized inflation ($d^2R/dt^2>0$) and, after reaching a maximum in dR/dt, gently evolves into a Friedmann-like period. The latter persists until the final point S'—a big crunch. To make transparent the above-mentioned Friedmann-like character, it suffices to superimpose Fig. 1 on Fig. 3. The latter shows the phase plane $(R,\dot{R})$ corresponding to the same set as above, but now with $\alpha=0$ (i.e., no bulk viscosity or particle production, Chen and Wu model [4]) for $\kappa>0$. For the sake of completeness, we also show (Fig. 4) the phase plane $(R,\dot{R})$ for $\alpha=0$ and $\kappa<0$. In this case, as in Fig. 2, a region of negative-energy density appears.

Because of the scarcity of accurate cosmological data, it is difficult to ascertain accurately which among the curves of Fig. 1 that start from O and end in S' correspond to our Universe. Whatever the curve is, it has to satisfy the requirement of beginning its Friedmann-like period in such a point of its radiation-dominated epoch that no conflict with the observed abundances of light elements arises. This is, admittedly, a delicate matter. However, since these solution curves make a continuous set, there is enough room to meet the mentioned requirement. The present status of our Universe $[H_0>0$, $(dH/dt)_0<0]$ is to be given by a point on one of these

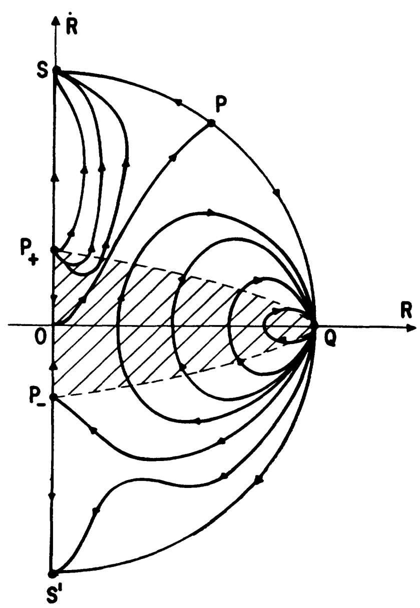

FIG. 2. Phase plane $(R,\dot{R})$ for $\gamma=1$, $\kappa=-\frac{1}{3}$, $-\frac{4}{3}$, and $\alpha>0$. The hatched region is of negative-energy density.

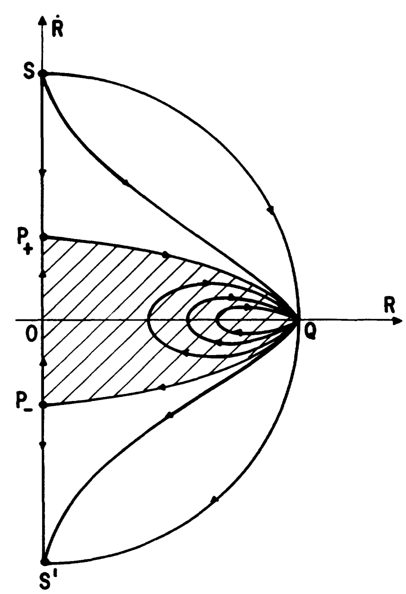

FIG. 4. Phase plane $(R,\dot{R})$ for $\gamma=1$, $\kappa=-\frac{1}{3}$, $-\frac{4}{3}$, and $\alpha=0$ [4]. The hatched region is of negative-energy density.

curves, somewhere between the beginning of its Friedmann-like evolutionary period and its crossing of the horizontal axis. [At the crossing ($\dot{R}=0$), the scale factor attains its maximum value.] In addition, the mentioned point has to pick up so that the time elapsed from the beginning of the latter period and the present epoch is of the order the inverse of the current value of Hubble's parameter.

III. IMPLICATIONS

From Eq. (6) and the definition of the deceleration parameter $q=-R\ddot{R}/\dot{R}^2$, a relationship for the present value of q can be obtained:

$$(2q_0-1)H_0^2=\frac{k-\gamma}{R_0^2}+9\alpha\left[\frac{\kappa}{R_0^2}+H_0^2\right]H_0 \ . \tag{12}$$

In deriving it, use was made of the condition $P_0=0$ (dust) as well as of Eqs. (10) and (5). This equation differs from its counterpart derived by Chen and Wu [see Eq. (13) in Ref. [4]] by the second term on the right-hand side.

One may think that Eq. (12) is too complicated to extract any useful information from it. However, it is possible to do that provided the following points are taken into account. (i) γ is a positive-definite dimensionless constant. (ii) By observation, $H_0 \ll 1$; hence, the second term on the right-hand side of (12) is negligible as compared with the first one. (iii) Because of the initial phase of generalized inflation ($d^2R/dt^2>0$), the quantity $(H_0R_0)^2$ has to be much greater than unity (note that the radius of the Universe, R_0, is greater than the radius of the observed Universe). As a consequence, $q_0 \simeq 0.5$.

The parameter Ω_0 can be cast into the form

$$\Omega_0=\frac{2q_0+2(\rho_{\text{vac},0}/\rho_{\text{cr},0})}{1+9\alpha H_0}, \quad \rho_{\text{cr},0}=\frac{3H_0^2}{8\pi G} \ . \tag{13}$$

Because of the restriction $H_0 \ll 1$, the last expression differs very little from the one given by Chen and Wu, $\Omega_0=2q_0+2(\rho_{\text{vac},0}/\rho_{\text{cr},0})$. Furthermore, it depends again on the ratio $\rho_{\text{vac},0}/\rho_{\text{cr},0}$. Since most of the vacuum-energy density [$\rho_{\text{vac}}=\Lambda/(8\pi G)$] must have gone into radiation and particles during the generalized inflation phase, it is not unreasonable to think that this ratio is negligible against q_0. This harmonizes with our earlier assumption of $(H_0R_0)^2$ being much higher than unity. Consequently, we obtain $\Omega_0 \simeq 1$. This result agrees very well with the observational estimation of Loh and Spillar [15] and the more recent one reported by Carlberg [16].

Our model predicts a generation rate of particles and radiation given by

$$\left.\frac{d(\rho R^3)}{R^3dt}\right|_0=2\rho_{\text{vac},0}H_0+9\alpha\rho_0H_0^2 \ . \tag{14}$$

This rate differs from expression (19) in Ref. [14] by the second term, which accounts for viscous dissipation and nonvacuum decaying-particle production. The presence of an additional H_0 factor in this second term makes it negligible as compared with the first one.

IV. CONCLUDING REMARKS

We have presented a homogeneous and isotropic cosmological model which incorporates a vacuum-energy-density term that decays with expansion and another term that takes into account viscous heating and/or particle production. A qualitative analysis of the differential equations governing the model selects naturally a closed cosmology ($\kappa>0$). The Universe starts from the initial singularity with a generalized inflation period and smoothly evolves into a Friedmann-like phase. The latter ends in a big crunch. The model does not conflict with observation and predicts $q_0 \simeq 0.5$, $\Omega_0 \simeq 1$.

To analyze the final stages of contraction (something that we do not attempt here), one should take into account the role played by astrophysical black holes since it seems to be of prime importance in closed evolutionary cosmologies [17].

Finally, we suggest two directions to generalize this work without disturbing isotropy nor homogeneity: first, to allow for values of m different from 1 in Eq. (11) and, second, to go over more general expressions than (2) for the bulk-viscous stress [10,11]. These generalizations are planned to be the subject of a future work.

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