Radiative angular distributions from charmonium states directly produced by $\bar{p}p$ annihilation

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We calculate the combined angular distributions of the photon and the electron in the cascade process $p\bar{p}\to\chi_J\to\psi\gamma\to(e^+e^-)\gamma$ (J=2,1,0), in terms of the helicity or the multipole transition amplitudes in $\chi_J\to\psi\gamma$. Our expressions for them differ from those found previously. We describe the origin of these differences. The effect of the motion of ψ in the χ_J rest frame on the angular-distribution functions is taken into account.

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The combined angular distribution functions of the photon and the electron in the cascade process $p\bar{p}\to\chi_J$ ($J=2,1,0$)$\to\psi\gamma\to(e^+e^-)\gamma$ have been discussed before [1–3]. We have rederived these angular distribution functions and we get different results. The correct expression is necessary if one is to extract the multipole amplitudes from the experimentally measured angular distribution for comparison with theoretical predictions [4].

In order to understand the origin of the difference between our expression and that of Ref. [3], we give a brief sketch of our derivation. A symbolic sketch of the cascade process is shown in Fig. 1. The probability amplitude for the process

$$\bar{p}(\lambda_1)p(\lambda_2)\to\chi_{J\nu}\to\psi_\sigma+\gamma_\mu\to e^-(\kappa_1)e^+(\kappa_2)+\gamma_\mu ,$$

where λ_1, λ_2, ν, σ, κ_1, and κ_2 are the particle helicities, can be written as the product of the amplitudes of three sequential events: $\bar{p}(\lambda_1)p(\lambda_2)\to\chi_{J\nu}$, $\chi_{J\nu}\to\psi_\sigma+\gamma_\mu$, and $\psi_\sigma\to e^-(\kappa_1)+e^+(\kappa_2)$.

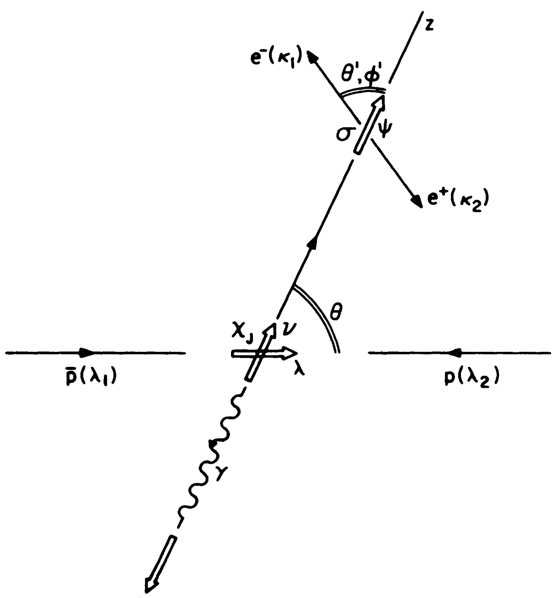

FIG. 1. Symbolic sketch of $\bar{p}(\lambda_1)p(\lambda_2)\to\chi_{J\nu}\to\psi_\sigma+\gamma_\mu\to e^-(\kappa_1)e^+(\kappa_2)+\gamma_\mu$ showing particle helicities. Note that $\nu=\sigma-\mu$.

If $|p,\theta,\phi;\lambda_1\lambda_2\rangle$ represents a two-particle helicity state in the zero-momentum (c.m.) frame, where p is the magnitude of either particle's momentum, and the angles (θ,ϕ) represent the direction of the first particle's momentum and λ_1, λ_2 the helicities of the two particles, then, using the notation of Jacob and Wick [5] and Martin and Spearman [6], we can write an expansion of the two-particle helicity state in terms of angular momentum states as

$$|p,\theta,\phi;\lambda_1\lambda_2\rangle = \sum_{J,M}\left[\frac{2J+1}{4\pi}\right]^{1/2} D^J_{M\lambda}(\phi,\theta,-\phi)|pJM;\lambda_1\lambda_2\rangle , \quad (1)$$

where ϕ, θ, and $-\phi$, are the three Euler angles. The ordering of the indices on the $(2J+1)$-dimensional rotation matrices D^J should be especially noted.

We will work in the χ_J rest frame with the Z axis taken to be in the direction of ψ. The $\bar{p}$ direction is in the X-Z plane, making an angle θ with the Z axis. Using Eq. (1), the amplitude for the process $\bar{p}(\lambda_1)p(\lambda_2)\to\chi_{J\nu}$ can be written as

$$\langle J\nu|B|\theta 0,\lambda_1\lambda_2\rangle = \left[\frac{2J+1}{4\pi}\right]^{1/2} B^J_{\lambda_1\lambda_2} d^J_{\nu\lambda}(\theta) , \quad (2a)$$

where

$$\lambda=\lambda_1-\lambda_2 , \quad (2b)$$

$$d^J_{\nu\lambda}(\theta)=D^J_{\nu\lambda}(0,\theta,0) , \quad (2c)$$

and B is a transition operator. The symbol $B^J_{\lambda_1\lambda_1}$ represents the partial-wave amplitude. The amplitude for the process $\chi_{J\nu}\to\psi_\sigma+\gamma_\mu$ with the quarkonium ψ and the photon γ moving along the $+Z$ and $-Z$ directions, respectively, can be written as

$$\langle 00,\sigma\mu|A|J\nu\rangle=\left[\frac{2J+1}{4\pi}\right]^{1/2}A'^J_{\sigma\mu}D^{J*}_{\nu\nu'}(0,0,0)\ , \tag{3a}$$

$$\nu'=\sigma-\mu\ . \tag{3b}$$

In the ψ rest frame, with the direction of the final electron's momentum specified by (θ',ϕ'), the amplitude for the process $\psi_\sigma\to e^-(\kappa_1)+e^+(\kappa_2)$ becomes

$$\langle\theta'\phi';\kappa_1\kappa_2|C|1\sigma\rangle=\left[\frac{3}{4\pi}\right]^{1/2}C_{\kappa_1\kappa_2}D^{1*}_{\sigma\kappa}(\phi',\theta',-\phi')\ , \tag{4a}$$

where

$$\kappa=\kappa_1-\kappa_2\ . \tag{4b}$$

If the e^+e^- system is produced by the process $q\bar{q}\to\gamma\to e^-e^+$, $C_{++}=C_{--}$ is of the order of $m/E\simeq3.3\times10^{-4}$ compared to C_{+-} or C_{-+} and can be safely neglected.

The amplitude $T_{\lambda_1\lambda_2\kappa_1\kappa_2\mu}$ for the process to go from the initial state of $\bar{p}(\lambda_1)p(\lambda_2)$ to the final state of $e^-(\kappa_1)e^+(\kappa_2)+\gamma_\mu$ through all possible helicity states ν of χ_J and σ of ψ is a sum of products of amplitudes of Eqs. (2a), (3a), and (4a). That is,

$$T^J_{\lambda_1\lambda_2\kappa_1\kappa_2\mu}=\left[\frac{3(2J+1)^2}{(4\pi)^3}\right]^{1/2}\sum_{\nu=-J}^{+J}\sum_{\sigma=\pm1,0}B^J_{\lambda_1\lambda_2}A'^J_{\sigma\mu}C_{\kappa_1\kappa_2}D^{1*}_{\sigma\kappa}(\phi',\theta',-\phi)D^{J*}_{\nu,\sigma-\mu}(0,0,0)d^J_{\nu\lambda}(\theta)\ . \tag{5}$$

Since

$$D^J_{\nu\nu'}(0,0,0)=\delta_{\nu\nu'}\ , \tag{6}$$

Eq. (5) becomes

$$T^J_{\lambda_1\lambda_2\kappa_1\kappa_2\mu}=\left[\frac{3(2J+1)^2}{(4\pi)^3}\right]^{1/2}B^J_{\lambda_1\lambda_2}C_{\kappa_1\kappa_2}\sum_{\nu(\mu)}A^J_{\nu\mu}D^{1*}_{\nu+\mu,\kappa}(\phi',\theta',-\phi')d^J_{\nu\lambda}(\theta)\ , \tag{7}$$

where ν takes the values 0 to $+J$ for $\mu=-1$ and $-J$ to 0 for $\mu=+1$. In Eq. (7) we have made the replacement $A^J_{\nu\mu}$ for $A'^J_{\nu+\mu,\mu}$. The probability for the cascade process when the initial $\bar{p}$ and p are unpolarized and the final polarizations of γ, e^-, and e^+ are not observed is obtained by squaring the absolute magnitude of Eq. (7) and summing over the final helicity indices κ_1, κ_2, and μ and averaging over the initial helicity indices λ_1 and λ_2. This probability will give the unnormalized angular distribution function to be

$$W(\theta;\theta',\phi')=\frac{1}{4}\frac{3(2J+1)^2}{(4\pi)^3}\sum_{\lambda_1\lambda_2\kappa_1\kappa_2\mu}|B^J_{\lambda_1\lambda_2}|^2|C_{\kappa_1\kappa_2}|^2$$
$$\times\sum_{\nu(\mu),\nu'(\mu)}A^{J*}_{\nu'\mu}A^J_{\nu\mu}d^J_{\nu'\lambda}(\theta)d^J_{\nu\lambda}(\theta)D^1_{\nu'+\mu,\kappa}(\phi',\theta',-\phi')D^{1*}_{\nu+\mu,\kappa}(\phi',\theta',-\phi')\ , \tag{8}$$

where the photon-polarization index μ can take only two values, $+1$ or -1.

Now we make use of the symmetry properties of the helicity amplitudes for two-body processes. By charge-conjugation invariance the amplitudes should satisfy [6]

$$M^J_{\lambda_1\lambda_2}=\eta_c(-1)^JM^J_{\lambda_2\lambda_1}\ , \tag{9}$$

where M is B or C, and η_c the charge-conjugation parity of the state under consideration. For quarkonium,

$$\eta_c=(-1)^{L+S}\ . \tag{10}$$

By parity invariance [6] for the amplitudes A, B, and C,

$$M^J_{\lambda_1\lambda_2}=\eta_p(-1)^JM^J_{-\lambda_1-\lambda_2}\ , \tag{11}$$

where M is A, B, or C, and η_p is the parity of the state in question. For quarkonium,

$$\eta_p=(-1)^{L+1}\ . \tag{12}$$

Using Eqs. (9) and (11),

$$\begin{aligned}|B^J_{+-}|^2&=|B^J_{-+}|^2=B_1^2\ ,\\ |B^J_{++}|^2&=|B^J_{--}|^2=\tfrac{1}{2}B_0^2\ ,\\ |C_{++}|^2&=|C_{--}|^2=\tfrac{1}{2}C_0^2\ ,\\ |C_{+-}|^2&=|C_{-+}|^2=C_1^2\ .\end{aligned} \tag{13}$$

By parity invariance [Eq. (11)],

$$A^J_{\nu,-1} = +A^J_{-\nu,1}(J=2,0)\ ,$$
$$A^J_{\nu,-1} = -A^J_{-\nu,1}(J=1)\ , \tag{14}$$

so we drop the index μ, the subscript giving the γ helicity from the amplitude A from now on. We also choose the normalizations

$$\sum_{\lambda_1\lambda_2} |B^J_{\lambda_1\lambda_2}|^2 = 2B_1^2 + B_0^2 = 1\ ,$$
$$\sum_{\kappa_1\kappa_2} |C_{\kappa_1\kappa_2}|^2 = 2C_1^2 + C_0^2 = 1 \simeq 2C_1^2\ , \tag{15}$$
$$\sum_{\nu=0}^{J} |A^J_\nu|^2 = 1\ .$$

Using Eqs. (13)–(15) in Eq. (8), we get

$$W(\theta,\theta'\phi') = \frac{3(2J+1)^2}{8(4\pi)^3} \sum_{\lambda=\pm1,0} |B^J_\lambda|^2 \sum_{\mu=\pm1} \sum_{\nu(\mu),\nu'(\mu)} A^{J*}_{\nu'} A^J_\nu d^J_{\nu'\lambda}(\theta) d^J_{\nu\lambda}(\theta) \sum_{\kappa=\pm1} D^1_{\nu'+\mu,\kappa}(\phi',\theta',-\phi') D^{1*}_{\nu+\mu,\kappa}(\phi',\theta',-\phi')\ , \tag{16a}$$

where

$$\nu(\mu) = -J \text{ to } 0 \quad \text{for } \mu = +1\ ,$$
$$\nu(\mu) = 0 \text{ to } +J \quad \text{for } \mu = -1\ . \tag{16b}$$

The ordering of the indices on the d^J functions in Eq. (16a) is opposite to that of Ref. [3]. Our ordering is that of Eq. (1), which is the same as that of Jacob and Wick [5], Martin and Spearman [6], and Jackson [8]. In the D^1 functions occurring in the $\psi \to e^- e^+$ process, however, Ref. [3] followed our convention. This is an inconsistent procedure, as explained below. First it should be noted [9] that

$$d^J_{mm'}(\theta) = (-1)^{m-m'} d^J_{m'm}(\theta) = d^J_{m'm}(-\theta) \tag{17}$$

and

$$D^J_{mm'}(\phi,\theta,-\phi) = (-1)^{m'-m} D^{J*}_{m'm}(\phi,\theta,-\phi)\ . \tag{18}$$

Because of Eq. (17), whenever $(\nu'-\nu)$ is an odd integer, we and the authors of [3] get opposite signs in the product function $d^{J*}_{\nu'\lambda} d^J_{\nu\lambda}$ of Eq. (16a). However, we get the same sign in the product function $D^1_{\nu'+\mu,\kappa} D^{1*}_{\nu+\mu,\kappa}$. Because of this situation, in terms of Eq. (16a) involving $A^{J*}_{\nu'} A^J_\nu$ where $(\nu'-\nu)$ is an odd integer, our signs are opposite those of Ref. [3]. If the authors of Ref. [3] followed our convention or the opposite convention consistently, we would have obtained the same sign for all the terms.

The normalized angular distribution function $\hat{W}(\theta;\theta',\phi')$ is

$$\hat{W}(\theta;\theta',\phi') = \frac{8\pi}{2J+1} W(\theta;\theta',\phi')\ . \tag{19}$$

The normalization is such that the integral $\hat{W}(\theta;\theta',\phi')$ over all angles is one. When the D^1 and the d^J functions [9] in Eq. (16a) are expressed in terms of the trigonometric functions, the normalized angular distribution functions for various values of J ($J=2,1,0$) take the following forms.

(1) $\bar{p}p \to \chi_2 \to \psi + \gamma \to (e^+e^-) + \gamma$. From Eq. (18), the normalized χ_2 distribution function is

$$\frac{64\pi^2}{15} \hat{W}(\theta;\theta',\phi') = K_1 + K_2\cos^2\theta + K_3\cos^4\theta + (K_4 + K_5\cos^2\theta + K_6\cos^4\theta)\cos^2\theta'$$
$$+ (K_7 + K_8\cos^2\theta + K_9\cos^4\theta)\sin^2\theta'\cos(2\phi') - (K_{10} + K_{11}\cos^2\theta)\sin(2\theta)\sin(2\theta')\cos\phi'\ . \tag{20}$$

In the last term on the right-hand side, Ref. [3] had a positive sign whereas we have a negative sign. This happens precisely because K_{10} and K_{11} when expressed in terms of the amplitudes A_0, A_1, and A_2 have only terms involving the products A_0A_1 and A_1A_2 so that $(\nu-\nu')$ is an odd integer in this case. The expressions for all the other K_i's involve products $A_\nu A_{\nu'}$ where $(\nu-\nu')$ is an even integer. We should note that our expressions for the K_i's in terms of A_0, A_1 and A_2 are exactly the same as those given in Ref. [3] (except for K_5) if we assume that all helicity amplitudes are real. In general, they are complex. In that case, every $A_\nu A_{\nu'}$ term in their expressions for the K_i's should be replaced by $\mathrm{Re}(A_\nu A_{\nu'*})$. Our final difference is in K_5. We get

$$\tfrac{4}{3}K_5 = -2A_0^2 - 4A_1^2 - A_2^2 + R(4A_0^2 + 6A_1^2 + A_2^2)\ , \tag{21}$$

where

$$R = \frac{2B_1^2}{2B_1^2 + B_0^2} = 2B_1^2\ . \tag{22}$$

Reference [3] had $2A_0^2$ instead of $4A_0^2$ multiplying the R. This is inconsistent with their own integrated angular distribution functions. We are in agreement with Ref. [3] on the partially integrated distribution functions. This happens because the term with the opposite signs in Eq. (20), when integrated over ϕ' or θ', gives zero.

(2) $\bar{p}p \to \chi_1 \to \psi + \gamma \to (e^+e^-) + \gamma$. In this case, where the intermediate state is χ_1, charge-conjugation invariance of Eq. (9) leads to

$$B^J_{\lambda_1\lambda_2} = (-1)^J B^J_{\lambda_2\lambda_1} . \tag{23}$$

For $J=1$, this leads to

$$B^1_{++} = -B^1_{++} = 0 \tag{24}$$

and

$$B^1_{--} = -B^1_{--} = 0 .$$

So,

$$B_0^2 = 2|B_{++}|^2 = 2|B_{--}|^2 = 0 , \tag{25}$$

which will imply

$$R = \frac{2B_1^2}{2B_1^2 + B_0^2} = 1 . \tag{26}$$

The normalized angular distribution function is

$$\frac{64\pi^2}{9}\hat{W}(\theta;\theta',\phi') = K_1 + K_2\cos^2\theta + (K_3 + K_4\cos^2\theta)\cos^2\theta' - K_5\sin(2\theta)\sin(2\theta')\cos\phi' . \tag{27}$$

In Eq. (27) the sign of the last term involving K_5 is opposite to that of Ref. [3] since K_5 involves only A_1A_0 so that $(\nu - \nu')$ is odd. The expressions for K_i's are exactly the same as in Ref. [3], but since $R=1$ in this case, the constants K_i take the simple forms

$$\begin{aligned} K_1 &= \tfrac{1}{2} , \\ K_2 &= \tfrac{1}{2}(2|A_1|^2 - 1) , \\ K_3 &= \tfrac{1}{2} - |A_1|^2 , \\ K_4 &= -\tfrac{1}{2} , \\ K_5 &= \tfrac{1}{4}\mathrm{Re}\, A_1^* A_0 . \end{aligned} \tag{28}$$

The expressions for the partially integrated angular distribution functions will remain the same as in Eqs. (17) of Ref. [3].

(3) $\bar{p}p \to \chi_0 \to \psi\gamma \to (e^+e^-)\gamma$. When the intermediate state is χ_0, the distribution function is

$$\frac{64\pi^2}{3}\hat{W}(\theta;\theta',\phi') = 1 + \cos^2\theta' , \tag{29}$$

since $R=0$ in this case [3].

The relationship between the helicity amplitudes and the multipole amplitudes will be given by the same orthogonal transformation as in Ref. [3]: namely,

$$A_\nu = \sum_k a_k \left[\frac{2k+1}{2J+1}\right]^{1/2} \langle kl;1,\nu-1|J\nu\rangle \tag{30}$$

or Eqs. (13) and (14) of Olsson and Suchyta [3].

In Eqs. (20), (27), and (29), the angles (θ',ϕ') give the direction of e^- in the ψ rest frame and θ gives that of $\bar{p}$ in the χ_J rest frame. There is no Lorentz frame where χ_J and ψ are both at rest. In the χ_J rest frame or the $\bar{p}p$ c.m. frame, ψ is moving with a velocity [7]

$$v = \beta \simeq 0.15 . \tag{31}$$

If the direction of e^- in the χ_J rest frame is given by (θ'',ϕ''), these angles are related to the angles (θ',ϕ') by the relations (to first order in β)

$$\begin{aligned} \cos\theta' &\simeq \cos\theta'' - \beta\sin^2\theta'' , \\ \sin\theta' &\simeq \sin\theta'' + \beta\sin\theta''\cos\theta'' , \\ \phi' &= \phi'' . \end{aligned} \tag{32}$$

Equations (32) must be used in Eqs. (20) and (27) to reexpress them in terms of (θ'',ϕ'') before applying them to determine the helicity and the multipole amplitudes from the experimentally measured angular distribution.

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