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Phase transition of the sine-Gordon theory at finite temperature

W. H. Kye, S. I. Hong, and J. K. Kim
Department of Physics, Korea Advanced Institute of Science and Technology, 373-1, Kusung-dong, Yusung-ku, Taejeon, Korea
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We investigate the critical behavior of the sine-Gordon theory at the one-loop level. We observe that the theory has a critical temperature β_c and restores the symmetry above β_c.

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The effective potential method for quantum field theory has been investigated by many authors. Jona-Lasinio pointed out that the method can be used for the study of the spontaneous symmetry-breaking behavior of the theory at the quantum level [1]. The diagrammatic evaluation method was developed by Jackiw [2].

Kirzhnits and Linde suggested that spontaneously broken symmetry can be restored above the critical temperature [3]. By the above suggestion, the study of the phase transition at finite temperature in quantum field theory becomes the matter of interest. The main motivations of this study are given by their consequences on cosmology and elementary particle physics [4]. Dolan and Jackiw pointed out that the critical temperature of the theory, which has local gauge symmetry, depends on the chosen gauge [5]. In a recent paper [6], the same facts have been reported in the Higgs model and the method by which one can find a gauge-invariant effective potential is suggested.

In the present Brief Report, we investigate the critical behavior of the sine-Gordon theory making use of the diagrammatic evaluation method. We evaluate the zero-temperature effective potential at the one-loop level and find the critical temperature β_c by the critical condition.

Consider the sine-Gordon system. Its action is given by

$$I\{\phi\}=\int dx^2\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{\lambda+\delta\lambda}{m}[1-\cos(m\phi)]\right], \quad (1)$$

where λ and m are constants, and $\delta\lambda$ is the counterterm ($\delta\lambda=\hbar\delta\lambda_1+\hbar^2\delta\lambda_2+\cdots$).

The potential has an infinitely degenerate vacuum at $\pm 2n\pi/m$, where n is an integer, and hence the system is located at the classically broken phase. To calculate the effective potential, we shift the field, following the suggested procedure [5], $\phi(x)\to\phi(x)+\hat{\phi}$, where the $\hat{\phi}$ is the space-time-independent constant. The shifted Lagrangian is given by

$$\hat{\mathcal{L}}=\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi-\tfrac{1}{2}m\lambda\cos(m\hat{\phi})+\hat{\mathcal{L}}_I\{\hat{\phi};\phi\}, \quad (2)$$

where the field-independent constant and linear terms are eliminated and $\hat{\mathcal{L}}_I$ is of higher order than the quadratic field term. The one-loop calculation can be performed exactly [5]:

$$V_1^0(\hat{\phi})=-\frac{1}{2}i\hbar\int\frac{dk^2}{(2\pi)^2}\ln\det(i\mathcal{D}^{-1}\{\hat{\phi};k\}), \quad (3)$$

where the superscript zero means zero temperature, the subscript one means a one-loop contribution, and $i\mathcal{D}^{-1}\{\hat{\phi};k\}=k^2-m\lambda\cos(m\hat{\phi})$. The integration has a logarithmic divergence. To regulate the divergence, we perform a "Wick rotation" to Euclidean space and introduce the spherical cutoff momentum Λ:

$$V_1^0\xrightarrow{\text{Wick}}\frac{\hbar}{8\pi}\int_0^{\Lambda^2}dk_E^2\ln(-k_E^2-\mu^2),$$
$$=\frac{\hbar}{8\pi}[\Lambda^2\ln(1+\mu^2/\Lambda^2)+\mu^2\ln(1+\Lambda^2/\mu^2)], \quad (4)$$

where $\mu^2=m(\lambda+\delta\lambda)\cos(m\hat{\phi})$ and a constant of order Λ^2, which is independent of all parameters, has been dropped. Expand the above results in terms of the $\Lambda^2\to\infty$ limit and then, by requiring the finiteness of the potential, we can perform renormalization. After fitting the counterterm, we obtain the renormalized zero-temperature effective potential

$$V_0^0+V_1^0=-\frac{\lambda}{m}[1-\cos(m\hat{\phi})]$$
$$-\hbar m\lambda\cos(m\hat{\phi})\ln[|m\cos(m\hat{\phi})|], \quad (5)$$

where $\delta\lambda_1=-m^2\ln(\Lambda^2/\lambda)$, we obtain the first-order counterterm. We can still see the vacuum degeneracy at $\hat{\phi}=\pm 2\pi n/m$. This represents the fact that the broken vacuum symmetry is not recovered by the one-loop correction.

The one-loop thermal contribution of the effective potential is given by Ref. [6] as

$$V_1^\beta=\frac{\hbar}{2\pi\beta^2}\int_0^\infty dx\ln[1-\exp(-\sqrt{x^2+\beta^2\mu^2})]. \quad (6)$$

This integration cannot be evaluated exactly. So we will consider the case of the high-temperature weak-coupling limit such that $\beta\sqrt{\lambda}\ll 1$. To find our critical point [4,5], we find the second derivative of the potential [7]:

$$\left[\frac{\partial^2 V_1^\beta}{\partial\hat{\phi}^2}\right]_{\hat{\phi}=0}=-\frac{m^3\lambda\hbar}{4\pi}\left[\frac{\pi}{2\beta\sqrt{\lambda m}}+\frac{1}{2}\ln\left[\frac{\beta\sqrt{\lambda m}}{4\pi}\right]+\frac{1}{2}\gamma+O((\beta\sqrt{\lambda})^2)\right], \quad (7)$$

where $\gamma(=0.577\cdots)$ is the Euler constant. By using the critical temperature condition, we can determine the critical temperature of the theory,

$$-\frac{M_R^2}{2}=\left[\frac{\partial^2 V_1^{\beta_c}}{\partial\hat{\phi}^2}\right]_{\hat{\phi}=0}, \tag{8}$$

where M_R^2 is the zero-temperature renormalized mass of the system, which is defined by

$$\left[\frac{\partial^2(V_0^0+V_1^0)}{\partial\hat{\phi}^2}\right]_{\hat{\phi}=0}=\lambda m+\hbar m^3\lambda[1+\ln(m)] \equiv M_R^2 . \tag{9}$$

If we retain only the sufficiently dominant term at the $\beta\sqrt{\lambda}\ll 1$ limit in Eq. (7),

$$\frac{1}{\beta_c}=\frac{2\sqrt{\lambda m}}{\pi}\left[\frac{2\pi M_R^2}{m^3\lambda\hbar}-\frac{\gamma}{2}-\frac{1}{2}\ln\left[\frac{\sqrt{m}}{4\pi}\right]\right]. \tag{10}$$

This is the critical temperature at which the sine-Gordon system restores the vacuum reflection symmetry $\hat{\phi}\to-\hat{\phi}$.

It is important to examine the existence of the instanton solution in order to see that our perturbative treatment is valid. By the virial theorem, there exists no soliton solution in the (2+1)-dimensional sine-Gordon theory. Hence there is not any instanton solution in (1+1)-dimensional sine-Gordon theory. There is no tunneling between the degenerated vacuum, thus symmetry will be spontaneously broken at the classical level. It is meaningful to expand perturbatively around the vacuum.

At zero temperature, the one-loop correction does not restore the spontaneously broken phase. Even though the classical potential is deformed by the one-loop correction, the vacuum degeneracy, at $\hat{\phi}=\pm 2\pi n/m$, was not affected. When we introduce the thermal bath, the new contribution given by Eq. (6) appears. This is the so-called temperature-dependent mass. We observe the fact that the system recovers the vacuum reflection symmetry above β_c.

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