Fermionic vortex solutions in Chern-Simons electrodynamics

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We present anyonic vortex solutions made of real electrons which could be interpreted as nontopological solitons of the $(2+1)$-dimensional Chern-Simons electrodynamics. The n-soliton solutions, which we obtain by imposing an effective axial symmetry on the $(3+1)$-dimensional quantum electrodynamics, have $4n$ real parameters which represent the position, size, and phase of each soliton.

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The axially symmetric vortex solutions which exist in $(3+1)$-dimensional gauge theories have played a very important role in physics. They describe the quantized magnetic flux lines in superconductivity [1], the string model in hadrodynamics [2], and the large-scale cosmic strings in cosmology [3]. Similar vortex solutions also exist in the $(2+1)$-dimensional Chern-Simons gauge theory [4]. In all these solutions, however, the presence of scalar fields has played a crucial role in providing the source of the vortices. So far no vortex solution made of a fermionic source has been constructed, although the fermionic bound states coupled to an arbitrary external magnetic vortex have been discussed by many authors [5]. The purpose of this paper is to show the existence of axially symmetric vortex solutions in which a fermion field provides the source of the vortices, and to discuss the physical implication of the solutions.

The system we discuss is the one derived from the $(3+1)$-dimensional quantum electrodynamics which has an effective axial symmetry described in the following. With the symmetry one can reduce the theory to the $(2+1)$-dimensional Maxwell electrodynamics which has two interacting fermionic sources: the right-handed and the left-handed fermions. Furthermore, when the effective axial symmetry is chosen in such a way to violate parity, one may add the Chern-Simons interaction to the theory. This is because the Chern-Simons interaction could be induced by the higher-order quantum correction of the fermions when a parity-violating interaction is present [6]. In this case the theory becomes Maxwell-Chern-Simons electrodynamics, but again with two fermionic sources. This means that, in the long-distance limit in which the Chern-Simons term dominates the Maxwell term, the theory can be approximated to an effective Chern-Simons electrodynamics. In this limit we show that the theory admits vortex solutions made of fermions.

Let us start with $(3+1)$-dimensional quantum electrodynamics. In the chiral representation in which γ_5 becomes diagonal one can describe the Dirac spinor Ψ with two two-components spinors

$$\Psi=\begin{pmatrix}\Psi_+\\ \Psi_-\end{pmatrix}, \tag{1}$$

where Ψ_+ and Ψ_- are the right-handed and the left-handed Weyl spinors. Now we impose the effective axial symmetry and assume that the Weyl spinors $\Psi_\pm$ are periodic in the z coordinate (with different periodicities),

$$\Psi_\pm=e^{ip_\pm z}\psi_\pm(t,x,y)\ , \tag{2}$$

but the gauge potential is independent of the z coordinate. With this effective axial symmetry one can easily reduce the theory to $(2+1)$-dimensional electrodynamics. After the dimensional reduction by integrating out the z dependence, we obtain the $(2+1)$-dimensional effective Lagrangian

$$\begin{aligned}\mathcal{L}&=-\tfrac{1}{4}F^{\alpha\beta}F_{\alpha\beta}+\psi_+^\dagger i(D_0+\sigma^1D_1+\sigma^2D_2)\psi_++\psi_-^\dagger i(D_0-\sigma^1D_1-\sigma^2D_2)\psi_-\\&\quad-p_+\psi_+^\dagger\sigma^3\psi_++p_-\psi_-^\dagger\sigma^3\psi_--m(\psi_+^\dagger\psi_-+\psi_-^\dagger\psi_+)\\&=-\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}+\bar\psi_+i\gamma_+^\alpha D_\alpha\psi_++\bar\psi_-i\gamma_-^\alpha D_\alpha\psi_--p_+\bar\psi_+\psi_+-p_-\bar\psi_-\psi_--m(\psi_+^\dagger\psi_-+\psi_-^\dagger\psi_+)\ ,\end{aligned} \tag{3}$$

where σ^i $(i=1,2,3)$ are the Pauli matrices, γ_+^α and γ_-^α $(\alpha=0,1,2)$ are two sets of $(2+1)$-dimensional γ matrices given by

$$\gamma_+^\alpha=(\sigma^3,i\sigma^2,-i\sigma^1),\quad \gamma_-^\alpha=(-\sigma^3,i\sigma^2,-i\sigma^1)\ ,$$

and m is the mass of the $(3+1)$-dimensional electron. Notice that here we have neglected the z component of the gauge potential, which is irrelevant for our purpose. At the same time we have kept the $(3+1)$-dimensional mass term for the generality, which must disappear when $p_+\neq p_-$. But of course here the momenta $p_\pm$, not m, play the role of the $(2+1)$-dimensional mass of the fermions.

Now, under the parity we have

$$(t,x,y)\to(t,-x,y)\ ,$$
$$\psi_\pm(t,x,y)\to\sigma^1\psi_\mp(t,-x,y)\ ,$$

so that after dimensional reduction ψ_+ and ψ_- become the parity partner of the other. But notice that the Lagrangian (3) becomes invariant under parity only when $p_+=p_-$. So one can legitimately add the parity-violating Chern-Simons interaction when $p_+\neq p_-$, which can be induced through quantum correction [6,7]. With the Chern-Simons interaction we obtain

$$\begin{aligned}\mathcal{L}=&-\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}+\frac{\mu}{4}\epsilon^{\alpha\beta\gamma}A_\alpha F_{\beta\gamma}\\&+\bar\psi_+ i\gamma_+^\alpha D_\alpha\psi_+ +\bar\psi_- i\gamma_-^\alpha D_\alpha\psi_-\\&-p_+\bar\psi_+\psi_+ -p_-\bar\psi_-\psi_- -m(\psi_+^\dagger\psi_- +\psi_-^\dagger\psi_+)\ ,\end{aligned}\tag{4}$$

where μ is the Chern-Simons coupling constant. From this we conclude that in the long-distance limit the theory reduces to Chern-Simons electrodynamics, but with two fermionic sources interacting with each other through the $(3+1)$-dimensional mass term.

In the absence of the Maxwell term the Lagrangian (4) gives the equations of motion

$$\begin{aligned}&\frac{\mu}{2}\epsilon^{\alpha\beta\gamma}F_{\beta\gamma}=e(\bar\psi_+\gamma_+^\alpha\psi_+ +\bar\psi_-\gamma_-^\alpha\psi_-)\ ,\\&(i\gamma_+^\alpha D_\alpha-p_+)\psi_+=m\sigma^3\psi_-\ ,\\&(i\gamma_-^\alpha D_\alpha-p_-)\psi_-=-m\sigma^3\psi_+\ .\end{aligned}\tag{5}$$

To obtain the desired solutions we choose the polar coordinates (t,ρ,φ) and the ansatz

$$\begin{aligned}&A_\alpha=\begin{cases}0, & \alpha=t,\rho,\\ A(\rho), & \alpha=\varphi,\end{cases}\\&\psi_+=e^{-iE_+t}\begin{pmatrix}f_+(\rho)e^{ik_+\varphi}\\ ig_+(\rho)e^{il_+\varphi}\end{pmatrix}\ ,\\&\psi_-=e^{-iE_-t}\begin{pmatrix}f_-(\rho)e^{ik_-\varphi}\\ ig_-(\rho)e^{il_-\varphi}\end{pmatrix}\ ,\end{aligned}\tag{6}$$

where $k_\pm$ and $l_\pm$ are integers. Now, when $m\neq 0$, Eq. (5) is reduced to

$$\begin{aligned}&k_+=l_+-1=k_-=l_--1\ ,\\&E_+=E_-=E,\quad p_+=p_-=p\ ,\\&-\frac{\mu}{\rho}\frac{dA}{d\rho}=e(|f_+|^2+|g_+|^2+|f_-|^2+|g_-|^2)\ ,\\&0=f_+g_+^*-f_-g_-^*\ ,\\&\frac{df_+}{d\rho}-\frac{k_++eA}{\rho}f_++(E+p)g_+=mg_-\ ,\\&\frac{dg_+}{d\rho}+\frac{l_++eA}{\rho}g_+-(E-p)f_+=-mf_-\ ,\\&\frac{df_-}{d\rho}-\frac{k_-+eA}{\rho}f_--(E-p)g_-=-mg_+\ ,\\&\frac{dg_-}{d\rho}+\frac{l_-+eA}{\rho}g_-+(E+p)f_-=mf_+\ .\end{aligned}\tag{7}$$

So with

$$E^2-p^2=m^2\tag{8}$$

we have the following solutions. When $\mu>0$ we have

$$\begin{aligned}&A=\frac{2(n+1)}{e}\frac{\rho^{2(eA_0+n+1)}}{\rho^{2(eA_0+n+1)}+\lambda^2}-A_0\frac{\rho^{2(eA_0+n+1)}-\lambda^2}{\rho^{2(eA_0+n+1)}+\lambda^2}\ ,\\&f_+=\frac{(eA_0+n+1)\lambda}{e}\left[2\mu\frac{E+p}{E}\right]^{1/2}\frac{\rho^{eA_0+n}}{\rho^{2(eA_0+n+1)}+\lambda^2}\ ,\\&f_-=\frac{E-p}{m}f_+\ ,\\&g_+=g_-=0\ ,\\&eA_0+n+1>0\ ,\end{aligned}\tag{9}$$

where $A_0=A(0)$, λ are the integration constants, and $n=k_+$. When $\mu<0$ we have

$$\begin{aligned}&A=\frac{2(n+1)}{e}\frac{\rho^{2(-eA_0+n+1)}}{\rho^{2(-eA_0+n+1)}+\lambda^2}\\&\qquad-A_0\frac{\rho^{2(-eA_0+n+1)}-\lambda^2}{\rho^{2(-eA_0+n+1)}+\lambda^2}\ ,\\&g_-=\frac{(-eA_0+n+1)\lambda}{e}\left[-2\mu\frac{E+p}{E}\right]^{1/2}\\&\qquad\times\frac{\rho^{-eA_0+n}}{\rho^{2(-eA_0+n+1)}+\lambda^2}\ ,\\&g_+=\frac{E-p}{m}g_-\ ,\\&f_+=f_-=0\ ,\\&-eA_0+n+1>0\ ,\end{aligned}\tag{10}$$

when $n=-l_+$. Notice that the solutions (9) and (10) remain valid even when m vanishes.

We have pointed out that the Chern-Simons interaction can be induced through quantum correction only when the axial symmetry (2) violates parity. So one may ask whether Chern-Simons electrodynamics admits any solution when $p_+\neq p_-$. To answer this notice that when $p_+\neq p_-$ the $(3+1)$-dimensional mass term in (3) must disappear, after one integrates out the z dependence and makes the dimensional reduction. Now, with $m=0$, Eq. (5) with (6) is reduced to

$$
\begin{aligned}
&k_+=l_+-1, \quad k_-=l_--1 ,\\
&-\frac{\mu}{\rho}\frac{dA}{d\rho}=e(|f_+|^2+|g_+|^2+|f_-|^2+|g_-|^2) ,\\
&0=f_+g_+^*-f_-g_-^* ,\\
&\frac{df_+}{d\rho}-\frac{k_++eA}{\rho}f_++(E_++p_+)g_+=0 ,\\
&\frac{dg_+}{d\rho}+\frac{l_++eA}{\rho}g_+-(E_+-p_+)f_+=0 ,\\
&\frac{df_-}{d\rho}-\frac{k_-+eA}{\rho}f_--(E_--p_-)g_-=0 ,\\
&\frac{dg_-}{d\rho}+\frac{l_-+eA}{\rho}g_-+(E_-+p_-)f_-=0 .
\end{aligned} \tag{11}
$$

Notice that in this case $E_+\neq E_-$ and $p_+\neq p_-$, in general. When $\mu>0$ the above equations can be reduced to the equations

$$
\begin{aligned}
&E_+-p_+=0, \quad E_-+p_-=0 ,\\
&-\frac{\mu}{\rho}\frac{dA}{d\rho}=e(1+c^2\rho^{2(k_+-k_-)})f_-^2 ,\\
&\frac{df_-}{d\rho}-\frac{k_-+eA}{\rho}f_-=0 ,\\
&f_\pm=c\rho^{k_+-k_-}f_-, \quad g_+=g_-=0 ,\\
&eA_0+k_++1>0, \quad eA_0+k_-+1>0 ,
\end{aligned} \tag{12}
$$

where c is an integration constant. The equations can easily be solved. When $k_+=k_-$, the solution becomes almost identical to the solution (9) with $m=0$. The only difference is that here one obtains the solution with $p_+\neq p_-$. In general with $k_+\neq k_-$, one can easily obtain a solution whose generic feature is very similar to the solution (9). When $\mu<0$ Eq. (11) can be reduced to

$$
\begin{aligned}
&E_++p_+=0, \quad E_--p_-=0 ,\\
&-\frac{\mu}{\rho}\frac{dA}{d\rho}=e(1+c^2\rho^{2(l_+-l_-)})g_+^2 ,\\
&\frac{dg_+}{d\rho}-\frac{l_++eA}{\rho}g_+=0 ,\\
&g_-=c\rho^{l_+-l_-}g_+, \quad f_+=f_-=0 ,\\
&eA_0+l_+-1<0, \quad eA_0+l_--1<0 ,
\end{aligned} \tag{13}
$$

which again admit a solution very similar to solution (10). This confirms the fact that Chern-Simons electrodynamics admits solutions very similar to (9) or (10), even when $p_+\neq p_-$.

Obviously the solutions describe vortices. The scale parameter λ determines the size of the vortices, because it determines the position of the maximum of the magnetic field (or equivalently the density of the electron wave function). To discuss the meaning of A_0, notice that when $A_0=0$ the solutions become regular everywhere, including the origin. However, A_0 can be nonvanishing if one is willing to allow a singularity at the origin. To see this notice that the magnetic flux $\Phi(\rho)$ passing through the area enclosed by the circle of radius ρ centered at the origin is given by

$$\Phi(\rho)=2\pi\int_{\rho'\leq\rho}\frac{dA(\rho')}{d\rho'}d\rho'=2\pi A(\rho) .$$

This means that when $A_0\neq 0$ the solution has a singular magnetic flux Φ_0 at the origin:

$$\Phi_0=2\pi A_0 . \tag{14}$$

Notice that the singularity becomes harmless and physically acceptable, as long as the fermion wave function vanishes at the origin. This is because with the boundary condition one could treat the singularity as an external magnetic flux. In this case, of course, we need an extra boundary condition: $eA_0+n>0$ for the solution (9) and $-eA_0+n>0$ for the solution (10). So, when $A_0\neq 0$, the solutions with the extra boundary condition could be interpreted to describe the motion of electrons around an external magnetic vortex line. Of course one could always require $A_0=0$ by choosing a proper gauge. However this is possible only with a singular gauge transformation which replaces Ψ with $e^{ieA_0}\Psi$, so that the gauge transformation does not change the physical nature of the singularity. For this reason we will keep the singularity in the following. The solutions are summarized in Fig. 1.

To discuss the physical content of the solutions notice that the total magnetic flux Φ and the electric charge q carried by the solution (9) is given by

$$
\begin{aligned}
&\Phi=2\pi A_\infty=-\Phi_0-4\pi\frac{n+1}{e}=-\frac{4\pi}{e}\left[\frac{\nu}{2}+n+1\right] ,\\
&q=-\mu\Phi ,
\end{aligned} \tag{15}
$$

where e is the charge of the electron and $\nu=eA_0$. To find the total energy and angular momentum notice that the energy-momentum tensor and the electron number density of the system are given by

$$
\begin{aligned}
T_{\alpha\beta}=&-\frac{i}{2}[\bar\psi_+(\gamma_{+\alpha}D_\beta+\gamma_{+\beta}D_\alpha)\psi_+\\
&+\bar\psi_-(\gamma_{-\alpha}D_\beta+\gamma_{-\beta}D_\alpha)\psi_-] ,\\
\mathcal{N}=&\psi_+^\dagger\psi_-+\psi_-^\dagger\psi_+ .
\end{aligned}
$$

So we have the following (2+1)-dimensional total energy $\mathcal{E}$, electron number N, and angular momentum J:

$$
\begin{aligned}
&\mathcal{E}=\int T^{00}d^2x=\frac{q}{e}E ,\\
&N=\int \mathcal{N}d^2x=\frac{q}{e}\frac{m}{E} ,\\
&J=\int \epsilon_{ij}x^iT^{0j}d^2x=\frac{\mu}{2e}\Phi\\
&\quad=-\frac{2\pi\mu}{e^2}\left[\frac{\nu}{2}+n+1\right]=-\frac{q}{2e} .
\end{aligned} \tag{16}
$$

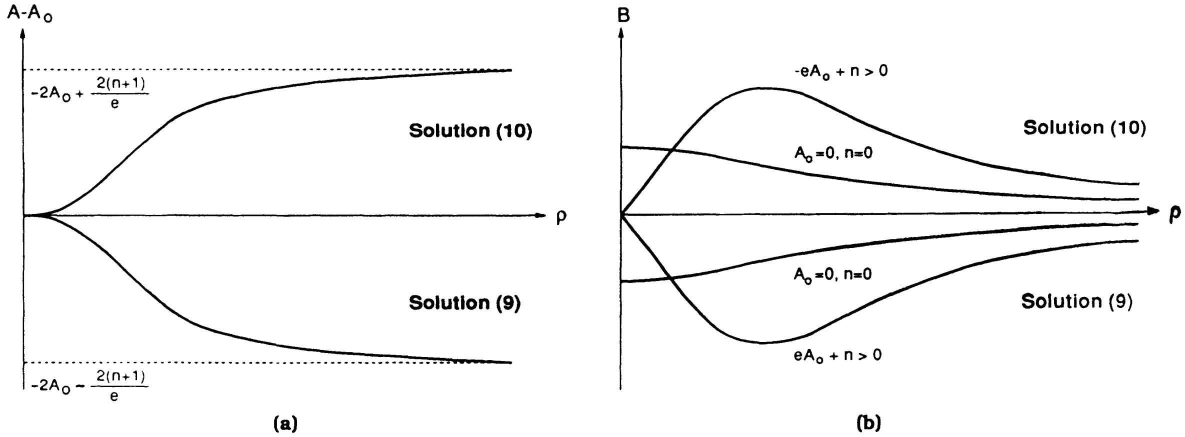

FIG. 1. The vortex solutions: The magnetic potential is shown in (a), and the magnetic field is shown in (b).

Observe that the angular momentum includes the intrinsic spin of the electron. For the solution (10) we have

$$\Phi=\frac{4\pi}{e}\left[-\frac{\nu}{2}+n+1\right],\quad q=-\mu\Phi\ ,$$

$$\mathcal{E}=\frac{q}{e}E,\quad N=\frac{q}{e}\frac{m}{E}\ , \tag{17}$$

$$J=-\frac{\mu}{2e}\Phi=-\frac{2\pi\mu}{e^2}\left[-\frac{\nu}{2}+n+1\right]=\frac{q}{2e}\ .$$

Remember that when $A_0=0$ the above solutions become regular everywhere. In this case the solution (9) has negative magnetic flux and angular momentum, but the solution (10) has positive magnetic flux and angular momentum. But both solutions carry positive electric charge. Also notice that the unit of magnetic flux of the solutions is $4\pi/e$, not $2\pi/e$. This should be contrasted with the vortex solutions made of scalar fields, where the unit of the magnetic flux quanta becomes $2\pi/e$.

Clearly the above solutions describe the charge-flux composite states made of real electrons. But notice that when $E=p$, (9) describes a solution in which $\psi_-=0$. Similarly when $E=p$, (10) describes a solution in which $\psi_+=0$. This means that when $E=p$, the solutions (9) and (10) become exact solutions of Chern-Simons electrodynamics which has only one fermionic source.

Recently Jackiw and Pi [8] have shown that the Chern-Simons-Higgs theory in its symmetric realization allows zero-mode soliton solutions in the nonrelativistic limit. It is very interesting to notice that, in spite of the fact that physically our solutions are totally different from theirs, mathematically they are closely related. The similarity follows from the fact that in both cases the main part of the equations of motion could be reduced to the Liouville equation with a proper ansatz. To see this let us choose a more general ansatz

$$\begin{aligned}&A_t=0,\quad \partial_t A_i=0\ ,\\&\psi_+=e^{-iF_+t}\begin{bmatrix}F_+(x,y)\\ iG_+(x,y)\end{bmatrix},\\&\psi_-=e^{-iE_-t}\begin{bmatrix}F_-(x,y)\\ iG_-(x,y)\end{bmatrix}.\end{aligned} \tag{18}$$

Then with

$$\begin{aligned}&E_+=E_-=E,\quad p_+=p_-=p\ ,\\&E^2-p^2=m^2,\quad F_-=\frac{E-p}{m}F_+,\quad G_+=\frac{E-p}{m}G_-\ ,\end{aligned} \tag{19}$$

Eq. (5) is reduced to

$$\begin{aligned}&\mu\epsilon^{ij}\partial_i A_j=-\frac{2eE}{E+p}(|F_+|^2+|G_-|^2)\ ,\\&(D_1+iD_2)F_\pm=0,\quad (D_1-iD_2)G_\pm=0\ .\end{aligned} \tag{20}$$

So with $F_\pm=0$ or $G_\pm=0$, the above equation becomes formally identical to the equation which describes the nonrelativistic solitons [8]. Nevertheless, it should be emphasized that our solutions are fundamentally different from theirs. First of all, our solutions are neither nonrelativistic nor the zero modes. They are completely relativistic, and carry a nontrivial energy. Second, our solutions are made of real electrons with intrinsic spin. So the total angular momentum in our case is given by $J=\mp(\mu/2e)\Phi$, which should be compared with $J=\mp(\mu/e)\Phi$ in their case. Finally, our solutions have a vanishing electric potential but theirs require a nontrivial electric field.

When the Chern-Simons interaction is induced by the quantum correction of the fermions the Chern-Simons coupling constant no longer remains an arbitrary parameter. Indeed, when $p_+p_-<0$ the quantum correction gives us [6]

$$\mu=\pm\frac{e^2}{2\pi}\ .$$

In this case from (9) one has

$$q=(\nu+2n+2)e, \quad J=-\frac{q}{2e} .$$

Similarly from (10) one has

$$q=(-\nu+2n+2)e, \quad J=\frac{q}{2e} .$$

Notice that the charge and the angular momentum of the solutions remain fractional when ν is so.

We conclude with the following remarks.

(1) When μ is arbitrary (or when ν is nonvanishing), our solutions could describe the anyons [9,10] made of real electrons. Probably the solutions constitute the first example of anyons made of fermions. Again, it is the Chern-Simons interaction which successfully provides the binding force for the charge-flux composite states. But remarkably the angular momentum of the anyons is no longer given by $(q/4\pi)\Phi$, which is the angular momentum of a quantum-mechanical charge-flux composite state bound by the Chern-Simons interaction [10].

(2) From the $(2+1)$-dimensional point of view our solutions clearly describe nontopological solitons, n identical solitons centered at the origin. But one could easily generalize the solutions to obtain multisoliton solutions in which each soliton locates at different points, with different sizes and phases. This is so because Eq. (20) can be reduced to the Liouville equation which admits an n-soliton solution which has $4n$ real parameters which determine the size, phase, and position of each soliton. The existence of the solitonic sector (which is made possible with the Chern-Simons interaction) demonstrates the fact that Chern-Simons interaction changes not only the statistics but also the dynamics of the theory.

(3) The solutions of the $(2+1)$-dimensional Dirac equation coupled to a background magnetic vortex has been discussed by many authors [5]. But notice that in our case the magnetic field (at least the regular part of the magnetic field) is generated by the electrons themselves. So our solutions describe the bound states of electrons in which a self-created (rather than independent) magnetic field, or more properly a nonlinear self-interaction of electrons, provides the binding force. Certainly one might wish to give a quantum-field-theoretic meaning to these solutions by quantizing them, treating them as $(2+1)$-dimensional solitons. The quantization will clarify the physical meaning of the solitonic sector.

(4) To obtain the above solutions we have neglected the Maxwell term for simplicity, which is well justified in the long-distance limit. However, it must be remembered that near the vortex the short-distance interaction becomes more important, so that the Maxwell term could alter the vortex solution significantly around the origin.

(5) It is quite possible that, with the realistic effective axial symmetry (2), our solutions could describe real physical objects. So they could play an important role in connection with the quantum Hall effect and the high-T_c superconductivity [11].

A more detailed discussion on the subject will be published elsewhere [12].

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