Sign of the color-hyperfine splitting in charmonium P states as a test of perturbative QCD

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The sign of the difference in energy between the center of gravity of the ^3P levels and the ^1P level in heavy quarkonium is a test of the α_s^2 terms in perturbative QCD with a fixed coupling constant. Whereas this energy difference is zero in the Fermi-Breit approximation and greater than zero in a generalized Fermi-Breit approximation with asymptotically free potentials, the energy difference is negative in perturbative QCD calculations to one-loop order with a fixed α_s. For potentials which reasonably agree with the spin-averaged data, the absolute magnitude of the splitting is calculated to be about 4 MeV or less in charmonium and 2 MeV or less in bottomonium. Since the energy of the ^3P center of gravity in charmonium is known, a measurement of the energy of the ^1P level is needed to discriminate between these different schemes. Although the ^1P levels in bottomonium will probably be harder to measure, we also present results for bottomonium.

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Experiment tells us that the spin-dependent forces acting between a quark and an antiquark in a meson are such that states having spin one and zero orbital angular momentum (^3S states) are higher in energy than the corresponding states with spin zero (^1S states). Thus, for example, the J/ψ, with a mass of 3097 MeV, lies higher than the η_c, with a mass of 2980 MeV. It is commonly believed that the same forces act to cause the energy $E(^3P)$ of the center of gravity of the ^3P levels to be higher than the energy $E(^1P)$ of the corresponding ^1P level. However, although this statement is true in some calculational schemes, it is false in others. Unfortunately, experimental evidence is lacking.

Our main purpose in this work is to compare two schemes: one of which predicts that in heavy quarkonia the energy difference $\Delta E_P = E(^3P) - E(^1P)$ is greater than or equal to zero and another which predicts that ΔE_P is less than zero. The first method we call the "generalized Fermi-Breit" method, while the second is an evaluation to one loop in QCD perturbation theory. The energy of the center of gravity of the $1\,^3P$ levels in charmonium and of the $1\,^3P$ and $2\,^3P$ levels in bottomonium have already been measured, and so measurements of the correspondingly ^1P levels can discriminate between these schemes.

Probably, the $1\,^1P$ level in charmonium is the best candidate for measurement, as we guess that the $1\,^1P$ and $2\,^1P$ levels in bottomonium will be harder to measure. (In light mesons the perturbation approximation on which the various schemes are based is not valid.)

De Rújula, Georgi, and Glashow [1] have pointed out that the one-gluon approximation to QCD leads to a color-hyperfine interaction analogous to the hyperfine interaction arising from one-photon exchange in QED. This approximation, in both QED and QCD, is known as the Fermi-Breit (FB) approximation. It gives rise to nonstatic corrections to the static Coulomb potential of QED or the static color-Coulomb potential of QCD. (By a static potential, we mean a potential which depends only on the distance r between the quark and antiquark.) The nonstatic corrections include the color-hyperfine interaction, a tensor interaction, and a spin-orbit interaction, plus nonstatic spin-independent terms, all of which must be evaluated (for consistency) in lowest-order perturbation theory.

It is well known [1] that the FB color-hyperfine interaction leads to a splitting of ^3S levels of quarkonium from the corresponding ^1S levels, with the ^3S levels lying higher. As we have noted, this prediction is in accord with observation. The contributions of the spin-orbit and tensor terms vanish in S states in lowest-order perturbation theory. Likewise, in lowest-order perturbation theory, the spin-orbit and tensor contributions cancel out of the difference $\Delta E_L = E(^3L) - E(^1L)$ if $E(^3L)$ is the center of gravity of the three triplet states with orbital angular momentum $L > 0$. Therefore, if lowest-order perturbation theory is a good approximation, a measurement of ΔE_L is a measurement of the expectation value of the color-hyperfine interaction.

In the FB approximation, the color-hyperfine interaction H' is given by

$$H'(\mathrm{FB}) = (8\pi\alpha_s/9m_1m_2)\delta(\mathbf{r})\sigma_1\cdot\sigma_2 , \quad (1)$$

where α_s is the strong-interaction coupling constant, m_1 and m_2 are the constituent masses of the quark and antiquark, and σ_1 and σ_2 are Pauli spin operators. This interaction leads to the following formula for ΔE_L:

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$$\Delta E_L(\mathrm{FB})=32\pi\alpha_s|\psi(0)|^2/(9m_1m_2)\ , \tag{2}$$

where $\psi(0)$ is the wave function at the origin. The value of $\psi(0)$ cannot be calculated perturbatively and must be calculated from a model, usually a potential model. It can be seen immediately from Eq. (2) that, independent of what potential is used, $\Delta E_L(\mathrm{FB})$ cannot be negative. In all potential models motivated by QCD (about which we will say more shortly), $|\psi(0)|>0$ in S states and $|\psi(0)|=0$ in states with $L\neq 0$. Thus, in the FB approximation, ΔE_S is predicted to be greater than zero, in accord with observation, while all other ΔE_L are predicted to be equal to zero.

The FB result follows from the fact that in the one-gluon approximation the static quark-antiquark potential is Coulomb-like, being of the form

$$V(r)=-4\alpha_s/(3r)\ . \tag{3}$$

A number of authors [2] have generalized the FB approximation so that it will apply to potentials which do not behave as $-1/r$. In the generalized Fermi-Breit (GFB) approximation (which is often just called the Fermi-Breit approximation), if the quark-antiquark potential $U(r)$ contains a part $V(r)$, which behaves as the zeroth component of a Lorentz four-vector, and another part $S(r)$, which behaves as a Lorentz scalar, then only the vector part $V(r)$ contributes to the color-hyperfine splitting. Here $U(r)=V(r)+S(r)$.

In the GFB approximation, H' is given by [2]

$$H'(\mathrm{GFB})=\nabla^2V(r)\sigma_1\cdot\sigma_2/(6m_1m_2)\ , \tag{4}$$

and leads to the following expression for ΔE_L:

$$\Delta E_L(\mathrm{GFB})=2\langle\nabla^2V(r)\rangle/(3m_1m_2)\ , \tag{5}$$

where the expectation value is taken with respect to the unperturbed wave function of the particular level. Although only $V(r)$ enters explicitly into the expression for the splitting in Eq. (5), both $V(r)$ and $S(r)$ are needed to calculate the wave function in order to obtain the expectation value. If $V(r)$ is given by Eq. (3), then the GFB result reduces to the usual FB formula.

Because of asymptotic freedom it is reasonable that $V(r)$ is less singular at the origin than $1/r$. One possibility is to replace the constant α_s in (3) by $\alpha_s(r)$, which behaves like $-1/\ln r$ as r goes to zero. For example, we may take $\alpha_s(r)=(\lambda r-1)/\ln\lambda r$, where λ is a positive constant [3]. Another possibility is to take $V(r)$ to be a power: $V(r)=-a/r^\beta$, where a and β are positive constants, with $\beta<1$. In either case it is easy to see that the GFB formula predicts that $\Delta E_L>0$ for all L and that, in particular, $\Delta E_P>0$.

More generally, for any potential $V(r)$ which satisfies

$$\nabla^2V(r)>0\ \ \text{all}\ \ r\ , \tag{6}$$

ΔE_L will be positive. Using the nonrelativistic Schrödinger equation, Baumgartner, Grosse, and Martin [4] have proved that if $\nabla^2U(r)$ has the same sign for all r, then the ordering of energy levels agrees with the ordering in charmonium and bottomonium only if $\nabla^2U(r)>0$. If the QCD static potential $V(r)$ also has this property, as we conjecture, then we have the general result that $\Delta E_L>0$ in the GFB approximation.

Eichten and Feinberg [5] and Gromes [6] have treated the spin-dependent interaction in quarkonium using perturbative QCD in an approximation which goes beyond the FB approximation. In the treatment of Eichten and Feinberg and Gromes, the color-hyperfine interaction is not written in terms of $V(r)$, but rather in terms of a new quantity, called $V_4(r)$, which can be evaluated in QCD perturbation theory. Gupta, Radford, and Repko [7] (GRR) and Pantaleone, Tye, and Ng [8] (PTN) have used the prescription of Eichten and Feinberg and Gromes to calculate the color-hyperfine splitting to order α_s^2. In their calculations the order-α_s contribution vanishes in P states (because they do not let α_s run). Thus their result to lowest order is the same as that obtained in the FB approximation, but not in the GFB approximation. It turns out that the order-α_s^2 (one-loop) contribution is negative. Therefore, GRR and PTN both obtain a sign for ΔE_P which is *opposite* to the sign obtained in the GFB approximation for potentials satisfying the inequality (6). (The calculations of GRR and PTN are not identical because they use different static potentials and different normalization prescriptions.) The expressions for ΔE_L in the GRR and PTN methods are rather lengthy, and we refer interested readers to the original papers.

If we compute ΔE_P using the GFB method for a potential satisfying (6), we will obtain a positive sign for ΔE_P, in contrast with the results of GRR and PTN. In order to obtain numerical results, we need to use a definite potential. It is clear that the exact value will depend on the potential we use. For illustrative purposes, just to get an order-of-magnitude estimate, we evaluate ΔE_P with the potential of Lichtenberg and co-workers [9], which provides rather good fits to the spin-averaged energy levels in charmonium and bottomonium. The potential of Lichtenberg and co-workers is representative of potentials which are less singular at the origin than $1/r$.

The potential of Lichtenberg and co-workers has the form

$$U(r)=-a/r^{3/4}+br^{3/4}+c\ , \tag{7}$$

where a and b are positive constants and the constant c may be of either sign. We call the potential $U(r)$ because it contains both a vector piece $V(r)$ and a scalar piece $S(r)$. We have numerically solved the Schrödinger equation with the potential of Lichtenberg and co-workers [9], varying the parameters so as to obtain a best fit to the charmonium and bottomonium spin-averaged data. With quark masses

$$m_c=1.82\ \mathrm{GeV},\quad m_b=5.18\ \mathrm{GeV}\ ,$$

the constants of the potential for charmonium are [9]

$$\begin{aligned} a&=0.751\ \mathrm{GeV}^{1/4}\ ,\\ b&=0.283\ \mathrm{GeV}^{7/4}\ ,\\ c&=-0.739\ \mathrm{GeV}\ , \end{aligned} \tag{8}$$

and for bottomonium are

$$a = 0.643 \text{ GeV}^{1/4},$$
$$b = 0.297 \text{ GeV}^{7/4}, \quad (9)$$
$$c = -0.795 \text{ GeV}.$$

If we use a flavor-independent potential (the same values of a, b, and c for both charmonium and bottomonium), we do not get quite so good agreement with the spin-averaged data [9], but it turns out that the values of ΔE_P are essentially unchanged.

In order to use the potential of Lichtenberg and co-workers of Eq. (7), we must divide it into vector and scalar parts. According to QCD perturbation theory, the potential behaves as a vector at short distances. There are indications from QCD lattice gauge calculations [10] that the confining part of the quark-antiquark potential behaves as a scalar. We are guided by the perturbative and lattice QCD results in splitting up the potential into vector and scalar parts, but understand that there is some ambiguity in this procedure. We take the vector and scalar parts of the potential of Lichtenberg and co-workers to be

$$V(r) = -a/r^{3/4}, \quad S(r) = br^{3/4} + c. \quad (10)$$

We have arbitrarily placed the constant c in $S(r)$, but this does not affect our result, as can be seen from Eq. (5). Note that the potential of Lichtenberg and co-workers satisfies (6) and thus leads to $\Delta E_P > 0$.

Our calculated values of ΔE_P (GFB) with the potential of Lichtenberg and co-workers are given in Table I. We note that a calculation of ΔE_S with the same potential gives $\Delta E_S = 144$ MeV for charmonium, in comparison with the experimental value of 117 MeV. Because the calculated value of ΔE_S is somewhat too large, we expect that our calculated values of ΔE_P might also be too large. However, the important point is that the sign is definitely positive.

With other potentials which agree reasonably well with the charmonium and bottomonium energy levels, we would get different values of the P-wave splitting, *but the sign and the order of magnitude would be the same as that with the potential of Lichtenberg and co-workers.* An exception is the potential of Eichten *et al.* [11] if the confining part is taken to be a scalar. Then the vector part of the potential of Eichten *et al.* goes like $-1/r$ and so gives $\Delta E_P = 0$ in the GFB scheme, just as in the FB case.

As we have remarked, GRR and PTN also compute the P-state color-hyperfine splitting. Pantaleone and Tye [12] (PT) specifically address this question in a later paper, using a method which yields similar, but not identical, results to those of PTN. In Table I we show for comparison the Fermi-Breit result and the results obtained by GRR and PT.

TABLE I. Calculated values of ΔE_P for charmonium and bottomonium in MeV using the potential of Lichtenberg and co-workers with the GFB method. For comparison, we also give the FB result and the values calculated by GRR [7] and PT [12] to order α_s^2.

	GFB	FB	GRR	PT
$1P\ c\bar{c}$	4	0	-2	-1.4
$1P\ b\bar{b}$	2	0	-1	-0.5
$2P\ b\bar{b}$	1	0	-1	-0.4

We can see from Table I that the color-hyperfine splitting of the charmonium P levels is typically 2–4 MeV, except that in the GFB approximation the singlet P state lies below the center of gravity of the triplet P states, while in the GRR and PT approximations the singlet P state lies above the center of gravity of the triplet P states. (Of course, in the FB approximation, the splitting is 0.) In bottomonium the qualitative features are the same, but the magnitudes of the splittings are less.

It is apparent from these results that measurements of the singlet P state in charmonium and bottomonium provide a test of whether the modern theory of the color-hyperfine interaction described by Eichten and Feinberg and Gromes and carried out in QCD perturbation theory to order α_s^2 with fixed α_s by GRR and PTN is in fact superior to the GFB approximation.

Other ways of treating the splitting have also appeared in the literature. Ono and Schöberl [13] include the Fermi-Breit term in the potential, rather than treating it as a perturbation. In order to get a finite result, they have to smear out the δ function in Eq. (1), thereby introducing another parameter. Their result for ΔE_P is positive. McClary and Byers [14] also use the Fermi-Breit Hamiltonian in a way which requires them to smear out the δ function. These authors state that, in their method, "For triplet states, the hyperfine interaction is repulsive. For singlet states it is attractive." However, in their treatment they find that ΔE_P is negative in both charmonium and bottomonium, as can be seen from Tables II and III of their paper. We guess that their result comes from including the tensor and spin-orbit forces nonperturbatively, in which case the effects of these interactions do not cancel when taking the center of gravity of the triplet states. Igi and Ono [15] have treated the P-state splitting using the method of GRR and PTN, but with a different potential, and, in agreement with GRR and PTN, obtain a negative result for ΔE_P (see the erratum to their paper).

Gupta, Repko, and Suchyta [16] do a one-loop perturbative calculation with fixed α_s, but nevertheless obtain a positive value for ΔE_P because they choose the confining potential to be part vector and part scalar. This is because the vector part of their potential contains a part (the confining part) which is less singular than $1/r$. Clearly, if the confining potential has a substantial vector part, despite the contrary indication from lattice QCD [10], then the sign of ΔE_P cannot be decided within the framework of perturbative QCD. But we prefer to take seriously the lattice QCD result, which says that the confining potential is scalar. Furthermore, we have no cogent reason to doubt the QCD result that α_s runs.

We expect that it will be easier to measure ΔE_P in charmonium than in bottomonium, so that the first test of QCD one-loop perturbation theory as calculated by GRR, PTN, and PT will come from charmonium. We

urge that such a measurement be given high priority. We note, however, that if an accurate measurement of ΔE_P could be made in bottomonium, the interpretation ought to be even cleaner than in charmonium, as perturbation theory should hold to a better approximation in the $b\bar{b}$ than in the $c\bar{c}$ system.

Finally, we have to mention a caveat. Other effects, which are difficult to calculate and which we have not taken into account, can also contribute to the P-wave splittings. For example, we have already mentioned that the tensor and spin-orbit contributions cancel only in lowest-order perturbation theory and contribute in higher order. Unfortunately, we do not know whether it is consistent to include these higher-order contributions in the generalized Fermi-Breit formalism. As another example, coupling to other channels can cause energy shifts of levels, but calculations of such shifts are highly model dependent. In any case we can say that with asymptotically free potentials the generalized Fermi-Breit contribution to ΔE_P is positive, in contrast with the calculations [7,8] performed to one-loop order with fixed α_s, which obtain a negative result.

Note added in proof. A recent preliminary measurement in E760 at Fermilab [17] has found $\Delta E_P \simeq -1$ MeV in charmonium, in qualitative agreement with one-loop QCD calculations.

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