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Reparametrization ghost contribution to superstring multiloop amplitudes

R. Pettorino
Dipartimento di Scienze Fisiche dell'Università di Napoli
and Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Mostra d'Oltremare Padiglione 19, I-80125 Napoli, Italy

F. Pezzella
Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Mostra d'Oltremare Padiglione 20, I-80125 Napoli, Italy

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We use the sewing procedure in the operator formalism to construct the N-point g-loop vertex $V_{N;g}$ for the (b,c) system in superstring theory. After computing explicitly the lowest picture-changed states, we saturate $V_{N;g}$ with them showing that the reparametrization ghost contribution to g-loop superstring amplitudes is just the same as in bosonic string theory.

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I. INTRODUCTION

One of the problems still open in string theory is to check its finiteness at any order of perturbation theory. In fact, many technical problems arise in writing out explicitly string amplitudes in the various formalisms proposed and developed for this purpose [1–14]. Recently, some interesting progress has been made in this direction [15].

A very straightforward approach in constructing multiloop amplitudes is based on the definition of a single global bralike operator $V_{N;g}$, the N-point g-loop vertex [5–8]. This latter encodes all the information about the g-loop scattering amplitudes involving N arbitrary physical particles; in fact, $V_{N;g}$ can be considered as the "generating functional" for those amplitudes since they are simply derived by saturating $V_{N;g}$ with the N physical states corresponding to the N particles. The starting point for constructing $V_{N;g}$ is the N-string vertex $V_{N;0}$ [4], which has the important property of reproducing the tree-level scattering amplitudes involving N physical particles, when it is saturated with the corresponding states. Indeed, $V_{N;g}$ is constructed by starting with the $(N+2g)$-string vertex and sewing together $2g$ legs, after the insertion of a suitable Becchi-Rouet-Stora-Tyutin-(BRST-)invariant propagator.

In superstring theory $V_{N;g}$ can be obtained, in principle, by the product of N-point g-loop vertices defined for each independent conformal sector of the theory: string coordinates, the (b,c) system of the reparametrization ghosts, and its supersymmetric partner, i.e., the (β,γ) system.

In [14], $V_{N;g}$ for the (b,c) system has been constructed for $g=2$ in its more general operator form necessary for computing superstring scattering amplitudes involving picture-changed states [16]. In superstring theory, in fact, to each physical state one can associate infinitely many vertices, differing from each other for values of the total ghost number $q+q'$, q $[q']$ being the eigenvalue of the ghost number associated with the (b,c) $[(\beta,\gamma)]$ system. Therefore, in order to saturate $V_{N;g}$ with such states, it is preliminary to compute explicitly picture-changed vertices and to examine their content in the ghost coordinates [17].

In this paper we generalize to g-loop amplitudes the partial result for $g=2$ obtained in [14].

After constructing $V_{N;g}$ for the (b,c) system through the above-mentioned sewing procedure, we saturate it with suitable picture-changed states. In so doing we conclude that the reparametrization ghost contribution to g-loop superstring amplitudes is just the same as in bosonic string theory.

The paper is organized as follows.

In Sec. II we compute the lowest picture-changing vertices: It turns out that their part with a (b,c) ghost number $q=1$, which is the only one to give a nonzero contribution to amplitudes, defines a primary field with the conformal dimension $\Delta=0$ having the same form $c(z)V(z)$ as the vertex operators in the bosonic string. The only difference is that, in the superstring case, the field $V(z)$ contains not only the orbital degrees of freedom, but also the superghost ones.

In Sec. III we use the sewing procedure in the operator formalism to construct $V_{N;g}$.

In Sec. IV we saturate $V_{N;g}$ with suitable picture-changed states, showing that it gives for superstring amplitudes the same g-loop contribution as in the bosonic string. We want to remark that use limited to picture-changed vertices up to $q+q'=3$ restricts the number N of the external states to be $\geq(g-1)$. However, this restriction vanishes if one assumes the above-mentioned property of the part with $q=1$ of picture-changed vertices valid in general.

II. PICTURE-CHANGED VERTICES

In this section we closely follow Ref. [17], summarizing the main results.

In superstring theory the orbital degrees of freedom of a physical state are carried by a supervertex operator that we will denote by

$$V(z,\theta)=V_0(z)+\theta V_1(z) \ . \tag{2.1}$$

The integral of (2.1) over superspace is required to be conformally invariant; therefore it must be a superconformal field with the conformal weight $\Delta=\frac{1}{2}$. The two components of the supervertex operator corresponding, for instance, to the tachyon and photon states are given by

$$\begin{aligned} &V_0^t(z)=:e^{ik\cdot x(z)}: \quad V_0^p=:\epsilon\cdot\psi(z)e^{ik\cdot x(z)}: \ , \\ &V_1^t(z)=:ik\cdot\psi(z)e^{ik\cdot x(z)}: \ , \\ &V_1^p(z)=:[\epsilon\cdot\partial x(z)+ik\cdot\psi(z)\epsilon\cdot\psi(z)]e^{ik\cdot x(z)}: \ , \end{aligned} \tag{2.2}$$

with $k^2=1$ for the tachyon and with $\epsilon\cdot k=0$ and $k^2=0$ for the photon. In a BRST-invariant formalism, the reparametrization ghosts (b,c) and the superconformal ones (β,γ) must be properly included in the definition of the vertex operators.

The fields b,c and β,γ are functions only of z and admit the holomorphic expansions[1]

$$b(z)=\sum_{n\in\mathbf{Z}} b_n z^{-n-2}, \quad c(z)=\sum_{n\in\mathbf{Z}} c_n z^{-n+1} \ , \tag{2.3}$$

$$\beta(z)=\sum_{n\in\mathbf{Z}} \beta_n z^{-n-3/2}, \quad \gamma(z)=\sum_{n\in\mathbf{Z}} \gamma_n z^{-n+1/2} \ , \tag{2.4}$$

where the oscillators b_n,c_n and β_n,γ_n satisfy, respectively, the canonical relations

$$\{b_n,c_m\}=\delta_{n+m,0}, \quad \{b_n,b_m\}=\{c_n,c_m\}=0 \ , \tag{2.5}$$

$$[\beta_n,\gamma_n]=\delta_{n+m,0}, \quad [\beta_n,\beta_m]=[\gamma_n,\gamma_m]=0 \ . \tag{2.6}$$

It turns out that for each physical state one can associate an infinite set of vertex operators corresponding to different values of the total ghost number $q+q'$, q and q' being the eigenvalues of the U(1) ghost-number currents associated, respectively, with the (b,c) and (β,γ) systems. Indeed, it is possible to transform BRST-invariant vertex operators to new ones carrying the same orbital degrees of freedom but a different eigenvalue of the total ghost number. This transformation is performed by the *picture-changing operation* [16].

Since both ghost-number currents are anomalous, in order to have nonzero results in the computation of the vacuum expectation values of vertices corresponding to string scattering amplitudes, one can use any vertex of the set, provided that the sums of the ghost numbers q_i and q_i' associated with the vertex labeled by i match the vacuum charge of the (b,c) and (β,γ) systems, respectively. In particular, in the superstring case one must require

$$\sum_i q_i=3, \quad \sum_i q_i'=2(g-1) \ .$$

[1]We write here definitions for the holomorphic sector of the theory only; similar expressions hold for the antiholomorphic sector.

The BRST invariance ensures that the result is independent from the particular vertex of the set that has been used.

The vertex operator with vanishing total ghost number is given by

$$V_{q+q'=0}(z)=c(z)\delta[\gamma(z)]V_0(z) \ . \tag{2.7}$$

It is a BRST-invariant and primary field with dimension $\Delta=0$. It is customary to proceed through the bosonization formalism [16], where

$$\beta(z)=:e^{-\phi(z)}:\partial\xi(z), \quad \gamma(z)=\eta(z):e^{\phi(z)}: \ .$$

In this formalism the vertex (2.7) can be written as

$$V_{q+q'=0}(z)=c(z):e^{-\phi(z)}:V_0(z) \ .$$

We want here to observe that one could also adopt the formalism in which the (β,γ) system is not bosonized [18], thus having a possibility of checking the expressions of the picture-changed vertices computed in the two cases [17]. In comparing those expressions in both formalisms, one obtains interesting bosonization rules, which we have listed in Table I. Here we have put $\delta'=\partial\delta[\beta(z)]/\partial\beta(z)$.

In the following we will limit ourselves to show our results only in the bosonization formalism.

We can construct the vertex with $q+q'=1$, by using the picture-changing operation [16]

$$V_{q+q'=1}(z)=[Q_{\mathrm{BRST}},\xi(z)V_{q+q'=0}(z)] \ , \tag{2.8}$$

with

$$Q_{\mathrm{BRST}}=\oint dZ\, j_{\mathrm{BRST}}(Z) \ , \tag{2.9}$$

$$\begin{aligned} j_{\mathrm{BRST}}=&C(Z)[T_{\mathrm{matter}}(Z)+\tfrac{1}{2}T_{\mathrm{ghost}}(Z)] \\ &-\tfrac{3}{4}D\{C(Z)[DC(Z)]B(Z)\} \ , \end{aligned} \tag{2.10}$$

where $Z\equiv(z,\theta)$, $dZ\equiv dz\,d\theta/2\pi i$, $D\equiv\partial_\theta+\theta\partial_z$, $T_{\mathrm{matter}}(Z)\equiv\frac{1}{2}G(z)+\theta L(z)$, T_{ghost} is the energy-momentum tensor relative to the ghost system [16], and $C(Z)$ and $B(Z)$ are the ghost superfields

$$C(Z)=c(z)+\theta\gamma(z), \quad B(Z)=\beta(z)+\theta b(z) \ .$$

After the integration over θ, we are left with

$$Q_{\mathrm{BRST}}=Q_{\mathrm{BRST}}^{(q=1)}+Q_{\mathrm{BRST}}^{(q=0)}+Q_{\mathrm{BRST}}^{(q=-1)} \ , \tag{2.11}$$

where

$$\begin{aligned} Q_{\mathrm{BRST}}^{(q=1)}=\oint\frac{dz}{2\pi i}[&-c(z)L(z)-c(z)\partial c(z)b(z) \\ &-\tfrac{1}{2}\partial\phi(z)\partial c(z) \\ &+c(z)(\tfrac{1}{2}\{[\partial\phi(z)]^2+\partial^2\phi(z)\} \\ &+:\eta(z)\partial\xi(z):)] \ , \end{aligned}$$

$$Q_{\mathrm{BRST}}^{(q=0)}=\tfrac{1}{2}\oint\frac{dz}{2\pi i}\eta(z):e^{\phi(z)}:G(z) \ ,$$

$$Q_{\mathrm{BRST}}^{(q=-1)}=-\tfrac{1}{4}\oint\frac{dz}{2\pi i}\partial\eta(z)\eta(z):e^{2\phi(z)}:b(z) \ .$$

The commutator in Eq. (2.8) can be more easily comput-

TABLE I. Bosonization rules for some quantities appearing in the expressions of the lowest picture-changed vertices.

$\beta(z)$	$:e^{-\phi(z)}:\partial\xi(z)$	$\gamma(z)$	$\eta(z):e^{\phi(z)}:$
$H[\beta(z)]$	$\xi(z)$	$\delta[\beta(z)]$	$:e^{\phi(z)}:$
$\delta[\gamma(z)]$	$:e^{-\phi(z)}:$	$\delta'[\beta(z)]$	$\eta(z):e^{2\phi(z)}:$
$\gamma^2(z)$	$-\partial\eta(z)\eta(z):e^{2\phi}(z):$	$\gamma(z)\beta(z)$	$\partial\phi(z)$
$\delta''[\beta(z)]$	$-:e^{3\phi(z)}:\partial\eta(z)\eta(z)$	$\delta[\beta(z)]\partial\beta(z)$	$-\partial\xi(z)$

$\gamma(z)\delta[\beta(z)]$	$\frac{1}{2}\eta(z)\partial(:e^{2\phi(z)}:)+\partial\eta(z):e^{2\phi(z)}:$
$\gamma(z)\delta'[\beta(z)]$	$-\frac{1}{2}\partial^2\eta(z)\eta(z):e^{3\phi(z)}:-\frac{1}{3}\partial\eta(z)\eta(z)\partial(:e^{3\phi(z)}:)$
$\partial\gamma(z)\beta(z)$	$\frac{1}{2}[\partial\phi(z)]^2+:\eta(z)\partial\xi(z):$
$-\frac{1}{2}\partial^2\delta[\beta(z)]+:\partial\beta(z)\delta[\beta(z)]\gamma(z):$	$-:\partial\xi(z)\eta(z)e^{\phi(z)}:$

ed in the following way:

$$\oint_w \frac{dz}{z-w} O_{\rm PC}(z)V_{q+q'=0}(w)\ , \tag{2.12}$$

where

$$O_{\rm PC}(w)\equiv\oint_w \frac{dz}{2\pi i}j_{\rm BRST}(z)\xi(w) = O_{\rm PC}^{(q=1)}(w)+O_{\rm PC}^{(q=0)}(w)+O_{\rm PC}^{(q=-1)}(w) \tag{2.13}$$

and

$$O_{\rm PC}^{(q=1)}(z)=-c(z)\partial\xi(z)\ , \tag{2.14}$$

$$O_{\rm PC}^{(q=0)}(z)=\tfrac{1}{2}:e^{\phi(z)}:G(z)\ , \tag{2.15}$$

$$O_{\rm PC}^{(q=-1)}(z)=\tfrac{1}{4}\partial[\eta(z):e^{2\phi(z)}:b(z)]+\tfrac{1}{4}\partial\eta(z):e^{2\phi(z)}:b(z)\ .$$

The computation of Eq. (2.12) gives

$$V_{q+q'=1}(z)=c(z)V_1(z)-\tfrac{1}{2}\eta(z):e^{\phi(z)}:V_0(z)\ . \tag{2.16}$$

The state including the insertion of one picture-changing operator is obtained from $V_{q+q'=1}$ through

$$|q+q'=1\rangle=\lim_{z\to 0} V_{q+q'=1}(z)|\text{vacuum}\rangle\ . \tag{2.17}$$

We want here to observe that both $V_{q+q'=0}$ and $V_{q+q'=1}$ are BRST-invariant and primary fields with the conformal dimension $\Delta=0$.

It is relevant to stress that the content of the reparametrization ghosts is limited for these vertices to the only field $c(z)$. This is a peculiar feature of BRST-invariant vertex operators associated with physical states in bosonic string theory, where they are indeed primary fields with dimension $\Delta=0$ having the form $c(z)V(z)$. Hence this form for the picture-changed vertices is preserved also in superstring theory, with the only difference that in this case the field $V(z)$ contains not only the orbital degrees of freedom, like in the bosonic string, but also the superghost ones. This peculiarity also holds for successive states.

Starting from the vertex $V_{q+q'=1}$, one can construct analogously $V_{q+q'=2}$. We obtain

$$V_{q+q'=2}(z)=V_{q+q'=2}^{(q=1)}(z)+V_{q+q'=2}^{(q=0)}(z)+V_{q+q'=2}^{(q=-1)}(z)\ ,$$

with

$$V_{q+q'=2}^{(q=1)}(z)=c(z):e^{\phi(z)}:V_{5/2}(z)+\tfrac{1}{2}c(z)\partial^2(:e^{\phi(z)}:)V_0(z)+c(z)\partial(:e^{\phi(z)}:)\partial V_0(z) +c(z):\partial\xi(z)\eta(z)e^{\phi(z)}:V_0(z)-\tfrac{1}{2}\partial^2c(z):e^{\phi(z)}:V_0(z)\ , \tag{2.18}$$

$$\begin{aligned}V_{q+q'=2}^{(q=0)}(z)={}&\tfrac{3}{4}\partial^2\eta:e^{2\phi(z)}:V_1(z)+\partial\eta(z)\partial(:e^{2\phi}:)V_1(z)+\tfrac{1}{4}\eta(z):e^{2\phi(z)}:[\partial\phi(z)]^2V_1(z)\\&+\tfrac{1}{4}\eta(z):e^{2\phi(z)}:\partial^2\phi(z)V_1(z)+\tfrac{1}{4}\eta(z)\partial^2(:e^{2\phi(z)}:)V_1(z)+\tfrac{1}{4}\eta(z)\partial(:e^{2\phi(z)}:)V_2(z)\\&+\tfrac{1}{2}\eta(z):e^{2\phi(z)}:V_3(z)+\tfrac{1}{2}\eta(z):e^{2\phi(z)}::\partial b(z)c(z):V_1(z)\\&+\tfrac{1}{2}\{\partial[\eta(z):e^{2\phi(z)}:]+\partial\eta(z):e^{2\phi(z)}:\}:b(z)c(z):V_1(z)\ ,\end{aligned} \tag{2.19}$$

$$\begin{aligned}V_{q+q'=2}^{(q=-1)}(z)={}&-\tfrac{1}{6}\partial^3\eta(z)\eta(z):e^{3\phi(z)}:b(z)V_0(z)-\tfrac{1}{4}\partial^2\eta(z)\eta(z)\partial(:e^{3\phi(z)}:)b(z)V_0(z)\\&-\partial\eta(z)\eta(z)\{:e^{3\phi(z)}:[\partial\phi(z)]^2+\tfrac{1}{2}:e^{3\phi(z)}:\partial^2\phi(z)\}b(z)V_0(z)\\&-\tfrac{3}{8}\partial^2\eta(z)\eta(z):e^{3\phi(z)}:\partial b(z)V_0(z)-\tfrac{1}{3}\partial\eta(z)\eta(z)\partial(:e^{3\phi(z)}:)\partial b(z)V_0(z)\\&-\tfrac{1}{4}\partial\eta(z)\eta(z):e^{3\phi(z)}:\partial^2b(z)V_0(z)\ .\end{aligned} \tag{2.20}$$

It can be proved that the vertex $V_{q+q'=2}$ is BRST invariant.

The vertex $V_{5/2}$ in (2.18) is defined as the finite part of the operator-product expansion (OPE):

$$G(z)V_1(w)=\frac{V_0(w)}{(z-w)^2}+\frac{\partial V_0(w)}{z-w}+V_{5/2}(w)+\ \cdots$$

and satisfies

$$L(z)V_{5/2}(w)=\frac{5}{2}\frac{V_0(w)}{(z-w)^4}+2\frac{\partial V_0(w)}{(z-w)^3}+\frac{5}{2}\frac{V_{5/2}(w)}{(z-w)^2}+\frac{\partial V_{5/2}(w)}{z-w}+\text{regular part} .$$

Furthermore, the vertices V_2 and V_3 in Eq. (2.19) are defined by the OPE

$$G(z)V_0(w)=\frac{V_1(w)}{z-w}+V_2(w)+V_3(w)(z-w)+\cdots$$

and, respectively, satisfy

$$L(z)V_2(w)=2\frac{V_1(w)}{(z-w)^3}+2\frac{V_2(w)}{(z-w)^2}+\frac{\partial V_2(w)}{z-w}+\cdots ,$$

$$L(z)V_3(w)=\frac{7}{2}\frac{V_1(w)}{(z-w)^4}+3\frac{V_2(w)}{(z-w)^3}+3\frac{V_3(w)}{(z-w)^2}+\frac{\partial V_3(w)}{z-w}+\cdots .$$

$V_{q+q'=2}$ can be rewritten in such a way that the term proportional to $\partial^2 c(z)$ in Eq. (2.18) can be canceled and substituted by terms containing only $c(z)$ [17]. By exploiting the BRST invariance of the vertex, this can be achieved by adding to it the commutator with $Q_{\rm BRST}$ of a suitable conformal field with dimension $\Delta=0$ and with $q+q'=1$, as follows:

$$V^{*(q=1)}_{q+q'=2}(z)=V^{(q=1)}_{q+q'=2}(z)-[Q^{(q=1)}_{\rm BRST},:e^{\phi(z)}:\partial V_0(z)] , \tag{2.21}$$

where

$$[Q^{(q=1)}_{\rm BRST},:e^{\phi(z)}:\partial V_0(z)]=-\tfrac{1}{2}\partial^2 c(z):e^{\phi(z)}:V_0(z)-c(z)\partial[:e^{\phi(z)}:\partial V_0(z)] , \tag{2.22}$$

finally obtaining

$$V^{*(q=1)}_{q+q'=2}(z)=c(z)[:e^{\phi(z)}:V_{5/2}+\tfrac{1}{2}\partial^2(:e^{\phi(z)}:)V_0(z)+2\partial(:e^{\phi(z)}:)\partial V_0(z)+:\partial\xi(z)\eta(z)e^{\phi(z)}:V_0(z)+:e^{\phi(z)}:\partial^2 V_0(z)] . \tag{2.23}$$

Analogously, one has

$$\begin{aligned}V^{*(q=0)}_{q=q'=2}(z)\equiv{}& V^{(q=0)}_{q+q'=2}(z)-[Q^{(q=0)}_{\rm BRST},:e^{\phi(z)}:\partial V_0(z)]\\ ={}&\partial^2\eta(z):e^{2\phi(z)}:V_1(z)+\tfrac{3}{2}\partial\eta(z)\partial(:e^{2\phi(z)}:)V_1(z)+\tfrac{1}{2}\eta(z):e^{2\phi(z)}:[\partial\phi(z)]^2V_1(z)+\tfrac{1}{2}\eta(z):e^{\phi(z)}:\partial^2\phi(z)V_1(z)\\ &+\tfrac{1}{4}\eta(z)\partial^2(:e^{2\phi(z)}:)V_1(z)+\tfrac{1}{4}\eta(z)\partial(:e^{2\phi(z)}:)V_2(z)+\tfrac{1}{2}\partial\eta(z):e^{2\phi(z)}:\partial V_1(z)+\tfrac{1}{4}\eta(z)\partial(:e^{2\phi(z)}:)\partial V_1(z)\\ &+\tfrac{1}{2}\eta(z):e^{2\phi(z)}:\partial V_2(z)+\tfrac{1}{2}\eta(z):e^{2\phi(z)}::\partial b(z)c(z):V_1(z)+\tfrac{1}{2}\{\partial[\eta(z):e^{2\phi(z)}:]+\partial\eta(z):e^{2\phi(z)}:\}:b(z)c(z):V_1(z) .\end{aligned} \tag{2.24}$$

In the same way one can get the expression for $V^{*(q=-1)}_{(q+q'=2)}$.

The vertex $V^{*}_{q+q'=2}$ results to be primary with dimension $\Delta=0$. Once again, we find the same structure $c(z)V(z)$ as the vertex operators in bosonic string theory.

This property holds also for the vertex $V_{q+q'=3}$. Without giving the complicated structure of the vertex, we will limit ourselves to show the essential steps leading to this result.

In computing $V_{q+q'=3}$ we are faced, first of all, with the problem of the existence of a term with $q=2$, we will denote by $V^{(q=2)}_{q+q'=3}$, deriving from the application of $O^{(q=1)}_{\rm PC}$, defined in Eq. (2.14), on the term

$$c(z):\partial\xi(z)\eta(z)e^{\phi(z)}:V_0(z) ,$$

in Eq. (2.23). Indeed, this yields

$$V^{(q=2)}_{q+q'=3}(z)=\tfrac{1}{2}\partial^2 c(z)c(z)\partial\xi(z):e^{\phi(z)}:V_0(z) . \tag{2.25}$$

However $V^{(q=2)}_{q+q'=3}$ can be written itself as the commutator of a conformal field with $Q_{\rm BRST}$, since

$$[Q_{\rm BRST},-3c(w)\partial\xi(w):e^{\phi(w)}:\partial V_0(w)]=\tfrac{1}{2}\partial^2 c(w)c(w)\partial\xi(w):e^{\phi(w)}:V_0(w) .$$

The elimination of $V^{(q=2)}_{q+q'=3}$ from the expression of $V_{q+q'=3}$ is therefore straightforward.

The part with $q=1$, $V^{(q=1)}_{q+q'=3}$, is generated by the application of $O^{(q=0)}_{\rm PC}$, defined by Eq. (2.15), on $V^{*(q=1)}_{q+q'=2}$ and from the application of $O^{(q=1)}_{\rm PC}$ on $V^{*(q=0)}_{q+q'=2}$. As regards the former, it generates only terms proportional to $c(z)$, as one can see from Eq. (2.23); for the latter we can say the same when it acts on the terms in Eq. (2.24) containing $:b(z)c(z):$ or $:\partial b(z)c(z):$. The other terms in (2.24) generate different pieces containing $\partial^4 c(z)$, $\partial^3 c(z)$, and $\partial^2 c(z)$; these can be eliminated by adding to $V^{(q=1)}_{q+q'=3}$ the commutator with $Q_{\rm BRST}$ of all the possible conformal fields with $\Delta=0$ and $q+q'=2$ that can be constructed with the field $:e^{2\phi(z)}:$ and the matter fields $V_1(z)$, $V_2(z)$, and $V_3(z)$.

Hence $V^{(q=1)}_{q+q'=0}$, $V^{(q=1)}_{q+q'=1}$, $V^{(q=1)}_{q+q'=2}$, and $V^{(q=1)}_{q+q'=3}$ can be finally put in the form $c(z)V(z)$.

This property plausibly remains true for higher picture-changed vertices, since the procedure we have followed is quite general and suitable to be extended, even if more and more painful at each step.

III. N-POINT g-LOOP VERTEX FOR THE (b,c) SYSTEM

The starting point for the construction of $V_{N;g}$ for the (b,c) system is the vertex $V_{N;0}$ [5]:

$$V_{N;0}=\prod_{I=1}^{N}({}_I\langle q=3|)\exp\left[-\sum_{\substack{I,J=1\\ I\neq J}}^{N}\sum_{n=2}^{\infty}\sum_{i=-1}^{\infty}c_n^{(I)}E_{ni}(U_IV_J)b_i^{(J)}\right]$$

$$\times\prod_{r=-1}^{1}\left[\sum_{I=1}^{N}\sum_{s=-1}^{1}E_{rs}(V_I)b_s^{(I)}\right]\prod_{s=1}^{N-3}\sum_{I=1}^{s+1}\sum_{n=-1}^{1}b_n^{(I)}e_n(U_{E_s}V_I)\ , \tag{3.1}$$

with

$$e_n(U_{E_s}V_I)\equiv E_{0n}(U_{E_s}V_I)-E_{-1n}(U_{E_s}V_I)\ , \tag{3.2}$$

where $U_I=\Gamma V_I^{-1}$. In Eq. (3.1), $\Gamma(z)=1/z$ and the matrix E_{nm} is defined by

$$\begin{aligned}E_{nm}(\gamma)&=\frac{1}{(m+1)!}\partial_z^{m+1}\{[\gamma(z)]^{n+1}[\gamma'(z)]^{-1}\}|_{z=0}\\&=\oint_0 dz\frac{1}{z^{m+2}}[\gamma(z)]^{n+1}[\gamma'(z)]^{-1}\ ,\end{aligned} \tag{3.3}$$

with $\gamma(z)=(az+b)/(cz+d)$ and $n,m=-2,-1,\ldots,\infty$. It is an infinite-dimensional representation of the projective group with conformal weight -1. Furthermore, $V_I^{-1}(z)$ is a projective transformation corresponding to a choice of local coordinates vanishing at the puncture z_I.

The matrix E_{nm} defined in (3.3) provides an infinite-dimensional representation of the projective group with conformal weight -1 also when the two indices n,m are either $n,m\geq 2$ or restricted to the zero-mode subspace, i.e., $-1\leq n,m\leq 1$. In particular, in this latter case, one has

$$E_{nm}(\gamma)=\frac{1}{ad-bc}\begin{bmatrix}d^2 & 2cd & c^2\\ bd & ad+bc & ac\\ b^2 & 2ad & a^2\end{bmatrix}\ . \tag{3.4}$$

We want to observe that det $E(\gamma)=1$ for any γ. From (3.4) one can straightforwardly compute $e_{-1}(\gamma)$, $e_0(\gamma)$, and $e_1(\gamma)$, defined in (3.2):

$$\begin{aligned}e_{-1}(\gamma)&=\frac{d(b-d)}{ad-bc}\ ,\\ e_0(\gamma)&=\frac{1}{ad-bc}[d(a-c)+c(b-d)]\ ,\\ e_1(\gamma)&=\frac{c(a-c)}{ad-bc}\ .\end{aligned} \tag{3.5}$$

These formulas will be useful in computing the zero-mode contribution to amplitudes.

The technique followed here for constructing $V_{N;g}$ is the same as the one discussed in great detail in Ref. [8]. The main difference in our computation is the use of the BRST-invariant twisted propagator [5]

$$P_t(x)=(b_0-b_1)P(x)=P(x)(b_0-b_{-1})\ , \tag{3.6}$$

where

$$P(x)=x^{L_0}\Omega(1-x)^W\ .$$

The twist operator Ω and W are given by

$$\Omega=e^{L_{-1}}(-1)^{L_0-p^2/2}$$

and

$$W=L_0-L_1\ .$$

The N-point g-loop vertex $V_{N;g}$ is obtained starting from $V_{N+2g;0}$, in which we will label the first N legs with an index I, running from 1 to N, while the remaining $2g$ legs will be distinguished into g "odd" legs and g "even" legs, labeled, respectively, by $N+2\mu-1$ and $N+2\mu$, $\mu=1,\ldots,g$ being a loop index. After the insertion of the propagator (3.6) on the "odd" leg $N+2\mu-1$, we identify the latter with the leg $N+2\mu$, following the notation of Fig. 1, in such a way that g loops are formed. In the following, for the sake of simplicity, we omit N in labeling both the "odd" and "even" legs.

$V_{N;g}$ is then defined

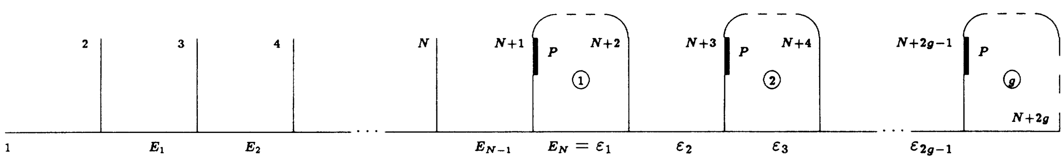

FIG. 1. Sewing procedure for the construction of the N-point g-loop vertex $V_{N;g}$.

$$V_{N;g}=\prod_{\mu=1}^{g}\mathrm{Tr}_{(2\mu-1,2,\mu)}[V^{\dagger}_{(N+2g),0}P(x)(b_0^{(2\mu-1)}-b_{-1}^{(2\mu-1)})], \qquad (3.7)$$

where $\mathrm{Tr}_{(2\mu-1,2\mu)}$ means a trace in the Fock spaces of legs $2\mu-1$ and 2μ [8]. $V^{\dagger}_{(N+2g);0}$ is obtained from $V_{(N+2g);0}$ by changing the vacuum and oscillators for the legs 2μ, according to

$$\begin{aligned}&{}_{2\mu}\langle q=3|\to|q=3\rangle_{2\mu}\,,\\ &b_n^{(2\mu)}\to b_{-n}^{(2\mu)},\quad c_n^{(2\mu)}\to c_{-n}^{(2\mu)}\,.\end{aligned} \qquad (3.8)$$

In addition, the insertion of the propagator on the leg $2\mu-1$ has the following effect:

$$V_{2\mu-1}\to\tilde V_{2\mu-1}=V_{2\mu-1}P(x),\quad U_{2\mu-1}\to\tilde U_{2\mu-1}=\Gamma(\tilde V_{2\mu-1})^{-1}\,.$$

By performing explicitly the trace in (3.7), we obtain

$$\begin{aligned}V_{N;g}=\prod_{\alpha}{}'\prod_{n=2}^{\infty}(1-k_\alpha^n)^2\prod_{I=1}^{N}({}_I\langle q=3|)\exp\left[\sum_{I=1}^{N}\sum_{n=2}^{\infty}\sum_{r,s=-1}^{1}c_n^{(I)}E_{nr}(U_I)E_{rs}(V_I)b_s^{(I)}\right]\\ \times\exp\left[-\sum_{I,J=1}^{N}\sum_{\alpha}{}'\sum_{n,m=2}^{\infty}\sum_{i=-1}^{\infty}c_n^{(I)}E_{nm}(U_IT_\alpha)E_{mi}(V_J)b_i^{(J)}\right]\mathcal{J}_{N;g}\,,\end{aligned} \qquad (3.9)$$

where, in the last line, T_α are elements of the Schottky group, generated by

$$S_\mu=\tilde V_{2\mu-1}U_{2\mu}\,,$$

k_α their multipliers, and $\sum_\alpha'$ means that the identity is excluded if $I=J$. The quantity $\mathcal{J}_{N;g}$ encloses the whole contribution of zero modes; it can be rewritten as an integral over fermionic Grassmann variables ϕ's, $s=-1,0,1$, in one-to-one correspondence with the oscillators b_s [8]. The introduction for each Schottky generator S_μ of a diagonalizing matrix L_μ, such that

$$L_\mu S_\mu L_\mu^{-1}=S_\mu^d,\quad E_{rs}(S_\mu^d)=k_\mu^s\delta_{rs}\,,$$

allows one to write $\mathcal{J}_{N;g}$ as

$$\mathcal{J}_{N;g}=\prod_{s=1}^{N-1}\sum_{I=1}^{s+1}\sum_{n=-1}^{1}b_n^{(I)}e_n(U_{E_s}V_I)\vartheta_{N;g}\,,$$

where

$$\vartheta_{N;g}=\int\prod_{r=-1}^{1}\prod_{\mu=1}^{g}d\phi_r^{(\mu)}\prod_{\mu=1}^{g}\delta_\mu\prod_{r=-1}^{1}\delta_r\prod_{\nu=1}^{2g-2}\Delta^\nu\exp\left[\sum_{I=1}^{N}\sum_{\mu=1}^{g}\sum_{\alpha}\sum_{rs=-1}^{1}\sum_{nm=2}^{\infty}c_n^{(I)}E_{nm}(U_IT_\alpha)E_{mr}(S_\mu)E_{rs}(L_\mu^{-1})\phi_s^{(\mu)}\right],$$

with

$$\delta_\mu = \sum_{r=-1}^{1} e_r(U_{2\mu}L_\mu^{-1})\phi_r^{(\mu)}, \quad \mu=1,2,\ldots,g ,$$

$$\delta_r = E_{r-1}(V_1)b_{-1}^{(1)} + \sum_{\mu=1}^{g}\sum_{s=-1}^{1} E_{rs}(L_\mu^{-1})(1-k_\mu^s)\phi_s^{(\mu)} , \quad r=-1,0,1 ,$$

$$\Delta^1 = \sum_{r=-1}^{1} [e_r(U_{\varepsilon_1}L_1^{-1})k_1^r\phi_r^{(1)}] ,$$

$$\Delta^2 = \sum_{r=-1}^{1} [e_r(U_{\varepsilon_2}L_1^{-1})(1-k_1^r)\phi_r^{(1)}] ,$$

$$\Delta^3 = \sum_{r=-1}^{1} [e_r(U_{\varepsilon_3}L_1^{-1})(1-k_1^r)\phi_r^{(1)} - e_r(U_{\varepsilon_3}L_2^{-1})k_2^r\phi_r^{(2)}] ,$$

$$\Delta^4 = \sum_{r=-1}^{1} [e_r(U_{\varepsilon_4}L_1^{-1})(1-k_1^r)\phi_r^{(1)} + e_r(U_{\varepsilon_4} - L_2^{-1})(1-k_2^r)\phi_r^{(2)}] ,$$

$$\Delta^{2g-3} = \sum_{s=-1}^{1} [e_s(U_{\varepsilon_{2g-3}}L_1^{-1})(1-k_1^s)\phi_s^{(1)} + e_s(U_{\varepsilon_{2g-3}}L_2^{-1})(1-k_2^s)\phi_s^{(2)} + \cdots - e_s(U_{\varepsilon_{2g-3}}L_{i-1}^{-1})k_{i-1}^s\phi_s^{(i-1)}] ,$$

$$\Delta^{2g-2} = \sum_{s=-1}^{1} [e_s(U_{\varepsilon_{2g-2}}L_1^{-1})(1-k_1^s)\phi_s^{(1)} + e_s(U_{\varepsilon_{2g-2}}L_2^{-1})(1-k_2^s)\phi_s^{(2)} + \cdots + e_s(U_{\varepsilon_{2g-2}}L_{g-1}^{-1})(1-k_{g-1}^s)\phi_s^{(g-1)}] .$$

We note that $\mathcal{J}_{N;g}$, being the integral of the product of $3g+1$ δ functions over $3g$ Grassmann variables, yields just one δ function; this, together with the other $N-1$ ones in $\mathcal{J}_{N;g}$, provides the right number of δ functions, i.e., one for each external leg, as required by the symmetry of $V_{N;g}$.

We want to remark here that when we add the μth loop we add three integration variables $\phi_s^{(\mu)}$, $s=-1,0,1$; furthermore, three new δ functions appear such that, once integrated, we are left with one overall; they are one extra δ_μ, $\Delta^{2\mu-3}$, and $\Delta^{2\mu-2}$.

We can observe that performing explicitly the integration over all the ϕ's will give to $\mathcal{J}_{N;g}$ the form

$$\mathcal{J}_{N;g} = K\left[\prod_{s=1}^{N-1}\sum_{I=1}^{s+1}\sum_{n=-1}^{1} b_n^{(I)}e_n(U_{E_s}V_I)\right]\delta\, e^{\mathcal{D}} , \qquad (3.10)$$

where K is a constant and δ is the only δ function left out by the integration over ϕ's, having the form

$$\delta = \sum_{I=1}^{N}\sum_{s=-1}^{1} b_s^{(I)}\mathcal{A}_s^{(I)} ,$$

while the exponent $\mathcal{D}$ in (3.10) has the following structure in terms of oscillators:

$$\mathcal{D} = \sum_{I,J=1}^{N}\sum_{n=2}^{\infty}\sum_{s=-1}^{1} c_n^{(I)}D_{ns}b_s^{(J)} ,$$

where the quantity D_{ns} results to be dependent on the sewing procedure, so making the vertex not dual as an operator, but as we will see shortly, the exponential of $\mathcal{D}$ will never give a contribution when the vertex is saturated on physical states.

IV. (b,c)-SYSTEM CONTRIBUTION TO g-LOOP AMPLITUDES

In order to compute the reparametrization ghost contribution to g-loop superstring amplitudes, we have to saturate $V_{N;g}$ on suitable picture-changed states, since the background anomaly for the superghost number must be considered. In particular, for N-point g-loop amplitudes, we have to insert $N+2(g-1)$ picture-changing operators. This follows from the following consideration. Since the condition

$$\sum_{i=1}^{N} q_i' = 2(g-1)$$

must be satisfied, we can, for example, saturate $V_{N;g}$ with $N-2$ states $|q'=0\rangle$ and with two states $|q'=g-1\rangle$; but a state $|q'=0\rangle$ is equivalent to the insertion of one picture-changing operator on an external state, while a state $|q'=g-1\rangle$ is equivalent to the insertions of g picture-changing operators, taking into account that the lowest state has $q'=-1$.

The insertions of the picture-changing operators can be accomplished in several ways, in principle all equivalent. However, we will consider picture-changed physical states up to $q+q'=3$, computed in Sec. III. Since the number of picture-changing operators to be inserted must be $\leq 3N$, the use limited to those vertices gives a restriction on the values of N, which must be $\geq(g-1)$. We stress again that this restriction disappears if one assumes as general the property of the part with $q=1$ of picture-changed vertices to be written in the form $c(z)V(z)$.

Because of the structure of the N fermionic δ functions present in (3.10), the terms having $q\neq 1$ in the picture-changed states never contribute when $V_{N;g}$ is saturated with them.

As we have already observed, since the terms $q=1$ contain only the $c(z)$ field, a picture-changed state corresponds to a physical state $|q=1\rangle = c_1|q=0\rangle$, just as in the bosonic string theory. Therefore this is equivalent to the purely bosonic string case, where the vertex is saturated on N physical states $|q=1\rangle$.

Saturating $V_{N;g}$ on N physical states $|q=1\rangle$ amounts to evaluating the contribution coming from $\mathcal{J}_{N;g}$. The terms which contribute are only those containing the oscillators $b_{-1}^{(I)}$; in particular, the exponential in (3.10) does not give any contribution. Therefore we have to evaluate the determinant of the $N\times N$ matrix built out of the coefficients of $b_{-1}^{(I)}$ in the δ functions:

$$\begin{vmatrix} e_{-1}(U_{E_1}V_1) & e_{-1}(U_{E_1}V_2) & 0 & \cdots & 0 & 0 \\ e_{-1}(U_{E_2}V_1) & e_{-1}(U_{E_2}V_2) & e_{-1}(U_{E_2}V_3) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ e_{-1}(U_{E_{N-1}}V_1) & e_{-1}(U_{E_{N-1}}V_2) & e_{-1}(U_{E_{N-1}}V_3) & \cdots & e_{-1}(U_{E_{N-1}}V_{N-1}) & e_{-1}(U_{E_{N-1}}V_N) \\ \mathcal{A}_{-1}^{(1)} & \mathcal{A}_{-1}^{(2)} & \mathcal{A}_{-1}^{(3)} & \cdots & \mathcal{A}_{-1}^{(N-1)} & \mathcal{A}_{-1}^{(N)} \end{vmatrix} .$$

Because of the fact that $e_{-1}(U_{E_r}V_1)=0$ for any r, it is easy to see that only the term $\mathcal{A}_{-1}^{(1)}$ will contribute to the final result. This means that, in order to compute this quantity, we can set from the beginning to zero the coefficients of $b_s^{(I)}$ with $s\neq -1$ or $I\neq 1$. By so doing, computing the integration over all the variables ϕ for $\mathcal{I}_{N;g}$ is drastically simplified. Performing such an integration means in fact using $3d$ of the $3g+1$ fermionic δ functions and solving them suitably with respect to the variables ϕ.

By using the results

$$e_0(U_{2\mu}L_\mu^{-1})=1,\quad e_1(U_{2\mu}L_\mu^{-1})=0,\quad e_1(U_{\varepsilon_{2\mu-1}}L_\mu)=0 ,$$

we finally get

$$\mathcal{A}_{-1}^{(1)}=E_{0-1}(L_gV_1)(1-k_g^{-1})\prod_{\mu=1}^{g}(1-k_\mu)$$
$$\times\prod_{\mu=2}^{g}\left[e_{-1}(U_{\varepsilon_{2\mu-3}}L_{\mu-1}^{-1})\frac{1}{k_{\mu-1}}-e_0(U_{\varepsilon_{2\mu-3}}L_{\mu-1}^{-1})e_{-1}(U_{2\mu-2}L_{\mu-1}^{-1})\right]e_1(U_{\varepsilon_{2\mu-2}}L_{\mu-1}^{-1}) .$$

Our aim is now to rewrite the result in terms of the variables z. We make use of the relations

$$E_{0-1}(L_gV_1)=\frac{z_2-z_{2g}}{z_1-z_2}\,\frac{z_{2g-1}-z_1}{z_{2g-1}-z_{2g}}\,\frac{1}{1-k_g} ,$$

$$e_{-1}(U_{\varepsilon_{2\mu-3}}L_{\mu-1}^{-1})=\frac{z_{2\mu-2}}{\gamma_{\mu-1}k_{\mu-1}-1}\,\frac{z_{2\mu-3}-z_{2\mu-2}}{z_{2\mu-1}-z_{2\mu-2}}\,\frac{z_1-z_{2\mu-1}}{z_1-z_{2\mu-2}}(k_{\mu-1}-\omega) ,$$

$$e_0(U_{\varepsilon_{2\mu-3}}L_{\mu-1}^{-1})=1 ,$$

$$e_{-1}(U_{\varepsilon_{2\mu-2}}L_{\mu-1}^{-1})=z_{2\mu-2}\frac{z_{2\mu-1}-z_{2\mu-3}}{z_{2\mu-2}-z_{2\mu-1}}\,\frac{1}{\gamma_{\mu-1}k_{\mu-1}-1} ,$$

$$e_1(U_{\varepsilon_{2\mu-2}}L_{\mu-1}^{-1})=\frac{z_1-z_{2\mu-2}}{z_1-z_{2\mu-1}}\,\frac{z_{2\mu-2}-z_{2\mu-1}}{z_{2\mu-3}-z_{2\mu-2}}\,\frac{1}{z_{2\mu-2}}\,\frac{\gamma_{\mu-1}k_{\mu-1}-1}{k_{\mu-1}-1} ,$$

where we use the notation

$$\gamma_\mu\equiv\frac{z_{2\mu-1}-z_{2\mu}}{z_{2\mu+1}-z_{2\mu}},\quad \omega\equiv\frac{z_1-z_{2\mu-3}}{z_1-z_{2\mu-1}}\,\frac{z_{2\mu-2}-z_{2\mu-1}}{z_{2\mu-2}-z_{2\mu-3}} .$$

We also note that, because of our definition of the sewed legs in the tree vertex,

$$\varepsilon_i\equiv E_{N-1+i} ;$$

therefore, in terms of the z's, we rename

$$z_{i+1}\to z_{N+i}\quad\text{and}\quad z_{i+2}\to z_{N+i+1} .$$

Then we have the following expression for $\mathcal{A}_{-1}^{(1)}$:

$$\mathcal{A}_{-1}^{(1)}=\frac{z_1-z_{N+1}}{z_1-z_2}\,\frac{z_2-z_{2g}}{z_1-z_{2g}}\prod_{\mu=1}^{g}\left[\frac{1-k_\mu}{k_\mu}\,\frac{z_{2\mu+1}-z_{2\mu}}{z_{2\mu}-z_{2\mu-1}}\right] . \tag{4.1}$$

We add now the first $N-1$ δ functions, and we evaluate the determinant yielding the complete zero-mode contribution to the string amplitudes:

$$\mathcal{C}_{N;g}=\prod_{i=1}^{N-1}e_{-1}(U_{E_i}V_{i+1})\mathcal{A}_{-1}^{(1)}$$
$$=\prod_{i=1}^{N}\frac{z_i-z_{i+2}}{z_i-z_{i+1}}\,\frac{z_1-z_2}{z_1-z_{N+1}}\,\frac{z_N-z_{N+1}}{z_N-z_{N+2}}\mathcal{A}_{-1}^{(1)} .$$

Finally, we get

$$\mathcal{C}_{N;g}=\frac{z_N-z_{N+1}}{z_N-z_{N+2}}\,\frac{z_2-z_{2g}}{z_1-z_{2g}}$$
$$\times\prod_{\mu=1}^{g}\frac{1-k_\mu}{k_\mu}\prod_{i=1}^{N}\frac{z_i-z_{i+2}}{z_i-z_{i+1}}\prod_{\mu=1}^{g}\frac{z_{2\mu+1}-z_{2\mu}}{z_{2\mu}-z_{2\mu-1}} . \tag{4.2}$$

We may still rewrite this result in order to compare it with the expression in [5]. By using

$$\alpha_\mu \equiv -\frac{(z_{2\mu-1}-z_{2\mu-2})(z_{2\mu}-z_{2\mu+1})}{(z_{2\mu-1}-z_{2\mu+1})(z_{2\mu}-z_{2\mu-2})},$$

we have

$$\prod_{\mu=1}^{g} \frac{z_{2\mu+1}-z_{2\mu}}{z_{2\mu}-z_{2\mu-1}} \frac{1}{-\alpha_\mu} = \prod_{i=N+1}^{N+2g} \frac{z_i-z_{i+2}}{z_i-z_{i+1}} \frac{z_N-z_{N+2}}{z_N-z_{N+1}} \frac{z_{N+2g}-z_1}{z_{N+2g}-z_2}.$$

This allows us to set, at the end,

$$\mathcal{C}_{N;g} = \prod_{\mu=1}^{g} (1-k_\mu) \left[-\frac{\alpha_\mu}{k_\mu} \right] \prod_{i=1}^{N+2g} \frac{z_i-z_{i+2}}{z_i-z_{i+1}}, \quad (4.3)$$

which reproduces exactly the (b,c)-system contribution to bosonic amplitudes in [5].

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