

Fixed sources in light-front dynamics and Wilson's model of coupling-constant renormalization

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We construct a fixed source model of nonperturbative coupling-constant renormalization in light-front dynamics as a limit of a theory of heavy fermionic sources when the fermion mass becomes very large. We begin with the canonical light-front Hamiltonian for Yukawa quantum field theory. We introduce cutoffs and consider the limit when the fermion mass is much larger than the momentum cutoffs. We then derive an effective Hamiltonian in the Fock space spanned by sectors with only one fermion and arbitrarily many bosons. The total momentum of the dressed fermionic source is separated from the internal dynamics of the system. The effective Hamiltonian for the internal dynamics is shown to be equivalent to the original fixed source model Hamiltonian considered by Wilson. Wilson's nonperturbative renormalization-group analysis applies to the light-front version of the model. The new feature of the light-front model is the appearance of manifest boost invariance, which allows one to study a heavy fermion source with arbitrary momentum. We discuss the generalization of the renormalization-group analysis to the case where the fermion mass is comparable to the boson mass.

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I. INTRODUCTION

Despite the great successes of the constituent quark model, Feynman's parton model, and perturbative QCD, adequate theoretical understanding of hadronic bound-state dynamics has not been achieved using QCD. Available models are not unified in a theory that allows accurate predictions for strong-interaction phenomena at hadronic scales. The major problem in understanding the nonperturbative dynamics of hadrons is that it seems to be unavoidable that one has to first solve for the vacuum state of QCD and only then is it possible to study hadrons as small excitations on top of the vacuum. One is stuck with a prohibitively complicated problem before being able to even think about deriving the simplest picture of hadrons and describing data from first principles. Fortunately, there is an alternative approach to QCD, based on light-front dynamics [1]. The light-front formulation of QCD is quite different from the customary equal-time formulation and involves new kinds of singularities. The nontrivial structure of the vacuum seems to be built into the theory in a qualitatively different way than in equal-time dynamics [2]. We may escape the necessity of constructing the vacuum, but we still need to understand how the vacuum affects hadronic structure. Wilson has suggested that if the severe singularities in canonical light-front QCD are properly renormalized, they may lead to an understanding of confinement and chiral-symmetry breaking. This suggests a new approach to computing hadronic structure that deserves investigation.

The primary tool for investigating the singular behav-

ior of quantum field theories is the renormalization group. However, light-front quantum field theory poses unusual renormalization problems that have received little attention. Wilson has pointed out that power counting on the light-front differs from equal-time power counting because transverse and longitudinal dimensions count differently, and new kinds of singular nonlocal interactions are allowed in renormalized light-front Hamiltonians. Even when one considers very much simplified bound-state problems using approximate Hamiltonian diagonalization, one discovers singularities that induce strong counterterms. These counterterms themselves may be a substantial part of observed strong interactions. Before this possibility can be explored with any rigor, we must first establish a nonperturbative renormalization theory for light-front Hamiltonians. Surprisingly little is known about nonperturbative renormalization theory for quantum fields on the light front. We have not found a single example of renormalization-group analysis for light-front Hamiltonians in the literature.

In this article we analyze a light-front version of an elementary model of coupling-constant renormalization that was originally studied by Wilson almost three decades ago using equal-time dynamics [3]. Our aim is to show how a similar example is constructed on the light front. Even though the model is rather unrealistic, it is instructive to see the full analysis and to appreciate the procedure required to find a renormalized Hamiltonian that can be used to calculate physical observables. Thus, although Wilson's model is not useful phenomenologically, it serves as a simple ground for the development of Hamiltonian-based renormalization-group analyses. We construct an analogous light-front model for this same purpose, as a preliminary step in the development of the light-front renormalization group.

In Sec. II we describe our construction of the light-front Hamiltonian for scalar bosons coupled to a single

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very heavy fermionic source. The reader may consider it profitable to consult Ref. [4] for a broad discussion of this type of problem. In Sec. III we sketch briefly how to obtain Wilson's results in light-front renormalization theory. We do not need to repeat Wilson's discussion, which is best presented in his original articles [3,5]. We conclude with some remarks on how to approach more

realistic examples of renormalized Hamiltonians for applications to hadronic bound states.

II. FIXED SOURCE LIGHT-FRONT HAMILTONIAN

In this section we derive the light-front Hamiltonian for a very heavy fermionic source in its rest frame. The canonical Hamiltonian for Yukawa theory is

$$\begin{aligned}
H_c = & \sum_{\lambda} \int [d^3p] \frac{m^2 + (p^{\perp})^2}{p^+} b_{p\lambda}^{\dagger} b_{p\lambda} + \int [d^3q] \frac{\mu^2 + (q^{\perp})^2}{q^+} a_q^{\dagger} a_q \\
& + g \sum_{\lambda_1} [d^3p_1] \int [d^3q] \sum_{\lambda_2} \int [d^3p_2] 2(2\pi)^3 \delta^3(P_{\text{created}} - P_{\text{annihilated}}) \bar{u}_{p_2\lambda_2} \Gamma u_{p_1\lambda_1} b_{p_2\lambda_2}^{\dagger} (a_q^{\dagger} + a_q) b_{p_1\lambda_1} \\
& + [\text{seagull terms with operators } b^{\dagger}, b, a^{\dagger}, \text{ and } a] \\
& + [\text{terms that change fermion number or involve antifermions}] .
\end{aligned} \tag{2.1}$$

The subscripts λ are spin and/or isospin indices and Γ is a spin and/or isospin matrix. We will suppress isospin indices in our initial discussion, since they are not explicitly required in the derivation of the fixed source model and are easily inserted later when required. For our initial analysis it is sufficient to focus on the case $\Gamma=1$. The creation and annihilation operators commute or anticommute to $2(2\pi)^3 p^+ \delta^3(p-p') \delta_{\lambda,\lambda'}$, and

$$\int [d^3p] = \int_0^{\infty} dp^+ \int d^2p^{\perp} \frac{1}{2(2\pi)^3 p^+} . \tag{2.2}$$

The fermion spinors are given by

$$u_{p\lambda} = \frac{1}{\sqrt{mp^+}} [\Lambda^-(m + \alpha^{\perp} p^{\perp}) + \Lambda^+ p^+] u_{\lambda} , \tag{2.3}$$

where u_{λ} denotes the Dirac spinor of a fermion at rest, normalized to $\bar{u}u=2m$, and $\Lambda^{\pm} = \frac{1}{2}(1 \pm \alpha^3)$. $P_{\text{created(annihilated)}}$ represents total momentum of particles created (annihilated) in a single vertex.

The seagull terms and the terms involving antifermions will turn out not to be important in our discussion of a fixed fermionic source, so we do not provide details.

Now we want to consider the limit in which the fermion mass tends to infinity, in order to discuss fixed source dynamics. However, the momentum integrals in the canonical Hamiltonian extend to infinity and must be replaced by limits of integrals over select finite ranges of momenta. We must carefully define how the mass and momentum limits are ordered.

The simplest way to consider the infinite fermion mass limit is to specify that the ratio of the fermion mass to the range of the momentum integrals becomes infinite. The

fermion mass then becomes the dominant scale in the Hamiltonian. Transverse momenta of all particles, and longitudinal momenta of bosons are negligible in comparison to m in the fermion rest frame. The bare fermion longitudinal momentum deviates from the fermion mass by relatively small amounts in the physical fermion rest frame. With these severe restrictions on the momentum integrals in the Hamiltonian one is able to discuss fixed source dynamics. However, taken literally, these restrictions would require us to limit the total momentum of the system. In an equal-time analysis one is forced to simultaneously limit both total and relative momenta in order to obtain the fixed source Hamiltonian. We will see that in light-front coordinates it is possible to exactly separate the total momentum of the dressed source from its internal dynamics. Therefore, we require only relative momenta to be negligible in comparison to the fermion mass, and can allow arbitrary motion of the source.

The seagull terms and the terms involving creation or annihilation of fermion-antifermion pairs are suppressed by one inverse power of the fermion mass. Since we have chosen to discuss the fermion source we can completely disregard antifermions.

Thus we arrive at a Hamiltonian that contains only the first three terms of Eq. (2.1), with integrals over *relative* momenta restricted to ranges small in comparison to the fermion mass. Eigenstates of different fermion number are widely separated in the spectrum because of the large fermion mass. We consider the effective Hamiltonian in the one-fermion sector.

The one-fermion eigenstates of our Hamiltonian have the following general form implied by light-front symmetries:

$$|P\lambda\rangle = \sum_{n=0}^{\infty} \sum_{\sigma} \int [d^3p] \int [d^3q_1] \cdots \int [d^3q_n] 2(2\pi)^3 P^+ \delta^3(P - p - q_1 - \cdots - q_n) \phi_{\lambda\sigma}^{(n)}(y_1, \kappa_1^{\perp}, \dots, y_n, \kappa_n^{\perp}) |p\sigma q_1 \cdots q_n\rangle , \tag{2.4}$$

where

$$|p_\sigma q_1 \cdots q_n\rangle = b_{p_\sigma}^\dagger a_{q_1}^\dagger \cdots a_{q_n}^\dagger |0\rangle. \quad (2.5)$$

P^+ and P^\perp are components of the total light-front three-momentum of the fermion eigenstate. The relative momenta, which are the arguments of the Fock-space wave functions $\phi^{(n)}$ are defined as

$$y_i = q_i^+ / P^+, \quad (2.6)$$

$$\kappa_i^\perp = q_i^\perp - y_i P^\perp, \quad (2.7)$$

so that the bare fermion momentum in the n th sector is

$$p_n^+ = x_n P^+, \quad (2.8)$$

$$p_n^\perp = x_n P^\perp - \kappa_1^\perp - \cdots - \kappa_n^\perp, \quad (2.9)$$

where

$$x_n = 1 - y_1 - \cdots - y_n. \quad (2.10)$$

The effective Hamiltonian for the fixed fermionic source is obtained by projecting the equation

$$H|P\lambda\rangle = \frac{(P^\perp)^2 + M^2}{P^+} |P\lambda\rangle \quad (2.11)$$

on the one-fermion Fock-space sectors

$$\langle p_\sigma q_1 \cdots q_n | H | P\lambda \rangle = 2(2\pi)^3 P^+ \delta^3(P - p - q_1 - \cdots - q_n) \frac{(P^\perp)^2 + M^2}{P^+} \phi_{\lambda\sigma}^{(n)}(y_1, \kappa_1^\perp, \dots, y_n, \kappa_n^\perp), \quad (2.12)$$

and evaluating explicitly the fermionic part of the Hamiltonian matrix elements for large m , leaving the bosonic part untouched. M is the physical fermion mass. One obtains

$$\frac{(P^\perp)^2 + M^2}{P^+} \phi_{\lambda\sigma}^{(n)}(y_1, \kappa_1^\perp, \dots, y_n, \kappa_n^\perp)$$

$$= \sum_{l=0}^{\infty} \int [d^3 k'_1] \cdots \int [d^3 k'_l] \sum_{\sigma'} \phi_{\lambda\sigma}^{(l)}(y'_1, \kappa'_1, \dots, y'_l, \kappa'_l)$$

$$\times \left\langle q_1 \cdots q_n \left| \left[\frac{(p_n^\perp)^2 + m^2}{p_n^+} \delta^{\sigma\sigma'} + P_{0,\text{bosons}}^- \delta^{\sigma\sigma'} \right] \right. \right.$$

$$\left. \left. + g \int [d^3 q] \left[\frac{\bar{u}_{p_n\sigma} u_{p_n+q,\sigma'}}{p_n^+ + q^+} a_q^\dagger + \frac{\bar{u}_{p_n\sigma} u_{p_n-q,\sigma'}}{p_n^+ - q^+} a_q \right] \right| \right| q'_1 \cdots q'_l \rangle.$$

(2.13)

In fact, for the fixed source Hamiltonian, there are only three terms on the right-hand side of this equation that survive: the free energy term with $\phi^{(n)}$ and two interaction terms, one with $\phi^{(n+1)}$ and one with $\phi^{(n-1)}$.

We are not forced to assume that P^+ differs little from M and that P^\perp/M is negligible, as one has to do in the equal-time analysis. Here, we can handle P^+ and P^\perp exactly, even in the large fermion mass limit, and separate the center-of-mass motion of the dressed fermionic source from its internal dynamics. This is done by expressing boson momenta in terms of the relative momenta in Eqs. (2.6) and (2.7). P^+ and P^\perp drop out of the eigenvalue equation for the physical fermion mass M^2 , as they clearly should. The light-front symmetries provide the opportunity to separate the “fixed source” motion from the dynamics of its bosonic cloud. Because of these symmetries one can obtain the boost-invariant results by considering only the special case $P^+ = M$ and $P^\perp = 0$.

One may notice at this point that boost invariance is usually violated by the introduction of momentum

cutoffs. Demanding that individual momenta of all particles be much smaller than the bare fermion mass prevents the fermion from moving with large velocity and implies that there cannot be boost invariance in the Hamiltonian spectrum. Fortunately, in the light-front form of dynamics we are not forced to impose cutoffs on the individual momenta of all particles. In fact, we want to stress that the light-front Hamiltonian can and should be constructed with cutoffs imposed on relative momenta, because it is these that are small in comparison to the fermion mass, and we can only maintain explicit boost invariance with these variables. Thus, we should also stress that it is deceptive to discuss a “fixed source” in light-front dynamics because our source can move with arbitrary velocity, and to be useful our model must allow this possibility. So far we have not specified the way cutoffs are imposed. We will discuss this point in more detail later.

Evaluating the spinor matrix elements and working in the rest frame of the physical fermion we see that

$$\begin{aligned}
M\phi_{\lambda\sigma}^{(n)}(1, \dots, n) &= \sum_{n'=n-1}^{n'=n+1} \int [1'] \cdots [n'] \phi_{\lambda\sigma}^{(n')}(1', \dots, n') \\
&\times \left\langle 1, \dots, n \left| \frac{m^2 + \left[\sum_{i=1}^n \kappa_i^\perp \right]^2}{\left[1 - \sum_{i=1}^n y_i \right] M} + \sum_{i=1}^n \frac{\mu^2 + (\kappa_i^\perp)^2}{y_i M} + 2g \int [d^3q] (a_q^\dagger + a_q) \right| 1', \dots, n' \right\rangle,
\end{aligned} \tag{2.14}$$

where we introduce an abbreviated notation for momentum variables, exhibiting only their subscripts. Note that we have distributed M in this equation to facilitate the limiting procedure. For eigenvalues of the form

$$M = m + E, \tag{2.15}$$

we assume that $E/m \ll 1$ and $\sum_{i=1}^n y_i \ll 1$, both of which are easily shown to hold for low-lying eigenstates *a posteriori*. Equation (2.14) becomes

$$\begin{aligned}
E\phi^{(n)}(1, \dots, n) &= \sum_{n'=n-1}^{n'=n+1} \int [1'] \cdots [n'] \phi^{(n')}(1', \dots, n') \\
&\times \left\langle 1, \dots, n \left| \sum_{i=1}^n \frac{1}{2} \left[y_i m + \frac{\mu^2 + (\kappa_i^\perp)^2}{y_i m} \right] + g \int [d^3q] (a_q^\dagger + a_q) \right| 1', \dots, n' \right\rangle.
\end{aligned} \tag{2.16}$$

This immediately implies that the effective light-front Hamiltonian for the fermionic source internal dynamics is

$$H_{\text{eff}} = \int [d^3q] \left[\frac{1}{2} \left[q^+ + \frac{\mu^2 + (q^\perp)^2}{q^+} \right] a_q^\dagger a_q + g(a_q^\dagger + a_q) \right], \tag{2.17}$$

where the momentum integration extends over a range of momenta that is negligible in comparison to the fermion mass. When the fermion mass becomes infinite we obtain the effective Hamiltonian of Eq. (2.17) without restrictions on the boson momenta. Note that the light-front energy is naturally replaced by the equal-time energy $(p^+ + p^-)/2$ when the fermion mass is large. In the rest frame of the dressed source the transverse meson momentum q^\perp coincides with the relative momentum κ^\perp and the longitudinal momentum q^+ coincides with ym , because the difference between m and $P^+ = M$ can be neglected in the product yP^+ .

III. RENORMALIZATION GROUP ON THE LIGHT FRONT

The fixed source light-front Hamiltonian of Eq. (2.17) leads to divergent results and therefore requires renormalization. Wilson found a way to define a class of renormalized Hamiltonians that correspond to the fixed source Hamiltonian in equal-time dynamics, and discovered the necessity to formulate the renormalization-group theory for quantum field Hamiltonians. His further work on the model produced the first nonperturbative renormalization-group analysis of coupling-constant renormalization [5].

In this section we briefly describe the situation on the

light front. We refer the reader to the two remarkable articles by Wilson [3,5], which provide the entire motivation for this section.

In order to define Wilson's model on the light front we need to introduce momentum ranges analogous to his equal-time energy shells [3]. Since we have only one unrestricted momentum integration to sample we can use our previous experience [6] and introduce the following regions of momenta:

$$\mu < \frac{1}{2} \left[q^+ + \frac{\mu^2 + (q^\perp)^2}{q^+} \right] < E_0, \tag{3.1}$$

$$\begin{aligned}
\sqrt{(\frac{1}{2}\Lambda^n k_0)^2 + \mu^2} &< \frac{1}{2} \left[q^+ + \frac{\mu^2 + (q^\perp)^2}{q^+} \right] \\
&< \sqrt{(\Lambda^n k_0)^2 + \mu^2}, \quad n \geq 1,
\end{aligned} \tag{3.2}$$

where

$$E_0 = \sqrt{\mu^2 + k_0^2}. \tag{3.3}$$

To show that the light-front problem is isomorphic to the equal-time problem, we next change variables from q^+ and q^\perp to k^3 and $k^\perp = (k^1, k^2)$ which form the three-vector \mathbf{k} :

$$q^\perp = k^\perp, \tag{3.4}$$

$$q^+ = \omega_\mu(\mathbf{k}) + k^3, \tag{3.5}$$

$$\omega_\mu(\mathbf{k}) = \sqrt{\mu^2 + (\mathbf{k})^2}, \tag{3.6}$$

$$[d^3q] = \frac{d^3k}{2\omega_\mu(\mathbf{k})(2\pi)^3} := [d^3k]. \tag{3.7}$$

Our Hamiltonian, Eq. (2.17), in terms of these variables is

$$H = \int_{\text{shells of } k} [d^3k] [\omega_\mu(\mathbf{k}) a_k^\dagger a_k + g(a_k^\dagger + a_k)]. \quad (3.8)$$

We can now step back to the initial theory, introduce two kinds of bosons (operators a and b) and isospin matrices τ^+ and τ^- , change the normalization of the creation and annihilation operators to match Wilson's convention, and let our coupling g be Wilson's bare coupling constant g_0 . After these steps our light-front fixed source Hamiltonian is identical with Wilson's model described in [3]. The whole discussion of Refs. [3] and [5] follows. In fact, this last step is a major one, but it is not necessary to repeat the details here and we cannot improve upon Wilson's original work. The construction of a light-front version of Wilson's model of the coupling-constant renormalization in quantum field theory is completed. To our knowledge this allows one to complete the first example of a nonperturbative renormalization-group analysis in light-front dynamics. Such an example provides a starting point for the study of significantly more complicated renormalization problems in light-front quantum field theories of elementary particles. On the basis of this model we can draw several conclusions concerning the renormalization-group analysis of relativistic Hamiltonians in light-front dynamics.

IV. CONCLUSION

The fixed source Hamiltonian model of nonperturbative coupling-constant renormalization, as constructed by Wilson in equal-time dynamics, can be equally well constructed in light-front dynamics and identical results can be obtained. The light-front construction is not merely a change of variables, it is actually both nontrivial and instructive.

In the equal-time analysis one employs momenta canonically conjugate to the equal-time spatial variables. One can simply place the fixed source at the origin of the coordinate system, and all boson coordinates are given relative to this position. The light-front analysis is necessarily more complicated, because a source fixed in light-front space moves at the speed of light in equal-time coordinates and has an infinite energy. To discover the analogue of the fixed source Hamiltonian, we began with the full Hamiltonian of the Yukawa quantum field theory and studied the limit as the fermion mass was taken to infinity. A similar type of analysis can be carried out in equal-time coordinates. However, in the light-front analysis the Poincaré generators of boosts do not contain interactions [1] and one is able to completely separate the total momentum from the problem, while in an equal-time analysis the boost operators contain interactions and the separation of the total momentum from the problem is not possible. In equal-time dynamics one is forced to assume that this momentum is small in comparison to the fermion mass in order to complete the analysis.

In the light-front model one automatically obtains a theory of heavy sources that can move with arbitrary velocity. The momentum \mathbf{k} (q^+ and q^\perp) is explicitly considered as a relative momentum of a boson and the center of mass of the *dressed* source. The motion of the dressed

source is separated from its internal dynamics.

For infinitely heavy fermions the distinction between relative and absolute spatial coordinates is not significant. However, for quarks (hadrons) coupled to gluons (mesons) there is no reason to believe that the difference does not matter. In order to understand relativistic bound-state dynamics in QCD or nuclear physics with nucleons and mesons, it is likely that one must understand the relative motion of constituents over many scales of momenta at once and it is not possible to employ nonrelativistic sources everywhere.

Such a situation invites a nonperturbative renormalization-group analysis and one must expect several important new features to appear in a renormalization-group analysis of relativistic Hamiltonians. In Wilson's original analysis the only interactions allowed are between a nonrelativistic source and relativistic bosons. In the light-front analysis one can see that it is the negligible recoil of the source that selects the momentum scales in Eqs. (3.1) and (3.2) as relevant to a renormalization-group analysis. These scales imply a relationship between the scaling of longitudinal and transverse momenta that is not surprising for extremely massive sources. However, we know of no reason for this relationship to hold for light sources and consider it an outstanding problem to discover the principle that will allow one to establish a set of scales appropriate for the study of relativistic sources analogous to those employed by Wilson for the study of a fixed source.

In the case of infinite fermion mass the key role is played by the meson mass parameter μ which is used in how one samples longitudinal and transverse momenta in the light-front renormalization-group analysis. The sampling is patterned after equal-time sampling of a free boson energy because the equal-time free boson energy is naturally selected in the light-front Hamiltonian for the internal dynamics of a heavy source, as displayed in Eq. (2.17). When fermion and boson masses are comparable, or when the bosons have self-interactions, the equal-time free boson energy does not naturally arise in a light-front analysis and a unique momentum sampling does not exist. Both fermion and boson masses need renormalization. One is forced to formulate an analysis that samples longitudinal and transverse relative momenta separately.

One is also forced to employ a Tamm-Dancoff restriction on the number of particles as a practical limitation on the analysis [7]. Such restrictions on the number of particles become a new argument of the renormalization-group transformation themselves. We expect that a considerable effort over a period of time will have to be devoted to the study of how the renormalization-group transformation depends on the Fock-space sectors considered before we will be able to firmly connect renormalized light-front Hamiltonians for QCD at hadronic scales with Feynman's parton model and perturbative QCD.

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