Electroweak phase transition in supersymmetry

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The electroweak phase transition in supersymmetric models is studied, analyzing the constraint on the Higgs-boson mass coming from the condition that the cosmic baryon asymmetry is not washed out soon after the phase transition. It is found that, in the minimal supersymmetric model, baryogenesis at the weak scale requires a Higgs boson lighter than about 50–55 GeV, as in the standard model. This result holds true also when the one-loop radiative corrections, which are important for a heavy top quark, are taken into account. On the other hand, in extended supersymmetric models, it is possible to have the lightest Higgs boson as heavy as 100 GeV and still satisfy the requirement of weak-scale baryogenesis.

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It has been by now recognized that baryon-number violation occurs rapidly at temperatures $T >> m_W/\alpha_W$ [1], due to anomalous electroweak processes. This has raised the interesting possibility that baryogenesis may take place at the electroweak phase transition [2], thus at temperatures very much lower than previously envisaged in conventional grand-unified-theory (GUT) schemes. The new mechanism of baryogenesis must also satisfy conditions of departure from thermal equilibrium and violation of both C and CP. Even if all these ingredients are present in the standard model, it seems very unlikely that a realistic cosmic asymmetry can be generated within its framework. Firstly, CP violation in the Kobayashi-Maskawa matrix, suppressed by mixing angles, is too small. Secondly, the requirement of a sufficiently strong first-order electroweak phase transition, where the coexistence of different phases can be achieved, implies an upper bound on the Higgs-boson mass of about 50 GeV [3] (or 55 GeV, if all uncertainties are conservatively taken into account [4]), a result barely compatible with the experimental searches, which presently give a lower bound of 48 GeV [5].

It is natural therefore to investigate what happens in extensions of the standard model. Supersymmetry seems a very promising candidate since it predicts a relatively light Higgs boson and new phases as sources of CP violation. In this paper we study the electroweak phase transition in supersymmetric models, focusing the analysis on the bound on the Higgs-boson mass imposed by low-temperature baryogenesis.

We will first consider the minimal version of the supersymmetric standard model (see, e.g., [6]), with two Higgs doublets H_1 and H_2 with opposite hypercharge:

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}. \tag{1}$$

The Higgs potential is

$$\begin{aligned} V = {} & m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1H_2 + \text{H.c.}) \\ & + \frac{g^2}{2}\left(H_1^\dagger \frac{\boldsymbol{\sigma}}{2} H_1 + H_2^\dagger \frac{\boldsymbol{\sigma}}{2} H_2\right)^2 \\ & + \frac{g'^2}{2}\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2, \end{aligned} \tag{2}$$

where the quartic terms are dictated by supersymmetry and m_1, m_2, m_3 are three mass parameters related to the supersymmetry-breaking terms. The theory greatly simplifies in the limit in which the mass scale of the supersymmetry-breaking terms, here generically denoted by $\tilde{m}$, is much larger than the weak scale [7]. All supersymmetric partners decouple with $\tilde{m}$, and only one combination of the two neutral Higgs bosons, that is

$$h = \sqrt{2}\cos\beta\ \text{Re}H_1^0 + \sqrt{2}\sin\beta\ \text{Re}H_2^0, \tag{3}$$

$$\text{Re}\langle H_1^0\rangle \equiv \frac{v_1}{\sqrt{2}}, \quad \text{Re}\langle H_2^0\rangle \equiv \frac{v_2}{\sqrt{2}}, \quad \tan\beta \equiv \frac{v_2}{v_1}, \tag{4}$$

remains light. Therefore, at energy scales less than $\tilde{m}$, the theory approximately reduces to the standard model with only one Higgs doublet. The neutral Higgs boson h, Eq. (3), has ordinary interactions with fermions, has a tree-level mass square $m_{H^0}^2 \equiv m_Z^2\cos^2 2\beta$, and a classical potential of the form

$$V_0 = \frac{m_{H^0}^2}{8v^2}\left(h^2 - v^2\right)^2, \quad v^2 \equiv v_1^2 + v_2^2, \tag{5}$$

where the Higgs vacuum expectation value v is related to the Z mass by $v^2 = 4m_Z^2/(g^2 + g'^2)$.

The one-loop potential at finite temperature V can be written as the sum of the classical potential V_0, Eq. (5), the Coleman-Weinberg [8] quantum corrections V_1, and a temperature-dependent part V_T [9]. The one-loop quantum corrections V_1 are given by [8]

$$V_1 = \sum_i \pm \frac{N_i}{64\pi^2} m_i^4(h) \ln m_i^2(h) + P(h^2), \tag{6}$$

where the sum extends over all bosons (+) and fermions (−) present in the theory with number of degrees of freedom N_i, and mass in the presence of a background field h, $m_i(h)$. In Eq. (6), $P(h^2)$ is a polynomial containing quartic and quadratic counterterms in h, which we fix by choosing the renormalization prescriptions:

$$\left.\frac{\partial V_1}{\partial h}\right|_{h=v} = 0, \tag{7}$$

$$\left.\frac{\partial^4 V_1}{\partial h^4}\right|_{h=\tilde{m}} = 0. \tag{8}$$

Equation (7) implies that the tree-level value of v is preserved at the one-loop level, while Eq. (8) implies that the Higgs quartic coupling constant at the scale $\tilde{m}$ is determined by the tree-level relation. This last condition stems from the fact that, neglecting threshold effects, the theory is exactly supersymmetric at energy scales larger than $\tilde{m}$ and V_1 identically vanishes. Taking now $m_i^2(h)$ of the form $\mu_i^2 + G_i h^2$ and imposing the renormalization conditions (7) and (8) on Eq. (6), we obtain

$$V_1 = \sum_i \pm \frac{N_i}{64\pi^2} \left[m_i^4(h) \ln \frac{m_i^2(h)}{m_i^2(\tilde{m})} - \frac{25}{6} m_i^4(h) + \left(\frac{11}{3} + \ln \frac{m_i^2(\tilde{m})}{m_i^2(v)} \right) 2 m_i^2(v) m_i^2(h) \right], \tag{9}$$

for $\tilde{m}^2 >> \mu_i^2$.

From the second derivative of $V_0 + V_1$ we can compute the Higgs-boson mass corrected at one loop:

$$m_H^2 = m_{H^0}^2 + \left.\frac{\partial^2 V_1}{\partial h^2}\right|_{h=v} = m_{H^0}^2 - \sum_i \pm \frac{N_i}{32\pi^2} \left(\frac{8}{3} + \ln \frac{m_i^2(\tilde{m})}{m_i^2(v)} \right) \left.\left(\frac{\partial m^2}{\partial h} \right)^2 \right|_{h=v} \tag{10}$$

Incidentally we note that the sum in Eq. (10) is dominated by the top quark, when its mass is large ($m_t >> m_W$), yielding

$$m_H^2 \simeq m_{H^0}^2 + \frac{3}{\pi^2} \frac{m_t^4}{v^2} \ln \frac{\tilde{m}}{v}, \tag{11}$$

which is the by now well-known result [7, 10] that the supersymmetric Higgs boson mass receives large quantum corrections for a heavy top quark.

If Eq. (10) is used to trade $m_{H^0}^2$ with the one-loop corrected Higgs-boson mass m_H^2, it is easy to verify that the one-loop scalar potential at $T = 0$, Eqs. (5) and (9), becomes

$$V_0 + V_1 = \frac{m_H^2}{8v^2} \left(h^2 - v^2\right)^2 + \sum_i \pm \frac{N_i}{64\pi^2} \left[m_i^4(h) \ln \frac{m_i^2(h)}{m_i^2(v)} - \frac{3}{2} m_i^4(h) + 2 m_i^2(h) m_i^2(v) \right], \tag{12}$$

and the dependence of the potential on $\tilde{m}$ has been reabsorbed[1] in m_H^2. Since the ultraviolet behavior of the theory is not affected by the finite-temperature terms, the counterterms, computed here at $T = 0$, are also valid for $T \neq 0$.

Finally we include the finite-temperature terms V_T. For $m/T < 1$, we use a high-temperature expansion [9]

$$V_T = \sum_i N_i \left(\frac{m_i^2(h) T^2}{24} - \frac{m_i^3(h) T}{12\pi} - \frac{m_i^4(h)}{64\pi^2} \ln \frac{m_i^2(h)}{A_b T^2} \right) \quad \text{for bosons}, \tag{13}$$

$$V_T = \sum_i N_i \left(\frac{m_i^2(h) T^2}{48} + \frac{m_i^4(h)}{64\pi^2} \ln \frac{m_i^2(h)}{A_f T^2} \right) \quad \text{for fermions}, \tag{14}$$

$$A_f = \pi^2 \exp(\tfrac{3}{2} - 2\gamma_E), \qquad A_b = 16 A_f, \qquad \gamma_E = 0.5772, \tag{15}$$

while for $m/T > 1$, the particle is Boltzmann suppressed and V_T becomes [9]

$$V_T = -\sum_i \frac{N_i T^2}{(2\pi)^{3/2}} m_i^2 \sqrt{\frac{T}{m_i}} e^{-m_i/T} \times \left[1 + \frac{15}{8} \frac{T}{m_i} + O\left(\frac{T^2}{m_i^2} \right) \right] \tag{16}$$

The complete one-loop potential is obtained by summing V_0, V_1, Eq. (12), and V_T, Eqs. (13)–(16), over the whole particle spectrum. We first sum over the standard-model particles, i.e., particles which do not decouple as $\tilde{m} \to \infty$. The contributions from the W, Z, Higgs boson and top quark (all other quarks and leptons can be safely neglected) give

$$V_{SM} = -\left(\frac{m_H^2}{2} - 4Bv^2 - \omega T^2 \right) \frac{h^2}{2} + \left(\frac{m_H^2}{2v^2} - 6B - 4\rho(T) \right) \frac{h^4}{4} - \delta T h^3 \tag{17}$$

[1] Of course, we could have directly obtained Eq. (12) by imposing, instead of Eqs. (7) and (8), the renormalization prescriptions $\partial(V_0 + V_1)/\partial h|_{h=v} = 0$, $\partial^2(V_0 + V_1)/\partial h^2|_{h=v} = m_H^2$, as is ordinarily done for the standard model. In this case, however, m_H^2 would be an independent parameter of the theory and not related to the other quantities, as it is in Eq. (10).

$$B \equiv \frac{3}{64\pi^2 v^4}\left(2m_W^4 + m_Z^4 - 4m_t^4\right),$$
$$\delta \equiv \frac{1}{4\pi v^3}\left(2m_W^3 + m_Z^3\right), \quad (18)$$

$$\omega \equiv \frac{1}{8v^2}\left(4m_W^2 + 2m_Z^2 + 4m_t^2 + m_H^2\right), \quad (19)$$

$$\rho(T) \equiv \frac{3}{64\pi^2 v^4}\left(2m_W^4 \ln\frac{m_W^2}{A_b T^2} + m_Z^4 \ln\frac{m_Z^2}{A_b T^2} - 4m_t^4 \ln\frac{m_t^2}{A_f T^2}\right), \quad (20)$$

where only the leading terms in m_H^2 have been retained. Equation (17) is the usual standard-model result. However, here the Higgs-boson mass m_H is not an independent parameter, but it is given in Eq. (10), where one-loop corrections have been included.

Next we compute the contribution of the supersymmetric particles to the effective potential. As we are considering the theory in the limit in which $\tilde{m}$ is much larger than the weak scale m_W, we can expand the mass squared of each supersymmetric particle in powers of $\tilde{m}^2$:

$$m_i^2(h) = \tilde{m}^2 + G_i h^2 + O\left(\frac{1}{\tilde{m}^2}\right) \quad (21)$$

Now we can plug expression (21) into Eqs. (12) and (16) and expand the result in powers of $\tilde{m}$, to obtain the leading supersymmetric contribution to the effective potential:

$$(\Delta V_1)_{\rm SUSY} = \sum_i \pm \frac{N_i}{64\pi^2}\frac{G_i^3}{3\tilde{m}^2}\left(h^2 - v^2\right)^3 \times \left[1 + O\left(\frac{1}{\tilde{m}^2}\right)\right], \quad (22)$$

$$(\Delta V_T)_{\rm SUSY} = \sum_i \frac{N_i}{2(2\pi)^{3/2}} G_i T^{3/2} \tilde{m}^{1/2} e^{-\tilde{m}/T} h^2 \times \left[1 + O\left(\frac{1}{\tilde{m}}\right)\right]. \quad (23)$$

Since in the following we will be interested in temperatures $T \sim m_W$, we have assumed that all supersymmetric particles are out of equilibrium and have used Eq. (16) for V_T.

The complete one-loop Higgs potential is given by the sum of the contributions from the standard model, Eq. (17), and supersymmetric particles, Eqs. (22) and (23). From it we can now compute the critical temperature for the electroweak phase transition, T_c, defined as the temperature at which the potential is flat at the origin:

$$T_c^2 = \frac{1}{\omega}\left[\frac{m_H^2}{2} - 4Bv^2 - \sum_i N_i G_i \left(\frac{T_c^{3/2}\tilde{m}^{1/2}}{(2\pi)^{3/2}} e^{-\tilde{m}/T_c} \pm \frac{G_i^2}{32\pi^2}\frac{v^4}{\tilde{m}^2}\right)\right], \quad (24)$$

and the vacuum expectation value $\sigma(T)$ of the Higgs field h at $T = T_c$ [for $\sigma^2(T_c) << v^2 \equiv \sigma^2(0)$]:

$$\frac{\sigma(T_c)}{T_c} = \frac{6\delta}{m_H^2/v^2 - 12B - 8\rho(T_c) - K}, \quad (25)$$

$$K \equiv \sum_i \pm \frac{N_i G_i^3}{8\pi^2}\frac{v^2}{\tilde{m}^2}. \quad (26)$$

A nonzero value of $\sigma(T_c)/T_c$ is a signal of a first-order phase transition. As pointed out in Ref. [3], the value of $\sigma(T_c)/T_c$ is constrained, in any baryogenesis scheme based on anomalous electroweak processes, by the requirement that the cosmic asymmetry is not washed out soon after the phase transition. In fact, if the sphaleron energy barrier is too small after the phase transition is completed, baryon violation can still occur and the baryon asymmetry will be erased. This implies a lower bound on $\sigma(T_c)/T_c$ of about 1.3 [3, 4], and consequently, from Eq. (25), an upper bound on the Higgs-boson mass. In the limit of very heavy supersymmetry ($\tilde{m} \to \infty$, $K \to 0$), this bound is the same as in the case of the standard model [3, 4], i.e., $m_H \lesssim 50$ GeV, for $m_t \lesssim 140$ GeV. For larger m_t the limit rapidly becomes much more stringent, as B becomes more and more negative. As shown in the careful analysis of Ref. [4], the uncertainties in the determination of the Higgs-boson-mass-bound amount to at least 10% of the value, due to the uncertainties in the computation of the rate of the baryon-number-violating processes.

We now want to include the contributions from the supersymmetric particles and investigate if they can relax the limit on the Higgs boson mass. As apparent from Eq. (25), this is possible if they amount to give a positive value for K. We first consider the extensively studied minimal supersymmetric model, using standard notations (see, e.g., Ref. [6]).

Expanding according to Eq. (21) the eigenvalues of the mass matrices for the two charged and four neutral supersymmetric fermions, we obtain the chargino-neutralino contribution to K:

$$K = \frac{2m_Z^6}{\pi^2 v^4 \mu^2}\left\{\cos^6\theta_W\left[\frac{(d+s)^3 d}{(1-d^2)^3} - 2\frac{1+(3-s^2)sd+d^2}{(1-d^2)^2}\right] + \sin^6\theta_W \frac{(d'+s)^3 d'}{(1-d'^2)^3} - \left[\frac{(1+s)(1-\sin^2\theta_W d - \cos^2\theta_W d')}{2(1-d)(1-d')}\right]^3 - \left[\frac{(1-s)(1+\sin^2\theta_W d + \cos^2\theta_W d')}{2(1+d)(1+d')}\right]^3\right\}, \quad (27)$$

$$d \equiv \frac{M}{\mu}, \quad d' \equiv \frac{M'}{\mu}, \quad s \equiv \sin(2\beta), \tag{28}$$

where M' and M are the supersymmetry-breaking masses for U(1) and SU(2) gauginos and μ is the Higgs superfield mixing parameter, all taken here to be of order $\tilde{m}$. The contribution to K from the new Higgs bosons (one charged, and two neutrals with opposite CP quantum numbers) is

$$K = \frac{1}{8\pi^2}\frac{m_Z^6}{v^4 m_a}\left[-\left(\frac{c^2}{2}\right)^3 + 2\left(\cos^2\theta_W - \frac{c^2}{2}\right)^3 + \left(1 - \frac{3}{2}c^2\right)^3\right], \tag{29}$$

$$c \equiv \cos(2\beta), \tag{30}$$

where m_a (the mass of the CP-odd Higgs boson at $T = 0$) is a parameter of order $\tilde{m}$. Finally, the contribution from the scalar partners of quarks and leptons is (assuming all Yukawa couplings are negligible)

$$K = \frac{9}{8\pi^2}\frac{m_Z^6}{v^4}c^3\sin^2\theta_W\left(\frac{1}{2} - \sin^2\theta_W\right)\left(\frac{1}{\tilde{m}_l^2} - \frac{1}{\tilde{m}_q^2}\right), \tag{31}$$

where two different supersymmetry-breaking masses for squarks and sleptons ($\tilde{m}_q$, $\tilde{m}_l$) have been assumed.

All these contributions, which are of order $K \sim m_Z^6/(\pi^2 v^4 \tilde{m}^2)$, can correspond to a change in the value of the bound on the Higgs-boson mass of at most a few percent and are therefore not very significant.

However, if the top quark is heavy, it is not possible to neglect its Yukawa coupling. For large m_t, the two scalar partners of the top quark have squared masses:

$$\tilde{m}^2 + A\tilde{m}m_t\frac{h}{v} + m_t^2\frac{h^2}{v^2}, \quad \tilde{m}^2 - A\tilde{m}m_t\frac{h}{v} + m_t^2\frac{h^2}{v^2}, \tag{32}$$

where A is a parameter of order 1, related to the supersymmetry-breaking terms. In this case, a linear term in h is present and Eq. (22), derived under the assumption (21) is no longer valid, should be replaced by

$$(\Delta V_1)_{\rm stop} = \frac{1}{32\pi^2}\left(2 - 3A^2 + A^4 - \frac{A^6}{10}\right) \times \frac{m_t^6}{v^6\tilde{m}^2}(h^2 - v^2)^3 + O\left(\frac{1}{\tilde{m}^4}\right). \tag{33}$$

Equation (25) is still valid and

$$K = \frac{3}{2\pi^2}\frac{m_t^6}{v^4\tilde{m}^2}\left(2 - 3A^2 + A^4 - \frac{A^6}{10}\right), \tag{34}$$

which is positive for $A^2 \lesssim 1$ and rapidly grows with m_t. However, if we take into account the m_t dependence contained in B, we see from Eq. (25) that the limit on the Higgs-boson mass can be relaxed only if the supersymmetric mass parameter $\tilde{m}$ is smaller than $0.8m_t$. This is unlikely in most realistic models, since all squarks (except the stop) would be lighter than the top quark.

Therefore, we can conclude that the change in the bound on the Higgs-boson mass due to the contribution of the minimal model supersymmetric particles is typically less than the uncertainties in the calculation [4] which, as we stated above, is about 10%. It is easy to check that this is true also in the case that the supersymmetric partners are much lighter than what we have considered here, and are in thermal equilibrium at $T = T_c$. On the other hand, the picture may change if more than one Higgs boson is light at $T = T_c$, since, in this case, the phase transition can be effectively driven by two Higgs vacuum expectation values, rather than only one, as we have considered above. Therefore we turn now to discuss this possibility by generalizing the procedure followed to obtain Eq. (25) for the case of two-Higgs-doublet potentials.

We start by choosing the following basis for the two-Higgs-doublet fields:

$$H_1 = e^{i\frac{\chi\cdot\sigma}{v}}\begin{pmatrix}\frac{1}{\sqrt{2}}(h_1 - i\sin\beta a)\\ -\sin\beta h^{+*}\end{pmatrix}, \quad H_2 = i\sigma_2 e^{-i\frac{\chi\cdot\sigma}{v}}\begin{pmatrix}\frac{1}{\sqrt{2}}(-h_2 + i\cos\beta a)\\ -\cos\beta h^{+}\end{pmatrix}, \tag{35}$$

where the would-be Goldstone bosons, absorbed by the W and Z gauge bosons, χ, explicitly disappear from the potential. In terms of the three real scalar fields h_1, h_2 (CP even), and a (CP odd), and the charged field h^+, the classical potential, Eq. (2), becomes

$$V_0 = \frac{m_1^2}{2}h_1^2 + \frac{m_2^2}{2}h_2^2 - m_3^2 h_1 h_2 + \left(\frac{1-c}{2}m_1^2 + \frac{1+c}{2}m_2^2 + sm_3^2\right)\left(\frac{a^2}{2} + |h^+|^2\right) + \frac{g^2 + g'^2}{8}\left[\frac{h_2^2 - h_1^2}{2} + c\left(\frac{a^2}{2} + |h^+|^2\right)\right]^2 + \frac{g^2}{4}|h^+|^2\left(\frac{1+c}{2}h_1^2 + \frac{1-c}{2}h_2^2 + sh_1h_2\right), \tag{36}$$

where $c = \cos(2\beta)$, $s = \sin(2\beta)$. It is convenient to express m_1^2, m_2^2, and m_3^2 in terms of the physical quantities of interest (which we choose to be $m_Z, \tan\beta, m_H$) using the tree-level relations:

$$\frac{m_1^2}{m_Z^2} = \frac{r(1-r) - c(c^2 - r^2)}{2(c^2 - r)}, \quad \frac{m_2^2}{m_Z^2} = \frac{r(1-r) + c(c^2 - r^2)}{2(c^2 - r)}, \quad \frac{m_3^2}{m_Z^2} = \frac{sr(1-r)}{2(c^2 - r)}, \tag{37}$$

where $r \equiv m_H^2/m_Z^2$ and m_H is the mass of the lightest CP-even neutral Higgs boson at zero temperature.

By taking second derivatives of the potential (36), we obtain the squared masses, in the presence of background fields h_1 and h_2, for the charged Higgs boson,

$$m_Z^2\left[\frac{c^2}{2}+\frac{r(1-r)}{c^2-r}+c\frac{(h_2^2-h_1^2)}{2v^2}\right]+\frac{m_W^2}{v^2}\left(\frac{1+c}{2}h_1^2+\frac{1-c}{2}h_2^2+sh_1h_2\right), \tag{38}$$

for the CP-odd neutral Higgs boson,

$$m_Z^2\left[\frac{c^2}{2}+\frac{r(1-r)}{c^2-r}+c\frac{(h_2^2-h_1^2)}{2v^2}\right], \tag{39}$$

and for the two CP-even neutral Higgs bosons,

$$\frac{m_Z^2}{2}\left\{\frac{r(1-r)}{c^2-r}+\frac{h_1^2+h_2^2}{v^2}\pm\sqrt{\left[\frac{c(c^2-r^2)}{c^2-r}+2\frac{(h_2^2-h_1^2)}{v^2}\right]^2+\left[\frac{sr(1-r)}{c^2-r}+2\frac{h_1h_2}{v^2}\right]^2}\right\}. \tag{40}$$

The one-loop-corrected potential (at zero temperature) for the fields h_1 and h_2 is V_0+V_1:

$$V_0=\frac{m_1^2}{2}h_1^2+\frac{m_2^2}{2}h_2^2-m_3^2h_1h_2+\frac{g^2+g'^2}{32}(h_2^2-h_1^2)^2, \tag{41}$$

$$V_1=\sum_i \pm\frac{N_i}{64\pi^2}m_i^4(h_1,h_2)\ln m_i^2(h_1,h_2)+P(h_1,h_2) \tag{42}$$

where m_1^2, m_2^2, and m_3^2 are given in Eq. (37), and Eq. (42) is the analogue of Eq. (6), with $P(h_1,h_2)$ being a fourth-order polynomial in h_1 and h_2. We choose to fix the quartic terms in P by requiring that the fourth derivatives of V_1 with respect to the Higgs fields vanish at the supersymmetric threshold ($h_1=h_2=\tilde{m}$). The quadratic terms in P are fixed by demanding that the tree-level values of v_1, v_2 and m_H are preserved by the one-loop correction

$$\left.\frac{\partial V_1}{\partial h_1}\right|_{\substack{h_1=v_1\\h_2=v_2}}=\left.\frac{\partial V_1}{\partial h_2}\right|_{\substack{h_1=v_1\\h_2=v_2}}=0, \tag{43}$$

$$\frac{1-c}{2}(c+r)^2\frac{\partial^2V_1}{\partial h_1\partial h_1}+\frac{1+c}{2}(c-r)^2\frac{\partial^2V_1}{\partial h_2\partial h_2}$$
$$\left.+s(c^2-r^2)\,\frac{\partial^2V_1}{\partial h_1\partial h_2}\right|_{\substack{h_1=v_1\\h_2=v_2}}=0. \tag{44}$$

The complete one-loop potential is $V=V_0+V_1+V_T$, where V_0 and V_1 are given in Eqs. (41) and (42) and V_T is the finite-temperature contribution, Eqs. (13)–(16). Given V, we can compute the critical temperature T_c

$$\det\left.\left(\frac{\partial^2V(h_1,h_2,T_c)}{\partial h_i\partial h_j}\right)\right|_{h_1=h_2=0}=0, \tag{45}$$

and $\sigma_1(T_c)$ and $\sigma_2(T_c)$, the vacuum expectation values of the Higgs fields at T_c:

$$\left.\frac{\partial V(h_1,h_2,T_c)}{\partial h_1}\right|_{\substack{h_2=\sigma_2(T_c)\\h_1=\sigma_1(T_c)}}=\left.\frac{\partial V(h_1,h_2,T_c)}{\partial h_2}\right|_{\substack{h_2=\sigma_2(T_c)\\h_1=\sigma_1(T_c)}}=0. \tag{46}$$

We have solved numerically Eqs. (45) and (46) including, in the effective potential, the contributions from W, Z gauge bosons, top and bottom quarks,[2], and the Higgs particles[3] with masses given in Eqs. (38)–(40). We have checked that by varying the parameters it is not possible to obtain $\sigma(T_c)/T_c>1.3$ ($\sigma^2\equiv\sigma_1^2+\sigma_2^2$), for $m_H>50$ GeV. For example, for $m_H=60$ GeV and $m_t=100$ GeV, $\sigma(T_c)/T_c=0.43, 0.39, 0.37$ (and $T_c=134, 145, 157$ GeV) for, respectively, $\tan\beta=3, 4, 8$. We can therefore conclude that, in the minimal supersymmetric model, baryogenesis at the electroweak scale implies a bound on the lightest Higgs boson of about 50 GeV, as in the standard model.

The main reason why the contribution of the supersymmetric particles is never large enough to strongly modify the Higgs-boson-mass bound is that, in the field-dependent masses, Eq. (21), G_i is almost always (except for the stop) of order g^2, and so K is too small, typically $K\sim m_Z^6/(\pi^2v^4\tilde{m}^2)$. The situation can be drastically different in extensions of the minimal model. If new large coupling constants are added to the theory, the contributions of the new particles to K can relax the bound on the Higgs-boson mass. As shown in Ref. [11], if a new scalar particle S with a coupling with the Higgs field of the form λS^2h^2 and a mass term $\tilde{m}^2S^2$ is introduced, its contribution to K is

$$K=\frac{\lambda^3}{4\pi^2}\frac{v^2}{\tilde{m}^2}. \tag{47}$$

[2] In this case we cannot neglect the contribution from the bottom quark, since for large $\tan\beta$, the bottom Yukawa coupling becomes large.

[3] Because of the contribution of the lightest Higgs boson, the effective potential contains an imaginary part for small values of the fields h_1 and h_2. As is usually done, we can neglect the imaginary part, since for the values of m_H under consideration, Im(V)<<Re(V).

For $\lambda^2/4\pi \sim 1$, and $\tilde{m} \sim 1$ TeV, the Higgs-boson mass bound is lifted to about 80 GeV. In supersymmetry, this can be explicitly realized if a new gauge singlet superfield N is introduced with interactions in the superpotential of the form[4]:

$$\lambda H_1 H_2 N + \frac{\rho}{3} N^3. \tag{48}$$

It is interesting to note that the bound on the Higgs-boson mass can be relaxed also by the contributions of new fermions. In the case of the superpotential of Eq. (48), the fermionic Higgs-gauge sector contains five Majorana neutral fermions and two charged fermions. Their contribution to K (neglecting terms of order g^2 and assuming $\langle N\rangle >> v$) is

$$K = -\frac{\lambda^4 v^2}{16\pi^2 \langle N\rangle^2} \frac{4\xi^2 + s(9-s^2)\xi + (1+3s^2)}{(1-\xi^2)^2}, \qquad \xi \equiv \frac{2\rho}{\lambda}, \tag{49}$$

which is always negative. However a positive value of K can be achieved if the theory is further extended. For instance, with two gauge singlet superfields N_1 and N_2 and a superpotential

$$\lambda_1 H_1 H_2 N_1 + \lambda_2 H_1 H_2 N_2 + \frac{\rho_1}{3} N_1^3 + \frac{\rho_2}{3} N_2^3, \tag{50}$$

the leading contribution to K from the Higgs-gauge fermion sector is (for $\tan\beta = 1$, $\lambda_1 = \lambda_2 \equiv \lambda$)

$$K = \frac{\lambda^4 v^2}{4\pi^2 x^2} \frac{(1-\xi_1)(\xi_1 - 4) + (1-\xi_2)(\xi_2 - 4)}{(1-\xi_1)^2 (1-\xi_2)^2}, \tag{51}$$

$$\xi_1 \equiv \frac{2\rho_1 \langle N_1\rangle}{\lambda x}, \quad \xi_2 \equiv \frac{2\rho_2 \langle N_2\rangle}{\lambda x}, \quad x \equiv \langle N_1\rangle + \langle N_2\rangle, \tag{52}$$

which can be positive. For instance, for $\xi_1 = 2$, $\xi_2 = 3$, $\lambda^2/4\pi = 1$, $x = 1.3$ TeV, the upper bound on the Higgs-boson mass becomes about 100 GeV.

In conclusion, we have investigated the electroweak phase transition in supersymmetric models by studying the bound on the lightest-Higgs-boson mass coming from the condition that the cosmic baryon asymmetry is not washed out soon after the phase transition, and we have obtained the following results. In the minimal supersymmetric model, the limit on the Higgs-boson mass m_H is numerically the same as in the standard model [3,4]. This is true for any top quark mass, when m_H is interpreted as the Higgs-boson mass corrected at one loop. The effect of the supersymmetric particles modifies the value of the limit on m_H by only a few percent, less than the uncertainty inherent in the calculation. On the other hand, it is not difficult to construct extended models, where no baryon wash-out occurs and all the Higgs bosons are heavier than 100 GeV, thus lying beyond the reach of the CERN e^+e^- collider LEP II.

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[4] Notice that the same interaction modifies also the tree-level bound on the lightest-Higgs-boson mass existing in minimal supersymmetry ($m_H^2 < c^2 m_Z^2$) to $m_H^2 < c^2 m_Z^2 + s^2 m_W^2 2\lambda^2/g^2$.

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