# Singularity-free decaying-vacuum cosmologies

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Decaying-vacuum singularity-free cosmological models based on the Chen-Wu ansatz of a cosmological term varying as $R^{-2}$ where $R$ is the scale factor of the Universe are introduced. They describe a closed ever-expanding universe of density parameter $\Omega \geq 1$ and with no entropy, horizon, or monopole problems. They include and extend the critical density cosmology of Özer and Taha. The Özer-Taha period of phase transition during part of which the pressure is negative occurs in these models. In its wake and throughout the radiation-dominated era the Universe is Einstein–de Sitter–like with $\Omega$ then and subsequently near unity. Nucleosynthesis proceeds as in the standard model. Consistency with the observed helium abundance and baryon asymmetry allows a maximum vacuum energy close to the radiation energy today. The presence of this vacuum energy could be detrimental to certain theories of galaxy formation. A specific model with initial conditions approaching the hot big bang is studied in detail, particularly as regards the entropy and flatness of the very early Universe.

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## I. INTRODUCTION

Astrophysical observations show that a nonvanishing cosmological constant must be extremely small. But the very early Universe [1] of standard cosmology [2] has probably gone through a series of phase transitions that should have left the cosmological constant larger than its observational upper bound by about 120 orders of magnitude.

Attempts [3] at a solution or gaining a better understanding of this problem have been numerous. In one classical approach a decaying cosmological "constant" is proposed. Generally a variable cosmological constant implies creation of radiation and matter and nonconservation of entropy [4]. A problem with the standard model concerns its inability to explain the generation of entropy in the Universe. The field equations of general relativity and the usual perfect-fluid form of the energy-momentum tensor imply that [2] entropy does not change throughout the reversible expansion of the Universe.

Recently Özer and Taha [5] have proposed a model in which the cosmological constant $\Lambda$ is time dependent and the cosmic density $\rho$ equals the Einstein–de Sitter critical density $\rho_c$. The condition $\rho=\rho_c$ and the requirement of an increasing entropy completely determine $\Lambda$ in terms of the Robertson-Walker scale factor $R$. In the resulting cosmology the Universe is closed, singularity-free, initially cold, and has no entropy, horizon, or monopole problems.

In a separate development independent of the critical density assumption, Chen and Wu [6] suggested, *a priori*, that $\Lambda \propto R^{-2}$. They have shown that such a behavior is deducible from simple general principles in line with quantum gravity. Their model is singular and preserves the standard picture of the early Universe. Its predictions for the fate of the Universe are, however, different.

Unless required by a hitherto unknown symmetry principle, the exact equality of the cosmic and critical densities, which underlies the Özer-Taha [5] model and is also favored by inflation [7], is hard to justify. On the other hand the Özer-Taha [5] model has the attractive feature of being free of the main cosmological problems. With all this in mind we have sought to generalize this model by abandoning the critical density assumption but using the Chen-Wu [6] prescription for $\Lambda$. The result is this paper.

In Sec. II we write the field equations and discuss the entropy problem. In Sec. III we show that the incorporation of the Chen-Wu ansatz ($\Lambda=3\gamma R^{-2}$ where $\gamma$ is a constant) into the field equations together with the requirements of entropy production and evasion of the initial singularity imply a closed universe.

In the present paper we follow Özer and Taha in postulating that the Universe has passed through three phases: a very early ($T \geq 10^{15}$ GeV) epoch of pure radiation; a subsequent phase-transition period of rest-mass or matter generation; and, lastly, an era of radiation and conserved matter reaching to the present. Except for the matter generation period we assume that $\Lambda$, or equivalently the vacuum, decays into thermal radiation with a Planck distribution.

Section IV discusses the pure radiation universe. It is noted that the time dependence of the scale factor restricts the Chen-Wu parameter $\gamma$ to the range $\frac{1}{2} < \gamma \leq 1$ and solves the horizon and monopole problems. It is also noted that the constraint on $\gamma$ implies that the density parameter $\Omega \geq 1$. The model of Özer and Taha corresponds to the limiting case $\gamma=\Omega=1$.

Sections V and VI generalize, respectively, sections 4 and 5 of Ref. [5]. They discuss respectively the matter and radiation and the rest-mass generation periods. Section VII explores observational implications of our work: cosmic helium synthesis, the baryon-to-photon and baryon-to-entropy ratios, and some possible consequences for galaxy formation. In Sec. VIII we study a specific model ($\gamma=\frac{2}{3}$). In Sec. IX we discuss our results and relate some of them to the cosmology of Freese *et al.* [8] and the recent work by Pavón [9]. Section X states our conclusion. The Appendix summarizes and generalizes the Özer-Taha arguments on the neutrino-to-photon temperature ratio in nonadiabatic models of the present type.

## II. VARIABLE-Λ COSMOLOGY

### A. Field equations

Einstein's field equations with a variable cosmological constant Λ are [10]

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R^{\sigma}{}_{\sigma}=8\pi G T_{\mu\nu}+\Lambda(x)g_{\mu\nu} . \tag{1}$$

In the homogeneous and isotropic Robertson-Walker spacetime these equations and the perfect-fluid matter energy-momentum tensor

$$T_{\mu\nu}=-p(t)g_{\mu\nu}+[\rho(t)+p(t)]u_{\mu}u_{\nu} \tag{2}$$

lead to

$$\frac{2\ddot{R}}{R}=\frac{2\Lambda}{3}-\alpha^{-1}(\rho+3p) , \tag{3}$$

$$\left[\frac{\dot{R}}{R}\right]^2=\alpha^{-1}\rho+\Lambda/3-k/R^2 , \tag{4}$$

where $\alpha\equiv 3/8\pi G$ and $\Lambda=\Lambda(t)$ because of the homogeneity and isotropy of the model. From Eqs. (3) and (4) follows the energy equation

$$\frac{d}{dt}(\rho R^3)+p\frac{d}{dt}R^3+\frac{\alpha}{3}R^3\frac{d}{dt}\Lambda=0 . \tag{5}$$

With a definite model for Λ and a choice of an appropriate equation of state, Eq. (5) determines $\rho$, leaving Eq. (4) to be solved for $R(t)$. Generally $\rho=\rho_m+\rho_r$, where $\rho_m$ is the rest-mass energy density and $\rho_r$ is the energy density of radiation and ultrarelativistic matter (henceforth to be referred to collectively as radiation-reserving "pure radiation" for photons and massless neutrinos).

The field equations suggest the correspondence

$$\rho_v\equiv\frac{\alpha}{3}\Lambda , \tag{6}$$

where $\rho_v$ is the vacuum energy density. Theoretically the implications of Eq. (5) would appear to depend on whether $\rho_v$ couples to radiation or to nonrelativistic matter [3,8].

### B. Entropy

Equation (5) may be rewritten as

$$-\frac{\alpha}{3}R^3 d\Lambda=T\,dS , \tag{7}$$

where $T$ and $S$ denote respectively the temperature and matter entropy of the Universe. In obtaining Eq. (7) we have identified the proper volume of the Universe with $R^3$. We shall return to this point later on.

In conventional general relativity $\Lambda=0$ so that $dS=0$, i.e., entropy is conserved. This is the entropy problem of standard cosmology. In variable-Λ cosmology the condition $dS/dt>0$ requires, as evident from Eq. (7), that $d\Lambda/dt<0$ for all $t>0$. Or, since $\dot{R}>0$, $d\Lambda/dR<0$ throughout cosmic expansion.

## III. SINGULARITY-FREE COSMOLOGIES WITH Λ VARYING AS $R^{-2}$

Chen and Wu [6] have introduced the ansatz [11]

$$\Lambda=3\gamma/R^2 , \tag{8}$$

where $\gamma$ is a model-dependent number. Phenomenologically $\gamma$ is expected to be of order 1 and positive in a flat universe [6].

We observe that the condition $d\Lambda/dR<0$ requires $\gamma>0$ independently of the curvature index $k$ and hence $\Lambda>0$ for all $t\geq 0$. With $\Lambda>0$ Eq. (3) shows that the existence of the initial singularity $R=0$ is not compelling. In particular it is possible for $R$ to have had a minimum nonvanishing initial value at $t=0$, say. The necessary condition for the existence of this minimum in an expanding universe is $\dot{R}=0$ at $t=0$. In what follows we explore this possibility.

Equations (4) and (8) and the assumption $\dot{R}=0$ at $t=0$ give (the subscript zero denotes values of the parameters at $t=0$)

$$\alpha^{-1}\rho_0 R_0^2=k-\gamma . \tag{9}$$

Then $\rho_0\geq 0$ implies that $k\geq\gamma>0$ so that $k=1$.

It is interesting to note that a closed universe is also a consequence of Eq. (4) with $\Lambda\geq 0$ in models where the present scale factor is taken to be maximum [12]. Such ideas were discussed in connection with possible modifications of the redshift-distance relation [13].

## IV. VERY EARLY UNIVERSE

Prior to the generation of rest mass, in the pure-radiation very early Universe, $p=\frac{1}{3}\rho$. Then use of the ansatz (8) in Eq. (5) leads to

$$\rho=\frac{\alpha\gamma}{R^2}\left[1-\frac{\gamma^{-1}(2\gamma-1)R_0^2}{R^2}\right] , \tag{10}$$

where $\rho_0$ was eliminated on using Eq. (9) (with $k=1$).

Substitution of Eqs. (8) and (10) in Eq. (4) gives

$$R^2\dot{R}^2=(2\gamma-1)(R^2-R_0^2) , \tag{11}$$

from which we deduce that $\gamma>\frac{1}{2}$ or, equivalently,

$$\rho_0<\frac{\alpha}{2R_0^2} . \tag{12}$$

Thus in the present cosmologies the Chen-Wu [6] parameter $\gamma$ is restricted to the range

$$\tfrac{1}{2}<\gamma\leq 1 , \tag{13}$$

independently of $R_0$ ($R_0\neq 0$).

When combined with Eqs. (4) and (8) the condition (13) leads to

$$0 \le \Omega - 1 < \frac{1}{2R^2H^2} , \tag{14}$$

where $H = \dot{R}/R$ is Hubble's constant and $\Omega = \rho/\rho_c$, with $\rho_c = \alpha H^2$ being the Einstein–de Sitter critical density.

The result (14) is valid throughout cosmic evolution with the upper bound and $\Omega$ approaching infinity as $t \to 0$. An infinite $\Omega$ at $t=0$ reflects a completely warped-up spacetime at the cosmic beginning. Thus in the present cosmologies $\Omega \ge 1$ with the upper bound on $\Omega - 1$ in Eq. (14) diminishing (increasing) with accelerated (decelerated) cosmic expansion. The identity $\Omega \equiv 1$ corresponds to the Özer-Taha [5] model.

The time dependence of $R$ follows on integrating Eq. (11). One has

$$R^2 = (2\gamma - 1)t^2 + R_0^2 . \tag{15}$$

The density $\rho$ attains a maximum value

$$\rho_{\max} = \frac{\alpha\gamma^2(2\gamma-1)^{-1}}{4R_0^2} \ge \frac{\alpha}{4R_0^2} , \tag{16}$$

corresponding to $R = R_{\rm mx}$ where

$$R_{\rm mx}^2 = 2\gamma^{-1}(2\gamma-1)R_0^2 \le 2R_0^2 . \tag{17}$$

The radiation temperature $T$ is assumed to be related to $\rho$ by $(h/2\pi = c = k_B = 1)$

$$\rho = \frac{\pi^2}{30} g_{\rm eff} T^4 , \tag{18}$$

where $g_{\rm eff}$ is the effective number of spin degrees of freedom, which we assume to be constant in the very early pure-radiation era [14]. Then,

$$T = \left[ \frac{30\alpha\gamma}{\pi^2 g_{\rm eff}} \right]^{1/4} \left[ 1 - \frac{\gamma^{-1}(2\gamma-1)R_0^2}{R^2} \right]^{1/4} R^{-1/2} , \tag{19}$$

with

$$T_{\max} = \left[ \frac{15\alpha\gamma^2}{2\pi^2 g_{\rm eff}(2\gamma-1)R_0^2} \right]^{1/4} \ge \left[ \frac{15\alpha}{2\pi^2 g_{\rm eff} R_0^2} \right]^{1/4} . \tag{20}$$

In terms of $t$,

$$\rho = \alpha\gamma(2\gamma-1)^{-1} \frac{t^2 + \gamma^{-1}(2\gamma-1)^{-1}(1-\gamma)R_0^2}{[t^2 + (2\gamma-1)^{-1}R_0^2]^2} , \tag{21}$$

and

$$T = \left[ \frac{30\alpha\gamma}{\pi^2 g_{\rm eff}(2\gamma-1)} \right]^{1/4} \times \frac{[t^2 + \gamma^{-1}(2\gamma-1)^{-1}(1-\gamma)R_0^2]^{1/4}}{[t^2 + (2\gamma-1)^{-1}R_0^2]^{1/2}} . \tag{22}$$

To obtain order-of-magnitude estimates of $R_0$ and hence $\rho_0$ we take $T_{\max} \sim M_{\rm Pl} = G^{-1/2}$, where $M_{\rm Pl}$ is Planck mass. This is natural since $M_{\rm Pl}$ is the only energy scale in the theory. Then according to Eq. (20), $R_0 \ge 5\times 10^{-36} g_{\rm eff}^{-1/2}$ m [using $G = 6.62\times 10^{-39}$ (GeV)$^{-2}$, 1 m $= 5\times 10^{15}$ (GeV)$^{-1}$]. It follows from Eq. (12) that $\rho_0 < 3.3\times 10^{96} g_{\rm eff}$ kg m$^{-3}$.

For a closed universe a time dependence of $R$ of the type exhibited by Eq. (15) solves the horizon and monopole problems (see Ref. [5] for details). In particular we find here that global causality is established at $t = t_{\rm caus}$ where

$$t_{\rm caus} = \frac{R_0}{(2\gamma-1)^{1/2}} \sinh \left[ \frac{\pi}{2}(2\gamma-1)^{1/2} \right] > \frac{\pi}{2} R_0 . \tag{23}$$

## V. RADIATION AND MATTER UNIVERSE

### A. Basic equations

In the wake of the pure radiation era, for $R_1 \le R \le R_2$, say, rest mass is generated. During this period the vacuum decayed (possibly into both matter and radiation) in accordance with Eq. (5). But the equation of state appropriate to this phase is not known. Nevertheless some general conclusions on the implications of Eq. (5) for the era of rest-mass generation can be reached. These are given in the next section.

For $R \ge R_2$, after creation of rest mass, we assume that the vacuum decayed into radiation only and that the rate of change of the rest-mass energy $E_m = \rho_m R^3$ was much smaller than that of the radiation energy $E_r = \rho_r R^3$. Then $E_m$ stayed constant so that (the subscript $p$ denotes present-day quantities)

$$E_m = \rho_m R^3 = E_{mp} = \rho_{mp} R_p^3 . \tag{24}$$

Furthermore we assume [5] that matter does not, under these conditions, contribute to pressure so that the equation of state is

$$P = \tfrac{1}{3}(\rho - E_{mp}R^{-3}), \quad R \ge R_2 . \tag{25}$$

We have previously defined $S$ and now also $E$ as the total entropy and total energy in a volume $R^3$, although the proper volume of a *closed* universe is $2\pi^2 R^3$. This is inconsequential because the measurable quantities are the entropy and energy per unit volume (entropy and energy densities).

Substituting Eqs. (24), (25), and (8) in Eq. (5) we arrive at

$$\rho_r = \frac{\alpha\gamma}{R^2} \left[ 1 + \frac{\omega R_p^2}{R^2} \right], \quad R \ge R_2 , \tag{26}$$

where

$$\omega + 1 = \alpha^{-1}\gamma^{-1}\rho_{rp}R_p^2 = \rho_{rp}/\rho_{vp} , \tag{27}$$

with $\rho_{rp}$ and $\rho_{vp}$ being the present radiation and vacuum energy densities, respectively.

From Eq. (26) the total radiation energy $E_r = \rho_r R^3$ is

$$E_r = \alpha\gamma R + \alpha\gamma\omega R_p^2 R^{-1} . \tag{28}$$

The equilibrium scale factor $R_{\rm eq}$ is defined as the value of $R$ at $t = t_{\rm eq}$, the beginning of the matter-dominated era of the Universe, when the radiation and matter energies were equal. Thus

$$E_r(R_{\rm eq})=E_{mp} \tag{29}$$

with

$$E_r(R)\gtrless E_{mp} \quad \text{when } R\lessgtr R_{\rm eq} \ .$$

This means that as $R$ rises to $R_{\rm eq}$ it must be small enough for $E_r$ to be decreasing at $R=R_{\rm eq}$ so that

$$\omega > R_{\rm eq}^2/R_p^2 \ . \tag{30}$$

This result was derived by Özer and Taha in the $\gamma=1$ critical density model. We note here that it holds for the whole class of cosmologies considered in this paper. It implies, from Eq. (27), that $\rho_{vp}<\rho_{rp}$.

Equation (29) yields two values for $R_{\rm eq}$, the smaller of which is

$$R_{\rm eq}=\frac{(1+\omega)^{3/2}\alpha^{1/2}\gamma^{1/2}\rho_{mp}}{2\rho_{rp}^{3/2}}\left[1-\left[1-\frac{4\omega\rho_{rp}^2}{(1+\omega)^2\rho_{mp}^2}\right]^{1/2}\right]. \tag{31}$$

With $g_{\rm eff}(T_{\rm eq})=g_{\rm eff}(T_p)$ one then deduces

$$T_{\rm eq}=2^{3/4}T_p\left[\frac{\rho_{rp}}{\rho_{mp}}\right]^{1/2}(1+\omega)^{-3/4} \times\left[1-\left[1-\frac{4\omega\rho_{rp}^2}{(1+\omega)^2\rho_{mp}^2}\right]^{1/2}\right]^{-3/4} . \tag{32}$$

The present radiation energy density $\rho_{rp}$ is calculable from

$$\rho_{rp}=\frac{\pi^2}{30}g_{\rm eff}(T_p)T_p^4 \ . \tag{33}$$

As shown in Ref. [5] and further explained in the Appendix, $g_{\rm eff}(T_p)=\frac{43}{11}$ in the present cosmologies (counting three neutrino types). Then the experimental value $T_p=2.7$ K produces

$$\rho_{rp}\approx 3.8\times 10^{-51}\ ({\rm GeV})^4 \ . \tag{34}$$

On the other hand the present total energy density is $\rho_p=\Omega_p\alpha H_p^2$. If we take $H_p=100h\,{\rm km\ s^{-1}\ Mpc^{-1}}$ $(0.5\le h\le 1)$, we have $\rho_p=8\times 10^{-47}\Omega_p h^2\ ({\rm GeV})^4$. Hence the Universe today is matter dominated.

The second root of Eq. (29), viz.,

$$R_{\rm eq}^{\rm II}=\frac{(1+\omega)^{3/2}\alpha^{1/2}\gamma^{1/2}\rho_{mp}}{2\rho_{rp}^{3/2}} \times\left[1+\left[1-\frac{4\omega\rho_{rp}^2}{(1+\omega)^2\rho_{mp}^2}\right]^{1/2}\right], \tag{35}$$

marks a second future equilibrium moment between radiation and matter. Thus in the models under consideration the dominance of matter today is not permanent.

Since [15] $\rho_{rp}/\rho_{mp}<2\times 10^{-4}$ we can approximate $R_{\rm eq}$, $T_{\rm eq}$, and $R_p$ by

$$R_{\rm eq}\approx\frac{\omega}{(1+\omega)^{1/2}}\frac{\gamma^{1/2}\alpha^{1/2}\rho_{rp}^{1/2}}{\rho_{mp}} \approx\frac{\omega}{(1+\omega)^{1/2}}\gamma^{1/2}\frac{3.3\times 10^{39}}{\Omega_p h^2}\ ({\rm GeV})^{-1} , \tag{36}$$

$$T_{\rm eq}\approx\frac{(1+\omega)^{3/4}}{\omega^{3/4}}T_p\frac{\rho_{mp}}{\rho_{rp}} \approx\frac{(1+\omega)^{3/4}}{\omega^{3/4}}5.6\times 10^4\Omega_p h^2\ {\rm K} , \tag{37}$$

and

$$R_p=\frac{(1+\omega)^{1/2}\gamma^{1/2}\alpha^{1/2}}{\rho_{rp}^{1/2}} \approx(1+\omega)^{1/2}\gamma^{1/2}0.7\times 10^{44}\ ({\rm GeV})^{-1} \ . \tag{38}$$

From Eqs. (38) and (36) one has [15]

$$\frac{\omega R_p^2}{R_{\rm eq}^2}\approx\frac{(1+\omega)^2}{\omega}\left[\frac{\rho_{mp}}{\rho_{rp}}\right]^2>8\times 10^8(\Omega_p h^2)^2 , \tag{39}$$

*independently of* $\omega$ *and* $\gamma$.

The result (39) implies that the $1/R^4$ term in the expression (26) for $\rho_r$ dominates throughout the period $R_2\le R\le R_{\rm eq}$. But this does not mean that the total entropy generated over the whole period $R_2\le R\le R_p$ is small. More precisely, consider Eqs. (7) and (8). They give

$$\frac{dS}{dR}=\frac{2\alpha\gamma}{T} \ . \tag{40}$$

Using Eqs. (18) and (26) to express $T$ in terms of $R$ and integrating lead to [16]

$$S(R_p)-S(R_2)\approx\frac{4}{3}\left[\frac{\pi^2}{30}g_{\rm eff}(T_p)\alpha^3\gamma^3\right]^{1/4} \times[(1+\omega)^{3/4}-\omega^{3/4}]R_p^{3/2} , \tag{41}$$

or

$$S(R_p)-S(R_2) \approx 7\gamma^{3/2}(1+\omega)^{3/4}[(1+\omega)^{3/4}-\omega^{3/4}]\times 10^{93} \ . \tag{42}$$

Note that the equations for $R_{\rm eq}$, $R_p$, and $S(R_p)$ reduce to those of Özer and Taha [5] on setting $\gamma=1$.

### B. $\Omega_p$, $q_p$, and $t_p$

From the inequality (14) and Eqs. (38) and (34) we have, for the density parameter $\Omega_p$ ($h=0.5$),

$$0\le\Omega_p-1<\frac{1}{2R_p^2H_p^2}=\frac{\rho_{rp}(1+\omega)^{-1}}{16\gamma h^2}\times 10^{47} \approx\gamma^{-1}(1+\omega)^{-1}\times 10^{-4}<2\times 10^{-4} \ . \tag{43}$$

It is interesting that we also had $\rho_{rp}/\rho_{mp}<2\times 10^{-4}$.

Now the exact Friedmann equation during the *whole* radiation and matter period $R\ge R_2$ is

$$\dot{R}^2=\alpha^{-1}\rho_{mp}R_p^3R^{-1}+\gamma\omega R_p^2R^{-2}+(2\gamma-1)\ . \qquad (44)$$

From this equation and the condition (39) we get, at [17] $t=t_{eq}$ $(0<2\gamma-1\le 1)$,

$$R_{eq}^2H_{eq}^2=\dot{R}_{eq}^2\approx\frac{2\gamma\omega R_p^2}{R_{eq}^2}+(2\gamma-1)>8\times 10^8(\Omega_p h^2)^2\ , \qquad (45)$$

so that $(\Omega_{eq}-1)<10^{-8}$ independently of $\omega$ and $\gamma$ and for $h\ge 0.5$. Tighter constraints on $\Omega-1$ result as $R$ is decreased retrogressively towards $R_2$.

According to Eq. (44) the deceleration parameter $q=-\ddot{R}R\dot{R}^{-2}$ today is

$$q_p=\frac{1/2+\xi}{1+\xi+\dfrac{(2\gamma-1)\rho_{rp}}{\gamma(1+\omega)\rho_{mp}}}\ , \qquad (46)$$

where

$$\xi=\frac{\omega}{(1+\omega)}\frac{\rho_{rp}}{\rho_{mp}}$$

and $0<(2\gamma-1)\gamma^{-1}\le 1$. Since $\rho_{rp}/\rho_{mp}<2\times 10^{-4}$, $q_p\approx\frac{1}{2}$.

From Eqs. (44) and (46) we may write, for the age of the Universe (taking approximately $t=t_2\approx 0$ when $R=R_2\approx 0$),

$$t_p=(2q_p)^{1/2}H_p^{-1}\int_0^1 dx\left[1-2q_p+2q_p/x+2\xi q_p/x^2\right]^{-1/2} \qquad (47)$$

so that with $q_p\approx\frac{1}{2}$, $t_p\approx\frac{2}{3}H_p^{-1}$.

The obtained $\Omega_p$, $q_p$, and $t_p$ values roughly match the $k=0$ standard-model predictions. By comparison Chen and Wu [6] find $\frac{2}{3}<\Omega_p<1$, $2q_p<\Omega_p$, and $\frac{2}{3}H_p^{-1}<t_p<H_p^{-1}$. But their model is singular at $t=0$ and they assume that the vacuum couples to matter in the matter-dominated universe. On the other hand according to Olson and Jordan [18] a vacuum energy density that decreases as $R^{-2}$ does not modify the standard-model predictions of the age of the Universe. Our result for $t_p$ is compatible with this view.

## VI. REST-MASS GENERATION

As mentioned previously, the present cosmologies postulate that a period of matter generation, corresponding to $R_1\le R\le R_2$, has preceded the radiation and matter era. Özer and Taha [5] have shown, for the critical density model with $\gamma=1$, that the cosmic pressure must have been negative during part of this period. We will now show that this feature, and other related results, are *independent of the critical density assumption provided* $\gamma$ *satisfies the constraint (13).*

Equation (5), with $\Lambda$ given by Eq. (8), may be written as

$$dE+p\,dR^3=2\alpha\gamma\,dR\ , \qquad (48)$$

a relation valid for all $R$. When integrated between $R_0$ and $R=R_2$, the beginning of the conserved matter phase, it leads to

$$3\int_{R_0}^{R_2}pR^2dR=2\alpha\gamma(R_2-R_0)+E_0-E_2\ , \qquad (49)$$

where

$$E_0=E(R_0)=\rho_0R_0^3\ , \qquad (50)$$

and

$$E_2=E(R_2)=E_m(R_p)+E_r(R_2)\ . \qquad (51)$$

Using Eqs. (24), (28), and (9) (with $k=1$) we rewrite Eq. (49) as

$$3\int_{R_0}^{R_2}pR^2dR=-\alpha\gamma R_2\left[(1+\omega)\frac{\rho_{mp}R_p}{\rho_{rp}R_2}-1\right]-\frac{\alpha\gamma\omega R_p^2}{R_2}-\alpha R_0(3\gamma-1)\ . \qquad (52)$$

Since $\rho_{mp}R_p>\rho_{rp}R_2$ and $3\gamma>\frac{3}{2}$ by Eq. (13) the right-hand side of Eq. (52) is negative and so is, therefore, also the integral on the left-hand side. But $p=\frac{1}{3}\rho>0$ for $R_0\le R\le R_1$. Therefore,

$$\int_{R_1}^{R_2}pR^2dR<0\ , \qquad (53)$$

implying the existence of at least one negative-pressure phase during the period of matter generation.

A negative-pressure regime in the very early Universe is commonly indicative of a phase transition. An example is the grand-unified-theory (GUT) phase transition at $T_{GUT}\approx 10^{15}$ GeV. We will estimate the time corresponding to this temperature in a particular model (with a definite $\gamma$) in Sec. VIII.

Before the matter-creation era, in the pure-radiation universe, Eq. (11) implies that $\ddot{R}>0$. Afterwards, for $R\ge R_2$, Eq. (44) implies that $\ddot{R}<0$. Thus the appearance of rest-mass ushers in a decelerated expansion phase extending to the present.

During creation of matter the generated energy is predominantly rest-mass energy produced at the rate

$$\frac{dE}{dR}=2\alpha\gamma-3pR^2\ge 0\ , \qquad (54)$$

with $E$ becoming maximum when

$$\frac{dE}{dR}=0\ \text{ or }\ p=\frac{2\alpha\gamma}{3R^2}>0\ .$$

Since $p$ is positive at $R=R_1$ and $R=R_2$ it must pass through at least two zero values in the region $R_1<R<R_2$. Thus $E$ becomes maximum in the later period of positive pressure within $(R_1,R_2)$. All these features, originally noted by Özer and Taha in their critical density model, are seen here to be independent of the critical density assumption.

## VII. OBSERVATIONAL CONSEQUENCES

### A. Cosmic helium synthesis

We now discuss primordial helium synthesis in the present cosmologies. Of particular interest will be the conditions under which the successful standard-model helium abundance prediction may be reproduced.

Progress of helium synthesis in the standard big-bang nucleosynthesis model is marked by the following temperatures: (i) $T_F$, the "freeze-out" temperature of the neutron-to-proton ratio $n/p$; (ii) a temperature which we denote by $T_*$ at which the weak-interaction rates $\lambda(p\to n)$, $\lambda(n+\nu\to p+e^-)$, and $\lambda(n+e^+\to p+\bar{\nu})$ become negligible compared to the neutron $\beta$-decay rate $\lambda(n\to p+e^-+\nu)$, and (iii) the temperature signaling the start of nucleosynthesis $T_N$ ($N$ for nucleosynthesis) when virtually all the residual neutrons surviving $\beta$ decay bind up into stable deuterons, almost all of which burn to helium. Denote the respective times by $t_F$, $t_*$, and $t_N$.

For $T\geq T_F$,

$$n/p=\exp(-Q/T)\ , \tag{55}$$

where $Q=1.293$ MeV is the $n$-$p$ mass difference. The limit temperature $T_F$ is determined by equating the weak-interaction transition rate [19] $[T_n=T/(10^n\ \mathrm{K})]$

$$\lambda(n\to p)\simeq\lambda(p\to n)=0.362T_{10}^5\ \mathrm{s}^{-1} \tag{56}$$

to the cosmic rate of expansion.

In the present cosmologies the expansion rate of the radiation and matter universe is given by Eq. (44). The substitution $\rho_{mp}R_p^3=\rho_{r\,\mathrm{eq}}R_{\mathrm{eq}}^3$, where $\rho_r$ is given by Eq. (26) and the transformation $R=yR_{\mathrm{eq}}$ change this equation to

$$\dot{y}^2=\frac{\gamma\omega R_p^2}{R_{\mathrm{eq}}^4}\left[\left(1+\frac{R_{\mathrm{eq}}^2}{\omega R_p^2}\right)y^{-1}+y^{-2}+\frac{2\gamma-1}{\gamma}\frac{R_{\mathrm{eq}}^2}{\omega R_p^2}\right]. \tag{57}$$

For $y\leq 1$ this equation reduces, because of condition (39), to the approximate form

$$\dot{y}^2\approx\frac{\gamma\omega R_p^2}{R_{\mathrm{eq}}^4}(y^{-1}+y^{-2})\ . \tag{58}$$

Assuming nucleosynthesis to have taken place at the early epoch $R\ll R_{\mathrm{eq}}$ we may neglect the $y^{-1}$ term. Then with

$$\rho_r\approx\frac{\alpha\gamma\omega R_p^2}{R^4},\quad R_2\leq R\leq R_{\mathrm{eq}} \tag{59}$$

we have

$$\dot{R}^2\approx\frac{\gamma\omega R_p^2}{R^2}\approx\alpha^{-1}\rho_r R^2 \tag{60}$$

at the time of nucleosynthesis.

Equation (60) is approximately the standard flat-space (Einstein–de Sitter) Friedmann equation. Here it implies that the universe of our models expanded at nucleosynthesis as if it were nearly of zero curvature and vanishing cosmological constant. Combined with Eq. (18) it gives

$$\frac{\dot{R}}{R}\approx\left[\frac{\pi^2 g_{\mathrm{eff}}}{30\alpha}\right]^{1/2}T^2=0.151g_{\mathrm{eff}}^{1/2}T_{10}^2\ \mathrm{s}^{-1}\ . \tag{61}$$

Taking the number of neutrino species $N_\nu=3$ (see the Appendix) and equating Eqs. (61) and (56) we find $T_{F10}\simeq 1.11$, the same as in the standard model.

On the other hand, from Eqs. (59) and (60),

$$\frac{4\dot{R}}{R}\approx-\frac{\dot{\rho}_r}{\rho_r}\approx 4(\alpha^{-1}\rho_r)^{1/2}\ , \tag{62}$$

so that

$$\begin{aligned}t&\approx\left[\frac{15\alpha}{2\pi^2 g_{\mathrm{eff}}}\right]^{1/2}T^{-2}+t_0\\&=3.31g_{\mathrm{eff}}^{-1/2}T_{10}^{-2}+t_0\ \ \text{(in seconds)}\ ,\end{aligned} \tag{63}$$

where $t_0$ is a constant. In the standard model the initial singularity at $t=0$ requires $t_0=0$. Here $t=0$ is in the pure radiation phase, which has a different temperature-time relation from Eq. (63) [see Eq. (22)] and is, in any case, separated from the region $R\geq R_2$ by the phase-transition period $R_1\leq R\leq R_2$. On the other hand Eq. (63) holds only for $R\ll R_{\mathrm{eq}}$, so one cannot relate [20] $t_0$ to $t_{\mathrm{eq}}$.

Equation (63) shifts the standard thermal history of the early matter and radiation universe: a temperature attained at time $t$ in the standard model is now reached at $(t+t_0)$. But this does not affect nucleosynthesis because what matters there are time differences. (See [20].)

Deferring for the time being discussion of the temperature $T_*$ we turn to the calculation of $T_N$. This temperature may be estimated from the equilibrium abundance ratio [21,22] ($T<T_*<T_F$):

$$\begin{aligned}\chi&\equiv\left[\frac{x_d}{x_n x_p}\right]_{\mathrm{eq}}\\&=\frac{3}{2}n_B\left[\frac{m_d}{m_N^2}\right]^{3/2}\left[\frac{2\pi}{T}\right]^{3/2}\exp(B_d/T)\ ,\end{aligned} \tag{64}$$

where $m_N$ and $m_d$ ($=2m_N$) are the nucleon and deuteron rest-masses, $n_B$ is the baryon-number density at temperature $T$, $B_d$ ($=2.23$ MeV) is the deuteron binding energy, and

$$x_n=\frac{n_n}{n_B},\quad x_p=\frac{n_p}{n_B},\quad x_d=\frac{2n_d}{n_B} \tag{65}$$

are the neutron, proton, and deuteron mass fractions, respectively ($x_n$ and $x_p$ are also the fractions by number). Conservation of baryon number throughout $R\geq R_2$ leads to

$$n_B=n_{Bp}R_p^3R^{-3} \tag{66}$$

with

$$n_{Bp}=\alpha m_N^{-1}\Omega_B H_p^2\ , \tag{67}$$

where $\Omega_B$ is the present baryonic fraction of the critical density.

With Eqs. (59), (38), and (18), Eq. (66) for $n_B$ yields the model expression

$$n_B \approx n_{Bp}\left[\frac{1+\omega}{\omega}\right]^{3/4}\left[\frac{g_{\rm eff}(T)}{g_{\rm eff}(T_p)}\right]^{3/4}\left[\frac{T}{T_p}\right]^3 \tag{68}$$

valid for $T$ in the range $T_{\rm eq} \le T \le T_2$.

If $\rho_{vp}$, the present vacuum energy density, is considerably smaller than the present radiation energy density $\rho_{rp}$, then $\omega$ will be large and Eq. (68) will approach the standard-model result (formally the transition to the standard model corresponds to $\omega \to \infty$). Note however that for finite $\omega$ Eq. (68) does not hold for $T < T_{\rm eq}$.

As stable deuterons begin to form, the equilibrium ratio $\chi$ in Eq. (64) is below unity. Subsequently it increases as the Universe cools, until a temperature is reached when $\chi = 1$. This temperature is defined [21] to be $T_N$. The helium abundance is assumed to be double the neutron mass fraction at [23] $T_N$.

Take $m_N = 0.94$ GeV, $T_p = 2.7$ K and $g_{\rm eff}(T_N) = g_{\rm eff}(T_p)$ ($T_N < T_\nu^d$, the temperature of neutrino decoupling). Then Eqs. (68), (67), and (64) at $T = T_N$ (in GeV) give

$$\frac{B_d}{T_N} + \frac{3}{2}\ln T_N + \ln(\Omega_B h^2) - 14.47 + \frac{3}{4}\ln\left[\frac{1+\omega}{\omega}\right] = 0\ . \tag{69}$$

The $\omega$-dependent logarithmic term in Eq. (69) represents the deviation from the standard model. Without it $T_N$ depends only on $\Omega_B h^2$. Arguments explaining the observed abundances of deuterium, $^3$He, and $^7$Li (assumed to be primordial relics) require [24] $0.010 \le \Omega_B h^2 \le 0.025$.

In the absence of the $\omega$-dependent term Eq. (69) yields values of $T_N$ in a range specified by the bounds on $\Omega_B h^2$. In the presence of this term the same range of $T_N$ values would obtain [25] if the sum of the third and fifth terms in Eq. (69) is limited by the bounds on $\Omega_B h^2$. This condition allows a maximum vacuum energy satisfying $\Omega_B h^2(1+\omega^{-1})^{3/4} \approx 0.025$ where $\Omega_B h^2 \approx 0.010$, or $\omega \approx 0.4$ corresponding to $\rho_{vp}/\rho_{rp} \approx 0.7$.

With $\Omega_B h^2(1+\omega^{-1})^{3/4} \approx 0.025$, iteration of Eq. (69) converges to $T_N = 68.54$ keV ($\equiv T_{N10} = 0.079$). Having known $T_F$, $T_N$, and the temperature-time relation, a rough estimate of the primordial helium abundance $Y_p$ may be made. Assuming that nucleosynthesis occurred suddenly and that all free neutrons at $t = t_N$ fused into $^4$He, $Y_p$ may be written approximately as [8]

$$Y_p = \left[\frac{2n}{n+p}\right]_{t_N} \approx \left[\frac{2n}{n+p}\right]_{t_F}\exp[-\lambda_n(t_N - t_F)]$$
$$= \frac{2}{1+\exp(Q/T_F)}\exp[-\lambda_n(t_N - t_F)]\ , \tag{70}$$

where $\lambda_n^{-1}\ln 2 = \tau_n$ is the neutron half-life. Taking $T_{F10} = 1.11$ ($N_\nu = 3$), $T_{N10} = 0.079$, and noting that $g_{\rm eff} = 2 + \frac{7}{11}N_\nu$ after neutrino decoupling (see the Appendix), we find from Eq. (63) that $t_N - t_F = 267$ s. Then for $\tau_n = 10.3$ min (lowest quoted [26] $\tau_n = 9.88$ min) Eq. (70) yields $Y_p \simeq 0.31$. Observationally [27] $0.23 \le Y_p \le 0.25$.

In the correct quantitative treatment of nucleosynthesis, the neutron fractional abundance [28] $x_n = n/(n+p)$ satisfies the differential equation

$$-\frac{dx_n}{dt} = \lambda(n \to p)x_n - \lambda(p \to n)(1 - x_n)\ , \tag{71}$$

where the transition rates $\lambda$ are given by [2] (see [19])

$$\lambda(n \to p) \equiv \lambda(n + \nu \to p + e^-) + \lambda(n + e^+ \to p + \bar\nu) + \lambda(n \to p + e^- + \bar\nu)$$
$$= A\int\left[1 - \frac{m_e^2}{(Q+q)^2}\right]^{1/2}(Q+q)^2 q^2 dq(1+e^{q/T_\nu})^{-1}(1+e^{-(Q+q)/T})^{-1} \tag{72}$$

and

$$\lambda(p \to n) \equiv \lambda(p + e^- \to n + \nu) + \lambda(p + \bar\nu \to n + e^+) + \lambda(p + e^- + \bar\nu \to n)$$
$$= A\int\left[1 - \frac{m_e^2}{(Q+q)^2}\right]^{1/2}(Q+q)^2 q^2 dq(1+e^{-q/T_\nu})^{-1}(1+e^{(Q+q)/T})^{-1} \tag{73}$$

in which $m_e$ is the electron mass, $T_\nu$ the neutrino temperature, and $m_e \le \epsilon(q+Q)(q+Q) \le \infty$. For $T \gg Q$ the rates reduce to Eq. (56).

The numerical solution of Eq. (71), for $T < T_F$, was performed in the standard model by Peebles [29,21,22]. His calculations show that at about $T_{10} \equiv T_{*10} = 0.13$ the rates $\lambda(p \to n)$, $\lambda(n + \nu \to p + e^-)$, $\lambda(n + e^+ \to p + \bar\nu)$ become negligible compared to $\lambda(n \to p + e^- + \bar\nu)$ ($\equiv \lambda_n$). Thus for $t \ge t_*$ neutrons ($\beta$) decay freely and $x_n$ decreases according to

$$x_n(t) = Ne^{-\lambda_n(t - t_*)},\quad t \ge t_*\ , \tag{74}$$

where [22] $N \simeq 0.142$.

In the present models, for $T < T_F$,

$$T_\nu = \zeta(\tfrac{4}{11})^{1/3}T,\quad \zeta = (\tfrac{11}{4})^{1/12} \tag{75}$$

(see the Appendix). This modification in the standard value of $T_\nu$ ($\zeta = 1$ in standard cosmology) alters the standard transitions rates $\lambda$ and thereby also the numerical integration of Eq. (71). Here we discuss qualitatively the effect of $\zeta \neq 1$.

Consider $\lambda(n \to p)$ in Eq. (72) and denote the contributions to it from the $q>0$ and $q<0$ regions of integration by $\lambda_+$ and $\lambda_-$, respectively. Clearly $\lambda_+$ is an increasing function of $\zeta$ whereas $\lambda_-$ is a decreasing function of $\zeta$. Since both are positive and $\zeta$ $(=1.088)$ does not differ substantially from unity, the effect on $\lambda(n \to p)$ is likely to be negligible. A similar argument goes for $\lambda(p \to n)$ where $\lambda_+$ $(\lambda_-)$ decreases (increases) with $\zeta$ and $\lambda_\pm > 0$ also.

Thus, retaining Eq. (74) with $N \simeq 0.142$ and with $t_*$ and $t_N$ corresponding to $T_{*10} \simeq 0.130$ and $T_{N10} \simeq 0.079$ respectively, we have from Eq. (63) $t_N - t_* = 169$ s, so that for $\tau_n = 10.3$ min, $Y_p = 2x_n(t_N) \simeq 0.235$.

For a given value of $\Omega_B h^2$, the temperature $T_N$ increases as $\omega$ decreases or as the vacuum energy increases. This would in turn raise $Y_p$. Taking as an observational limit $Y_p = 0.25$ we deduce from Eq. (74), with $\tau_n = 10.3$ min, that $t_N - t_* = 114$ s, and hence from Eq. (63) that $T_{N10} = 0.089$. With $\Omega_B h^2 = 0.010$ we therefore have from Eq. (69) $\omega \simeq 3.5 \times 10^{-3}$, or $\rho_{vp}/\rho_{rp} \approx 0.9965$. Note that in this paper the present vacuum energy cannot exactly equal or exceed the radiation energy [see Eq. (30)]. Henceforth we shall take $\omega \geq 3.5 \times 10^{-3}$ as our nucleosynthesis constraint.

The nucleosynthesis scenario has implications for the ratio of nucleons (baryons) to photons. As stable deuterons begin to form, the number density per neutron of thermal photons capable of breaking up the deuterons must be low [28], i.e., $n(\tilde{\gamma})/n_n = n(\tilde{\gamma})/[x_n(t) n_B] \sim 1$, where $\tilde{\gamma}$ denotes photons with energy exceeding $B_d$. Now the fraction $n(\tilde{\gamma})/n_\gamma$ of photons of Planckian spectrum at a temperature $T \ll B_d$ $(\equiv 2.58 \times 10^{10}$ K) is approximately given by [30]

$$\frac{n(\tilde{\gamma})}{n_\gamma} = \frac{n(\tilde{\gamma})}{n_B}\frac{n_B}{n_\gamma} = 0.42 e^{-B_d/T}\left[\left[\frac{B_d}{T}\right]^2 + 2\frac{B_d}{T} + 2\right] . \quad (76)$$

In our models photons are continuously produced by the decaying vacuum energy. Because it is assumed that these photons are produced with a Planckian thermal distribution, the ratio of the number of photons created with energy in a certain range at a certain instant to the total number of photons created at that instant will still be given by Eq. (76). Thus Eq. (76) is applicable in the present nonadiabatic models. Then for $2x_n = Y_p = 0.25 - 0.23$, corresponding to temperatures in the range $T_{10} \approx 0.09 - 0.08$ (the deuteron photodissociation temperature $T_d^\gamma \sim 10^9$ K) we obtain $\eta \equiv n_B/n_\gamma \approx 10^{-9} - 4 \times 10^{-11}$.

### B. Baryon-to-photon ratio

In the standard model the baryon-to-photon ratio $\eta = n_B/n_\gamma$ is constant and falls at nucleosynthesis in the range $10^{-10} \leq \eta \leq 10^{-9}$. Yet at $T \simeq 20$ MeV, well before nucleosynthesis has commenced, the standard model gives [31] $\eta \approx 10^{-18}$, in disagreement with the observed baryon asymmetry of the Universe.

In the present work the assumption that the vacuum decays into radiation with a Planck distribution implies [8] that the radiation energy per particle must redshift like the temperature, so that

$$\rho_\gamma/n_\gamma \sim T \sim \left[\frac{30}{\pi^2 g_{\text{eff}}}\right]^{1/4} \rho_\gamma^{1/4} . \quad (77)$$

Hence

$$n_\gamma \propto \left[\frac{\pi^2 g_{\text{eff}}}{30}\right]^{1/4} \rho_\gamma^{3/4} . \quad (78)$$

For $R_2 \leq R \leq R_{\text{eq}}$, $\rho_r$ is given to a very good approximation [see Eq. (39)] by Eq. (59). Then ($g_{\text{eff}} = 2$ for photons; $\rho_{\gamma p} \approx \rho_{rp}$)

$$\eta = \frac{n_B}{n_\gamma} \approx C\alpha \left[\frac{30}{\pi^2 g_{\text{eff}}}\right]^{1/4} \frac{(1+\omega)^{3/4}}{\omega^{3/4}} \frac{\Omega_B H_p^2}{m_N \rho_{rp}^{3/4}} , \quad (79)$$

where $C$ is a hitherto arbitrary dimensionless constant and we have used Eqs. (66), (67), and (38). Equation (79) shows that $\eta$ is approximately constant in the early Universe (for $R_2 \leq R \leq R_{\text{eq}}$), despite entropy generation.

With $\rho_{rp}$ given by Eq. (34), we find

$$\eta \approx C \frac{(1+\omega)^{3/4}}{\omega^{3/4}} (\Omega_B h^2) \times 6.5 \times 10^{-9} . \quad (80)$$

Taking $\Omega_B h^2 = 0.010$ we have $\eta > 0.65 C \times 10^{-10}$ so that $\eta > 6.5 \times 10^{-11}$ at nucleosynthesis if $C \gtrsim 1$. Note that the present models and standard cosmology are on par in that neither explains the origin of the nucleosynthesis value of $\eta$.

More generally, for the whole matter and radiation epoch $R_2 \leq R \leq R_p$, $\rho_r$ is given by Eq. (26). Then Eqs. (78), (66), and (67) lead to

$$\eta \approx \frac{\omega^{3/4}}{(\omega + R^2/R_p^2)^{3/4}} \eta_{\text{early}} , \quad (81)$$

where $\eta_{\text{early}}$ is the value of $\eta$ in the early matter and radiation universe $R_2 \leq R \leq R_{\text{eq}}$ [Eq. (79)]. In particular, at present,

$$\eta_p \approx \frac{\omega^{3/4}}{(\omega+1)^{3/4}} \eta_{\text{early}} . \quad (82)$$

Using the nucleosynthesis constraint $\omega \geq 3.5 \times 10^{-3}$ in this equation we find $0.014 \leq \eta_p/\eta_{\text{early}} \leq 1$. Thus if $\eta_{\text{early}} \sim 10^{-10}$ then $10^{-12} \leq \eta_p \leq \eta_{\text{early}}$. A lower bound as low as $10^{-12}$ strains agreement with observation and may be replaced by $10^{-11}$ on requiring that $\eta$ has not dropped by more than one order of magnitude. Taking this as a constraint in Eq. (82) we get $\omega \geq 4.9 \times 10^{-2}$ or $\rho_{vp}/\rho_{rp} \leq 0.95$. Note that the change in entropy since the beginning of the matter and radiation era is very large for $\omega \approx 0.049$: from Eq. (42) we obtain in this case $S(R_p) - S(R_2) \approx 7\gamma^{3/2} \times 10^{93} (\frac{1}{2} < \gamma \leq 1)$.

### C. Baryon-to-entropy ratio

A parameter for which the effect of entropy generation in the present models can be directly investigated is the baryon-to-entropy ratio $n_B/(SR^{-3})=n_B/\sigma$ ($\equiv B$, after Kolb and Turner [32]; $\sigma=SR^{-3}$ is the entropy density). In standard cosmology where entropy is conserved $B\approx\frac{1}{7}\eta=\text{const}$. Here the rate of change of $B^{-1}$ is calculable from Eq. (40) and reads

$$\frac{dB^{-1}}{dR}=\frac{\alpha^{3/4}\gamma^{3/4}}{n_{Bp}R_p^3}\left[\frac{8g_{\text{eff}}}{15}\right]^{1/4}\frac{R}{(R^2+\omega R_p^2)^{1/4}}\ . \tag{83}$$

Integrating and using Eq. (38) we obtain

$$B^{-1}=B_p^{-1}-\frac{2m_N}{3\alpha\Omega_B H_p^2}\left[\frac{8g_{\text{eff}}}{15}\right]^{1/4}\times\rho_{rp}^{3/4}\left[1-\frac{(\omega+R^2/R_p^2)^{3/4}}{(\omega+1)^{3/4}}\right], \tag{84}$$

where [33] $g_{\text{eff}}\equiv g_{\text{eff}}(T_p)=\frac{43}{11}$. Denoting $B$ for $R\ll R_{\text{eq}}$ by $B_{\text{early}}$ we have

$$B_p^{-1}-B_{\text{early}}^{-1}\approx\Delta(\omega)\ , \tag{85}$$

where

$$\Delta(\omega)=\frac{2m_N}{3\alpha\Omega_B H_p^2}\left[\frac{8g_{\text{eff}}}{15}\right]^{1/4}\rho_{rp}^{3/4}\left[1-\frac{\omega^{3/4}}{(1+\omega)^{3/4}}\right]=\left[1-\frac{\omega^{3/4}}{(1+\omega)^{3/4}}\right]g_{\text{eff}}^{1/4}(\Omega_B h^2)^{-1}\times10^8\ . \tag{86}$$

Hence ($\Omega_B h^2=0.010$)

$$0<B_p^{-1}-B_{\text{early}}^{-1}<10^{10}\ . \tag{87}$$

This result is compatible with the (common) view that the observed baryon asymmetry of the Universe [32] ($B_p\sim10^{-11}$) originated in the early matter and radiation era. Because entropy increases continuously in the present models, this conclusion is significant.

### D. Possible consequences for galaxy formation

Detailed investigations of the effect of the vacuum energy on growth of density perturbations for $t>t_{\text{eq}}$ require a solution of the differential equation for the density contrast in the linearized approximation. In the present work this entails knowledge of the time dependence of $\dot{R}/R$ in Eq. (44). An exact analytic solution does not seem to be possible and it might be necessary to make approximations and numerical integrations. (We will attempt to address this issue in a subsequent article.) Yet one can still infer, in the manner of Freese *et al.* [8], information on the problem without solving for the density contrast. We elaborate next.

In standard cosmology the equilibrium redshift is defined by ("barred" quantities are of the standard model)

$$1+\bar{z}_{\text{eq}}=R_p/R_{\text{eq}}=\left[\frac{\rho_{r\,\text{eq}}}{\rho_{mp}}\right]^{1/3}=\left[\frac{R_p^4\rho_{rp}}{R_{\text{eq}}^4\rho_{mp}}\right]^{1/3}=\rho_{mp}/\rho_{rp}=2.1\times10^4\Omega_p h^2\ . \tag{88}$$

On the other hand Eqs. (38) and (36) here yield

$$1+z_{\text{eq}}=R_p/R_{\text{eq}}=\frac{1+\omega}{\omega}\left[\frac{\rho_{mp}}{\rho_{rp}}\right]=\frac{1+\omega}{\omega}(1+\bar{z}_{\text{eq}})\ , \tag{89}$$

implying $z_{\text{eq}}>\bar{z}_{\text{eq}}$. Clearly $z_{\text{eq}}$ could be much larger than $\bar{z}_{\text{eq}}$ if the vacuum energy is very close to the radiation energy today. The increase in $z_{\text{eq}}$ as a consequence of the presence of a decaying vacuum energy was noted by Freese *et al.* [8]. However, the size of the effect is model dependent.

Now simultaneous compatibility of the present models with baryon asymmetry and helium synthesis requires $\omega\geq4.9\times10^{-2}$ (or $\rho_{vp}/\rho_{rp}\leq0.95$). Then for $\Omega_p h^2\approx0.25$ (here and below we take $h=\frac{1}{2}$ and $\Omega_p\approx1$), Eqs. (88) and (89) give $z_{\text{eq}}\leq1.1\times10^5$, with an upper bound considerably higher than $\bar{z}_{\text{eq}}=5\times10^3$ of standard cosmology. $z_{\text{eq}}\approx1.1\times10^5$ can also be reached in the cosmologies of Freese *et al.*, where it is induced by a vacuum-to-radiation energy density ratio $\rho_{vp}/\rho_{rp}\approx0.07$ (allowed there by the observational constraints). But this ratio is $<\frac{1}{13}$ the corresponding value here ($\rho_{vp}/\rho_{rp}\approx0.95$) and produces in our case an equilibrium redshift of about $5.7\times10^3$ only. Quite generally much larger vacuum energies than are allowed (and sufficient) in the cosmologies of Freese *et al.* are needed (and allowed) here in order to increase $z_{\text{eq}}$ significantly over its standard value.

A larger $z_{\text{eq}}$ implies an earlier $t_{\text{eq}}$, providing more time for post-equilibrium protogalactic structure to grow. The relation of $t_{\text{eq}}$ to $z_{\text{eq}}$ is readily obtainable in the present models. Integrating Eq. (58), assuming the approximate boundary condition $y=y_2=R_2R_{\text{eq}}^{-1}\approx0$, we have

$$t_{\text{eq}}\approx\left[\frac{R_{\text{eq}}^4}{\gamma\omega R_p^2}\right]^{1/2}\int_0^1 y(1+y)^{-1/2}dy=\frac{2}{3}(2-\sqrt{2})\left[\frac{R_{\text{eq}}^4}{\gamma\omega R_p^2}\right]^{1/2}\ . \tag{90}$$

Using Eq. (36) we then get

$$t_{\text{eq}}(1+z_{\text{eq}})\approx\frac{2}{3}(2-\sqrt{2})\left[\frac{\omega}{1+\omega}\right]^{1/2}\frac{\alpha^{1/2}\rho_{rp}^{1/2}}{\rho_{mp}}\approx\left[\frac{\omega}{1+\omega}\right]^{1/2}(\Omega_p h^2)^{-1}\times8.3\text{ Mpc}\ . \tag{91}$$

Equation (91) has consequences for models of galaxy formation with a (Harrison-Zeldovich) spectrum of cold-dark-matter adiabatic density perturbations [8,34]. It implies that the scale $\lambda_{\text{eq}}\equiv t_{\text{eq}}(1+z_{\text{eq}})$ below which the spectrum flattens can drop well below the normalization scale $\lambda_n\simeq8h^{-1}$ Mpc if the present vacuum energy is sufficiently large. In particular, for $\omega=4.9\times10^{-2}$ or $\rho_{vp}/\rho_{rp}\approx0.95$, $\lambda_{\text{eq}}\approx7$ Mpc$\approx0.44\lambda_n$. In such a circumstance small-scale perturbation amplitudes and small-angle cosmic microwave background (CMB) anisotropy will become important. But in our models $\lambda_{eq} \approx \lambda_n$ for $\rho_{vp}/\rho_{rp} \approx 0.77$ compared to as low a ratio as 0.06 only in the models of Freese *et al.* Thus, although damping of large-scale perturbation amplitudes and large-angle CMB anisotropy occurs in our cosmologies, it requires, to become appreciable, the presence of a vacuum energy much higher than is needed (or even allowed) in the models of Freese *et al.*

## VIII. PARTICULAR MODEL: $\gamma=\frac{2}{3}$

From among the class of models that we studied there is one that deserves additional attention. It has $\gamma=\frac{2}{3}$ and describes a universe in which the initial cosmic density and temperature are maximum but finite. Within the class of models considered, it is thus the closest, regarding the initial conditions, to the big-bang model. Therefore it is interesting to analyze further the pure-radiation equations in this particular case.

### A. Basic equations

For $\gamma=\frac{2}{3}$ the general equations of Sec. IV give $R_{\rm mx}=R_0$, $\rho_{\rm max}=\alpha/(3R_0^2)=\rho_0$, and $R^2=\frac{1}{3}t^2+R_0^2$. In terms of $R$ one has

$$\rho=\frac{2\alpha}{3R^2}\left[1-\frac{R_0^2}{2R^2}\right], \tag{92}$$

$$T=\left[\frac{20\alpha}{\pi^2 g_{\rm eff}}\right]^{1/4}\left[1-\frac{R_0^2}{2R^2}\right]^{1/4}R^{-1/2}, \tag{93}$$

with the maximum temperature given by

$$T_{\rm max}=\left[\frac{10\alpha}{\pi^2 g_{\rm eff} R_0^2}\right]^{1/4}. \tag{94}$$

Using this expression for $T_{\rm max}$ we estimate (see Sec. IV) that $R_0 \sim 5.7\times10^{-36} g_{\rm eff}^{-1/2} m$, $\rho_0 \sim 1.7\times10^{96} g_{\rm eff}$ kg m$^{-3}$.

In terms of $t$,

$$\rho=\frac{\alpha(2t^2+3R_0^2)}{(t^2+3R_0^2)^2}, \tag{95}$$

and

$$T=\left[\frac{30\alpha}{\pi^2 g_{\rm eff}}\right]^{1/4}\left[\frac{2t^2+3R_0^2}{(t^2+3R_0^2)^2}\right]^{1/4}. \tag{96}$$

For $t/\sqrt{3} \gg R_0$, i.e., $t \gg 3\times10^{-44} g_{\rm eff}^{-1/2}$ s, Eqs. (95) and (96) reduce to [35]

$$\rho=\frac{2\alpha}{t^2}, \tag{97}$$

and

$$T=\left[\frac{30\alpha}{\pi^2 g_{\rm eff}}\right]^{1/4}\frac{1}{(t/\sqrt{2})^{1/2}}. \tag{98}$$

Corresponding to Eqs. (97) and (98) one has, in the $k=0$ standard model,

$$\bar{\rho}=\frac{\alpha}{(2t)^2} \tag{99}$$

and

$$\bar{T}=\left[\frac{30\alpha}{\pi^2 g_{\rm eff}}\right]^{1/4}\frac{1}{(2t)^{1/2}}, \tag{100}$$

and in the Özer-Taha model,

$$\rho_{\rm OT}=\frac{\alpha}{t^2} \tag{101}$$

and

$$T_{\rm OT}=\left[\frac{30\alpha}{\pi^2 g_{\rm eff}}\right]^{1/4}\frac{1}{t^{1/2}}. \tag{102}$$

Thus for $t \gg R_0$ in the pure-radiation era, a temperature $T$ attained at time $t$ in the standard model is reached at $2\sqrt{2}t$ in the present model and at $2t$ in the Özer-Taha model. So despite the very different time dependences of the scale factor in the pure-radiation universe in the standard model on the one hand ($\bar{R}=\text{const}\times t^{1/2}$) and in the other two models on the other, and the different initial conditions in all three models, the thermal history of the early Universe in the three cases is, very soon after $t=0$, essentially the same. For example the GUT temperature $T_{\rm GUT}\approx10^{15}$ GeV occurs at $\bar{t}\sim2.5\times10^{-36} g_{\rm eff}^{-1/2}$ s, $t\sim7\times10^{-36} g_{\rm eff}^{-1/2}$ s, or $t_{\rm OT}\sim5\times10^{-36} g_{\rm eff}^{-1/2}$ s.

With $\gamma=\frac{2}{3}$, the time marking the onset of global causality is given from Eq. (23) by

$$t_{\rm caus}=\sqrt{3}R_0 \sinh\left[\frac{\pi}{2\sqrt{3}}\right]=1.8R_0. \tag{103}$$

Taking $R_0\sim5.7\times10^{-36} g_{\rm eff}^{-1/2}$ m and [14] $g_{\rm eff}\approx100$, we have $t_{\rm caus}\sim2\times10^{-45}$ s, of the order of Planck's time $t_{\rm Pl}$. Thus the Universe in this model, which has no initial cold era as in the $\gamma=1$ model, became causally connected at a very early time.

Lastly, the cosmic expansion rate, as deduced from Eq. (4) or directly from the equation for $R(t)$, is ($t \gg R_0$) $\dot{R}/R=t^{-1}$, which is double the corresponding rate in the standard model. But observe that this rate is only valid for $t \gg t_P$. In particular it does not hold at $t=0$.

### B. Entropy

For $\gamma=\frac{2}{3}$ and $t \gg R_0$, Eqs. (40) and (93) may be combined to express $dS/dR$ in terms of $R$; then integrate to obtain

$$S=\frac{4\sqrt{2}}{9}\left[\frac{\pi^2 g_{\rm eff}\alpha^3}{5}\right]^{1/4}R^{3/2}+S_0, \tag{104}$$

where $S_0$ is a dimensionless constant denoting the initial entropy of the Universe.

Bekenstein [36] has proposed that the entropy $S$ and energy $E$ of a system which may be enclosed in a sphere of radius $R$ satisfy

$$S < 4\pi RE \ . \tag{105}$$

Although this result was criticized by several authors [37], Bekenstein [38] has replied by stressing that it holds for a complete system.

Here we apply Bekenstein's theorem to the *closed* universe of our model at $t=0$. With $\rho_0 = \alpha/(3R_0^2)$, we deduce from the inequality (105) and Eq. (94) ($T_{\max} \sim G^{-1/2}$) that

$$S_0 \leq \left[ \frac{5\alpha^3}{18 g_{\text{eff}}} \right]^{1/4} R_0^{3/2} \ . \tag{106}$$

Denoting the first term on the right-hand side (RHS) of Eq. (104) by $\tilde{S}$ we see that ($g_{\text{eff}} \approx 100$, $t \gg R_0$)

$$\tilde{S}/S_0 \geq \frac{4\sqrt{2}}{9} \left[ \frac{2\pi^2}{75} g_{\text{eff}}^2 \right]^{1/4} t^{3/2} R_0^{-3/2}$$
$$\approx 4.5 t^{3/2} R_0^{-3/2} \gg 1 \ . \tag{107}$$

For example, at the time of the GUT transition, $\tilde{S}/S_0 \sim 10^{13}$. Thus for $t \gg t_p$ the initial entropy $S_0$ may be neglected. (Note that in the $\gamma = 1$ model $S_0 = 0$.) Then we have from Eqs. (104) and (93), with $R \gg R_0$,

$$\sigma = \frac{2\pi^2}{45} g_{\text{eff}} T^3, \quad T \ll T_{\max} \ , \tag{108}$$

which coincides in form with the corresponding standard-model *exact* relation.

### C. Flatness

Observations today indicate that [39] $0.1 \leq \Omega_p \leq 4$, i.e., $|\Omega_p - 1|$ is of order 1. In $k = \pm 1$ standard cosmology this requires [40] that $\Omega$ in the early Universe had been extraordinarily finely tuned near the "flat" value $\Omega = 1$, e.g., $(\Omega - 1)_{\text{GUT}} \approx 4 \times 10^{-51} (\Omega_p - 1)$.

In the present model we have, directly from Eqs. (4) and (8) ($x = R R_0^{-1}$),

$$\Omega - 1 = x^2 (x^2 - 1)^{-1} \ , \tag{109}$$

in the pure-radiation universe. In particular $\Omega = \infty, 3, \frac{7}{3}$ when $x = 1, \sqrt{2}, 2$, respectively, in accordance with the condition (14). ($x = \sqrt{2}$ corresponds to $t = \sqrt{3} R_0 = t_{\text{caus}} \operatorname{csch}[\pi/(2\sqrt{3})] \approx t_{\text{Pl}}$; see Eq. (103) and ensuing remarks.) For $x \gg 1$, $\Omega = 2$. In fact for $x > 1.05$, $\Omega - 1$ is of order 1 throughout the pure-radiation era.

On the other hand, for $R \gg R_p$ we have from Eq. (44) $\dot{R}^2 = (2\gamma - 1)$ so that Eqs. (4) and (8) imply $\Omega - 1 = (1-\gamma)/(2\gamma - 1) = 1$ for $\gamma = \frac{2}{3}$. So in the very distant future also, $\Omega = 2$. But from Eq. (44) $\ddot{R} \leq 0$ for all $R \geq R_2$. Hence by the inequality (14), $\Omega - 1$ in the $\gamma = \frac{2}{3}$ model is of order 1, except for the Planck and rest-mass generation eras.

## IX. SUMMARY AND DISCUSSION

The cosmologies studied in this paper are based on the Chen-Wu [6] ansatz for a decaying cosmological constant: $\Lambda = 3\gamma R^{-2}$. Requiring that the standard cosmological initial singularity be avoided and that cosmic entropy increase restricts the parameter $\gamma$ to the range $\frac{1}{2} < \gamma \leq 1$, giving rise to a class of models that include and generalize the critical density cosmology of Özer and Taha [5]. The constraint on $\gamma$ implies that the density parameter $\Omega \geq 1$ in these models.

The Universe is taken to have passed through three distinct epochs with different equations of state: (i) a very early era of pure radiation; (ii) a phase transition period of rest-mass generation, and (iii) an epoch of radiation and matter extending to the present.

In the pure radiation era the time dependence of the scale factor solves the horizon and monopole problems. The proof is readily constructable following the steps of Özer and Taha [5].

The rest-mass generation period is characterized by the occurrence of negative pressure and the appearance of decelerated expansion. These features were also previously noted by Özer and Taha [5] in their critical density model. Here we have shown that they are independent of the critical density assumption. But a better understanding of the rest-mass generation period, which possibly corresponds to the standard model's early Universe phase transition(s) (GUT, Weinberg-Salam, . . .) is imperative. It is essential to propose and test appropriate equations of state for this epoch.

The matter and radiation period following rest-mass generation can be further subdivided into radiation-dominated and matter-dominated phases, respectively preceding and succeeding the equilibrium of radiation and matter. The Universe in the radiation-dominated phase is nearly Einstein–de Sitter-like: approximately described by a $k = 0$ standard model Friedmann equation [Eq. (60)]. Throughout, the density parameter $\Omega$ exceeds but stays very close to unity. The Friedmann equation for the matter-dominated phase [Eq. (44)] is on the other hand too complicated to solve analytically. However since $2\gamma > 1$ in the present work it implies that the Universe will expand forever. Chen and Wu [6] show that the fate of the universe of their model depends on the value of $\gamma$: If $\gamma \geq k/3$ (or $\gamma < k/3$) then the universe will continuously expand (or will eventually collapse). This is not contradicted by our models. But Chen and Wu [6] assume that the vacuum energy associated with $\Lambda$ decays into matter in the matter-dominated universe. We assume that it decays into thermal radiation of the Planckian spectrum throughout the matter and radiation period.

The present matter-dominated era is not everlasting in the studied cosmologies. The $1/R^4$ term in Eq. (26) for the radiation density, although dominant for $R \leq R_{\text{eq}}$ [see Eq. (39)], will eventually become, for sufficiently large $R \gg R_p$, negligible as compared to the $1/R^2$ term. Then the radiation density will redshift like $1/R^2$ while the matter density continues to redshift, because of conservation of matter, like $1/R^3$. Thus according to the present models radiation is destined to dominate again [when $R$ is given by Eq. (35)].

Assuming primordial nucleosynthesis occurred early in the radiation-dominated era of the matter and radiation period, i.e. when the Universe was Einstein–de Sitter-like, we noted that the neutron-proton freeze-out temperature remains as in the standard model, and so does also the temperature-time relation, up to an inconsequential additive constant. Under these conditions the models are consistent with Peebles [29,21,22] helium abundance calculations, provided the present vacuum energy density $\rho_{vp}$ is, at most, just under the radiation energy density $\rho_{rp}$. Specifically we have the constraint $\rho_{rp}/\rho_{vp}-1\equiv\omega\geq 3.5\times 10^{-3}$. In the present cosmologies the vacuum energy today cannot reach the radiation energy in any case. The sensitivity of helium synthesis to small variations in $\omega$ for a given $\Omega_B h^2$ can be estimated: From Eq. (69),

$$\frac{\delta T_N}{T_N}=-\frac{3}{4(1+\omega)}\left[\frac{B_d}{T_N}-\frac{3}{2}\right]^{-1}\frac{\delta\omega}{\omega} \tag{110}$$

so that for $\omega\ll 1$, $B_d\equiv T_{10}=2.58$ and $T_{N10}=0.089$, $\delta T_N/T_N\approx -0.03\delta\omega/\omega$. This leads via Eqs. (63) and (74) to

$$\frac{\delta Y_p}{Y_p}=6.62\lambda_n g_{\rm eff}^{-1/2}T_{N10}^{-2}\frac{\delta T_N}{T_N} \approx -0.01\frac{\delta\omega}{\omega}\,, \tag{111}$$

independently of the constant $N$ in Eq. (74). Thus sensitivity of $Y_p$ to small variations in $\omega$ is low and, as Eq. (110) shows, decreases with increasing $\omega$ or decreasing vacuum energy.

We have used in the helium synthesis calculation $\Omega_B h^2=0.010$, which is widely taken [24,41] as a reasonably firm lower bound on this parameter. For $T_{N10}$ fixed at 0.089 by helium abundance the lower bound on $\Omega_B h^2$ corresponds to the minimum $\omega$ or maximum admissible vacuum energy. Sensitivity of $\omega$ to small variations in $\Omega_B h^2$ can also be estimated: With $T_N$ fixed in Eq. (69) one has

$$\frac{\delta\omega}{\omega}=\frac{4}{3}(1+\omega)\frac{\delta(\Omega_B h^2)}{\Omega_B h^2}\,. \tag{112}$$

Thus the relative change in $\omega$ is of the same order as that in $\Omega_B h^2$ when $\omega$ is small and increases with increasing $\omega$.

We have confined the nucleosynthesis discussion to $^4$He because of the high precision with which its abundance is known. Still, a more complete analysis should take account of other light elements, e.g., residual deuterium, $^3$He, and $^7$Li. Freese *et al.* [8] have addressed this issue in their cosmology. As we will explain shortly there is a sense in which we can relate our nucleosynthesis work to theirs. In their model nucleosynthesis requires $\rho_{vp}/\rho_{rp}\leq\frac{1}{9}$.

In standard cosmology the baryon-to-photon ratio $\eta$ is constant. Here it turns out to be approximately constant in the early matter and radiation universe and that, for $\omega\geq 3.5\times 10^{-3}$, its value now $\eta_p$ could have dropped from its early Universe value ($\sim 10^{-10}$) by as much as two orders of magnitude. This admits $\eta_p\sim 10^{-12}$, a too low value observationally. Requiring $\eta_p(\min)\sim 10^{-11}$ imposes the constraint $\omega\geq 4.9\times 10^{-2}$ or $\rho_{vp}/\rho_{rp}\leq 0.95$. This latter condition is quite close to the nucleosynthesis constraint $\rho_{vp}/\rho_{rp}\leq 0.99$ although the corresponding limits on $\omega$ differ by one order of magnitude. For if we denote $\rho_{vp}/\rho_{rp}$ by $r$ we have from Eq. (27) $\delta\omega/\omega=-(1-r)^{-1}\delta r/r\approx -20\delta r/r$ for $r\approx 0.95$. So a small change in $r$ ($r$ near 1) induces a much larger one in $\omega$. Moreover from Eq. (82) (with $\eta_p/\eta_{\rm early}\equiv\bar\eta$) $\delta\bar\eta/\bar\eta=\frac{3}{4}(1+\omega)^{-1}\delta\omega/\omega$, implying that when $\omega$ is small $\eta_p/\eta_{\rm early}$ is, like $\omega$, very sensitive to small changes in the vacuum energy in the vicinity of $\rho_{vp}/\rho_{rp}\approx 1$. The desired compatibility of $\eta_p$ with observation can therefore be obtained without a drastic change in the nucleosynthesis condition on $\rho_{vp}/\rho_{rp}$.

We have also considered the baryon-to-entropy ratio ($B$) which, in the adiabatic standard model, is proportional to $\eta$. We have derived an inequality connecting the early Universe and present values of $B^{-1}$ [Eq. (87)]. It is consistent with attributing the presently observed cosmic baryon asymmetry to processes in the early Universe. These results for $\eta$ and $B$ hold despite the production of a large amount of entropy ($\approx 10^{93}$) between the early Universe and now. (Recall that in the standard model the present (constant) entropy of the Universe is about [41] $10^{87}$.) Thus the present models are consistent with baryon asymmetry and helium synthesis at the same time for $\rho_{vp}/\rho_{rp}\leq 0.95$.

An important problem which we have touched upon but plan to pursue in detail elsewhere is the question of galaxy formation and cosmic microwave background (CMB) radiation anisotropies. Here, we have shown, in the manner of Freese *et al.* [8], that saturation of the upper bound on the present vacuum energy ($\rho_{vp}/\rho_{rp}\leq 0.95$) enhances small-scale adiabatic density perturbations and small-angle CMB anisotropies in cold-dark-matter models of galaxy formation. The suppression of large-scale perturbations might curtail the ability of these models to produce large-scale structure [8]. However, for this effect to become significant the present vacuum energy must be larger than that sufficient in the cosmology of Freese *et al.* [8].

In our work the present value of the density parameter $\Omega$ is found to be unity to within two parts in $10^{-4}$ [Eq. (43)]. Since current estimates [42] place the upper bound on $\Omega_B h^2$ at no higher than 0.035 some sort of dark matter is needed to reconcile $\Omega_B$ (0.04–0.14) with $\Omega_p$.

The near equality of the cosmic and critical densities is not a distinguishing feature of the present Universe. As we go back in time the deviation $\Omega-1$ decreases becoming $\lesssim 10^{-8}$ at the equilibrium of radiation and matter and continues to diminish with the approach to the beginning of the matter and radiation era. Thus $\Omega\approx 1$ (with varying precision) characterized the Universe ever since it emerged from the matter generation phase and till today. To this extent our models have no flatness problem. In the pure-radiation universe, however, the issue is model dependent: Eqs. (4), (8), and (11) give ($x=RR_0^{-1}$)

$$\Omega-1=\frac{(1-\gamma)x^2}{(2\gamma-1)(x^2-1)}\,, \tag{113}$$

so in order to estimate the RHS a particular model (definite $\gamma$) should be chosen. Similarly for $R\gg R_p$ in the distant future Eq. (44) reduces to $\dot{R}^2=(2\gamma-1)$ so that Eqs. (4) and (8) lead to $(\Omega-1)=(1-\gamma)/(2\gamma-1)$.

The critical density model ($\gamma=1$) that motivated this work is very different in its initial state (cold beginning) from the standard model. The class of cosmologies includes, however, an interesting model ($\gamma=\frac{2}{3}$) of a universe with *maximum* but finite initial density. In this respect it is the closest model in the class to big-bang cosmology. We have studied the pure-radiation equations of this model in some detail. In particular we have written an upper bound on the entropy at $t=0$ and have shown that $\Omega-1$ is of order 1 for $t\gtrsim t_{\rm Pl}$ throughout the pure-radiation era. In fact this result holds for all $t$ not in the Planck or rest-mass generation periods (see the end of Sec. VIII).

Freese *et al.* [8] postulate that the vacuum and radiation energy densities redshift at the same rate for large $R$ so that the parameter $x\equiv\rho_v/(\rho_r+\rho_v)$ is a constant independent of time. In the pure-radiation eras of our cosmology and the cosmology of Chen and Wu [6] $\rho_v/\rho_r\rightarrow 1$ so that $x\rightarrow\frac{1}{2}$. [See Eqs. (6), (8), and (10).] In this case the postulate of Freese *et al.* emerges as a mere by-product of the present approach. This point was recently emphasized by Pavón [43] in connection with the $\gamma=1$ and Chen-Wu models. Note, however, that the result is independent of $\gamma$.

On the other hand, in the matter and radiation universe we have, from Eqs. (6), (8), and (26),

$$x=\frac{1}{2}\left[1+\frac{\omega R_p^2}{2R^2}\right]^{-1}, \tag{114}$$

with $x_p=(2+\omega)^{-1}$ and $x\rightarrow\frac{1}{2}$ for $R\gg R_p$. But for $R\leq R_{\rm eq}$ the inequality (39) implies that $x\lesssim 10^{-8}$.

This constraint on $x$ provides an alternative (albeit formal) way that leads to our nucleosynthesis conclusions. For since the universe of our models at the time of nucleosynthesis is almost Einstein–de Sitter-like, our helium results can be understood in the context of the nucleosynthesis code computations of Freese *et al.* [8]. There $x\lesssim 10^{-8}$ simply implies a nucleosynthesis scenario indistinguishable from the standard picture. Note, however, that it would not be correct in our case to require $x_p\lesssim 10^{-8}$ also since $x$ is not constant over the interval $t_2\leq t\leq t_p$.

The present models do not explain the physical origin of the initial energy density $\rho_0$. We remark that the idea of a limiting initial density has been discussed by several authors [44]. Quite recently Israelit and Rosen [45] have proposed a nonsingular cosmological model in which the Universe has an initial energy density of the order of the Planck density [$\sim 10^{97}$ kg/m$^3\sim\rho_0$; see the remark after Eq. (94)].

Although the dynamics of vacuum decay into radiation is not specified, the fluctuations in the vacuum energy flux $\dot{\rho}_v$ may be discussed. It has been shown by Pavón [9] that the ratio of the flux fluctuation to the average flux in the Özer-Taha and Chen-Wu models diminishes with cosmic expansion. This is essential or else the radiation produced in vacuum decay will not retain a Planck distribution and the CMB will be distorted beyond the observational limits. The fact that the Özer-Taha and Chen-Wu models exhibit this feature has been regarded by Pavón [9] as strong evidence of an $R^{-2}$ behavior of decaying vacuum energy. Inspection of Pavón's work reveals that the damping of the flux-fluctuation-to-average-flux ratio occurs in all the present models, *independently* of the observational constraints on $\omega$.

If the vacuum decays into matter in the matter-dominated universe then $E_m=\rho_m R^3\neq$const. Decay of the vacuum into matter would produce, through the creation and subsequent annihilation of baryon-antibaryon pairs, an observable $\gamma$-ray flux. It was noted [4,6,8] in different models that observations so restrict such a flux as to justify regarding $E_m$ constant [46].

## X. CONCLUSION

A class of nonsingular decaying-vacuum cosmological models free of the main cosmological problems was presented. They are compatible with the principal constraints of observational cosmology provided the vacuum energy today is, at most, just under the cosmic radiation energy.

## APPENDIX

This appendix gives the relation between the neutrino and photon temperatures today in the present nonadiabatic models. It rehashes and straightforwardly extends (beyond the critical density case) the corresponding appendix of Özer and Taha [5].

After neutrinos decouple at $T=T_\nu^d$ they leave $e^\pm$ in equilibrium with photons. Assuming different components of the radiation redshift at the same rate we have, from Eq. (59),

$$\rho_\nu\approx\frac{\gamma_\nu\alpha\omega R_p^2}{R^4},\quad R_\nu^d\leq R\leq R_{\rm eq}\ , \tag{A1}$$

$$\rho_{\gamma e^+e^-}\approx\gamma_{\gamma e^+e^-}\frac{\alpha\omega R_p^2}{R^4},\quad R_\nu^d\leq R\leq R_{e^+e^-}^d\ , \tag{A2}$$

where $R_\nu^d$ and $R_{e^+e^-}^d$ are, respectively, the values of $R$ when $T=T_\nu^d$ and when $T=T_{e^+e^-}^d$, the temperature at which $e^\pm$ decouple from photons. $\gamma_\nu$ and $\gamma_{\gamma e^+e^-}$ are the partial couplings of the decaying vacuum to neutrinos and the $\gamma e^\pm$ radiation "soup," respectively.

After $e^\pm$ drop out of equilibrium with photons we have, from Eq. (26) for $R\geq R_{e^+e^-}^d$,

$$\rho_\gamma=\frac{\gamma_\gamma\alpha}{R^2}\left[1+\frac{\omega R_p^2}{R^2}\right], \tag{A3}$$

in addition to

$$\rho_\nu=\frac{\gamma_\nu\alpha}{R^2}\left[1+\frac{\omega R_p^2}{R^2}\right]. \tag{A4}$$

From Eqs. (A1), (A2), and (59) ($\rho=\rho_\nu+\rho_{\gamma e^+e^-}$ for

$R \le R^{d}_{e^+e^-}$) one has

$$\gamma = \gamma_\nu + \gamma_{\gamma e^+e^-} , \tag{A5}$$

whereas Eqs. (A3), (A4), and (26) ($\rho_\nu + \rho_\gamma = \rho$ for $R \ge R^{d}_{e^+e^-}$) give

$$\gamma_\nu + \gamma_\gamma = \gamma . \tag{A6}$$

Hence

$$\gamma_{\gamma e^+e^-} \equiv \gamma_\gamma . \tag{A7}$$

Now (suppressing the subscript "eff" on $g$ temporarily for convenience),

$$\rho_r = \frac{1}{30}\pi^2 g_r T_r^4, \quad r = \nu, \gamma, \gamma e^+e^- . \tag{A8}$$

Thus from Eqs. (A8), (A4), and (A3) at $T = T_p$,

$$\frac{g_\nu}{g_\gamma}\frac{T_{\nu p}^4}{T_p^4} = \frac{\gamma_\nu}{\gamma_\gamma} . \tag{A9}$$

But from Eqs. (A1), (A2), (A7), and (A8) at $T = T_\nu^d$,

$$\frac{\gamma_\nu}{\gamma_\gamma} = \frac{g_\nu}{g_{\gamma e^+e^-}} . \tag{A10}$$

Therefore from Eqs. (A9) and (A10) we obtain

$$\frac{T_{\nu p}}{T_p} = \left[\frac{g_\gamma}{g_{\gamma e^+e^-}}\right]^{1/4} . \tag{A11}$$

Now $g_\gamma = 2$ and $g_{\gamma e^+e^-} = 2 + \frac{7}{8}(2+2) = \frac{11}{2}$. Hence

$$\frac{T_{\nu p}}{T_p} = \left(\tfrac{4}{11}\right)^{1/4} , \tag{A12}$$

compared to $(\frac{4}{11})^{1/3}$ in the standard model. Then

$$g_{\rm eff}(T_p) = 2 + \tfrac{7}{11} N_\nu , \tag{A13}$$

where $N_\nu$ is the number of neutrino species.

With $g_\nu = \frac{7}{4} N_\nu$ Eq. (A10) becomes

$$\frac{\gamma_\nu}{\gamma_\gamma} = \frac{7}{22} N_\nu . \tag{A14}$$

If the vacuum-neutrino coupling is not stronger than the vacuum-photon coupling then $\gamma_\nu/\gamma_\gamma \le 1$, implying $N_\nu \le \frac{22}{7}$, i.e., $N_\nu \le 3$, so there are *no* more than three neutrino types. This would agree with current experimental indications, e.g., results from the 1989 $Z^0$ width measurements at the CERN $e^+e^-$ collider LEP.

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[10] Our notation is such that the flat-space metric has $\eta_{00} = 1$.

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[15] $\Omega > 1$ together with Eq. (34) and $0.5 \le h \le 1$ imply that $\rho_{mp} \approx 8\times 10^{-47}\Omega_p h^2\,(\text{GeV})^4 > \frac{1}{2}\times 10^4 \rho_{rp}$.

[16] We made two approximations in Eq. (41). First $R_2/R_p \approx 0$ since $R_2 \ll R_p$. Second, although $g_{\rm eff}(T)$ for $T > T_\nu^d$, the temperature at which neutrinos decoupled (when $R = R_\nu^d$), exceeds $g_{\rm eff}(T_p)$, we have, because $R_\nu^d \sim R_2 \approx 0$, neglected the entropy change for $R_2 \le R \le R_\nu^d$ and took $g_{\rm eff} = g_{\rm eff}(T_p)$ throughout.

[17] Here we use Eqs. (26) and (39) to put $\alpha^{-1}\rho_{mp} R_p^3 R_{\rm eq}^{-1} \approx \gamma \omega R_p^2 R_{\rm eq}^{-2}$.

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[20] If the temperature immediately after the end of an early universe phase transition (e.g., GUT) is comparable in order of magnitude to its value at the beginning (see A. H. Guth, in *The Very Early Universe* [1], p. 171) then since $T = 10^{15}$ GeV $\approx 10^{28}$ K Eq. (63) gives ($t_E \equiv$ time marking end of transition) $t_E \approx 3.31 g_{\rm eff}^{-1/2} \times 10^{-36} + t_0$ (seconds), so that $t_E \ll 1$ sec implies $t_0 \ll 1$ sec.

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[25] We already noted the identity of $T_F$ and the $T$-$t$ relation (up to an additive constant) with those in the standard model. Then $T_N$ must not depart significantly from $T_N$ (standard) if the present account of nucleosynthesis is to approach the successful standard one.

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[27] See Dominguez-Tenreiro and Quirós, *Cosmology and Particle Physics* [14], p. 92.

[28] $x_n$ represents the neutron fractional abundance in some region of space (see Ref. [21]). For a spatial region of unit volume $x_n = n_n/(n_n + n_p) = n_n/n_B$ [see Eq. (65)].

[29] P. J. E. Peebles, Astrophys. J. **146**, 542 (1966).

[30] K. Krane, *Modern Physics* (Wiley, New York, 1983), pp. 480–481.

[31] See Dominguez-Tenreiro and Quirós, *Cosmology and Particle Physics* [14], p. 270.

[32] E. W. Kolb and M. S. Turner, Annu. Rev. Nucl. Part. Sci. **33**, 645 (1983).

[33] Before neutrinos decouple at $R = R_\nu^d$, $g_{\text{eff}} > g_{\text{eff}}(T_p)$. However, in the interval $R_2 \le R \le R_\nu^d$ the change

$$\Delta B^{-1} \propto \left[1 + \frac{(R_\nu^d)^2}{\omega R_p^2}\right]^{3/4} - \left[1 + \frac{R_2^2}{\omega R_p^2}\right]^{3/4} \approx 0$$

because of Eq. (39).

[34] G. R. Blumenthal, S. M. Faber, J. R. Primack, and M. J. Rees, Nature (London) **311**, 517 (1984).

[35] It is interesting to note that $\rho$ of Eq. (97) [i.e., $\rho = 3/(4\pi G t^2)$] coincides with the Machian energy density of Whitrow and Randall: G. Whitrow, Nature (London) **158**, 165 (1946); G. Whitrow and D. Randall, Mon. Not. R. Astron. Soc. **111**, 455 (1951); M. S. Berman and M. M. Som, Gen. Relativ. Gravit. **22**, 625 (1990); see also Berman [4].

[36] J. D. Bekenstein, Phys. Rev. D **23**, 287 (1981).

[37] W. G. Unruh and R. M. Wald, Phys. Rev. D **25**, 942 (1982); S. D. Unwin, *ibid.* **26**, 944 (1982); D. N. Page, *ibid.* **26**, 947 (1982).

[38] J. D. Bekenstein, Phys. Rev. D **26**, 950 (1982).

[39] P.D. B. Collins, A. D. Martin, and E. J. Squires, *Particle Physics and Cosmology* (Wiley, New York, 1989), p. 390.

[40] J. V. Narlikar and T. Padmanabhan, *Gravity, Gauge Theories and Quantum Cosmology* (Reidel, Dordrecht, Holland, 1986), pp. 266–268.

[41] See Dominguez-Tenreiro and Quirós [14], p. 248.

[42] M. J. Rees, J. Astrophys. Astron. **5**, 331 (1984).

[43] Pavón [9] states incorrectly that, for large $R$, $x \to (16\pi G)^{-1}$ in the Özer-Taha model. The correct (dimensionless) limit is $\frac{1}{2}$. This correction does not affect the rest of Pavón's paper.

[44] E. B. Gliner, Zh. Eksp. Teor. Fiz. **49**, 542 (1965) [Sov. Phys. JETP **22**, 378 (1966)]; Dokl. Akad. Nauk SSSR **192**, 771 (1970) [Sov. Phys. Dokl. **15**, 559 (1970)]; M. A. Markov, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 214 (1982) [JETP Lett. **36**, 265 (1982)]; N. Rosen, Astrophys. J. **297**, 347 (1985).

[45] M. Israelit and N. Rosen, Astrophys. J **342**, 627 (1989).

[46] But see Ref. [8] for possible, albeit theoretically implausible, loopholes in the reasoning underlying this argument. Obviously the analysis justifying $E_m = \text{const}$ will also hold if the vacuum couples to both matter and radiation.