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Nucleation of strange matter in dense stellar cores

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We investigate the nucleation of strange quark matter inside hot, dense nuclear matter. Applying Zel'dovich's kinetic theory of nucleation we find a lower limit of the temperature T for strange-matter bubbles to appear, which happens to be satisfied inside the Kelvin-Helmholtz cooling era of a compact star life but not much after it. Our bounds thus suggest that a prompt conversion could be achieved, giving support to earlier expectations for nonstandard type-II supernova scenarios.

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I. INTRODUCTION

Considerable attention has been recently devoted to the astrophysical consequences of the strange-quark-matter (SQM) hypothesis [1]. Particularly, it has been suggested [2] that SQM may play a key role in type-II supernova events as subnuclear energy is released from the conversion of neutron matter (NM) into SQM after a core bounce. This process should result in an explosive expansion mediated by a detonation front [2,3], a phenomenon that would be important for the fate of the collapsed star which would be otherwise in serious danger of becoming a black hole.

Regardless of the actual evolution of the phase-change front, it is obviously necessary for SQM to appear in the first place. Several scenarios for the start of this conversion have been advanced [4], but no compelling argument showing the occurrence of any particular process can be made yet. One of the physically simplest possibilities is the spontaneous nucleation of SQM inside homogeneous NM as a result of fluctuations in the former medium, and we shall address it in the following.

As SQM is a low-entropy configuration, the quark gas is not a lower-free-energy state than a nucleon gas at intermediate temperatures. Compression (i.e., baryochemical potential $\mu \neq 0$) is therefore needed to compensate the $-TS$ term in the free energy if SQM is to be preferred to NM at $T > 2$ MeV [5]. These are precisely the physical conditions generally believed to exist in young protoneutron stars [6] immediately after the passage of the prompt hydrodynamical shock [7] in type-II supernovas, i.e., inside the Kelvin-Helmholtz epoch of the compact object life. The opposite case of NM nucleation inside SQM lumps has been widely discussed [8,9] in relation to the boiling of quark nuggets at intermediate temperatures and low degeneracies. Both hypothetical processes thus follow different (almost orthogonal) paths in the T-μ plane [5].

We shall discuss in the present work the appearance of SQM inside degenerate NM, which is characteristic of protoneutron stars. In Sec. II we apply the thermodynamic theory of classical nucleation to give a rough semiquantitative estimate for the plausibility of the referred process. A refined calculation based on a kinetic approach to the problem [10] is given in Sec. III. Finally, a brief discussion and conclusions are presented in Sec. IV.

II. NUCLEATION OF SQM BUBBLES: THERMODYNAMIC ESTIMATES

The "boiling" of quark nuggets at intermediate temperatures and $\mu \simeq 0$ has been previously discussed [8,9], in relation to the problem of their survival as massive relics, in the framework of classical nucleation theory [11]. This simple approach is based on the calculation of the formation rate of critical bubbles (also termed "nuclei" hereafter) of the stable phase into the metastable one in terms of equilibrium quantities. Surface effects disfavor the survival of small bubbles below a certain critical radius r_c, which is nothing but the value that extremizes the thermodynamical work W necessary to create the bubbles. A suitable form of W for strongly degenerate matter is [12]

$$W = -(P_i - P_e)\tfrac{4}{3}\pi r^3 + 4\pi\sigma r^2 + n(\mu_i - \mu_e)\tfrac{4}{3}\pi r^3 \; , \quad (1)$$

where $P_{i,e}$ are the pressures internal (i) and external (e) to the bubble (assumed to be spherical), σ is the surface tension, n the particle-number density, and $\mu_{i,e}$ are the chemical potentials of each phase. This expression generalizes the one for $\mu = 0$ used in Refs. [8,9]. Requiring W to be an extreme, $\partial W / \partial r = 0$, yields the value of the critical radius r_c,

$$r_c = \frac{2\sigma}{\Delta P(1+n\,\Delta\mu/\Delta P)} , \qquad (2)$$

and we have defined $\Delta P = P_i - P_e$ and $\Delta\mu = \mu_e - \mu_i$ for convenience, both being positive quantities in the SQM hypothesis framework. Substituting in Eq. (1) the critical work results:

$$W_c = \frac{16\pi\sigma^3}{3(\Delta P)^2(1+n\,\Delta\mu/\Delta P)^2} . \qquad (3)$$

Once W_c is calculated, the rate of critical bubble formation [8], $\xi \simeq T^4 \exp(-W_c/T)$, can be computed. To obtain the net number of SQM bubbles, the rate ξ must be multiplied by the time interval available for prompt nucleation, $\Delta t \simeq 1$ s, and by the volume where the nucleation can take place in the dense core, $V_0 \simeq (1\ \text{km})^3$. We shall impose that at least one SQM bubble appears (which would suffice to convert the whole neutronized core), which is expressed by the condition

$$\Delta t\, V_0 T^4 \exp\left[-\frac{16\pi\sigma^3}{3T(\Delta P)^2(1+n\,\Delta\mu/\Delta P)^2}\right] \geq 1 . \qquad (4)$$

In principle, given suitable models of SQM and NM, we could calculate the thermodynamic quantities appearing in the argument of the exponential (including the surface tension σ) and thus obtain a lower bound on the physical temperature T which makes the nucleation viable (see, for example, Ref. [9] for self-consistent calculations of this type). In practice, it is not easy to extract a sensible result because of the well-known uncertainties in both phases, particularly at high densities. In other words, the inequality (4) is very sensitive to poorly determined features such as the stiffness of the NM equation of state. As a first estimate, we set $\Delta P \simeq 10$ MeV fm^{-3}, $\Delta\mu \simeq 20$ MeV, and $n \simeq 0.8$ fm^{-3} and scale everything to $\sigma_{100} = \sigma/(100\ \text{MeV})^3$. These are not the results of a particular model calculation, but rather reflect reasonable expectations for the actual physical conditions. A lower bound for T is then

$$T \geq 4.3\sigma_{100}\ \text{MeV}. \qquad (5)$$

To perform a comparison with the numerical results of the protoneutron-star birth, it is important to note that the reference value of $(100\ \text{MeV})^3$ for σ is probably an extreme upper bound [13]. Detailed numerical [9,13] and analytical calculations [14], as well as phenomenological fittings to detailed models [15,16], certainly do not favor $\sigma > (70\ \text{MeV})^3$ for any value of the strange quark mass m_s and temperature range. Thus, accepting the latter upper bound, we may state $T \geq 1.55$ MeV for the nucleation to occur. This is certainly much lower than the expected post-shock temperatures in type-II supernovas and indicates the plausibility of the nucleation scenario.

III. NUCLEATION KINETICS

The results of the preceding section, even though encouraging, suffer from a series of drawbacks which do not allow us to state firmer conclusions. In addition to the above-mentioned problems connected with the particular choice of each side description, it is clear that the very assumption of a description in terms of equilibrium values may be a rough one for the problem at hand. It is then desirable to tackle the nucleation from a kinetic point of view, which can provide more reliable quantitative answers. A suitable formalism for this purpose is Zel'dovich's theory of nucleation, which describes the growth of the nuclei (assumed to contain a sufficiently large number of particles) by means of a Fokker-Planck equation

$$\frac{\partial f}{\partial t} = -\frac{\partial \xi}{\partial r} , \qquad (6)$$

where $f(r,t)$ is the time-dependent size distribution of nuclei. Imposing appropriate boundary conditions, an integration of Eq. (6) yields the result [10]

$$\xi = 2(\sigma/T)^{1/2} B(r_c) f_0(r_c) , \qquad (7)$$

for the nucleation rate. The quantity $f_0(r_c)$ is the equilibrium distribution function evaluated at $r = r_c$, which is expressed as [10]

$$f_0(r_c) = r_c^2 n_q n_n \exp(-4\pi\sigma^2/3T) , \qquad (8)$$

with n_q and n_n the number densities in SQM and NM, respectively. We shall also need an explicit expression for the size diffusion coefficient $B(r_c)$, which is linear in the derivative dr/dt calculated for nuclei beyond the critical range (i.e., those governed by macroscopic equations) and inversely proportional to the difference $(r - r_c)$ in this approximation. Since $B(r = r_c)$ must be free of singularities and recalling that the growth of the SQM should be limited by the relevant strangeness-changing reactions operating on a weak-interaction time scale τ_w, we have set $dr/dt = (r - r_c)/\tau_w$ as a reasonable choice. Thus $B(r_c)$ turns out to be

$$B(r_c) = T/8\pi\sigma\tau_w . \qquad (9)$$

Assuming that the newly formed SQM bubble can be described by a simple bag model containing N_q ultrarelativistic quarks [17] [i.e., the bubble energy given by $E(r) = \frac{4}{3}\pi r^3 B + 2.04 N_q/r - Z_0/r$, with $Z_0 \simeq 2$ a zero-point correction], we may replace $r \simeq 0.85 N_q^{1/4}$ fm/$(B^{1/4}/145\ \text{MeV})$ everywhere. (Strictly speaking, the bag is not in a vacuum, but compressed by a finite pressure P_e; while this should change the numerical coefficient of r, we have not attempted to include such a correction. Thus we are overestimating the bag size, which is on the conservative side for the numbers given below.) The formula for ξ now reads

$$\xi = 2.2\times 10^{-2} (T/\sigma)^{1/2} N_{qc}^{3/4} \tau_w^{-1} \exp(-3.1 N_{qc}^{1/2}\sigma/T) . \qquad (10)$$

In order to know the nucleation rate as a function of T, we must still specify the baryon-number threshold $A_c = N_{qc}/3$ above which SQM is stable and will inevitably grow further by converting the surrounding NM. A_c is presently unknown with any accuracy, and it is expected to be in the range 10–100 [15], i.e., $N_{qc} \leq 300$.

The larger A_c is, the smaller ξ will result, reflecting the improbable case for a simultaneous high-order weak-interaction fluctuation. This can be seen from Eq. (10) by the presence of the $N_{qc}^{1/2}$ in the dominating exponential factor.

We now proceed as in Sec. II and require the product $\xi\,\Delta t\,V_0$ to be ≥ 1 for the phase change to start. Adopting, as before, $\Delta t \simeq 1$ s and $V_0 \simeq (1\ \text{km})^3$ and imposing $\tau_w \simeq 10^{-8}$ s and $N_{qc}=100$, we find the lower bound for T to be

$$T \geq 6.2\sigma_{100}\ \text{MeV} \tag{11}$$

or, imposing again $\sigma \leq (70\ \text{MeV})^3$ because of the reasons stated in Sec. II,

$$T \geq 2.1\ \text{MeV}\ , \tag{12}$$

which constitutes the main result of the present work.

IV. DISCUSSION AND CONCLUSIONS

We have applied the Zel'dovich kinetic theory of nucleation to the problem of SQM appearance in dense/hot stellar cores. The adopted formalism is, however, valid if the dominating mechanism for nucleation is *thermal* (negligible quantum fluctuations). If we denote by τ the characteristic time for the change of a relevant physical quantity (to be identified with the strangeness $-S$ in our problem), it is easily shown [11] that the condition for dominating thermal fluctuations in $-S$ is $\tau=\tau_w \gg T^{-1}$, which is satisfied for any reasonable physical temperature found inside a neutron star along its life. However, nucleation of SQM is strongly suppressed as T decreases [Eqs. (4) and (10)] as should have been expected, in complete analogy with the boiling of water in physically equivalent conditions. Therefore we can assert that, if thermal nucleation is the dominating mechanism for SQM appearance in dense matter, it should form only if Eq. (12) can be satisfied. These conditions are certainly met in the prompt-shock aftermath in type-II supernovas, but not after approximately minutes of the neutron-star existence. Peak temperatures at the former situation are typically an order of magnitude higher than the bound of Eq. (12), $T_{\text{peak}} \simeq 20-30$ MeV. Because the prompt shock forms at a mass coordinate $\sim 0.7 M_\odot$ (i.e., away from the center [7]), the temperature profile is relatively flat near the center where the density is highest. Detailed models show that, in addition to being compressed by the core deleptonization, which causes a decrease of the ultrarelativistic particle pressure, heat diffuses inward on a time scale ~ 0.5 s [6], thus heating the central region and increasing the nucleation probability. Empirically, a lower bound for T has been established by the determination of the effective temperature of the neutrino emission in SN 1987A, namely, $T_{\text{eff}} \simeq 4$ MeV [18], suggesting that the higher physical temperatures are more than enough to satisfy the nucleation conditions derived above (which are actually quite conservative).

It should be kept in mind that the discussed scenario is not the only one which can give rise to SQM bubbles. Conversion via two-flavor quark-matter formation [4,19] or the presence of strangelets in the supernova progenitor becoming active after neutronization [20] are likely alternatives (and there may be other ones as well; see [4]). Irrespective of the *specific* mechanism for SQM appearance, we note that, for any of the mentioned possibilities, the favorable conditions are achieved on ≤ 1 s time scales and thus the existence of a mixed population of neutron and strange stars [4,21] is difficult to accommodate [22] (in other words, we believe there is no reason to expect any *delay* to the conversion events). Needless to say, a prompt conversion may be very important for the supernova outburst itself [2,3,23].

It has been recently claimed [24] that a strange ground state is ruled out by the lack of glitch mechanisms in homogeneous strange stars [25]. This problem has been previously discussed and gave rise to preliminary models of strange pulsars [26], and the corollary is that the referred results should be interpreted rather as evidence of our incomplete knowledge of "strange nuclear physics" (see also Ref. [22] for a discussion on this point).

The formation of SQM bubbles in cold NM has been previously addressed by Slominski [16], who compared the Gibbs free energies of NM containing one SQM bubble to the same NM model without any bubbles. Those results also show that SQM should form inside NM relatively easily, but far away from the star center. We believe, however, that this behavior is induced by the employed Friedmann-Pandharipande NM equation of state which produces a nonmonotonic dependence of the Gibbs-free-energy difference ΔG with the pressure (this has been already noted by the author [16]). Adopting realistic models for SQM and NM should cure this illness, which we have also encountered in the approach given in Sec. II. On the other hand, it would be interesting to extend his analysis to the case $T\neq 0$ and confirm in detail the bound given in Eq. (12), as the very existence of suitable fluctuations may depend critically on this.

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