Spontaneous breaking of flavor symmetry and parity in lattice QCD with Wilson fermions

S. Aoki
Institute of Physics, University of Tsukuba, Ibaraki 305, Japan

A. Gocksch
Physics Department, Brookhaven National Laboratory, Upton, New York 11973
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By adding a symmetry-breaking source term of the form $Hi\bar{\psi}\gamma_5\tau_3\psi$ to the Wilson fermion action for two degenerate flavors of quarks, we provide evidence for the existence of a phase at large values of the hopping parameter κ in which both parity *and* flavor symmetries are spontaneously broken and which is separate from the "high-temperature" phase. This is done by means of numerical simulations of lattice QCD employing the hybrid Monte Carlo algorithm.

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INTRODUCTION

The Wilson lattice fermion is defined through the action

$$S_f=\sum_{x,i}\bar{\psi}^i(x)\psi^i(x)+\sum_{x,i,\mu}\frac{1}{2a}[\bar{\psi}^i(x)\gamma_\mu U_\mu(x)\psi^i(x+\hat{\mu})-\bar{\psi}^i(x+\hat{\mu})\gamma_\mu U^\dagger_\mu(x)\psi^i(x)]$$
$$-\sum_{x,i,\mu}\frac{r}{2a}[\bar{\psi}^i(x)U_\mu(x)\psi^i(x+\hat{\mu})+\bar{\psi}^i(x+\hat{\mu})U^\dagger_\mu(x)\psi^i(x)-2\bar{\psi}^i(x)\psi^i(x)]\ , \qquad (1)$$

where $x_\mu=n_\mu a$, $\sum_x=a^4\sum_n$ and i is a flavor index. The last term in this equation (the "Wilson term") is of order $O(a)$ in the naive continuum limit but nevertheless it has a pronounced effect on the theory. It gives the fermionic doublers masses on the order of the momentum-space cutoff $O(r/a)$ and also serves to produce the correct anomaly of the flavor-singlet axial-vector current in the continuum limit [1]. In accordance with a well-known theorem [2] chiral symmetry, however, must be broken for all nonzero values of the lattice spacing a. In the case at hand this is self-evident since the Wilson term explicitly breaks the symmetry. Nevertheless there seems to be a rather widespread misconception that one can somehow tune the mass in Eq. (1) in such a way as to cancel the effect of the Wilson term even at *finite a*. While this might be true for certain amplitudes there will always be others in which chiral symmetry is broken by terms vanishing with the lattice spacing. To illustrate the above point consider the extreme strong-coupling limit $\beta=6/g^2=0$ of lattice QCD. This is a theory with an explicit cutoff which can be given any value (in "physical units") and has nothing to do with continuum QCD. In the limit of an infinite number of colors N_c the theory can be solved and one finds the following [3]: Whereas the pion mass can be made to vanish by tuning $M\to M_c$ the $\pi\pi$ scattering amplitude does *not* vanish at zero momentum in that limit. Hence while one hallmark of *spontaneously* broken chiral symmetry seems to emerge at strong coupling another one does not. As we explained above this is not unexpected. Note that there is no contradiction here with the perturbative analysis of Bochiccio *et al.* [4]. In the continuum limit we expect to recover spontaneously broken chiral symmetry in all its glory. At finite lattice spacing, however, this leaves us with something to be explained: Everything that might erroneously be attributed to the "spontaneous breaking" of chiral symmetry now calls out for an alternate explanation. In particular, since chiral symmetry is *explicitly* broken at finite a one must explain why the pion becomes massless at M_c without recourse to chiral symmetry. Such an explanation was given by one of us some time ago [5]. It is the purpose of the present paper to give further evidence for the correctness of the ideas put forth in Ref. [5], continuing our previous work on the same subject [6]. We will be concerned with two degenerate flavors of Wilson fermions. In order to conform with more standard notation we now set the lattice spacing and r to one, rescale the fields and introduce the hopping parameter $\kappa=1/2(m+4)\equiv 1/2M$:

$$S_f=\sum_{x,i}\bar{\psi}^i(x)\psi^i(x)-\kappa\sum_{x,i,\mu}[\bar{\psi}^i(x)(1-\gamma_\mu)U_\mu(x)\psi^i(x+\hat{\mu})+\bar{\psi}^i(x+\hat{\mu})(1+\gamma_\mu)U^\dagger_\mu(x)\psi^i(x)]\ . \qquad (2)$$

To understand the main idea of Ref. [5] it is sufficient to first consider the case of one-flavor QCD. In this case we are searching for an explanation of the fact that the pion, which is created from the vacuum by the operator $i\bar{\psi}\gamma_5\psi$, becomes massless as $\kappa\to\kappa_c$. The explanation of Ref. [5] is that κ_c is a point of a conventional second-order phase transition of the lattice model at which *parity is spontaneously broken.* The masslessness of the pion in lattice units expresses nothing but the fact that there is a diverging correlation length in the system which is associated with the order parameter of the transition, $\langle i\bar{\psi}\gamma_5\psi\rangle$. Note that the pion is *not* a Goldstone boson in this picture as the symmetry being broken is a discrete one. The relation $m_\pi^2 \sim m$ merely expresses the fact that the transition has mean-field critical exponents (up to possible logarithmic corrections). At infinite coupling the existence of the parity-breaking phase for $\kappa > \kappa_c$ can be demonstrated using the strong-coupling expansion. At intermediate coupling we gave good evidence for its existence using Monte Carlo simulations in the quenched approximation [6]. Also, studies of the eigenvalue distribution of the Wilson-Dirac operator suggest that parity is broken above κ_c in the quenched approximation [7]. Returning now to the two-flavor case at hand the situation is slightly more complicated. Here there are two possible scenarios: Either there is a phase in which $\langle i\bar{\psi}\gamma_5 1\psi\rangle \neq 0$ and $\langle i\bar{\psi}\gamma_5\tau_3\psi\rangle = 0$ or vice versa. The second case, which breaks both parity and flavor symmetries, is clearly the more attractive one since it has the potential of "solving" the lattice U(1) problem. The reason is the following: At κ_c the π^0 which is associated with the operator $i\bar{\psi}\gamma_5\tau_3\psi$ becomes massless just as in the one-flavor case only. For $\kappa \neq \kappa_c$ this mode is massive since it is associated with the diverging correlation length only at the critical point. The π^+ and π^- mesons are massless for $\kappa > \kappa_c$—they are the Goldstone modes of the spontaneously broken flavor symmetry. The flavor-singlet η meson, however, which is associated with the operator $i\bar{\psi}\gamma_5 1\psi$, always stays *massive* since according to this scenario the flavor-singlet symmetry remains unbroken. The strong-coupling expansion suggests that this is actually what happens [5]. In the present paper we would like to investigate the issue numerically at intermediate couplings.

Before we come to a discussion of our numerical techniques and the data we would like to discuss one more point which might be confusing to the reader. There are proofs by Vafa and Witten that in vectorlike theories (such as QCD) neither parity [8] nor flavor [9] are spontaneously broken. It is easy to see though that in our case one or more of the assumptions going into these proofs are violated. The idea of Ref. [8] is to study the vacuum energy in the presence of the (in Euclidean space) imaginary source term $i\lambda\mathcal{O}$ for the parity-breaking operator $\mathcal{O}$.[1] If the measure is positive then the oscillations introduced into the path integral for the vacuum energy by the source term can only increase the energy and hence the vacuum expectation value of $\mathcal{O}$ must vanish. In the

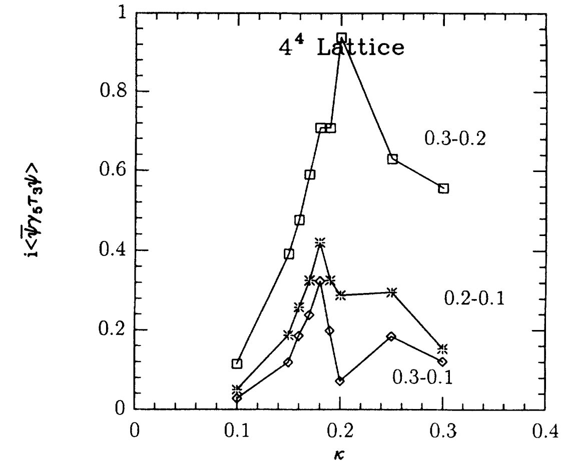

FIG. 1. The expectation value of $i\bar{\psi}\gamma_5\tau_3\psi$ on a 4^4 lattice. The symbols used in all subsequent figures are shown here and correspond to different magnetic fields used in the extrapolation (see text).

quenched approximation one cannot define a vacuum energy so the proof does not apply. In the case of one flavor it is easy to see that for $\kappa > \kappa_c$ the fermionic determinant and hence the measure can become negative. For two flavors adding the operator $\mathcal{O} = i\bar{\psi}\gamma_5\tau_3\psi$ leads to a real integrand in the partition function and hence the proof of Ref. [8] does not apply. The proof of no violation of flavor in vectorlike theories does not go through for Wilson fermions because of the modification of the Dirac operator by the Wilson term. We now turn to a discussion of our numerical calculation.

SIMULATION

The strong-coupling analysis of Ref. [5] shows that it is the operator $i\bar{\psi}\gamma_5\tau_3\psi$ which develops an expectation value for κ greater than some critical value κ_c. To see if this result also holds at some intermediate value of the

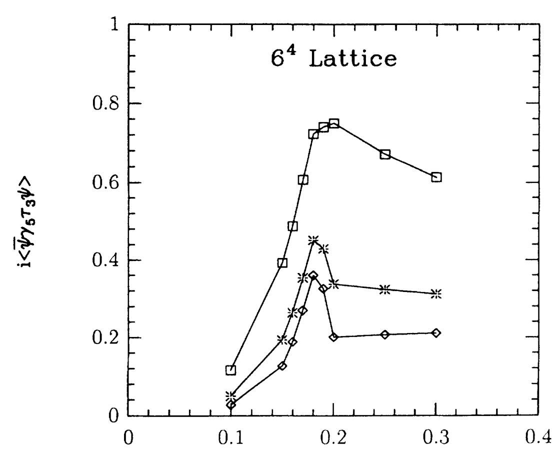

FIG. 2. The expectation value of $i\bar{\psi}\gamma_5\tau_3\psi$ on a 6^4 lattice.

[1]A note on conventions for Hermitian conjugation in Euclidean space at this point: The operator $i\bar{\psi}\gamma_5\psi$ is Hermitian in Minkowski space. The (Hermitian) γ_5 picks up an i under Wick rotation. Nevertheless it is the operator $i\bar{\psi}\gamma_5\psi$ with $\gamma_5 = -\gamma_1\gamma_2\gamma_3\gamma_4$ with Hermitian (Euclidean) γ_μ which is real in Euclidean space [10].

TABLE I. Compilation of our simulation parameters. The meaning of the parameters is explained in the text.

κ	H	ϵ	n_{MD}	V	n_{conf}	Acc
0.1	0.1	0.08	10	4^4	8	0.72
0.1	0.1	0.06	13	6^4	8	0.66
0.1	0.2	0.08	10	4^4	10	0.68
0.1	0.2	0.08	8	$4^2\times 6^2$	8	0.62
0.1	0.2	0.06	10	6^4	8	0.65
0.1	0.3	0.08	10	4^4	8	0.72
0.1	0.3	0.08	10	$4^2\times 6^2$	8	0.61
0.1	0.3	0.07	9	6^4	8	0.52
0.15	0.1	0.08	10	4^4	8	0.68
0.15	0.1	0.05	16	6^4	8	0.70
0.15	0.2	0.08	10	4^4	8	0.66
0.15	0.2	0.06	11	$4^2\times 6^2$	8	0.69
0.15	0.2	0.06	11	6^4	8	0.60
0.15	0.3	0.08	10	4^4	8	0.71
0.15	0.3	0.07	10	$4^2\times 6^2$	8	0.62
0.15	0.3	0.06	10	6^4	8	0.58
0.16	0.1	0.08	10	4^4	8	0.64
0.16	0.1	0.05	15	6^4	8	0.64
0.16	0.2	0.08	10	4^4	8	0.66
0.16	0.2	0.06	11	$4^2\times 6^2$	8	0.71
0.16	0.2	0.05	12	6^4	8	0.68
0.16	0.3	0.08	10	4^4	8	0.68
0.16	0.3	0.07	10	$4^2\times 6^2$	8	0.63
0.16	0.3	0.06	14	6^4	8	0.59
0.17	0.1	0.08	10	4^4	8	0.62
0.17	0.1	0.05	14	6^4	8	0.64
0.17	0.2	0.08	10	4^4	8	0.62
0.17	0.2	0.06	11	$4^2\times 6^2$	8	0.65
0.17	0.2	0.05	13	6^4	8	0.69
0.17	0.3	0.08	10	4^4	8	0.69
0.17	0.3	0.07	9	$4^2\times 6^2$	8	0.62
0.17	0.3	0.06	13	6^4	8	0.59
0.18	0.1	0.07	8	4^4	8	0.60
0.18	0.1	0.05	15	6^4	8	0.59
0.18	0.2	0.07	9	4^4	12	0.63
0.18	0.2	0.06	11	$4^2\times 6^2$	8	0.63
0.18	0.2	0.05	14	6^4	8	0.68
0.18	0.3	0.08	9	4^4	8	0.66
0.18	0.3	0.07	9	$4^2\times 6^2$	8	0.55
0.18	0.3	0.06	13	6^4	8	0.57
0.19	0.1	0.06	12	4^4	8	0.68
0.19	0.1	0.05	14	6^4	8	0.50
0.19	0.2	0.07	9	4^4	8	0.63
0.19	0.2	0.06	11	$4^2\times 6^2$	8	0.62
0.19	0.2	0.05	12	6^4	8	0.59
0.19	0.3	0.07	9	4^4	8	0.69
0.19	0.3	0.06	11	$4^2\times 6^2$	8	0.67
0.19	0.3	0.05	14	6^4	8	0.69
0.20	0.1	0.06	12	4^4	8	0.66
0.20	0.1	0.04	15	6^4	8	0.62
0.20	0.2	0.07	9	4^4	8	0.65
0.20	0.2	0.05	13	$4^2\times 6^2$	8	0.68
0.20	0.2	0.05	13	6^4	8	0.61
0.20	0.3	0.07	9	4^4	8	0.63
0.20	0.3	0.06	11	$4^2\times 6^2$	8	0.72
0.20	0.3	0.05	12	6^4	8	0.63

TABLE I. (*Continued*).

κ	H	ϵ	$n_{\rm MD}$	V	$n_{\rm conf}$	Acc
0.25	0.1	0.06	12	4^4	8	0.59
0.25	0.1	0.04	16	6^4	8	0.61
0.25	0.2	0.06	12	4^4	8	0.65
0.25	0.2	0.05	13	$4^2\times 6^2$	12	0.65
0.25	0.2	0.05	13	6^4	8	0.51
0.25	0.3	0.07	9	4^4	12	0.64
0.25	0.3	0.06	11	$4^2\times 6^2$	8	0.61
0.25	0.3	0.05	12	6^4	8	0.59
0.30	0.1	0.06	12	4^4	8	0.53
0.30	0.1	0.04	15	6^4	8	0.55
0.30	0.2	0.06	12	4^4	12	0.60
0.30	0.2	0.05	13	$4^2\times 6^2$	8	0.58
0.30	0.2	0.04	15	6^4	8	0.61
0.30	0.3	0.07	10	4^4	12	0.56
0.30	0.3	0.06	11	$4^2\times 6^2$	8	0.54
0.30	0.3	0.04	15	6^4	8	0.62

coupling we have performed hybrid Monte Carlo simulations [11,12] on small lattices at $\beta=5.0$ with two flavors of dynamical Wilson fermions. The essential trick in our simulation is to add, just as we did earlier in our quenched simulations [6], a symmetry-breaking term $Hi\bar{\psi}\gamma_5\tau_3\psi$ to S_f. Note that it is easy to show [6] that in the absence of the magnetic field H, $\langle i\bar{\psi}\gamma_5\tau_3\psi\rangle=0$ even without averaging over the gauge fields. Hence in order to see whether or not symmetry breaking occurs we must add H and then extrapolate to zero field. In practice, depending on the volume there will be a minimum value of H from which meaningful results can be obtained. Note

TABLE II. $\langle i\bar{\psi}\gamma_5\tau_3\psi\rangle$ for three values of the external field.

κ	V	$H=0.3$	$H=0.2$	$H=0.1$
0.10	4^4	0.7485(2)	0.5374(2)	0.2823(1)
0.15	4^4	1.2923(23)	0.9921(18)	0.5555(14)
0.16	4^4	1.4181(31)	1.1043(27)	0.6447(20)
0.17	4^4	1.5445(29)	1.2262(70)	0.7318(35)
0.18	4^4	1.6602(77)	1.3430(60)	0.8333(101)
0.19	4^4	1.7547(74)	1.4059(121)	0.8021(93)
0.20	4^4	1.8416(86)	1.5403(166)	0.8059(81)
0.25	4^4	2.0516(210)	1.5776(272)	0.8808(396)
0.30	4^4	2.1030(260)	1.5873(145)	0.8541(144)
0.10	6^4	0.7483(0.2)	0.5373(0.1)	0.2822(0.3)
0.15	6^4	1.2914(12)	0.9917(5)	0.5590(3)
0.16	6^4	1.4133(19)	1.1048(11)	0.6466(9)
0.17	6^4	1.5372(24)	1.2272(31)	0.7482(21)
0.18	6^4	1.6538(33)	1.3434(36)	0.8517(42)
0.19	6^4	1.7737(64)	1.4290(70)	0.8775(89)
0.20	6^4	1.8704(68)	1.4967(89)	0.08486(118)
0.25	6^4	2.0613(173)	1.5979(69)	0.9024(82)
0.30	6^4	2.1505(232)	1.6382(275)	0.9245(100)
0.10	$4^2\times 6^2$	0.7484(2)	0.5374(2)	
0.15	$4^2\times 6^2$	1.2939(16)	0.9905(16)	
0.16	$4^2\times 6^2$	1.4139(16)	1.1064(16)	
0.17	$4^2\times 6^2$	1.5400(26)	1.2259(25)	
0.18	$4^2\times 6^2$	1.6464(73)	1.3436(74)	
0.19	$4^2\times 6^2$	1.7696(101)	1.4373(116)	
0.20	$4^2\times 6^2$	1.8627(85)	1.4955(127)	
0.25	$4^2\times 6^2$	2.0458(238)	1.5776(100)	
0.30	$4^2\times 6^2$	2.1222(153)	1.6152(129)	

that if we write the action for one flavor $S_f = \bar{\psi} D(H)\psi$ integrating out the fermions leads to the determinant $\det D(H) \det D(-H)$ which is *real* because

$$D(H)^\dagger = \gamma_5 D(-H)\gamma_5 \ . \tag{3}$$

The simulations were done on lattices of size 4^4, $4^2\times 6^2$, and 6^4 for values of κ ranging between 0.1 and 0.3. The magnetic field H took on the values 0.1, 0.2, and 0.3. In Table I we summarize our simulation parameters. In the tables ϵ denotes the step size, $n_{\rm MD}$ the number of molecular-dynamics steps used before refreshing the momenta and $n_{\rm conf}$ is the number of configurations used for averaging. The Monte Carlo acceptance rate is denoted by "Acc" in the tables. We allowed 400 trajectories for thermalization and separated configurations used for measurements by 50 trajectories. The stopping condition on the conjugate-gradient inversion was $r = 4\times 10^{-5}$ in the update and $r = 10^{-5}$ in the fermionic measurement. For the measurement of the fermionic observations we used two source points and averaged over all color and spinor components.

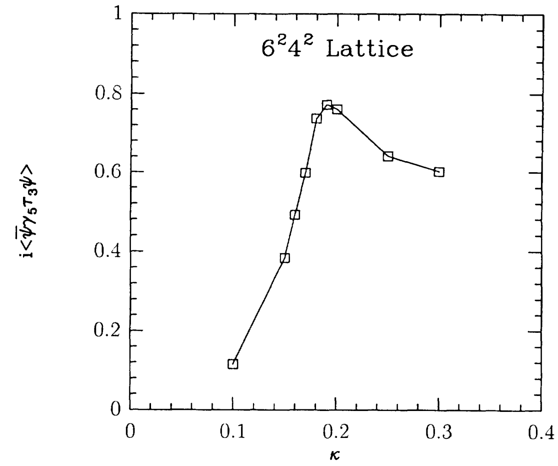

FIG. 3. The expectation value of $i\bar{\psi}\gamma_5\tau_3\psi$ on a $6^2\times 4^2$ lattice.

DATA

Apart from $\langle i\bar{\psi}\gamma_5\tau_3\psi\rangle$ we also monitored $\langle\bar{\psi}\psi\rangle$, the Wilson line $\mathrm{Re}\langle L\rangle$, and the average plaquette $\langle S_g\rangle$. The gluonic operators were measured in order to be able to differentiate between the flavor- and parity-breaking transition and the finite-temperature transition on our small lattice. The fact that it is difficult to approach κ_c in the "confining" phase was first pointed out by Fukugita, Ohta, and Ukawa [13]. Before κ_c is reached from below the system switches into the "would be" chirally symmetric phase [14]. It now appears that for lattices of size 12^4 at $\beta = 5.4$ the two transitions approach each other

TABLE III. $\langle\bar{\psi}\psi\rangle$ for three values of the external field.

κ	V	$H=0.3$	$H=0.2$	$H=0.1$
0.10	4^4	2.1266(5)	2.2614(6)	2.3554(7)
0.15	4^4	2.9323(52)	3.1890(61)	3.3840(55)
0.16	4^4	3.0327(62)	3.3040(83)	3.5553(89)
0.17	4^4	3.1148(48)	3.3893(139)	3.6440(95)
0.18	4^4	3.1727(103)	3.4633(117)	3.6890(258)
0.19	4^4	3.1966(111)	3.4269(150)	3.5782(135)
0.20	4^4	3.1827(74)	3.4895(189)	3.5478(187)
0.25	4^4	3.0658(112)	3.2280(130)	3.3723(212)
0.30	4^4	2.8464(140)	2.9503(135)	3.0361(193)
0.10	6^4	2.1263(3)	2.2610(5)	2.3550(4)
0.15	6^4	2.9232(29)	3.1889(27)	3.3935(49)
0.16	6^4	3.0259(50)	3.2960(53)	3.5421(96)
0.17	6^4	3.1043(51)	3.3876(89)	3.6670(93)
0.18	6^4	3.1631(49)	3.4512(88)	3.6874(145)
0.19	6^4	3.2073(104)	3.4304(141)	3.6334(166)
0.20	6^4	3.2186(113)	3.4254(102)	3.5430(122)
0.25	6^4	3.0358(113)	3.2226(74)	3.3560(172)
0.30	6^4	2.8227(154)	2.9482(198)	3.0476(150)
0.10	$4^2\times 6^2$	2.1261(4)	2.2618(6)	
0.15	$4^2\times 6^2$	2.9316(39)	3.1842(51)	
0.16	$4^2\times 6^2$	3.0325(40)	3.3030(69)	
0.17	$4^2\times 6^2$	3.1129(61)	3.3944(93)	
0.18	$4^2\times 6^2$	3.1603(92)	3.4582(106)	
0.19	$4^2\times 6^2$	3.2100(133)	3.4720(132)	
0.20	$4^2\times 6^2$	3.2300(95)	3.4439(127)	
0.25	$4^2\times 6^2$	3.0524(177)	3.2449(135)	
0.30	$4^2\times 6^2$	2.8463(108)	2.9954(240)	

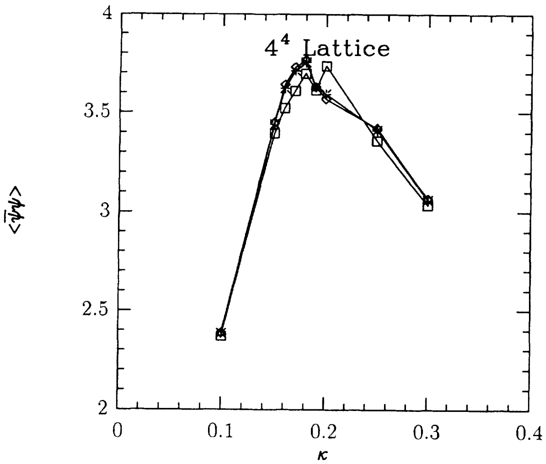

FIG. 4. The expectation value of $\bar{\psi}\psi$ on a 4^4 lattice.

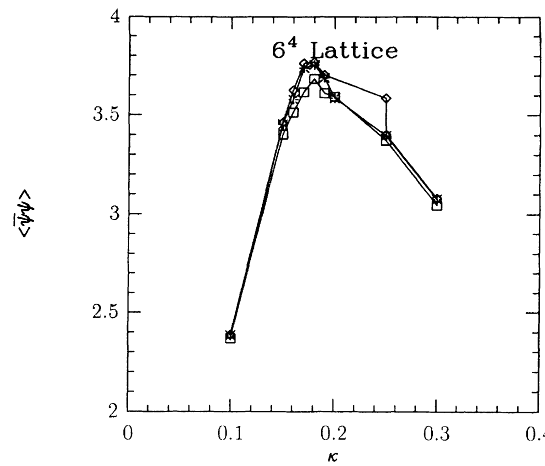

FIG. 5. The expectation value of $\bar{\psi}\psi$ on a 6^4 lattice.

[15]. Hence on somewhat bigger lattices one hopefully will be able to approach κ_c before the system changes over into the deconfined phase. Our lattice size and corresponding value of β is of course very different from the numbers mentioned above. Our raw (not extrapolated) data can be found in Tables II–V. We give all the data so that the interested reader can see for himself whether our extrapolations, to be shown in the figures later on, are meaningful. The error quoted in the tables is always the "naive" error—the standard deviation of the mean.

As was mentioned above, $\langle i\bar{\psi}\gamma_5\tau_3\psi\rangle$ vanishes in the limit $H\rightarrow 0$. Hence in order to extract an estimate of the infinite value of this quantity we must extrapolate. Ideally this should be done for various values of the volume. Depending on the volume there will be a minimum value of the field that can be used. Because of our limited resources we could only study three different lattice sizes. Going from the 4^4 to the $6^2\times 4^2$ lattice increases the spa-

TABLE IV. The average plaquette $\langle S_g\rangle$ for three values of the external field.

κ	V	$H=0.3$	$H=0.2$	$H=0.1$
0.10	4^4	0.3996(1)	0.4026(9)	0.4005(14)
0.15	4^4	0.4101(16)	0.4116(24)	0.4137(12)
0.16	4^4	0.4178(15)	0.4192(22)	0.4209(14)
0.17	4^4	0.4245(15)	0.4334(18)	0.4392(15)
0.18	4^4	0.4333(25)	0.4433(19)	0.4592(40)
0.19	4^4	0.4470(32)	0.4619(34)	0.5239(23)
0.20	4^4	0.4553(26)	0.4780(55)	0.5429(12)
0.25	4^4	0.5363(23)	0.5612(8)	0.5779(8)
0.30	4^4	0.5668(6)	0.5826(6)	0.5959(9)
0.10	6^4	0.4035(6)	0.4041(6)	0.4038(7)
0.15	6^4	0.4138(10)	0.4167(8)	0.4176(7)
0.16	6^4	0.4205(9)	0.4238(5)	0.4249(8)
0.17	6^4	0.4243(7)	0.4319(10)	0.4380(7)
0.18	6^4	0.4345(8)	0.4453(12)	0.4567(10)
0.19	6^4	0.4414(6)	0.4610(13)	0.4979(10)
0.20	6^4	0.4527(8)	0.4806(18)	0.5288(8)
0.25	6^4	0.5228(13)	0.5570(5)	0.5747(5)
0.30	6^4	0.5616(5)	0.5805(2)	0.5950(3)
0.10	$4^2\times 6^2$	0.4015(5)	0.4022(12)	
0.15	$4^2\times 6^2$	0.4124(8)	0.4136(8)	
0.16	$4^2\times 6^2$	0.4182(12)	0.4210(12)	
0.17	$4^2\times 6^2$	0.4244(18)	0.4265(16)	
0.18	$4^2\times 6^2$	0.4314(17)	0.4397(14)	
0.19	$4^2\times 6^2$	0.4384(12)	0.4532(15)	
0.20	$4^2\times 6^2$	0.4513(16)	0.4785(26)	
0.25	$4^2\times 6^2$	0.5354(9)	0.5588(7)	
0.30	$4^2\times 6^2$	0.5624(9)	0.5805(7)	

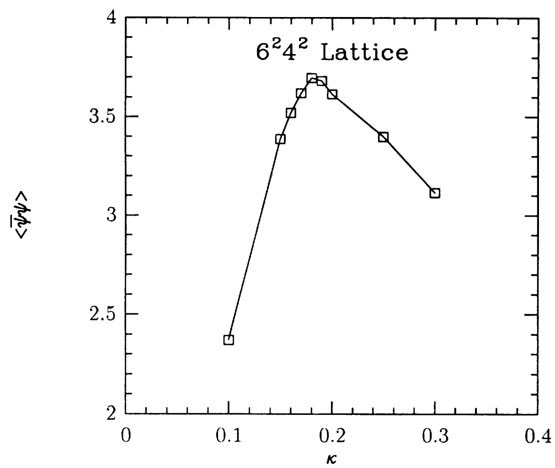

FIG. 6. The expectation value of $\bar{\psi}\psi$ on a $6^2\times 4^2$ lattice.

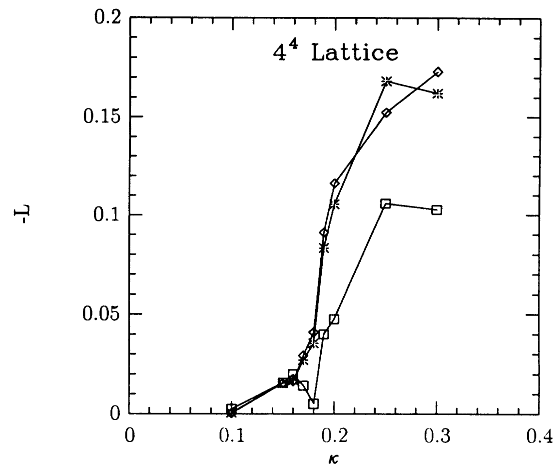

FIG. 7. The expectation value of (minus) the Wilson line on a 4^4 lattice.

tial volume by a factor of 2.25 while keeping n_t, the number of time slices, constant. On the other hand, our biggest, the 6^4 lattice, also increases n_t, effectively decreasing the temperature. Rather than giving tables we show our extrapolated results in the figures. In the case of $\langle i\bar{\psi}\gamma_5\tau_3\psi\rangle$ we used a linear extrapolation; in all other observables a quadratic extrapolation was employed. Actually there is not much of a difference between linear and quadratic extrapolation here. Nevertheless our choice is based on theoretical prejudice coming from the strong coupling expansion [6]. In the figures the symbols used

TABLE V. The Wilson line $\langle L\rangle$ for three values of the external field.

κ	V	$H=0.3$	$H=0.2$	$H=0.1$
0.10	4^4	−0.0029(24)	−0.0026(17)	−0.0008(35)
0.15	4^4	−0.0090(23)	−0.0127(38)	−0.0147(33)
0.16	4^4	−0.0066(31)	−0.0140(29)	−0.0157(24)
0.17	4^4	−0.0112(21)	−0.0129(31)	−0.0252(33)
0.18	4^4	−0.0190(37)	−0.0113(37)	−0.0336(60)
0.19	4^4	−0.0197(54)	−0.0310(57)	−0.0761(63)
0.20	4^4	−0.0164(41)	−0.0337(65)	−0.0955(65)
0.25	4^4	−0.0779(37)	−0.0934(108)	−0.1375(62)
0.30	4^4	−0.1141(52)	−0.1077(71)	−0.1566(51)
0.10	6^4	−0.0009(17)	−0.0015(17)	+0.0000(17)
0.15	6^4	+0.0002(10)	+0.0013(10)	−0.0011(11)
0.16	6^4	−0.0025(13)	−0.0019(13)	−0.0015(17)
0.17	6^4	+0.0018(7)	−0.0018(7)	−0.0028(18)
0.18	6^4	−0.0014(10)	−0.0032(10)	−0.0024(13)
0.19	6^4	−0.0004(16)	−0.0013(16)	−0.0020(14)
0.20	6^4	−0.0030(19)	−0.0011(19)	−0.0175(19)
0.25	6^4	−0.0028(45)	−0.0188(45)	−0.0476(45)
0.30	6^4	+0.0715(35)	−0.0663(35)	−0.0484(55)
0.10	$4^2\times 6^2$	−0.0037(23)	+0.0022(27)	
0.15	$4^2\times 6^2$	−0.0068(19)	−0.0113(23)	
0.16	$4^2\times 6^2$	−0.0113(23)	−0.0122(20)	
0.17	$4^2\times 6^2$	−0.0074(20)	−0.0122(21)	
0.18	$4^2\times 6^2$	−0.0105(28)	−0.0170(33)	
0.19	$4^2\times 6^2$	−0.0141(21)	−0.0235(29)	
0.20	$4^2\times 6^2$	−0.0167(22)	−0.0307(31)	
0.25	$4^2\times 6^2$	−0.1004(34)	−0.1207(39)	
0.30	$4^2\times 6^2$	−0.1224(47)	−0.1251(82)	

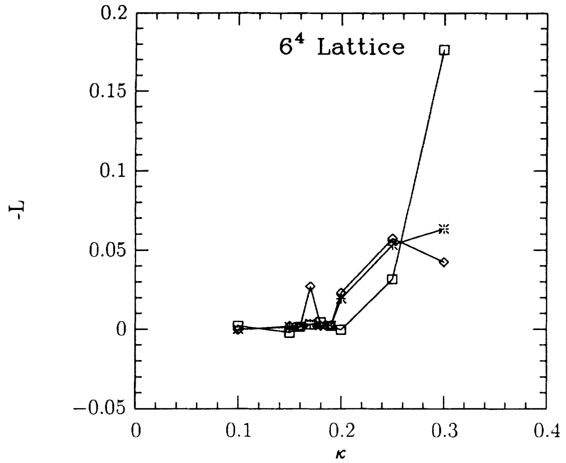

FIG. 8. The expectation value of (minus) the Wilson line on a 6^4 lattice.

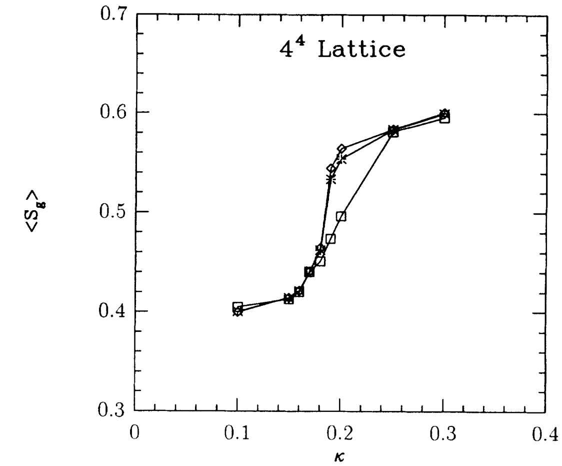

FIG. 10. The expectation value of the plaquette on a 4^4 lattice.

always denote the same mode of extrapolation: All three values of the field included (◇), $H=0.2$ and $H=0.1$ only (∗) and $H=0.3$ and $H=0.2$ only (□).

DISCUSSION

Looking first at Fig. 1 we clearly see the rapid rise of $\langle i\bar{\psi}\gamma_5\tau_3\psi\rangle$. If the scenario advocated in the Introduction were correct, then this quantity would be the order parameter of a second-order phase transition in the infinite-volume limit. As such we expect it to have a characteristic behavior near the transition point parametrized by a critical exponent β. At strong coupling one finds [5] the mean-field behavior $\langle i\bar{\psi}\gamma_5\tau_3\psi\rangle \sim (\kappa-\kappa_c)^{1/2}$. At large values of κ we observe a drop in the order parameter which is probably due to the finite-size (or -temperature) effect of our lattice. This is supported by the fact that on the bigger lattice (Fig. 2) the drop is much less pronounced. We believe that the data in Figs. 1–3 support the picture advocated in the Introduction. We cannot, however, make a definite claim based on our data because of the obvious extrapolation dependence of the result. In order to make a definite conclusion, we will have to do a finite size scaling study of this order parameter, which is planned to be done in the near future. It is probably true that $H=0.1$ is already too small a field on our lattices. This is consistent with the fact that increasing the volume seems to improve the situation somewhat. In any case, no matter how the extrapolation is performed we obtain a finite value for the order parameter. Note that in Ref. [15] no evidence for parity breaking was claimed in the case of one Wilson-flavor QCD. We would like to point out here that their claim was based on the quantity $\langle \bar{\psi}\gamma_5\psi\rangle$ which is purely imaginary (see the footnote).

In Figs. 4–6 we show the behavior of $\langle \bar{\psi}\psi\rangle$. The peak in this quantity is actually anticipated [5,6] and we use it to determine an approximate value for κ_c. A rough estimate is $\kappa_c \sim 0.180(5)$. Ukawa [16] finds $\kappa_c=0.187(1)$. Note that the value of κ_c determined by $\langle \bar{\psi}\psi\rangle$ is almost

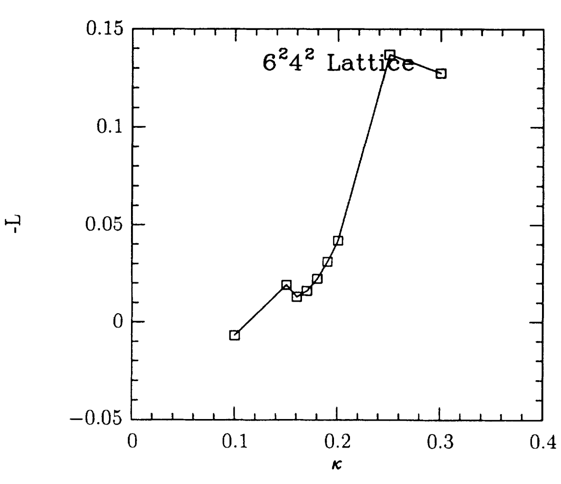

FIG. 9. The expectation value of (minus) the Wilson line on a $6^2\times 4^2$ lattice.

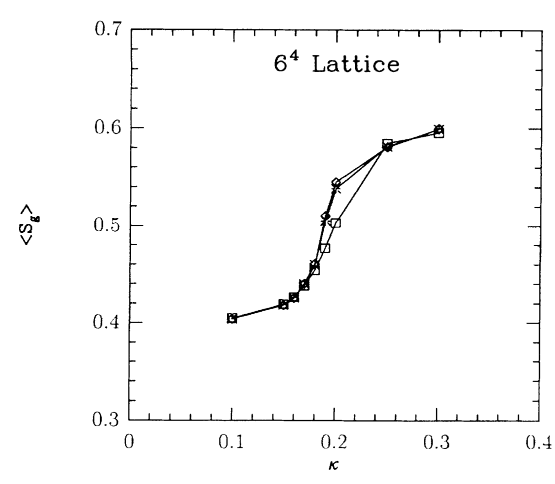

FIG. 11. The expectation value of the plaquette on a 6^4 lattice.

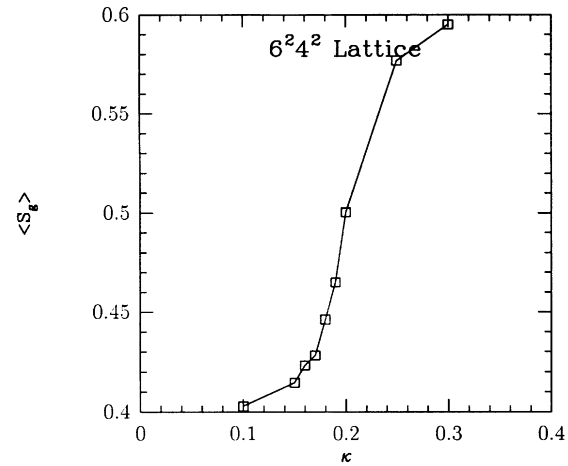

FIG. 12. The expectation value of the plaquette on a $6^2 \times 4^2$ lattice.

independent on the lattice size.

Judging from Figs. 7–9 there is clear evidence for a finite-temperature transition on our lattices. On the 4^4 lattice the "critical" value $\kappa_T \sim 0.19$, which is close to but larger than $\kappa_c \sim 0.18$, although it is of course clear that it is extremely ambiguous to try to read off a value from a plot such as Fig. 7. On the $n_t = 6$ lattice (Fig. 8) the transition shifts to a higher value, $\kappa_T \geq 0.25$, so it seems that the flavor and parity breaking occurs at smaller value of κ than that of the finite-temperature transition on a large enough lattice. The average plaquette shown in Figs. 10–12 follows the Wilson line quite closely. Again $\kappa_T \sim 0.19$ on the 4^4 lattice and $\kappa_T \geq 0.25$ on the 6^4 lattice appear likely.

To summarize, we have studied two-flavor QCD with Wilson fermions and have provided evidence that there is a phase of the theory which breaks both flavor and parity symmetries and that this phase transition can be separated from the finite-size (or -temperature) transition. Clearly, only time will tell if this result survives further scrutiny. Clearly simulations on larger lattices with better statistics are called for.

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