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Dynamics of plane-symmetric thin walls in general relativity

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Plane walls (including plane domain walls) without reflection symmetry are studied in the framework of Einstein's general relativity. Using the distribution theory, all the Einstein field equations and Bianchi identities are split into two groups: one holding in the regions outside of the wall and the other holding at the wall. The Einstein field equations at the wall are found to take a very simple form, and given explicitly in terms of the discontinuities of the metric coefficients and their derivatives. The Bianchi identities at the wall are also given explicitly. Using the latter, the interaction of a plane wall with gravitational waves and some specific matter fields is studied. In particular, it is found that, when a gravitational plane wave passes through a wall, if the wall has no reflection symmetry, the phenomena, such as reflection, stimulation, or absorption, in general, occur. It is also found that, unlike for gravitational waves, a massless scalar wave or an electromagnetic wave continuously passes through a wall without any reflection. The repulsion and attraction of a plane wall are also studied. It is found that the acceleration of an observer who is at rest relative to the wall usually consists of three parts: one is due to the force produced by the wall, the second is due to the force produced by the space-time curvature, which is zero if the wall has reflection symmetry, and the last is due to the accelerated motion of the wall. As a result, a repulsive (attractive) plane wall may not be repulsive (attractive) at all. Finally, the collision and interaction among the walls are studied.

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I. INTRODUCTION

Topological defects, such as domain walls, cosmic strings, monopoles, and textures, formed before or during the inflationary epoch of the Universe are usually believed to have been inflated away. The only relics that we could be able to observe today are those formed after inflation. However, quite recently Linde and Lyth [1], and Basu, Guth, and Vilenkin [2] have argued that, due to quantum-mechanical tunneling, such defects could be formed during inflation. Thus, the defects corresponding to preinflationary phase transitions can still be present after inflation. Unlike the ones formed after inflation, the ones formed during inflation could be exponentially large and heavy [1].

On the other hand, the newly proposed inflationary models [3] have been becoming involved with more and more matter fields in order to overcome some flaws inhabited in the old ones [4]. Once they are formed, the topological defects will interact with those matter fields as well as the gravitational fields generated by them. Thus, the significance of studying these interactions is becoming more evident.

Nevertheless, because of the mathematical complexity of the problems concerned, the work in this direction is frequently restricted to some very simple cases. For example, for plane domain walls one usually assumes that the walls are planar [5], while for spherically symmetric domain walls or bubbles, the metrics outside and inside of a bubble are either the Schwarzschild, de Sitter, or Reissner-Nordström ones [6]. However, these simplications are based on technical reasons rather than on physical ones. Therefore, it is important to investigate these defects in a more general case.

As a first step to the above problem, recently we have studied a quite general class of space-times with plane symmetry, in which a plane thin wall has been assumed present [7]. It has been found that a thin plane domain wall is always repulsive whether or not the space-time out of the wall is curved. It has been also found that such a domain wall does not absorb or reflect any gravitational radiation. A gravitational wave just simply passes through a wall, as though the wall did not exist at all. However, we have found that, when a matter wave passes through a domain wall, the stimulation, reflection, or absorption, in general, indeed occur.

Considering the above problems more carefully, we will find that in deriving the above conclusions we had made several assumptions, which can be summarized as follows: (a) First, the space-time was assumed to be plane symmetric. (b) Second, the thickness of the walls was assumed to be negligible compared with any other physical sizes involved. Consequently, the "thin-wall" approximation was used. The justification and applicability of this assumption were studied by several authors [8], and some perturbation theories were developed [9]. (c) Third, the wall was reflection symmetric. (d) Fourth, the wall was "static." By "static" we mean that the wall is located on a fixed hypersurface during all the time of its evolution, and does not have accelerated motion in the direction perpendicular to the wall. In review of the above assumptions, one might argue that the conclusions obtained in Ref. [7] perhaps are due to the high symmetries assumed.

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In this paper we shall study plane walls without the last two assumptions mentioned above. It should be noted that a particular solution representing a static planar domain wall without reflection symmetry was found recently by Tomita [10]. It was shown that, unlike the reflection-symmetric case [11], non-reflection-symmetric domain walls can be obtained by "gluing" two static vacuum Kasner metrics together with different Kasner exponents. In his paper, a generalization of the solution to the case where the space-time contains more than one wall was also discussed.

The rest of the paper proceeds as follows. The properties of the space-times with plane symmetry are briefly reviewed in Sec. II. The space-times containing a thin plane wall without reflection symmetry are studied in Sec. III, and the Einstein field equations at the wall are given explicitly in terms of the discontinuities of the metric coefficients and their derivatives. In Sec. IV, the repulsion and attraction of a plane wall are investigated. Following it, i.e., in Sec. V, the "generalized" Bianchi identities are studied. In particular, they are divided into two groups, one of which holds in the regions out of the wall, the other holds on the wall. It is the latter that enables us to study the interaction of a plane wall with gravitational waves and matter fields. Because of the great importance of domain walls to the early Universe, in Sec. VI we investigate the interaction of a plane domain wall with gravitational fields and several specific matter fields, which are very interesting from the point of view of physics. In Sec. VII, the previous studies are extended to the space-times containing more than one wall, so that the collision and interaction among the walls can be studied. Finally, our main results obtained in this paper are summarized in Sec. VIII.

The notation and convention to be used in this paper will closely follow those specified in Ref. [7]. For example, the units will be chosen so that $8\pi G=1=c$, where G is the gravitational constant, and c the velocity of light.

II. THE SPACE-TIMES WITH PLANE SYMMETRY

To facilitate our discussions, it is useful first to review some general properties of the space-times with plane symmetry. For the details, one may see Refs. [7,12] and the references therein.

The metric of the space-time with two commuting spacelike Killing vectors can be cast into the form [13]

$$ds^2=2e^{-M}du\,dv-e^{-U}(e^{V}\cosh W\,dx^2-2\sinh W\,dx\,dy+e^{-V}\cosh W\,dy^2)\ , \tag{2.1}$$

where M, U, V, and W are functions of only the null coordinates u and v, the two Killing vectors are ∂_x and ∂_y, and $\{x^\mu\}\equiv\{u,v,x,y\}$, where $\mu=0,1,2,3$ and $u,v,x,y\in(-\infty,+\infty)$.

When $V=0=W$, the corresponding space-times are often said to have planar symmetry [14].

It is convenient to introduce a timelike and a spacelike coordinate via the relations

$$u=\frac{t+z}{\sqrt{2}},\quad v=\frac{t-z}{\sqrt{2}}\ . \tag{2.2}$$

The metric (2.1) has been intensively studied recently, and often used to describe cosmological models [15] as well as interacting gravitational plane waves [12,13,16].

Choosing a null tetrad as the one given by Eqs. (5) and (6) in Ref. [7] (hereafter, it will be referred to as paper I), we find that the only nonvanishing Weyl and Ricci scalars are Ψ_0,Ψ_2,Ψ_4, and $\Phi_{00},\Phi_{02},\Phi_{11},\Phi_{22},\Lambda$, respectively. The former represents the gravitational fields, while the latter the matter fields. The relations between the Ψ_A's and the Weyl conformal tensor $C_{\mu\nu\lambda\rho}$ have been given in Ref. [12], and the ones between the Φ_{ij}'s, Λ, and the Einstein tensor $G_{\mu\nu}$ given in paper I. The Einstein field equations

$$G_{\mu\nu}=\kappa T_{\mu\nu} \tag{2.3}$$

can be written in terms of the Φ_{ij}'s and Λ, once the matter fields are specified, where $\kappa=8\pi G/c^4=1$ in the present case.

In addition to the Einstein field equations (2.3), we also have the so-called conservation equations of the energy and stress of sources

$$T_{\mu\nu;\lambda}g^{\nu\lambda}=0\ , \tag{2.4}$$

which is a direct consequence of the combination of Eq. (2.3) and the Bianchi identities [17]

$$C_{\mu\nu\lambda\rho}{}^{;\rho}=j_{\mu\nu\lambda}\ , \tag{2.5}$$

where $j_{\mu\nu\lambda}$ is defined as

$$j_{\mu\nu\lambda}\equiv R_{\lambda[\mu;\nu]}-\tfrac{1}{6}g_{\lambda[\mu}R_{;\nu]}\ , \tag{2.6}$$

and satisfies the "conservation" equations

$$j_{\mu\nu\lambda}{}^{;\lambda}=0\ . \tag{2.7}$$

A semicolon denotes covariant differentiation.

Equation (2.5) represents the true interaction between gravitational fields $C_{\mu\nu\lambda\rho}$ and the matter fields $R_{\mu\nu}$ [17].

Our main task in this paper is to exploit Eqs. (2.3)–(2.6) for the space-times containing plane walls in some details by using the distribution theory.

Before proceeding, we first note the following facts. The Einstein field equations contain the second-order derivatives of the metric coefficients, while the Bianchi identities contain the third-order derivatives. So, to make these equations meaningful, one usually requires the metric coefficients to be at least C^3, and any coordinate transformations to be at least C^4. However, these requirements are too strong for our present problem. In the next section, we shall consider the metric (2.1) in the case where we are allowed to study plane walls as well as their interaction with gravitational waves and surrounding matter fields.

III. PLANE-SYMMETRIC SPACE-TIMES CONTAINING PLANE WALLS

As noted previously, to have plane-symmetric space-times containing plane walls, we need to relax some restrictions imposed usually on the metric coefficients [18,19]. First, instead of requiring that they be at least C^3 in the whole space-time, we require that they be piecewise C^3, and across some hypersurfaces be C^0. For the sake of convenience, in this section and Secs. IV–VI, we shall consider the cases where there is only one such hypersurface in a space-time, and in Sec. VII we will come back to this issue.

Let Σ denote the hypersurface across which the metric coefficients are C^0, and be described by

$$\varphi(t,z)\equiv z-P(t)=0\ , \tag{3.1}$$

where $P(t)$ is a smooth function of only t. Then, the normal vector to the surface can be defined by

$$\zeta_\mu\equiv\frac{\partial\varphi(t,z)}{\partial x^\mu}=\frac{1}{\sqrt{2}}\{1-\dot{P},-(1+\dot{P}),0,0\}\ , \tag{3.2}$$

where an overdot denotes an ordinary differentiation with respect to t.

Since we are concerned with plane walls, we assume that the hypersurface Σ is timelike, i.e., $\zeta_\lambda\zeta^\lambda<0$, which is equivalent to

$$1-\dot{P}^2>0\ . \tag{3.3}$$

We also assume that the hypersurface Σ divides the space-time Ω into two parts Ω^+ and Ω^-, where $\Omega^+\equiv\{x^\mu:\varphi\geq 0\}$ and $\Omega^-\equiv\{x^\mu:\varphi\leq 0\}$. With this notation, the above assumptions now are equivalent to saying that the restrictions $g^{\pm}_{\mu\nu}=g_{\mu\nu}|_{\Omega^\pm}$ are at least C^3, and across Σ $g_{\mu\nu}$ is at least C^0.

Under the above assumptions, it can be shown that the Einstein field equations (2.3) and the Bianchi identities (2.5), as well as their contractions (2.4), hold in the sense of distributions [7,18,20].

Following paper I, we write any function $f(t,z)$, which is C^3 in $\Omega^\pm$ and C^0 at Σ as

$$f(t,z)=f^+(t,z)H(\varphi)+f^-(t,z)[1-H(\varphi)]\ , \tag{3.4}$$

where $H(\varphi)$ denotes the Heaviside function, defined by

$$H(\varphi)\equiv\begin{cases}1, & \varphi\geq 0,\\ 0, & \varphi<0\ .\end{cases} \tag{3.5}$$

Then, it can be shown that the following holds in the sense of distributions [7]:

$$\begin{aligned}
&\frac{\partial f(t,z)}{\partial u}=f^D_{,u}(t,z),\quad \frac{\partial f(t,z)}{\partial v}=f^D_{,v}(t,z)\ ,\\
&\frac{\partial^2(t,z)}{\partial u^2}=f^D_{,uu}(t,z)+\frac{1}{2}(1-\dot{P})^2[f_{,\varphi}]^-\delta(\varphi)\ ,\\
&\frac{\partial^2 f(t,z)}{\partial u\partial v}=\frac{\partial^2 f(t,z)}{\partial v\partial u}\\
&\qquad=f^D_{,uv}(t,z)-\frac{1}{2}(1-\dot{P}^2)[f_{,\varphi}]^-\delta(\varphi)\ ,\\
&\frac{\partial^2(t,z)}{\partial v^2}=f^D_{,vv}(t,z)+\frac{1}{2}(1+\dot{P})^2[f_{,\varphi}]^-\delta(\varphi)\ ,
\end{aligned} \tag{3.6}$$

where $\delta(\varphi)$ denotes the Dirac delta-function distribution with support on Σ, and

$$f^D_{,u}(t,z)\equiv f^+_{,u}(t,z)H(\varphi)+f^-_{,u}(t,z)[1-H(\varphi)]\ , \tag{3.7a}$$

$$[f_{,\varphi}]^-\equiv\lim_{\varphi\to 0^+}f^+_{,\varphi}(t,z)-\lim_{\varphi\to 0^-}f^-_{,\varphi}(t,z)\ , \tag{3.7b}$$

etc.

Note that in deriving Eq. (3.6) we had used the relations

$$\frac{\partial H(\varphi)}{\partial x^\mu}=\zeta_\mu\delta(\varphi)\ , \tag{3.8}$$

$$[f_{,u}]^-=\frac{1-\dot{P}}{\sqrt{2}}[f_{,\varphi}]^-,\quad [f_{,v}]^-=-\frac{1+\dot{P}}{\sqrt{2}}[f_{,\varphi}]^-\ . \tag{3.9}$$

Inserting Eq. (3.6) into those expressions for the Weyl and Ricci scalars given in Refs. [20,21], we find that they can be written in the form

$$\Psi_A(t,z)=\Psi^D_A(t,z)+\Psi^{\rm im}_A\delta(\varphi)\quad (A=0,2,4) \tag{3.10a}$$

and

$$\begin{aligned}
&\Lambda(t,z)=\Lambda^D(t,z)+\Lambda^{\rm im}\delta(\varphi)\ ,\\
&\Phi_{ij}(t,z)=\Phi^D_{ij}(t,z)+\Phi^{\rm im}_{ij}\delta(\varphi)\quad (i,j=0,1,2)\ ,
\end{aligned} \tag{3.10b}$$

where $\Psi^{\pm}_A(t,z)$, $\Phi^{\pm}_{ij}(t,z)$, and $\Lambda^{\pm}(t,z)$ are the regular part of the Weyl and Ricci scalars, calculated, respectively, in regions $\Omega^\pm$, and $\Psi^{\rm im}_A$, $\Phi^{\rm im}_{ij}$, and $\Lambda^{\rm im}$ the distribution part with support on Σ, and defined by

$$\begin{aligned}
&\Psi^{\rm im}_0\equiv-\frac{B^2}{4}(1+\dot{P})^2(\cosh W[V_{,\varphi}]^--i[W_{,\varphi}]^-)\ ,\\
&\Psi^{\rm im}_4\equiv-\frac{A^2}{4}(1-\dot{P})^2(\cosh W[V_{,\varphi}]^-+i[W_{,\varphi}]^-)\ ,\\
&\Psi^{\rm im}_2\equiv\frac{AB}{12}(1-\dot{P}^2)([U_{,\varphi}]^--[M_{,\varphi}]^-)\quad(\varphi=0)\ ,
\end{aligned} \tag{3.11a}$$

and

$$\begin{aligned}
&\Phi^{\rm im}_{00}\equiv\frac{B^2}{4}(1+\dot{P})^2[U_{,\varphi}]^-,\quad \Phi^{\rm im}_{22}\equiv\frac{A^2}{4}(1-\dot{P})^2[U_{,\varphi}]^-\ ,\\
&\Phi^{\rm im}_{02}=-\frac{AB}{4}(1-\dot{P}^2)(\cosh W[V_{,\varphi}]^--i[W_{,\varphi}]^-)\ ,\\
&\Lambda^{\rm im}\equiv\frac{AB}{24}(1-\dot{P}^2)(2[U_{,\varphi}]^-+[M_{,\varphi}]^-)\ ,\\
&\Phi^{\rm im}_{11}\equiv-\frac{AB}{8}(1-\dot{P}^2)[M_{,\varphi}]^-\quad(\varphi=0)\ .
\end{aligned} \tag{3.11b}$$

Inserting Eqs. (3.10b) and (3.11b) into the expression for the Einstein tensor, $G_{\mu\nu}$, given by [7]

$$\begin{aligned}
G_{\mu\nu}=2[&\Phi_{00}n_\mu n_\nu+\Phi_{22}l_\mu l_u+\overline{\Phi}_{02}m_\mu m_\nu+\Phi_{02}\overline{m}_\mu\overline{m}_\nu\\
&+(\Phi_{11}+3\Lambda)(l_\mu n_\nu+n_\mu l_\nu)\\
&+(\Phi_{11}-3\Lambda)(m_\mu\overline{m}_\nu+m_\nu\overline{m}_\mu)]\ ,
\end{aligned} \tag{3.12}$$

we find that $G_{\mu\nu}$ takes the form

$$G_{\mu\nu}=G^+_{\mu\nu}(t,z)H(\varphi)+G^-_{\mu\nu}(t,z)[1-H(\varphi)]+\gamma_{\mu\nu}\delta(\varphi)\ , \tag{3.13}$$

where $\gamma_{\mu\nu}$ denotes the distribution part of $G_{\mu\nu}$ with support on Σ, and $G^{\pm}_{\mu\nu}$ the regular part defined, respectively, in regions $\Omega^{\pm}$.

Combining Eq. (3.13) with the Einstein field equations (2.3), we find that these equations can be divided into two groups:

$$G^{\pm}_{\mu\nu}=\kappa T^{\pm}_{\mu\nu} , \tag{3.14a}$$

$$\gamma_{\mu\nu}=\kappa\tau_{\mu\nu} , \tag{3.14b}$$

where $T^{\pm}_{\mu\nu}$ are the energy-stress tensor defined, respectively, in regions $\Omega^{\pm}$, and $\tau_{\mu\nu}$ the surface energy-stress tensor with support on Σ. Once the matter fields $T^{\pm}_{\mu\nu}$ are specified, Eq. (3.14a) will give us the regular part of the Einstein field equations, which can be easily found in Refs. [13,20,21]. So, in the following we shall concentrate ourselves only on Eq. (3.14b).

Following paper I, let us consider the surface energy-stress tensor given by

$$\tau_{\mu\nu}=\sigma u_\mu u_\nu+\tau(h_{\mu\nu}-u_\mu u_\nu) , \tag{3.15}$$

where σ denotes the surface energy density of the wall, τ the tension, $h_{\mu\nu}$ the three-metric, and u_μ the four-velocity, given by

$$u_\mu=\frac{1}{\sqrt{2(1-\dot{P}^2)AB}}\{1-\dot{P},1+\dot{P},0,0\} \quad (u^\mu u_\mu=1) , \tag{3.16a}$$

$$h_{\mu\nu}=g_{\mu\nu}-\frac{\zeta_\mu\zeta_\nu}{\zeta_\lambda\zeta^\lambda} . \tag{3.16b}$$

From Eqs. (3.11b), (3.13), and (3.15), and taking into account the fact that in this paper the units are chosen so that $\kappa=1$, we find that the Einstein field equations (3.14b) read

$$e^M(1-\dot{P}^2)[U_{,\varphi}]^-=\sigma , \tag{3.17a}$$

$$e^M(1-\dot{P}^2)[M_{,\varphi}]^-=2\tau-\sigma , \tag{3.17b}$$

$$[V_{,\varphi}]^-=0=[W_{,\varphi}]^- \quad (\varphi=0) . \tag{3.17c}$$

Equations (3.14a) and (3.17) are the basic differential equations for a plane wall space-time. However, as usual, to completely solve these equations, one needs to specify the matter fields in $\Omega^{\pm}$, the corresponding equations of state, and the equation of state of the wall.

Ipser [22] and Garfinkle and Vuille [23] have studied the so-called Γ walls with the equation of state of the wall given by

$$\tau=\Gamma\sigma , \tag{3.18}$$

where Γ is a constant subject to the restriction $\Gamma\leq 1$. The case with $\Gamma=1$ corresponds to domain walls, the one with $\Gamma=\frac{1}{2}$ to walls consisting of isotropically distributed cosmic strings, and the one with $\Gamma=0$ to dust walls.

For a domain wall, Eqs. (3.17a) and (3.17b) give

$$[U_{,\varphi}]^-=[M_{,\varphi}]^- . \tag{3.19}$$

Then, from Eqs. (3.11a), (3.17c), and (3.19) we can see that for such a wall the Weyl scalars are absent of the impulsive part, while for a general wall only the Ψ_2^{im} term is different from zero [7].

IV. REPULSION AND ATTRACTION OF A PLANE WALL

One of the most remarkable features of plane walls is that the gravitational force generated by a plane domain wall is repulsive [7,11,24,25], whether or not the space-time out of the wall is curved. However, as mentioned previously, these conclusions are obtained under some assumptions. Now, it is natural to ask do the above conclusions still hold for a general plane wall without those symmetries? To answer this question, let us consider the acceleration of an observer who is at rest related to the wall. Then, the acceleration of the observer is given by

$$A_\mu=u_{\mu;\nu}u^\nu , \tag{4.1}$$

where the covariant differentiation should be taken with respect to the connection coefficients [7,18]

$$\Gamma^\mu_{\nu\lambda}(t,z)=\Gamma^{+\mu}_{\nu\lambda}(t,z)H(\varphi)+\Gamma^{-\mu}_{\nu\lambda}(t,z)[1-H(\varphi)] . \tag{4.2}$$

It is convenient to introduce a new quantity θ via the differential equations

$$\frac{\partial\theta}{\partial x^\mu}=\frac{\alpha(t,z)}{\sqrt{2}}\{1-\dot{P},1+\dot{P},0,0\} , \tag{4.3}$$

where the function $\alpha(t,z)$ is a solution of the equation

$$\alpha_{,z}+\dot{\alpha}\dot{P}+\alpha\ddot{P}=0 . \tag{4.4}$$

From Eq. (4.3) it is easy to show that

$$g^{\mu\nu}\frac{\partial\theta}{\partial x^\mu}\frac{\partial\theta}{\partial x^\nu}>0, \quad g^{\mu\nu}\frac{\partial\varphi}{\partial x^\mu}\frac{\partial\theta}{\partial x^\nu}=0 . \tag{4.5}$$

Thus, φ and θ can be taken, respectively, as a new spacelike coordinate and a new timelike coordinate.

Taking Eqs. (3.2), (4.2), and (4.3) into account, we find that the four-acceleration of the observer is given by

$$A_\mu=\{A_u,A_v,0,0\} , \tag{4.6}$$

where

$$\begin{aligned} A_u &\equiv \frac{1-\dot{P}}{2\sqrt{2}}\{M^+_{,\varphi}H(\varphi)+M^-_{,\varphi}[1-H(\varphi)]\} \\ &\quad -\frac{\ddot{P}}{\sqrt{2}(1+\dot{P})(1-\dot{P}^2)} , \\ A_v &\equiv -\frac{1+\dot{P}}{2\sqrt{2}}\{M^+_{,\varphi}H(\varphi)+M^-_{,\varphi}[1-H(\varphi)]\} \\ &\quad +\frac{\ddot{P}}{\sqrt{2}(1-\dot{P})(1-\dot{P}^2)} . \end{aligned} \tag{4.7}$$

Hence, if the observer just hovers off the wall, the perpendicular component of the acceleration to the wall is given by

$$A_\lambda \xi^\lambda = -\frac{1}{2}e^M(1-\dot{P}^2)\{M^+_{,\varphi}H(\varphi)+M^-_{,\varphi}[1-H(\varphi)]\} + e^M\frac{\ddot{P}}{1-\dot{P}^2} . \tag{4.8}$$

From Eq. (4.8) it is easy to find that in each side of the wall the perpendicular component reads

$$A_\lambda \xi^\lambda|_{\varphi=0^+} = -\frac{1}{4}(2\tau-\sigma) - \frac{1}{4}e^M(1-\dot{P}^2)[M_{,\varphi}]^+ + e^M\frac{\ddot{P}}{1-\dot{P}^2} , \tag{4.9a}$$

$$A_\lambda \xi^\lambda|_{\varphi=0^-} = \frac{1}{4}(2\tau-\sigma) - \frac{1}{4}e^M(1-\dot{P}^2)[M_{,\varphi}]^+ + e^M\frac{\ddot{P}}{1-\dot{P}^2} , \tag{4.9b}$$

where

$$[M_{,\varphi}]^+ \equiv \lim_{\varphi\to 0^+} M^+_{,\varphi} + \lim_{\varphi\to 0^-} M^-_{,\varphi} . \tag{4.10}$$

Comparing Eqs. (4.9) with Eq. (48) in paper I, we find that there are two extra terms adding to the expressions of $A_\mu\xi^\mu|_{\varphi=0^\pm}$, and that when the wall is "static" and has reflection symmetry, these two extra terms vanish identically. In the following, we shall consider these terms separately.

Let us first consider the two first terms, each of which appears, respectively, in the right-hand sides of Eqs. (4.9a) and (4.9b). These two terms have opposite signs, but the same amplitude, and represent the acceleration produced by the wall. For $\tau > \sigma/2$ it is repulsive, and for $\tau < \sigma/2$ it is attractive. When $\tau = \sigma/2$, which corresponds to the walls consisting of isotropically distributed cosmic strings, it is zero. The two second terms appearing, respectively, in the right-hand sides of Eqs. (4.9a) and (4.9b) have the same sign, the same amplitude, and are proportional to $[M_{,\varphi}]^+$. When the wall has reflection symmetry, they vanish. Therefore, the appearance of these two terms is due to the asymmetry of the space-time in both sides of the wall. We interpret these two terms as representing the acceleration generated by the space-time curvature. When the wall has reflection symmetry, the force generated by the curvature in one side of the wall balances the one generated in the other side of the wall, so, the total effect of this force on the observer is zero. When the space-time does not have such a symmetry, we would expect that this effective force is different from zero, and given by the second terms. The two last terms also have the same sign and the same amplitude. Hence, they should have the same properties as the second ones. However, the origin of this force is different from the last one, and clearly is due to the accelerated motion of the wall itself.

Therefore, an observer who is at rest related to the wall usually feels a force acting on him. But this force, in general, is the sum of the above three different forces, produced, respectively, by the wall, the space-time curvature, and the accelerated motion of the wall. As a result, an attractive (repulsive) wall may not be attractive (repulsive) at all, because of the action of the last two forces discussed above.

V. INTERACTION OF A PLANE WALL WITH SURROUNDING MATTER FIELDS AND GRAVITATIONAL PLANE WAVES

In this section, we shall study the interaction of a plane wall with surrounding matter fields and gravitational plane waves.

Following paper I, we first split the Bianchi identities into two groups, one holding in regions $\Omega^\pm$ and the other holding on the wall.

Let us first note that for any function $f(t,z)$, which is C^3 in $\Omega^\pm$ and C^0 at Σ, we have

$$f^+_{,\theta}(t,z) = f^-_{,\theta}(t,z) \equiv f_{,\theta}(t,z) . \tag{5.1}$$

On the other hand, from Eq. (3.17c) we find

$$\begin{aligned} V^+_{,\varphi}(t,z) &= V^-_{,\varphi}(t,z) \equiv V_{,\varphi}(t,z) , \\ W^+_{,\varphi}(t,z) &= W^-_{,\varphi}(t,z) \equiv W_{,\varphi}(t,z) . \end{aligned} \tag{5.2}$$

From Eqs. (3.10) and (3.11), we also find

$$\Psi^{\text{im}}_{2,u} = \frac{\alpha(1-\dot{P})}{\sqrt{2}}\Psi^{\text{im}}_{2,\theta}, \quad \Psi^{\text{im}}_{2,v} = \frac{\alpha(1+\dot{P})}{\sqrt{2}}\Psi^{\text{im}}_{2,\theta} , \tag{5.3}$$

$$\begin{aligned} \Psi_{2,u}(t,z) &= \Psi^D_{2,u}(t,z) + \frac{1-\dot{P}}{\sqrt{2}}\{[\Psi_2]^-\delta(\varphi) + \alpha\Psi^{\text{im}}_{2,\theta}\delta(\varphi) + \Psi^{\text{im}}_2\delta'(\varphi)\} , \\ \Psi_{2,v}(t,z) &= \Psi^D_{2,v}(t,z) - \frac{1+\dot{P}}{\sqrt{2}}\{[\Psi_2]^-\delta(\varphi) - \alpha\Psi^{\text{im}}_{2,\theta}\delta(\varphi) + \Psi^{\text{im}}_2\delta'(\varphi)\} , \end{aligned} \tag{5.4}$$

and so on, where a prime denotes ordinary differentiation with respect to the indicated argument. Combining Eqs. (5.1)–(5.4) and the facts (in the sense of distributions, see Appendix)

$$H(\varphi)\delta(\varphi) = \tfrac{1}{2}\delta(\varphi), \quad [1-H(\varphi)]\delta(\varphi) = \tfrac{1}{2}\delta(\varphi) , \tag{5.5}$$

we find that the Bianchi identities given by Eqs. (35) and (36) in paper I remain the same in regions $\Omega^\pm$, while on the wall we have

$$(1-\dot{P})[\Psi_0]^- - (1+\dot{P})[\Phi_{02}]^- = -\tfrac{1}{4}(1+\dot{P})[2\alpha(\sigma_0-\tau_0)(\cosh W V_{,\theta} - iW_{,\theta}) + (2\sigma_0-\tau_0)(\cosh W V_{,\varphi} - iW_{,\varphi})] , \tag{5.6a}$$

$$(1+\dot{P})[\Psi_4]^- - (1-\dot{P})[\Phi_{20}]^- = \tfrac{1}{4}(1-\dot{P})[2\alpha(\sigma_0-\tau_0)(\cosh W V_{,\theta} + iW_{,\theta}) - (2\sigma_0-\tau_0)(\cosh W V_{,\varphi} + iW_{,\varphi})] , \tag{5.6b}$$

$$[\Psi_2 - \Phi_{11} - \Lambda]^- = \tfrac{1}{8}\sigma_0[U_{,\varphi}]^+ , \tag{5.6c}$$

$$(1+\dot{P})^2[\Phi_{22}]^- +(1-\dot{P})^2[\Phi_{00}]^- -2(1-\dot{P}^2)[\Phi_{11}+3\Lambda]^- =\tfrac{1}{4}(1-\dot{P}^2)\{\sigma_0[M_{,\varphi}]^+ +2\tau_0[U_{,\varphi}]^+ -4\sigma_0\ddot{P}(1-\dot{P}^2)^{-2}\} , \quad (5.6d)$$

$$(1+\dot{P})^2[\Phi_{22}]^- -(1-\dot{P})^2[\Phi_{00}]^- =\tfrac{1}{2}\alpha(1-\dot{P}^2)[2\sigma_{0,\theta}+\sigma_0 M_{,\theta}-2(\sigma_0-\tau_0)U_{,\theta}] \quad (\varphi=0) , \quad (5.6e)$$

where

$$\sigma_0\equiv e^{-M}\sigma, \quad \tau_0\equiv e^{-M}\tau . \quad (5.7)$$

Equations (5.6) represent the interaction of a plane wall with gravitational and matter fields.

It should be noted that all the Weyl and Ricci scalars appearing in Eq. (5.6) correspond to the "scale-invariant" ones, defined by Eq. (34) in paper I. From now on, we shall continuously use these "scale-invariant" ones, unless some specific statements are made.

When the wall has reflection symmetry, it is easy to show that the following is true:

$$f^+{}_{,\varphi}|_{\varphi=0^+} = -f^-{}_{,\varphi}|_{\varphi=0^-} . \quad (5.8)$$

Consequently, we have

$$[M_{,\varphi}]^+ =0=[U_{,\varphi}]^+ , \quad (5.9a)$$

$$V_{,\varphi}=0=W_{,\varphi} \quad (\varphi=0) . \quad (5.9b)$$

In obtaining Eq. (5.9b) we had used Eq. (5.2).

Therefore, for a plane domain wall ($\sigma=\tau$) with reflection symmetry, Eqs. (5.6) become

$$(1-\dot{P})[\Psi_0]^- =(1+\dot{P})[\Phi_{02}]^- ,$$
$$(1+\dot{P})[\Psi_4]^- =(1-\dot{P})[\Phi_{20}]^- , \quad (5.10a)$$

$$[\Psi_2]^- =[\Phi_{11}+\Lambda]^- , \quad (5.10b)$$

$$(1+\dot{P})^2[\Phi_{22}]^- +(1-\dot{P})^2[\Phi_{00}]^- -2(1-\dot{P}^2)[\Phi_{11}+3\Lambda]^- =-\sigma_0\ddot{P}(1-\dot{P}^2)^{-1} , \quad (5.10c)$$

$$(1+\dot{P})^2[\Phi_{22}]^- -(1-\dot{P})^2[\Phi_{00}]^- =\tfrac{1}{2}\alpha(1-\dot{P}^2)(2\sigma_{0,\theta}+\sigma_0 M_{,\theta}) \quad (\varphi=0,\sigma=\tau) , \quad (5.10d)$$

which shows that such a plane wall does not interact with any gravitational waves (Ψ_0 and Ψ_4), although it does with surrounding matter fields and the "Coulomb-like" gravitational field Ψ_2. The latter is carried out through Eqs. (5.10c) and (5.10d). This is consistent with our earlier conclusions obtained in paper I.

On the other hand, from Eqs. (2.4) and (3.14) we find

$$T_{\mu\nu;\lambda}g^{\nu\lambda}=T^D_{\mu\nu;\lambda}g^{\nu\lambda}+\{\zeta^\lambda[T_{\mu\lambda}]^- +\tau_{\mu\nu;\lambda}g^{\nu\lambda}\}\delta(\varphi) +\tau_{\mu\lambda}\zeta^\lambda\delta'(\varphi)=0 , \quad (5.11)$$

or equivalently

$$T^\pm_{\mu\nu;\lambda}g^{\nu\lambda}=0 , \quad (5.12a)$$

$$\tau_{\mu\lambda}\zeta^\lambda=0 , \quad (5.12b)$$

$$\tau_{\mu\nu;\lambda}g^{\nu\lambda}=-\zeta^\lambda[T_{\mu\lambda}]^- , \quad (5.12c)$$

where the covariant differentiation of $\tau_{\mu\nu}$ is taken with respect to the connection coefficients [see Eqs. (5.5)]

$$\Gamma^\mu_{\nu\lambda}=\tfrac{1}{2}\{\Gamma^{+\mu}_{\nu\lambda}(t,z)|_{\varphi=0^+}+\Gamma^{-\mu}_{\nu\lambda}(t,z)|_{\varphi=0^-}\} . \quad (5.13)$$

It is easy to show that Eq. (5.12b) is satisfied identically while Eq. (5.12c) gives the last two equations of Eqs. (5.6). This is what we expect, since Eq. (2.4) is the direct result of the combination of Eqs. (2.3) and (2.5).

VI. THE INTERACTION OF A PLANE DOMAIN WALL WITH GRAVITATIONAL FIELD AND SOME SPECIFIC MATTER FIELDS

Because of the particular importance of domain walls to the early Universe, we devote this section to consider the interaction of a plane domain wall with a gravitational field and some specific matter fields, which are very interesting from the point of view of physics.

A. When the matter fields on both sides of the wall are vanishing

When the matter fields on both sides of the wall vanish, we have

$$\Phi^\pm{}_{ij}=0=\Lambda^\pm \quad (i,j=0,1,2) . \quad (6.1)$$

Then, Eqs. (5.6) reduce to

$$(1-\dot{P})[\Psi_0]^- =-\tfrac{1}{4}(1+\dot{P})\sigma_0(\cosh W\, V_{,\varphi}-iW_{,\varphi}) , \quad (6.2a)$$

$$(1+\dot{P})[\Psi_4]^- =-\tfrac{1}{4}(1-\dot{P})\sigma_0(\cosh W\, V_{,\varphi}+iW_{,\varphi}) , \quad (6.2b)$$

$$[\Psi_2]^- =\tfrac{1}{8}\sigma_0[U_{,\varphi}]^+ , \quad (6.2c)$$

$$[M_{,\varphi}]^+ +2[U_{,\varphi}]^+ =4\ddot{P}(1-\dot{P}^2)^{-2} , \quad (6.2d)$$

$$2\sigma_{0,\theta}+\sigma_0 M_{,\theta}=0 \quad (\varphi=0) . \quad (6.2e)$$

Equations (6.2a)–(6.2c) show that, due to the nonreflection symmetry of the space-time, the gravitational field interacts with the wall.

To be more specific, let us consider the Ψ_0 term, which represents the transverse gravitational-wave component propagating in the n^μ direction [12]. Without loss of generality, we assume that at the beginning the gravitational wave is in the region Ω^+, and moving towards the wall. As long as the velocity of the wall is less than the velocity of light, at some moment, say, t_0, the gravitational wave will reach the wall at $z=P(t_0)$, and then pass through it into the region Ω^-. According to Eq. (6.2a), the amplitude $(\Psi_0\bar{\Psi}_0)^{1/2}$ of the gravitational wave [12] is not equal in both sides of the wall. The difference, obviously, is partially due to the reflection of the wall to this gravitational wave, and partially to the fact that when the gravitational wave passes through the wall it may stimulate gravitational radiation or be partially absorbed.

On the other hand, from Eq. (6.2d) we can see that, because of the nonreflection symmetry, the space-time curvature produces an effective force acting on the wall so that the wall has to have accelerated motion.

However, when the wall has reflection symmetry, Eqs. (6.2) will read

$$[\Psi_0]^- = [\Psi_2]^- = [\Psi_4]^- = 0 , \tag{6.3a}$$

$$2\sigma_{0,\theta} + \sigma_0 M_{,\theta} = 0 , \tag{6.3b}$$

$$\ddot{P} = 0 \quad (\varphi = 0) . \tag{6.3c}$$

That is, the gravitational field in this case is continuous across the wall, and no reflection, stimulation, or absorption occurs. From Eq. (6.3c), on the other hand, we can see that the wall does not have accelerated motion, and

$$P(t) = u_0 t , \tag{6.4}$$

where u_0 is an arbitrary constant, but when Eq. (3.3) is taken into account we must have $u_0^2 < 1$. Thus, u_0 can be interpreted as the velocity of the wall.

Combining Eqs. (4.3), (4.4), and (6.4), we find that the function θ takes the form

$$\theta(t,z) = t - u_0 z . \tag{6.5}$$

In terms of φ and θ, the metric (2.1) takes exactly the form used in paper I. Consequently, all the results obtained in paper I are applicable to this special case.

B. When a plane domain wall is coupled with a perfect fluid

The energy-stress tensor of a perfect fluid and the corresponding nonvanishing Ricci ("scale-invariant") scalars are given, respectively, by Eqs. (4.13) and (4.16) in Ref. [12].

Across the hypersurface Σ where the wall is located, the fluid is usually required to satisfy the so-called Rankine-Hugoniot equations [26]

$$[\rho v^\lambda \xi_\lambda]^- = 0 , \tag{6.6a}$$

$$[T_{\mu\lambda} \xi^\lambda]^- = 0 , \tag{6.6b}$$

where ρ denotes the rest particle density of the fluid, and ξ^μ is the normal vector to the wall defined by Eq. (3.2). Equations (6.6a) and (6.6b) are the generalization of the equation of the conservation of the particle number, and of the energy and stress of the fluid, respectively.

On the other hand, the combination of Eqs. (3.12), (3.14a), and (6.6b) yields

$$\begin{aligned} (1-\dot{P})[\Phi_{00}]^- &= (1+\dot{P})[\Phi_{11}+3\Lambda]^- , \\ (1+\dot{P})[\Phi_{22}]^- &= (1-\dot{P})[\Phi_{11}+3\Lambda]^- . \end{aligned} \tag{6.7}$$

Substituting Eq. (6.7) into Eqs. (5.6), we find that the reduced equations take exactly the same form as Eqs. (6.2) for the vacuum case, except Eq. (6.2c) now must be replaced by

$$[\Psi_2 - \Phi_{11} - \Lambda]^- = \tfrac{1}{8}\sigma_0 [U_{,\varphi}]^+ . \tag{6.8}$$

Then, it follows that the nonreflection symmetry of the wall is partially preserved by the wall's reflecting, and or absorbing gravitational radiation. The wall interacts with the fluid through the components Φ_{11}, and Λ via Eq. (6.8).

When the space-time of the wall has reflection symmetry, the wall does not interact either with the gravitational field or the fluid. As a result, all such walls can only have uniform motion [see Eq. (6.3c)].

Note that when the fluid in both sides of the wall has a different "chemistry," then we have detonation waves [27].

C. When a plane domain wall is coupled with null dust

The energy-stress tensor for a null dust field can be written as a sum of two pure radiation fields [28–30], moving in opposite directions [see Eq. (5.15) in Ref. [12]].

It is easy to show that in this case the only nonvanishing Ricci scalars in regions $\Omega^\pm$ are $\Phi_{00}^\pm$ and $\Phi_{22}^\pm$, given by Eq. (5.16) in Ref. [12].

Setting $\Phi_{02}^\pm$, $\Phi_{11}^\pm$, and $\Lambda^\pm$ equal to zero in Eqs. (5.6), we find

$$(1-\dot{P})[\Psi_0]^- = -\tfrac{1}{4}(1+\dot{P})\sigma_0(\cosh W\, V_{,\varphi} - iW_{,\varphi}) , \tag{6.9a}$$

$$(1+\dot{P})[\Psi_4]^- = -\tfrac{1}{4}(1-\dot{P})\sigma_0(\cosh W\, V_{,\varphi} + iW_{,\varphi}) , \tag{6.9b}$$

$$[\Psi_2]^- = \tfrac{1}{8}\sigma_0[U_{,\varphi}]^+ , \tag{6.9c}$$

$$\begin{aligned} &(1+\dot{P})^2[\Phi_{22}]^- + (1-\dot{P})^2[\Phi_{00}]^- \\ &\quad = \tfrac{1}{4}(1-\dot{P}^2)\sigma_0\{[M_{,\varphi}]^+ + 2[U_{,\varphi}]^+ - 4\ddot{P}(1-\dot{P}^2)^{-2}\} , \end{aligned} \tag{6.9d}$$

$$\begin{aligned} &(1+\dot{P})^2[\Phi_{22}]^- - (1-\dot{P})^2[\Phi_{00}]^- \\ &\quad = \tfrac{1}{2}\alpha(1-\dot{P}^2)(2\sigma_{0,\theta} + \sigma_0 M_{,\theta}) \quad (\varphi=0) . \end{aligned} \tag{6.9e}$$

It is clear that in the present case domain walls interact with both gravitational and matter fields. Similar to the vacuum and perfect-fluid cases, the interaction of the gravitational field with a domain wall is entirely due to the nonreflection symmetry of the wall. Otherwise, the right-hand sides of Eqs. (6.9a)–(6.9c) vanish, and we have

$$[\Psi_0]^- = [\Psi_2]^- = [\Psi_4]^- = 0 , \tag{6.10}$$

which means that there is no interaction between the gravitational field and domain walls. However, in contrast with the vacuum and perfect-fluid cases, even when the space-time has reflection symmetry, $\ddot{P}$ does not vanish. Hence, the wall in this case can still have an accelerated motion due to the interaction of it with the pure radiation fields.

D. When a plane domain wall is coupled with a massless scalar field

The energy-stress tensor for a massless scalar field ϕ is given by Eq. (5.19) in Ref. [12], and ϕ satisfies the massless Klein-Gordon equation given by Eq. (5.21). For the following discussions, we quote it here:

$$2\phi_{,uv}-U_{,u}\phi_{,v}-U_{,v}\phi_{,u}=0\ . \tag{6.11}$$

Note that the energy-stress tensor $T_{\mu\nu}$ in this case includes the quadratic terms of $\phi_{,u}$ and $\phi_{,v}$. Thus, to be physically meaningful, ϕ must be at least C^0 across the wall. Otherwise, $T_{\mu\nu}$ will contain the squares of the Dirac delta function, which is physically unacceptable. Since Eq. (6.11) is satisfied in regions $\Omega^{\pm}$, we can see that ϕ is at least C^2 in these regions. Following the discussions carried out in Sec. III, we can write $\phi(t,z)$ as

$$\phi(t,z)=\phi^{+}(t,z)H(\varphi)+\phi^{-}(t,z)[1-H(\varphi)]\ . \tag{6.12}$$

Hence, we have

$$\begin{aligned}&\phi_{,u}(t,z)=\phi^{D}_{,u}(t,z),\quad \phi_{,v}(t,z)=\phi^{D}_{,v}(t,z)\ ,\\ &\phi_{,uv}(t,z)=\phi^{D}_{,uv}(t,z)-\tfrac{1}{2}(1-\dot{P}^2)[\phi_{,\varphi}]^{-}\delta(\varphi)\ .\end{aligned} \tag{6.13}$$

Inserting Eq. (6.13) into Eq. (6.11), and taking into account the fact that $\phi^{\pm}$ satisfy Eq. (6.11), respectively, in regions $\Omega^{\pm}$, we find that

$$[\phi_{,\varphi}]^{-}=0\ . \tag{6.14}$$

That is, across the hypersurface Σ not only is ϕ necessarily continuous, but also its first derivatives $\phi_{,\mu}$. The continuities of ϕ_{μ} across Σ imply that the reflectivity of a plane domain wall to this massless scalar field is zero. If an observer hovers just off the wall, he will find that an incident massless scalar wave continuously passes through the wall without reflection.

Note that the above conclusions are also true for a general plane wall, since in obtaining them we did not use the equation of state of the wall.

On the other hand, the nonvanishing Ricci scalars in this case are those given by Eq. (5.22) in Ref. [12]. By the fact that $\phi_{,\mu}$ are C^0 across Σ, it is easy to show that

$$[\Phi_{00}]^{-}=[\Phi_{11}]^{-}=[\Phi_{22}]^{-}=[\Lambda]^{-}=0\ , \tag{6.15}$$

which means that the massless scalar field is "inert" to plane domain walls.

Substituting Eq. (6.15) into Eqs. (5.6), we find that the reduced equations are exactly the same as those given by Eqs. (6.2) for the vacuum case. Consequently, the conclusions obtained in that subsection are also true for a massless scalar field.

It should be mentioned that when $\phi_{,\mu}$ is timelike, the corresponding ϕ field is energetically equivalent to a "stiff" fluid with its energy density μ, pressure p, and four-velocity v_μ given, respectively, by [30,31]

$$\mu=p=\frac{1}{2}\phi_{,\lambda}\phi^{,\lambda},\quad v_{\mu}=(\phi_{,\lambda}\phi^{,\lambda})^{-1/2}\phi_{,\mu}\ . \tag{6.16}$$

When $\phi_{,\mu}$ is spacelike, on the other hand, it is equivalent to an anisotropic fluid with a vanishing heat-flow vector [30,31].

There also exist the cases in which the sign of $\phi_{,\mu}$ changes from one region to another, and the hypersurfaces separating those regions are the ones on which $\phi_{,\lambda}\phi^{,\lambda}$ vanishes [30].

E. When a plane domain wall is coupled with an electromagnetic field

When the space-time is filled with an electromagnetic field, the energy-stress tensor is given by Eq. (5.27) in Ref. [12] with the field tensor $F_{\mu\nu}$ satisfying the Maxwell equations

$$F_{\mu\nu;\lambda}g^{\nu\lambda}=-j_{\mu},\quad F_{[\mu\nu;\lambda]}=0\ , \tag{6.17}$$

where j_μ denotes the density flow vector, and satisfies the conservation equation of charge

$$j_{\nu;\lambda}g^{\nu\lambda}=0\ . \tag{6.18}$$

Introducing the Maxwell scalars as those given by Eq. (5.29) in Ref. [12], we find that the *sourceless* ($j_\mu=0$) Maxwell equations read

$$\begin{aligned}&\Phi^{(0)}_{1,v}=(U_{,v}-\tfrac{1}{2}M_{,v})\Phi^{(0)}_{1},\\ &\Phi^{(0)}_{1,u}=(U_{,u}-\tfrac{1}{2}M_{,u})\Phi^{(0)}_{1}\ ,\\ &2\Phi^{(0)}_{0,u}=(U_{,u}-i\sinh W\,V_{,u})\Phi^{(0)}_{0}\\ &\qquad-(\cosh W\,V_{,v}-iW_{,v})\Phi^{(0)}_{2}\ ,\\ &2\Phi^{(0)}_{2,v}=(U_{,v}+i\sinh W\,V_{,v})\Phi^{(0)}_{2}\\ &\qquad-(\cosh W\,V_{,u}+iW_{,u})\Phi^{(0)}_{0}\ ,\end{aligned} \tag{6.19}$$

where $\Phi^{(0)}_{i}$ are defined by

$$\begin{aligned}&\Phi^{(0)}_{0}\equiv B^{-1}\Phi_0\ ,\\ &\Phi^{(0)}_{1}\equiv(AB)^{-1/2}\Phi_1\ ,\\ &\Phi^{(0)}_{2}\equiv A^{-1}\Phi_2\ .\end{aligned} \tag{6.20}$$

Since the energy-stress tensor $T_{\mu\nu}$ contains the quadratic terms of $\Phi^{(0)}_{i}$, as argued in the last subsection, we must assume that $\Phi^{(0)}_{i}$ at most have the H-function discontinuities across Σ. Thus, combining the assumption that the sourceless Maxwell equations are satisfied in both sides of the wall, we can write $\Phi^{(0)}_{i}$ in the same form as $\phi(t,z)$ in Eq. (6.12). Then, we have

$$\begin{aligned}&\Phi^{(0)}_{i,u}=\Phi^{(0)^{D}}_{i,u}(t,z)+\frac{1-\dot{P}}{\sqrt{2}}[\Phi^{(0)}_{i}]^{-}\delta(\varphi)\ ,\\ &\Phi^{(0)}_{i,v}=\Phi^{(0)^{D}}_{i,v}(t,z)-\frac{1+\dot{P}}{\sqrt{2}}[\Phi^{(0)}_{i}]^{-}\delta(\varphi)\ .\end{aligned} \tag{6.21}$$

Inserting Eq. (6.21) into Eq. (6.19) we find

$$[\Phi^{(0)}_{i}]^{-}=0\ , \tag{6.22}$$

which means that, to guarantee the *sourceless* Maxwell equations to be satisfied in the whole space-time including the hypersurface Σ, the Maxwell scalars $\Phi^{(0)}_{i}$ must be continuous across Σ.

The continuities of the $\Phi^{(0)}_{i}$'s, or equivalently, of the antisymmetric tensor $F_{\mu\nu}$, across Σ imply that the wall is completely transparent to this field, too. Similar to the last case, this conclusion holds also for a general wall.

On the other hand, the nonvanishing "scale-invariant" Ricci scalars in this case are given by

$$\Phi_{ij}=\Phi_i^{(0)}\Phi_j^{(0)} \quad (i,j=0,1,2) . \tag{6.23}$$

Using Eq. (6.22), it is easy to show

$$[\Phi_{ij}]^-=0 . \tag{6.24}$$

Thus, like the massless scalar field, the electromagnetic field is also "inert" to plane thin domain walls.

VII. THE COLLISION AND INTERACTION AMONG PLANE WALLS

In the previous sections, we have considered the space-time in which there exists only one single plane wall. However, in reality a space-time may contain many such walls, and during their evolution they may collide and interact with each other [10]. Thus, in order to cover the latter case, we have to consider the space-time containing more than one C^1 hypersurface; crossing it $g_{\mu\nu}$ is C^0.

The generalization of the treatment given in Sec. III to the present case is straightforward. For the sake of completeness, in the following we give a brief outline.

Let Σ_a denote the ath hypersurface and be described by

$$\varphi_a(t,z)\equiv z-P_a(t)=0 \quad (a=1,2,\ldots,N) , \tag{7.1}$$

where N is the number of the walls contained in the space-time considered. Then, for each of these surfaces we define a normal vector $\zeta_{(a)\mu}$ by

$$\zeta_{(a)\mu}\equiv\frac{\partial\varphi_a(t,z)}{\partial x^\mu}=\frac{1}{\sqrt{2}}\{1-\dot{P}_a,-(1+\dot{P}_1),0,0\} . \tag{7.2}$$

The condition that guarantees Σ_a to be timelike now reads

$$1-\dot{P}_a^2>0 \quad (a=1,2,\ldots,N) . \tag{7.3}$$

Similar to the one-wall case, we assume that each hypersurface Σ_a divides a region of the space-time into two subregions $\Omega_{(a-1)}$ and $\Omega_{(a)}$, where $\Omega_{(a-1)}$ denotes the region between the hypersurfaces Σ_{a-1} and Σ_a and includes Σ_{a-1} and Σ_a, and $\Omega_{(a)}$ the region between Σ_a and Σ_{a+1} and also includes them. The restrictions $g_{(a)\mu\nu}=g_{\mu\nu}|_{\Omega_{(a)}}$ are assumed to be at least C^3, while across Σ_a $g_{\mu\nu}$ at least to be C^0.

With the above assumptions, it is easy to see that the treatment for each of these hypersurfaces is exactly the same as that done for a one-wall case in Sec. III. In particular, the Einstein field equations on each of these walls take the form

$$e^M(1-\dot{P}_a^2)[U_{,\varphi_a}]^-=\sigma_a , \tag{7.4a}$$

$$e^M(1-\dot{P}_a^2)[M_{,\varphi_a}]^-=2\tau_a-\sigma_a , \tag{7.4b}$$

$$[V_{,\varphi_a}]^-=0=[W_{,\varphi_a}]^- , \tag{7.4c}$$

where σ_a and τ_a denote the surface energy density and tension of the ath wall, respectively.

A similar set of equations to Eqs. (4.9) also holds, after σ, τ, and $P(t)$ are replaced by σ_a, τ_a, and $P_a(t)$.

At a moment $t=t_0$, the ath wall may collide with the bth wall on the hypersurface $z=P_a(t_0)=P_b(t_0)$, where $a\neq b$. Afterwards, they may be part away from each other, and behave like two solitons, or may exchange stress and energy and form a "bounded" state [32].

An interesting case is that the space-time contains only two walls described, respectively, by $\varphi_1=z-P(t)$ and $\varphi_2=z+P(t)$. Obviously, in this case the two walls have the same velocity but move in opposite directions. If the equation $P(t)=0$ has one real root, say, t_0, then we can see that the two walls will collide at the moment $t=t_0$ on the hypersurface $z=0$. If the equation $P(t)=0$ has more than one real root, it means that they will collide more than one time on the hypersurface $z=0$ before they move away from each other.

VIII. CONCLUSIONS

In the previous sections, a plane thin wall without reflection symmetry has been discussed by using the distribution theory, which has shown to be a very powerful tool to this problem. It has been found that the Einstein field equations take a very simple form on the wall, and are given explicitly by Eqs. (3.17). The repulsion and attraction of a plane wall have been studied in Sec. IV. In particular, it has been shown that an observer who is just next to the wall, in general, feels a force acting on himself. This force is produced not only by the wall, but also by the space-time curvature due to the nonreflection symmetry of the wall. Thus, the force this observer feels is the sum of the above two different ones. In addition, if the observer wants to remain at rest related to the wall, he has to have another force acting on him in order to have the same acceleration as the wall does. As a result, an attractive (repulsive) plane wall may not be attractive (repulsive) at all.

Later on, the Bianchi identities have been split into two groups: one of which holds in the space-time out of the wall, the other holds on the wall. It is this that has enabled us to study the interaction of a plane thin wall with gravitational fields and surrounding matter fields. In particular, it has been found that plane thin walls (including domain walls) are completely transparent to a massless scalar or an electromagnetic field. The situation, however, is quite different when a gravitational wave passes through such a thin plane wall. If the wall has no reflection symmetry, we have found that the phenomena such as reflection, stimulation, or absorption of the wall to a gravitational wave occur. In Sec. VII, the discussions given in the previous sections have been extended to the case where a space-time contains more than one plane wall, so that the collision and interaction among the walls can be studied.

A natural generalization of the above studies is, by using the same method used in this paper, to consider space-times which have spherical or cylindrical symmetries. The former corresponds to spherical domain walls or bubbles, while the latter to cylindrical walls. The work in this direction will be published in the future.

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APPENDIX: THE PROOF OF Eq. (5.5)

To prove Eq. (5.5), it is useful first to construct a function whose limit is the Dirac delta-function distribution.

Without loss of generality, let us consider the problem in a $D1$ space. Construct a function $\delta_\epsilon(\varphi)$ by

$$\delta_\epsilon(\varphi)\equiv\begin{cases}\dfrac{1}{2\epsilon} & (-\epsilon\le\varphi\le\epsilon),\\ 0 & (-\infty<\varphi<-\epsilon,\epsilon<\varphi<+\infty)\ ,\end{cases}\tag{A1}$$

where ϵ is a positive number. Then, for any test function $f(\varphi)$ we have

$$\int_{-\infty}^{+\infty}\delta_\epsilon(\varphi)f(\varphi)d\varphi=\frac{1}{2\epsilon}\int_{-\epsilon}^{\epsilon}f(\varphi)d\varphi=f(\zeta)\ ,\tag{A2}$$

where $\zeta\in(-\epsilon,\epsilon)$. In the last step of the above equation, we had used the mean-value theorem for integrals. As $\epsilon\rightarrow 0$, we have $f(\zeta)\rightarrow f(0)$. Then, from Eq. (A2) we find

$$\lim_{\epsilon\rightarrow 0}\int_{-\infty}^{+\infty}\delta_\epsilon(\varphi)f(\varphi)d\varphi=f(0)=\langle\delta(\varphi),f(\varphi)\rangle\ .\tag{A3}$$

That is, the Dirac delta-function distribution $\delta(\varphi)$ can be considered as a limit of the function $\delta_\epsilon(\varphi)$ defined by Eq. (A1).

In review of Eqs. (A1)–(A3), now it is easy to show that

$$\langle H(\varphi)\delta(\varphi),f(\varphi)\rangle=\lim_{\epsilon\rightarrow 0}\int_{-\infty}^{+\infty}\delta_\epsilon(\varphi)H(\varphi)f(\varphi)d\varphi=\langle\tfrac{1}{2}\delta(\varphi),f(\varphi)\rangle\ ,\tag{A4a}$$

$$\langle[1-H(\varphi)]\delta(\varphi),f(\varphi)\rangle=\lim_{\epsilon\rightarrow 0}\int_{-\infty}^{+\infty}\delta_\epsilon(\varphi)[1-H(\varphi)]\times f(\varphi)d\varphi=\langle\tfrac{1}{2}\delta(\varphi),f(\varphi)\rangle\ .\tag{A4b}$$

Thus, in the sense of distributions Eq. (5.5) holds.

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