General-relativistic model of a spinning cosmic string

Bjørn Jensen and Harald H. Soleng
Institute of Physics, University of Oslo, P.O. Box 1048 Blindern, N-0316 Oslo 3, Norway
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We investigate the infinite, straight, rotating cosmic string within the framework of Einstein's general theory of relativity. A class of exact interior solutions is derived for which the source satisfies the weak and the dominant energy conditions. The interior metric is matched smoothly to the exterior vacuum. A subclass of these solutions has closed timelike curves both in the interior and the exterior geometries.

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I. INTRODUCTION

The geometry outside a straight, static cosmic string with vanishing radius [1–3] represents a four-dimensional extension of the point-particle solution of (2+1)-dimensional Einstein gravity [4]. In three spacetime dimensions the Riemann curvature depends algebraically on the Einstein curvature. Hence, it can be expressed entirely in terms of the local distribution of matter and energy. This makes vacuum solutions of three-dimensional Einstein theory differ from Minkowski spacetime only in their global topological properties [5].

Deser and Jackiw [6] have studied new solutions in (2+1)-dimensional gravity that describe geometries generated by static string sources. As opposed to (2+1)-dimensional particles and (3+1)-dimensional strings, the exterior geometry of a static (2+1)-dimensional string with tension always corresponds to maximum angular deficit. Although these solutions are quite different from four-dimensional string solutions, they may find useful applications in studies of domain-wall dynamics. Further solutions, representing spinning strings, were found by Grignani and Lee [7] and by Clément [8].

Just as a static three-dimensional point-particle solution becomes a static four-dimensional cosmic-string model, the three-dimensional "Kerr" metric [4,5] may be interpreted as a vacuum solution outside a cosmic string carrying angular momentum [9,10]. This geometry has attracted considerable interest because of its nontrivial global topology. In addition to the conical topology of the usual string solution, there is a helical structure of time which gives rise to the possibility of closed timelike curves (CTC's) near the source, as well as a gravitational time delay [11].

Semiclassical gravitational effects on a spinning cone have been considered by Matsas [12], who showed that the vacuum expectation value of the angular momentum of a massless conformally coupled scalar field is nonzero in this background geometry. But in quantum theory the causality-violating region of such a "spinning cone" gives rise to apparently pathological features such as unitarity problems [13], and it is responsible for making the Dirac Hamiltonian lose its self-adjoint character [14, 15]. Hartle [16] has, however, shown that the Hamiltonian formulation of quantum mechanics is ill suited to this type of spacetime. Instead one should apply the sum-over-histories formulation of quantum mechanics. In this formalism the Cauchy problem is well posed, but the evolution is nonunitary in the region with CTC's [17]. Deutsch [18] has reached the same result in a quantum computation model. Maybe nonunitarity is a general property of quantum mechanics with time machines.

Nevertheless, a more serious problem remains: What happens to the vacuum expectation value of the energy-momentum tensor on the Cauchy horizon? In the wormhole models of time machines it diverges [19, 20], and Hawking [21] has argued that quantum effects therefore would prevent the formation of CTC's. Others, e.g., Kim and Thorne [20], hold that the divergence is too weak: It produces fluctuations of the geometry which are smaller than the quantum gravity fluctuations and hence the gravitational effect of the vacuum polarization is unimportant [20]. In the present paper we will consider a purely classical model of a spinning string in 3+1 dimensions, without addressing the above issue.

Recently a number of authors have pointed out that spinning point particles (in 2+1 dimensions), or strings (in 3+1 dimensions), behave as gravitational analogues to anyons — gravitational anyons [22–26]. Such particles, which display fractional statistics, were first shown to exist within the framework of quantum mechanics in two space dimensions [27].

Because of the singularities associated with the idealized infinitely thin string model, one should construct a more realistic model where the tip of the cone is smoothed out. Such a nonsingular spacetime manifold is the proper setting for the study of semiclassical gravitational effects outside a spinning cosmic string. The singular tip of the cone could be replaced by an extended interior region with a nonvanishing Einstein curvature [28–30]. Hence, the tip is replaced by a smooth cap, producing a "ballpoint pen" model [31]. Alternatively the source could be concentrated on a ring of finite radius. Then the geometry is that of a "flower pot" [31]; see Fig. 1.

It has been shown that, when regarded as a line singularity, the spacetime geometry for a spinning string exhibits torsion at the location of the source [32]. A

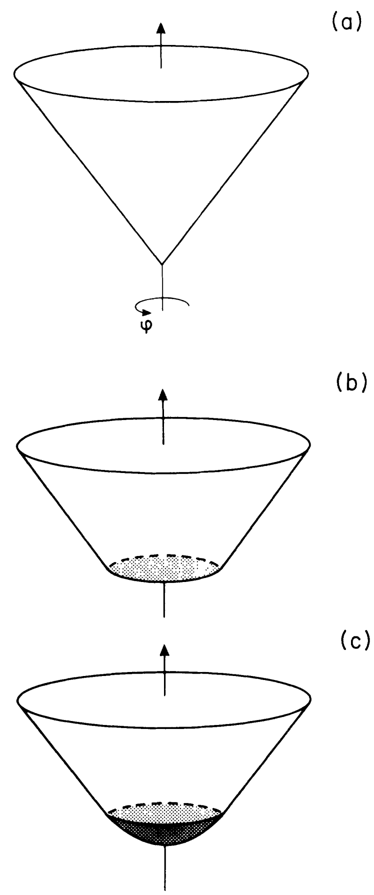

FIG. 1. The figures show two-dimensional projections of the string geometry. (a) The source is a point singularity on the tip of a cone. (b) In the "flower-pot" model, the source is a ring at a finite radius. (c) In the "ballpoint pen" model the Einstein curvature is distributed over a smooth cap surface.

more realistic model could be composed in two ways: either by considering an extended source with torsion or a rotating source. A "ballpoint pen" source of the first kind has been found within the Einstein-Cartan theory [33]. In this model the source is a homogeneous cylinder with spin polarization along the axis of symmetry. In the present paper we will construct a source of the second kind.

The belief that physically realistic sources will not produce CTC's is often encountered. In [4], referring to three-dimensional spacetime, it is stated that "One can show that such closed time-like contours are not possible in a space with n moving spinless particles (...)." Waelbroeck [34] has shown that a pair of cosmic strings with relative angular momentum does not have CTC's. A more general conviction is found in Ref. [35]: "We believe that this is a general result: closed time-like curves are absent for reasonable geometries." In Ref. [36] entitled "Physical cosmic strings do not generate closed time-like curves" it is proved, in the case of point particles, that "there are no CTC if the spacetimes have physically acceptable global structure, which they do for physically acceptable sources." A four-dimensional rotating dust cylinder can produce closed timelike curves [37, 38], however. This solution must be dismissed, it might be argued, since the exterior is not asymptotically flat, and hence unphysical. On the other hand it would be surprising if the causal structure of a realistic spinning cosmic-string model differs fundamentally from that of a spinning dust cylinder. After all, the existence of CTC's is more of a topological issue than a geometrical one.

Here we consider both types of extended string models. We find that a rotating cosmic string can be given by a rotating "flower pot," with the ring source being a (2+1)-dimensional spinning string [8]. For the "ballpoint pen," we find new exact solutions of Einstein's field equations. In the latter model we show that closed timelike curves can exist even if both the weak and the dominant energy conditions hold in the interior of the string. The loophole in the proof of the nonexistence of CTC's in Ref. [36] is the restriction to a source of n point particles, combined with the lack of gravitational attraction between such particles (strings) in three (four) spacetime dimensions. It could well be argued that a fluid model in which *gravitational attraction* appears is more realistic than the point-particle (string) model. After all the vacuum-polarization effect does liberate the graviton and introduces gravitational attraction on the semiclassical level [39–41].

II. GEOMETRY OF THE SPINNING STRING

The line element in both the exterior and interior regions will be assumed to be of the form

$$ds^2 = -(F dt + M d\phi)^2 + A^2 d\phi^2 + dz^2 + dr^2, \qquad (1)$$

where F, M, and A are functions of r only. Note that Lorentz invariance along the z axis implies that any solution of the four-dimensional field equations may also be interpreted as a solution of the corresponding (2+1)-dimensional equations.

In the exterior region, the metric is a flat vacuum solution [4]. Here

$$F = 1, \quad M = m, \quad \text{and} \quad A = B(r + r_0) \ , \qquad (2)$$

where m, B, and r_0 are constants. Locally this is the Minkowski metric in disguise ($t \equiv T - m\phi$), but the global topology is different. Firstly $m \neq 0$ induces a helical structure of time, and secondly $B < 1$ produces a conical topology. The constant m is determined by the angular momentum per length, J, by

$$m = 4GJ \ . \qquad (3)$$

B is a measure of angle deficit of the cone, which is determined by the mass per length, μ, by

$$B \equiv 1 - 4G\mu \ . \qquad (4)$$

Note that if $r + r_0 < m/B$, the ϕ coordinate becomes timelike, and because it is periodic, closed timelike curves do exist here. r_0 is a constant determining the origin of the exterior radial coordinate, so that the radial coordinates coincide in the interior and exterior coordinate systems.

For later convenience we define an orthonormal tetrad frame by

$$\omega^0 = F dt + M d\phi \ ,$$
$$\omega^1 = A d\phi \ ,$$
$$\omega^2 = dz \ ,$$
$$\omega^3 = dr \ . \tag{5}$$

We will match the interior region to the exterior vacuum, and thereby determine the properties of the string surface by means of Israel's formalism of singular surfaces [42]. The energy-momentum surface density of this surface is given by Lanczos energy-momentum tensor:

$$8\pi G S^i{}_j = [K^i{}_j] - \delta^i{}_j [K] \ , \tag{6}$$

where $K^i{}_j$ are components of the external curvature tensor, x^i are the coordinates in the hyperspace (t, ϕ, z), and square brackets signify the discontinuity at the junction radius.

Since we are using a radial coordinate where $g_{rr} = 1$, the exterior curvature is given simply by $K_{ij} = -\frac{1}{2} g_{ij,r}$.

Transforming K_{ij} to the tetrad basis (5), we get the following energy density and pressures[1]

$$8\pi G S^0{}_0 = \left[\frac{A'}{A}\right], \quad 8\pi G S^1{}_1 = \left[\frac{F'}{F}\right],$$
$$8\pi G S^2{}_2 = \left[\frac{A'}{A} + \frac{F'}{F}\right], \tag{7}$$

and the energy current

$$8\pi G S^0{}_1 = -8\pi G S^1{}_0 = \left[\frac{F'M}{2AF} - \frac{M'}{2A}\right] \ . \tag{8}$$

III. THE "FLOWER-POT" MODEL

The spacetime region inside a uniformly rotating infinitely thin hollow cylinder is flat [43, 44]. Therefore the interior spacetime geometry can be described by the metric of a rotating disk [8]

$$F^2 = \gamma^2 - \omega^2 r^2, \quad M = \omega r^2 / F, \quad A^2 = r^2 + M^2. \tag{9}$$

A nonzero ω is necessary to allow a continuous matching to the exterior vacuum solution.

The projection of the metric into the junction surface must be a continuous function over the junction. This gives

$$\gamma^2 = 1 + \omega^2 r_s^2, \quad \omega r_s^2 = m, \quad r_s^2 + m^2 = B^2 (r_s + r_0)^2, \tag{10}$$

[1]Throughout the paper a prime denotes a derivative with respect to r.

where the subscript s denotes the value of the radial coordinate r at the junction surface.

Inserting the metric functions of the exterior and interior solutions into Eqs. (7) and (8), we get the Lanczos tensor

$$8\pi G S^0{}_0 = \frac{1}{(r_s^2 + m^2)^{1/2}} \left(1 - 4G\mu - \frac{(r_s^2 + m^2)^{3/2}}{r_s^3}\right), \tag{11}$$

$$8\pi G S^1{}_1 = \frac{1}{(r_s^2 + m^2)^{1/2}} \left(\frac{m^2 (r_s^2 + m^2)^{1/2}}{r_s^3}\right), \tag{12}$$

$$8\pi G S^2{}_2 = \frac{1}{(r_s^2 + m^2)^{1/2}} \left(1 - 4G\mu - \frac{(r_s^2 + m^2)^{1/2}}{r_s}\right), \tag{13}$$

and

$$8\pi G S^0{}_1 = \frac{1}{(r_s^2 + m^2)^{1/2}} \left(\frac{m(m^2 + r_s^2)}{r_s^2}\right). \tag{14}$$

This shows that the "flower-pot" model is a possible source of the spinning cosmic string. Note that causality violation is excluded in this model because $\omega r_s = m/r_s < 1$. We now turn to a "ballpoint pen" model.

IV. THE "BALLPOINT PEN" MODEL

Here we set $F \equiv 1$. Then with the definition

$$\Omega \equiv \frac{M'}{2A} \ , \tag{15}$$

Einstein's field equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ take the form

$$8\pi G \rho = 3\Omega^2 - \frac{A''}{A}, \tag{16}$$
$$8\pi G p_\phi = \Omega^2, \tag{17}$$
$$8\pi G p_z = -\Omega^2 + \frac{A''}{A}, \tag{18}$$
$$8\pi G p_r = \Omega^2, \tag{19}$$
$$8\pi G q_\phi = \Omega', \tag{20}$$

where ρ, p, and q are the energy density, the pressure and the heat flow relative to the reference frame (5), respectively. Because of Lorentz invariance along the z axis, the same field equations, except (18), are valid in 2+1 dimensions. Note that it is the rotation which allows us to construct a finite (2+1)-dimensional model with pressure. In the hydrostatic case such models do not exist [45].

Petti [46] has shown that discrete rotating masses in general relativity imply the presence of translational holonomy which is transformed into torsion by the limiting process which transforms discrete masses into continuous matter fields. According to the Einstein-Cartan theory, it is the *canonical* energy-momentum tensor which gives the correct local description of the energy and momentum of matter, whereas the *combined* energy-momentum tensor is the source of the metric field [47]. In simple classical spin fluid models one finds that the effective gravitational mass and pressure densities are the canonical ones minus the spin density squared. By Petti's correspondence one should expect that the exterior gravitational field of a limited rotating mass distribution should be determined by the *combined* density and pressures $\bar{\rho} = \rho - \Omega^2$ and $\bar{p} = p - \Omega^2$. These quantities imply the proper equation of state for a cosmic string:

$$\bar{\rho} = -\bar{p}_z \quad \text{and} \quad \bar{p}_\phi = \bar{p}_r = 0 \ . \tag{21}$$

Note that if we calculate the Tolman mass using these quantities we get zero, which is consistent with a flat exterior geometry.

To be able to integrate the field equations, we assume that the energy density is of the simple form

$$8\pi G\rho = \lambda + 3\Omega^2 \ , \tag{22}$$

where λ is a positive constant. Then it follows from Einstein's field equations that the weak energy condition is satisfied.

With the condition that the metric is Minkowskian on the axis, the solution for A is

$$A = \frac{1}{\sqrt{\lambda}} \sin(\sqrt{\lambda} r) \ . \tag{23}$$

Without further specifications of the physical properties of the string, Ω remains undetermined. One alternative is to select a simple form of q_ϕ, and integrate the field equations. Choosing q_ϕ = const, and demanding that the dominant energy condition

$$-T^0{}_0 - |T^0{}_1| \geq 0 \tag{24}$$

be satsified, the energy flux is restricted by $|8\pi G q_\phi| = |\Omega'| \leq \lambda$. Thus

$$\Omega' = -\alpha\lambda \ , \tag{25}$$

where $\alpha \leq 1$.

To match this solution with an external vacuum we must demand that the radial pressure vanish at the junction radius. Hence, if we let r_s stand for the value of r at the radius of the string, we have to set $\Omega(r_s) = 0$ or

$$\Omega = \alpha\lambda(r_s - r) \ . \tag{26}$$

With the definition (15), and the condition that $M(0) = 0$, we find

$$M(r) = 2\alpha\left((r - r_s)\cos(\sqrt{\lambda} r) - \frac{1}{\sqrt{\lambda}}\sin(\sqrt{\lambda} r) + r_s\right) \ . \tag{27}$$

The condition for the existence of closed timelike curves is $g_{\phi\phi} < 0$ for some values of r. Then the periodic coordinate ϕ becomes timelike. Hence, we must demand

$$A^2 - M^2 < 0 \ . \tag{28}$$

By choosing the parameters α and r_s one may construct models with and without CTC's, which satisfy the weak and the dominant energy conditions. In Fig. 2 we present a graph of $g_{\phi\phi}$ as a function of r for a model with CTC's.

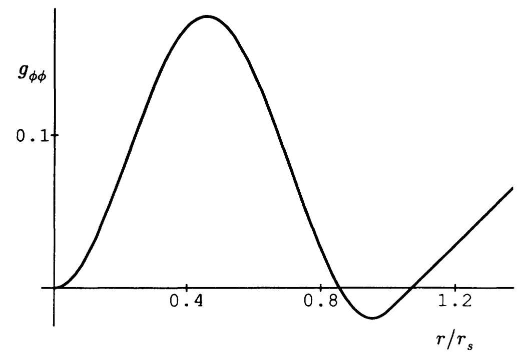

FIG. 2. The figure shows $g_{\phi\phi}$ in units of λ^{-1} as a function of r/r_s with the parameter values $\alpha = 1$, and $\sqrt{\lambda} r_s = 3/2$. There are CTC's in the region where $g_{\phi\phi}$ is negative.

The total *effective* mass per unit length, μ, and angular momentum per unit length, J, are given by the integrals

$$\mu = \frac{1}{8\pi G}\int_0^{2\pi}\int_0^{r_s} \lambda\omega^1\omega^2 \tag{29}$$

and

$$J = \int_0^{2\pi}\int_0^{r_s} \sigma\omega^1\omega^2, \tag{30}$$

where $\sigma = \Omega/4\pi G$ is the angular momentum density, and $\omega^1\omega^2 = A\,dr\,d\phi$ is the area element of the string section. This gives

$$\mu = \frac{1}{4G}[1 - \cos(\sqrt{\lambda} r_s)] \tag{31}$$

and

$$J = \frac{1}{4G}M(r_s) \ . \tag{32}$$

This solution is to be matched to an exterior vacuum solution. The metric projected into the junction surface has to be a continuous function of r. This yields

$$M(r_s) = m \quad \text{and} \quad A(r_s) = B(r_s + r_0) \ . \tag{33}$$

Also the first derivatives of the metric must be continuous. Hence, the solution must also satisfy the conditions

$$[A'] = 0 \quad \text{and} \quad [M'] = 0 \ . \tag{34}$$

The conditions (33) and (34) are met provided

$$r_0 = \left(\frac{[1 - (1 - 4G\mu)^2]^{1/2}}{(1 - 4G\mu)\arccos(1 - 4G\mu)} - 1\right) r_s \ . \tag{35}$$

We have constructed a physical source for all the three constants of the exterior vacuum solution. $B = 1 - 4G\mu$ is a measure of the total mass per length of the string, and the spin parameter $m = 4GJ$ measures the total angular momentum per unit length of the string.

In the limit $m \to 0$ and $\Omega \to 0$, the model reduces to the Gott-Linet-Hiscock model [28–30].

V. CONCLUSION

The fact that the exterior metric is flat is consistent with the fact that the Tolman mass of the string is zero. Therefore, a string as discussed above has no gravitational mass, and all the gravitational effects are of topological origin. Hence, a freely falling test particle feels no gravitational attraction outside the string, but because of the conical topology, two parallel straight lines passing on different sides of the string will converge and cross each other.

As opposed to the Einstein-Cartan spinning string model there is no homogeneous interior solution in Einstein's theory. Both in Einstein's theory and in the Einstein-Cartan theory there is no fundamental law which forbids causality-violating regions in the spacetime of a spinning cosmic string. This, however, does not mean that such objects really exist in nature. On the contrary, for realistic strings the angular momentum density is too small for the phenomenon to exist [33].

But even if the angular momentum density is large enough to produce CTC's there are other mechanisms present which could prevent their formation. There are for instance indications that rotating objects will radiate away sufficient angular momentum to prevent CTC's from coming into existence [48]. It may also be that quantum distortions of the classical spacetime prevent the creation of CTC's [21]. Furthermore, as pointed out in [35], it is reasonable to believe that the history of formation of a system should be given by a Cauchy evolution of spacelike surfaces. This puts severe restrictions on the possibility of the occurrence of CTC's as shown by Tipler [49]; "(...) closed timelike lines cannot evolve from regular initial data in a singularity-free asymptotically flat spacetime" (which satisfies the weak energy condition and the generic condition). On the other hand, despite the appeal of the idea that the history of any system should be constructable by a Cauchy evolution, there does not exist a compelling reason based on physical arguments that the Universe really admits a Cauchy surface [50]. One should also note that Tipler's theorem does not hold for the spinning string, because the generic condition states that any nonspacelike geodesics must feel tidal forces in at least one point. This condition cannot be satisfied in the spacetime exterior to a finite matter distribution in three-dimensional spacetime, because of the complete absence of tidal forces in a vacuum. An infinite straight string is of course an idealization. It is tempting to assume that spacetime close to a very long string does not differ much from the spacetime outside an infinte one. But it is risky to claim that properties of an infinite cylinder also hold true for a finite one [51]. This is particularly evident here, since the spacetime geometry exterior to a *finite* string is curved, and therefore satisfies the generic condition.

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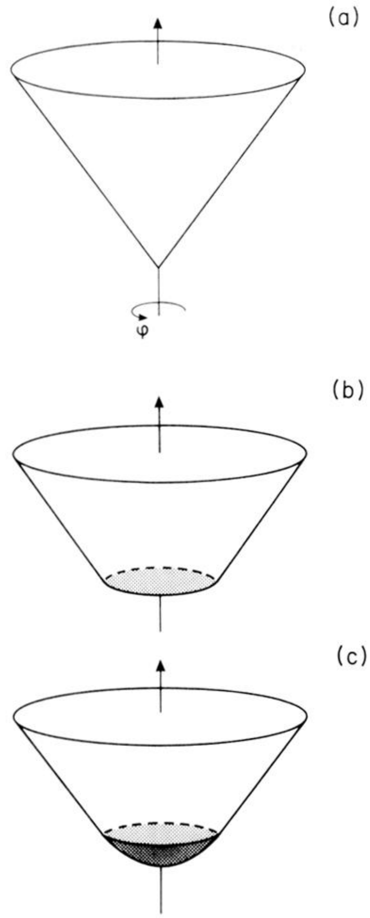

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