Effective potential at finite temperature in the standard model

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There has been much recent interest in the nature of the electroweak phase transition. This information is of importance in the context of the sphaleron models that have recently been proposed to explain the observed net baryon number in the Universe. The presence of a term that is cubic in the Higgs condensate in the one-loop effective potential appears to indicate a first-order phase transition. However, the infrared singularities inherent in massless models produce cubic terms that are of the same order in the coupling. In this paper, we include these terms and show that the standard model has a first-order phase transition.

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I. INTRODUCTION

It was first observed by Kirzhnits and Linde [1] that spontaneously broken symmetries are usually restored at high temperature. Such phase transitions may be relevant for the study of high-energy particle collisions [2], neutron stars [3], and the early Universe [4]. In particular, there has been much recent interest in the nature of the electroweak phase transition. This information is important in the context of attempts to explain the observed net baryon number in the Universe in terms of baryon-number violation in the electroweak theory [5], and cosmological models of baryogenesis [6].

We consider models in which the symmetry is spontaneously broken by a negative-mass-squared term in the Lagrangian. In most simple models, the symmetry is restored at sufficiently high temperatures [7]. The nature of the symmetry-restoring phase transition is determined by the behavior of the effective potential. The fact that the symmetry is restored at high temperatures is a result of the $T^2m^2(v)$ term that occurs in the one-loop effective potential [$m^2(v)$ is a mass-squared parameter equal to the sum of the bare mass squared and a term proportional to the square of the expectation value of some classical scalar field v times a coupling constant λ]. This term is the leading-order contribution from the thermal fluctuations of the field. As the temperature is increased, the contribution from thermal fluctuations will eventually dominate the negative-mass-squared term in the tree potential and symmetry will be restored.

These conclusions depend on the validity of the approximations used in the calculation of the effective potential. It was shown by Dolan and Jackiw that there are imaginary terms in the scalar model effective potential at one-loop [7(a)]. These terms are proportional to $m^3(v)T$ and are imaginary when the mass squared is negative. Weinberg and Wu have shown that imaginary terms in the effective potential indicate a physical instability and that the imaginary part of the effective potential can be interpreted as a decay rate per unit volume [8]. Takahashi has suggested that the appearance of imaginary terms in the one-loop effective potential indicates the breakdown of the semiclassical loop expansion through infrared singularities. He has shown that, for the simple scalar model, the imaginary terms, which are proportional to $m^3(v)T$, cancel when the dominant infrared contributions from higher-order diagrams are included [9]. Effectively the term $\sim m^3(v)T$ is replaced by a term $\sim[m^2(v)+c\lambda T^2]^{3/2}T$, where c is a constant of order one, which is real for T large enough. We note that in the standard model some of the fields have zero bare mass and non-negative mass squared for all v. These fields contribute terms $\sim m^3(v)T$ to the effective potential, which are always real; for these fields, the cancellation of the terms that are cubic in the mass does not occur and is not related to the issue of the complexity of the effective potential [10].

There is an additional reason to include the infrared contributions from higher-order diagrams. These diagrams contribute terms that are cubic in v and of the same order in the coupling as the term $\sim m^3(v)T$ from the one-loop graph. Since a cubic term in the effective potential could generate a first-order phase transition, it is crucial that we include all the leading-order terms, which are cubic in the field.

In this paper we calculate the effective potential for the standard model consistently within the loop expansion. We work to order $\lambda^{3/2}$, g^3, and g'^3, where λ is the Higgs coupling and g and g' are the SU(2) and U(1) couplings, respectively. The paper is organized as follows. In Sec. II we discuss the simple scalar theory. In Sec. III we present our calculation of the effective potential in the standard model, and in Sec. IV we discuss our results.

II. THE SCALAR THEORY

The one-loop effective potential of the simple scalar theory at finite temperature has a leading-order term $\sim\lambda v^2T^2$ and a next-to-leading-order term $\sim\lambda^{3/2}v^3T$, where v is the expectation value of the scalar field. This v^3 term could give rise to a first-order phase transition. However, the ring diagram contributions are also of order $\lambda^{3/2}$, and they exactly cancel the cubic term from the one-loop graph in the scalar model.

We note that the "ring diagrams" will be understood to include the dominant part of the two-loop graph. The two-loop graph is λ times the square of the one-loop graph, which is $\sim[T^2+m(v)T+\cdots]$. The first term in the one-loop result is the leading-order term and the second term comes entirely from the zero-frequency term in the Matsubara sum. Squaring this result, the two-loop graph is $\sim\lambda T^4[1+m(v)/T+\cdots]$. The v-independent T^4 term is proportional to the square of the leading-order contribution to the one-loop polarization tensor. We do not consider this term since it does not depend on v. The leading-order v-dependent term is $\sim\lambda^{3/2}T^3 v$ and comes from the graph with one loop given by the leading-order contribution to the one-loop polarization tensor, and the other loop calculated at zero frequency. This contribution is included in what we will call the ring diagrams. We drop the next-to-leading-order term from the two-loop graph, which is $\sim\lambda^2 v^2 T^2 \ln(\lambda v^2/T^2)$.

Effectively, the inclusion of the ring diagrams replaces the $\lambda^{3/2}v^3 T$ term from the one-loop graph with a term of the form $\lambda^{3/2}[m^2(v)+c\lambda T^2]^{3/2}T$, where c is some constant of order one. As discussed in the Introduction, this term is real for T large enough. It is not obvious that a term of this form gives a first-order phase transition. It has been shown that the first-order phase transition does occur in the scalar theory; it is the purpose of this paper to see if this is the case in the standard model. In this section we study the calculation of the effective potential for the scalar model in order to understand in detail how the cancellation of the cubic term occurs. We discuss how to calculate consistently within the loop expansion. We obtain the ring diagrams as first-order corrections to the mean-field result at one loop. Shifting both the one-point and two-point functions and extremizing the effective potential to find the equilibrium values is equivalent to performing the resummation of infrared-divergent graphs. The result is the usual diagrammatic expansion for the effective potential with the bare masses shifted by the infrared limit of the polarization tensor.

We introduce the following notation. Greek indices indicate Lorentz four-vectors and vectors with indices $i,j,\ldots$ are three-space vectors. The product of three-vectors is denoted $\mathbf{k}^2$. In Minkowski space, the product of two four-vectors is written k^2. In Euclidean space, the product of two four-vectors is indicated by k_E^2 at zero temperature, and by $\omega_n^2+\mathbf{k}^2$ at finite temperature.

The Lagrangian is

$$\mathcal{L}=\tfrac{1}{2}(\partial_\mu\phi)^2+\tfrac{1}{2}c^2\phi^2-\tfrac{1}{4}\lambda\phi^4 \ . \tag{1}$$

This Lagrangian is symmetric under the transformation $\phi\to-\phi$. However, the vacuum state is not symmetric under this transformation; the vacuum has a degeneracy that leads to what is called spontaneous symmetry breaking. We select a vacuum about which we can do perturbative calculations by shifting the field about the classical value v: we write $\phi=v+\chi$. After performing this shift, the mass of the field χ is $m^2(v)=3\lambda v^2-c^2$, and the new Lagrangian does not have the original symmetry. For constant v, the thermodynamic potential gives the potential energy of the field v and is called the effective potential. We will calculate the effective potential $V(v)$ and extremize with respect to v to find the equilibrium value $\langle v\rangle$. If $\langle v\rangle\to 0$ at some finite temperature T_c, then the system has a phase transition between the broken-symmetry phase ($\langle v\rangle\neq 0$) and full symmetry phase ($\langle v\rangle=0$). We want to determine the order of this phase transition.

We use the loop expansion. To lowest order we have the tree potential

$$V(v)_{\text{tree}}=-\tfrac{1}{2}c^2v^2+\tfrac{1}{4}\lambda v^4 \ . \tag{2}$$

Extremizing at this level of approximation gives the classical minimum $\langle v\rangle_0=c/\lambda^{1/2}$, and the classical mass $m^2(\langle v\rangle_0)=2\lambda\langle v\rangle_0^2=2c^2$.

To next order, we consider only terms in the action which are quadratic in the fluctuations χ about the classical field v. This approximation gives the one-loop contribution, which is given by the familiar expression [11] (see Fig. 1)

$$V_1(v)=\frac{1}{2(2\pi)^4}\int d^4k \ln[k_E^2+m^2(v)] \ . \tag{3}$$

At finite temperature, we rewrite the k_4 integral as a sum over Matsubara frequencies:

$$\int\frac{dk_4}{2\pi}f(k_4)\to T\sum_n f(k_4=i\omega_n), \quad \omega_n=2\pi nT \ ,$$

and replace the frequency sum by a contour integral in the usual way [12]. We can separately evaluate the zero-temperature and the finite-temperature contributions.

The zero-temperature part is given by

$$V_1^{(0)}(v)=\frac{1}{2(2\pi)^3}\int d^3\mathbf{k}[\mathbf{k}^2+m^2(v)]^{1/2} \ , \tag{4}$$

and represents the shift in the vacuum energy from zero-point oscillations. The integral is divergent and must be cut off at some Λ. The result can be renormalized by introducing the counterterms

$$\mathcal{L}_{\text{ct}}=\frac{Av^2}{2}+\frac{Bv^4}{4}+C \ . \tag{5}$$

C is a constant that can be used to cancel the v-independent part of the vacuum energy. We determine A and B by requiring that the position of the minimum and the mass remain at their classical values:

$$\left[\frac{dV^{(0)}(v)}{dv}\right]_{v=\langle v\rangle_0}=0, \quad \left[\frac{d^2V^{(0)}(v)}{dv^2}\right]_{v=\langle v\rangle_0}=2c^2 \ ,$$

where $V^{(0)}(v)=V(v)_{\text{tree}}+V_1^{(0)}(v)$. The result is

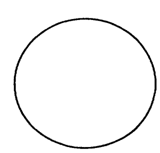

FIG. 1. One-loop contribution to the effective potential for the scalar model.

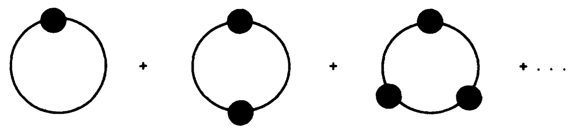

FIG. 2. Ring diagrams.

$$V^{(0)}(v)=-\frac{c^2v^2}{2}+\frac{\lambda v^4}{4}+\frac{1}{64\pi^2}m^4(v)\ln\frac{m^2(v)}{2c^2}+\frac{21\lambda c^2v^2}{64\pi^2}-\frac{27\lambda^2v^4}{128\pi^2}\ . \tag{6}$$

The finite-temperature part of Eq. (3) is

$$V_1^{(T)}(v)=\frac{T}{2\pi^2}\int dk\, k^2\ln(1-\exp\{-\beta[k^2+m^2(v)]^{1/2}\})\ , \tag{7}$$

where $\beta=1/T$. In the limit that $m(v)/T$ is small this can be expanded as

$$V_1^{(T)}(v)=T^4\left[-\frac{\pi^2}{90}+\frac{m^2(v)}{24T^2}-\frac{m^2(v)}{12\pi T^3}-\frac{m^4(v)}{32\pi^2T^4}\ln\frac{m(v)}{4\pi T}+O\left[\frac{m^4(V)}{T^4}\right]\right]\ . \tag{8}$$

We note that the term proportional to $m^4(v)\ln[m(v)]$ cancels between the $V_1^{(0)}(v)$ and $V_1^{(T)}(v)$ contributions. This cancellation also occurs in the standard model.

Now we will show that the next-higher-order correction is not the two-loop term but the ring diagram contribution (Fig. 2), which is of order $\lambda^{3/2}$. It is well known that the ring diagrams give such a contribution for massless fields, and therefore it seems likely that an order-$\lambda^{3/2}$ term will contribute in the limit of small $m(v)/T$. Physically, the ring diagrams give contributions from long-distance effects. In this problem they can be obtained by going beyond the mean-field approximation at the one-loop level.

Previously, we shifted the one-point function $\phi(x)=v+\chi(x)$, where $\phi(x)$ and $\chi(x)$ are quantum fields and v is a c number. If we stop at this point, we can extremize the effective potential with respect to v and obtain the mean-field value of $\langle v\rangle$:

$$\left[\frac{\delta V(v)}{\delta v}\right]_{v=\langle v\rangle}=0\ .$$

To go beyond the mean-field level, we need to consider the effective potential as a function of both v and the two-point function $\pi(\omega_n,\mathbf{p})$, and extremize with respect to v and $\pi(\omega_n,\mathbf{p})$. The result will be the reexpression of the perturbation theory in terms of the full propagator [13].

At the one-loop level we obtain

$$\begin{aligned}V(v)&=V(v)_{\rm tree}+\tfrac{1}{2}T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln[\beta^2D^{-1}(\omega_n,\mathbf{p},\langle\pi(\omega_n,\mathbf{p})\rangle)]\\&=V(v)_{\rm tree}+\tfrac{1}{2}T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln\{\beta^2[\omega_n^2+\mathbf{p}^2+m^2(v)+\langle\pi(\omega_n,\mathbf{p})\rangle]\}\ .\end{aligned} \tag{9}$$

$\langle\pi(\omega_n,\mathbf{p})\rangle$ is to be obtained self-consistently as the choice of $\pi(\omega_n,\mathbf{p})$ that extremizes the effective potential. This extremization condition is just the Dyson equation, which is shown in Fig. 3 to one loop. From now on, we suppress the bra and ket and write the solution to the Dyson equation as $\pi(\omega_n,\mathbf{p})$.

We will obtain $\pi(\omega_n,\mathbf{p})$ self-consistently within perturbation theory. For a massless theory, the dominant contribution at order λ^N to the polarization tensor comes from the infrared-divergent diagram shown in Fig. 4. The small bubbles are one-loop polarization tensors in the infrared limit:

$$\pi^{(1)}(\omega_n,\mathbf{k})=\pi^{(1)}(0)=3\lambda T\sum_n\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{\omega_n^2+\mathbf{q}^2}=\lambda\frac{T^2}{4}\ .$$

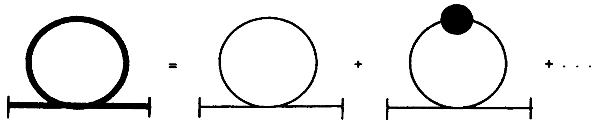

FIG. 3. One-loop Dyson equation.

In this problem we take the infrared limit by setting the zeroth component of the external momentum to zero and taking the limit that the spatial components approach zero: $\pi^{(1)}(p_0=0,p\to 0)\equiv\pi^{(1)}(0)$ (note that in the simple scalar model the infrared limit is trivial). The infrared limit of the one-loop polarization tensor is momentum independent, and we can explicitly sum over N. The result is an expression for the polarization tensor $\pi(\omega_n,\mathbf{p})$ as a function of the propagator which has an effective-mass-squared $\pi^{(1)}(0)$. Finally, the dominant contribution to the ring diagrams (Fig. 6) comes from the infrared limit of this result for $\pi(\omega_n,\mathbf{p})$:

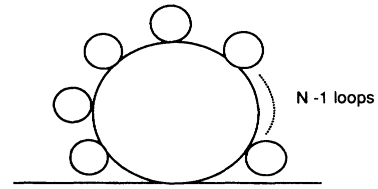

FIG. 4. Contribution to the polarization tensor of order λ^N.

$$\pi(\omega_n,\mathbf{p})=\pi(0)=3\lambda T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\omega_n^2+\mathbf{p}^2+\pi^{(1)}(0)}\ .$$

In our problem, where the field is not massless, we expect to get a nonzero contribution from the ring diagrams in the ultrarelativistic limit $m(v)/T\ll 1$. We will proceed by analogy with the massless case and obtain the polarization tensor as a function of the propagator, which has an effective-mass-squared $m^2(v)+\pi^{(1)}(0)$ (see Fig. 5). We will show explicitly that the ring diagram contribution is small unless $m(v)/T\ll 1$, and, therefore, that it is sufficient to take the limit $m(v)/T\ll 1$ in $\pi^{(1)}(0)$ and $\pi(0)$.

The one-loop polarization tensor is given by

$$\pi^{(1)}(\omega_n,\mathbf{k})=\pi^{(1)}(0)=3\lambda T\sum_n\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{\omega_n^2+\mathbf{q}^2+m^2(v)}$$
$$=\lambda\frac{T^2}{4}\left[1+O\left[\frac{m(v)}{T}\right]\right]\ . \quad (10)$$

Summing over N we obtain

$$\pi(\omega_n,\mathbf{k})=\pi(0)$$
$$=3\lambda T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\omega_n^2+\mathbf{p}^2+m^2(v)+\pi^{(1)}(0)}\ . \quad (11)$$

From Eqs. (10) and (11) it is clear that it is sufficient to drop terms of $O(m(v)/T)$ in $\pi^{(1)}(0)$. These terms are not important unless $m(v)/T\sim 1$, but in that case $\pi^{(1)}(0)\sim\lambda m^2(v)$, which is small compared to $m^2(v)$. Therefore, these corrections do not contribute to Eq. (11). With $\pi^{(1)}(0)=\lambda T^2/4$, we can expand Eq. (11) in $m(v)/T$ to obtain

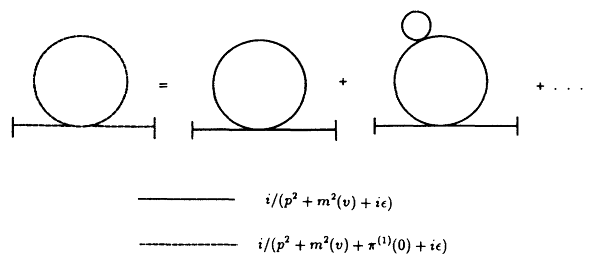

FIG. 5. Resummed polarization tensor.

$$\pi(0)=\lambda\frac{T^2}{4}\left[1+O\left[\frac{m(v)}{T}\right]\right]\ . \quad (12)$$

Finally, substituting Eq. (12) into Eq. (9) gives the result for the effective potential. After making this substitution, we see that we can take the zeroth-order result in Eq. (12), as well. The argument of the logarithm in Eq. (9) is

$$\beta^2\left\{\omega_n^2+\mathbf{p}^2+m^2(v)+\lambda\frac{T^2}{4}\left[1+O\left[\frac{m(v)}{T}\right]\right]\right\}\ .$$

As in the previous discussion about $\pi^{(1)}(0)$, if $m(v)/T\sim 1$ so that the corrections are not small, then $\pi(0)\sim\lambda m^2(v)$, which is small relative to $m^2(v)$ and does not contribute to the argument of the logarithm in Eq. (9). In this case, Eq. (9) becomes $V(v)_{\mathrm{eff}}\sim V(v)_{\mathrm{tree}}+V_1(v)$ [see Eqs. (2) and (3)]. Thus, if $m(v)/T\sim 1$, then $V(v)_{\mathrm{ring}}\sim 0$. We conclude that we can use the zeroth-order result for the polarization tensor $\pi(0)=\lambda T^2/4$ everywhere in our calculation of the ring diagrams, since this approximation is good in the regime in which the ring diagrams give a nonzero contribution. Thus we have

$$V(v)=V(v)_{\mathrm{tree}}+\tfrac{1}{2}T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln\left[\beta^2\left[\omega_n^2+\mathbf{p}^2+m^2(v)+\lambda\frac{T^2}{4}\right]\right]\ . \quad (13)$$

It is straightforward to verify that we have obtained the sum of the tree potential, and the one loop and the ring diagram contributions to the potential. We separate these contributions analytically by writing

$$D^{-1}(\omega_n,\mathbf{p})=\bar{D}_0^{-1}(\omega_n,\mathbf{p})+\pi(0)=\bar{D}_0^{-1}(\omega_n,\mathbf{p})[1+\bar{D}_0(\omega_n,\mathbf{p})\pi(0)]\ ,$$
$$\bar{D}_0^{-1}(\omega_n,\mathbf{p})=\omega_n^2+\mathbf{p}^2+m^2(v)\ .$$

Equation (9) becomes

$$V(v)=V(v)_{\mathrm{tree}}+\tfrac{1}{2}T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\{\ln[\beta^2\bar{D}_0^{-1}(\omega_n,\mathbf{p})]+\ln[\beta^2(1+\bar{D}_0(\omega_n,\mathbf{p})\pi(0))]\}$$
$$=V(v)_{\mathrm{tree}}+V_1(v)-\tfrac{1}{2}T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\sum_{N=1}^{\infty}\frac{1}{N}[-\bar{D}_0(\omega_n,\mathbf{p})\pi(0)]^N$$
$$=V(v)_{\mathrm{tree}}+V_1(v)+V(v)_{\mathrm{ring}}\ , \quad (14)$$

where

$$V(v)_{\mathrm{ring}}=-\tfrac{1}{2}T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\sum_{N=1}^{\infty}\frac{1}{N}[-\bar{D}_0(\omega_n,\mathbf{p})\pi(0)]^N \quad (15)$$

(see Fig. 6).

Doing the integrations in Eq. (15), we obtain

$$V(v)_{\mathrm{ring}}=-\frac{T}{12\pi}\{[m^2(v)+\pi(0)]^{3/2}-m^3(v)\}\ . \quad (16)$$

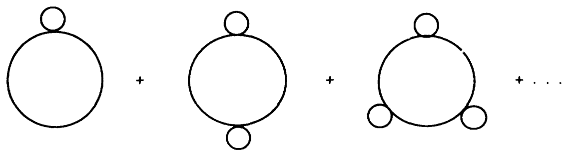

FIG. 6. Ring diagrams to leading order.

From Eqs. (12) and (16) we see explicitly that $V(v)_{\rm ring} \sim 0$ if $m(v)/T \sim 1$. If $m(v)/T \sim 1$, then $\pi(0) \sim \lambda m^2(v)$, which is small compared to $m^2(v)$ in Eq. (16). Thus, $V(v)_{\rm ring} \sim 0$ if $m(v)/T \sim 1$. In conclusion, therefore, if we want to relax the assumption $m(v)/T \ll 1$, we must perform the $V_1^{(T)}(v)$ integration in Eq. (7) numerically, but $V(v)_{\rm ring}$ can always be evaluated with the zeroth-order result $\pi(0) = \lambda T^2/4$.

Combining Eqs. (6), (8), and (16) and dropping the v-independent term, we get, in the limit $m(v)/T \ll 1$,

$$\begin{aligned} V(v) &= V(v)_{\rm tree} + V_1^{(0)}(v) + V_1^{(T)}(v) + V(v)_{\rm ring} \\ &= -\frac{c^2 v^2}{2} + \frac{\lambda v^4}{4} + \frac{21\lambda c^2 v^2}{64\pi^2} - \frac{27\lambda^2 v^4}{128\pi^2} \\ &\quad + \frac{T^2}{24} m^2(v) - \frac{T}{12\pi}\left[m^2(v) + \lambda \frac{T^2}{4} \right]^{3/2} \\ &\quad + O(m^4(v)) \ . \end{aligned} \tag{17}$$

Note that the terms proportional to $m^4(v)\ln[m(v)]$ have canceled between $V_1^{(0)}(v)$ [Eq. (6)] and $V_1^{(T)}(v)$ [Eq. (8)]. In addition, the terms proportional to $m^3(v)$ have canceled between $V_1^{(T)}(v)$ [Eq. (8)] and $V(v)_{\rm ring}$ [Eq. (16)]. This cancellation is significant, since $m^3(v)$ is imaginary for $m^2(v) = 3\lambda v^2 - c^2 < 0$ or for $v^2 < c^2/3\lambda$. Note that the cancellation of the $m^3(v)$ terms is expected from Eq. (13), which tells us that, when long-distance effects are included, the effective potential is a function of the shifted mass $m^2(v) + \pi(0)$.

Equation (17) contains the tree potential plus the first-order corrections to it, the usual leading-order one-loop term, and a contribution from the ring diagrams, which is $\sim v^3$ and is real for T large enough. Specifically, the argument of the cube root is real for

$$m^2(v) + \lambda T^2/4 = 3\lambda v^2 - c^2 + \lambda T^2/4 \geq 0 \ .$$

This condition is satisfied for all v if $T \geq T_1 = 2c/\lambda^{1/2} = 2\langle v \rangle_0$.

The effective mass is also real if $T \geq T_1$. We define the effective mass squared as

$$m_{\rm eff}^2 = \left[\frac{\partial^2 V_{\rm eff}}{\partial v^2} \right]_{v=0} \ . \tag{18}$$

From Eqs. (17) and (18) we have, to lowest order, $m_{\rm eff}^2 = -c^2 + \lambda T^2/4$. Thus, both the effective mass and the effective potential are imaginary when $T < T_1$.

Since the quadratic part of the effective potential is zero at $T = T_1$ to lowest order, the leading-order term at small v is the cubic term, which is $-T_1(3\lambda v^2)^{3/2}/12\pi$. At large v the quartic term in the tree potential dominates. The effective potential has the shape shown in Fig. 7 at $T = T_1$. When the temperature is increased above T_1, the coefficient of the quadratic term and thus the curvature at the origin becomes positive. Equivalently, the potential develops a local minimum at the origin. This indicates a first-order phase transition. It has been shown numerically that when the temperature is increased above T_1, a first-order transition does occur [9]. In the next section we will perform the same calculation for the standard model.

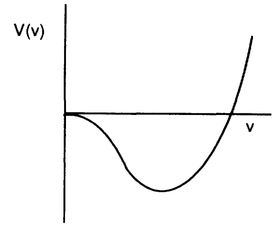

FIG. 7. Effective potential at $T = T_1$.

III. THE STANDARD MODEL

A. Notation

In this section we describe the calculation of the effective potential in the standard model. As in the case of the scalar model, the ring diagrams contribute terms that are cubic in the condensate and are of the same order in the coupling as the $m^3(v)$ term from the one-loop graph. In the standard model, however, the $m^3(v)$ terms do not cancel exactly in the gauge-boson sector. As discussed in the Introduction, for fields with zero bare mass this cancellation is not necessary to avoid imaginary terms in the effective potential. We will show that when all terms of order $\lambda^{3/2}$ are included, the phase transition is first order in the weak-coupling expansion.

We will use the following notation for indices. $a,b,\ldots$ have the values 1,2,3 and refer to the SU(2) fields. We define a vector of gauge bosons $(A_a^\mu, B^\mu) = A_A^\mu$ so that the indices $A,B,\ldots$ run from 1 to 4 in the gauge-boson space. The indices $l,m,\ldots$ have values 1,2 and refer to the two components of the complex Higgs doublet, and $i,j,\ldots$ run from 1 to 4 and refer to the four real Higgs bosons. Vector products are written as in Sec. II.

The Lagrangian is

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\rm gauge\ field} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm fermions} \\ + \mathcal{L}_{\rm Yukawa} + \mathcal{L}_{\rm gauge\ fixing} \ . \end{aligned} \tag{19}$$

Each of these terms is described below. The fields are eigenstates of isospin and hypercharge (denoted Y) and the Lagrangian is symmetric under $SU(2) \times U(1)_Y$ transformations. The vacuum state is infinitely degenerate and not invariant under $SU(2) \times U(1)_Y$ transformations. This degeneracy gives rise to spontaneous symmetry breaking.

The gauge-field kinetic terms are given by

$$\mathcal{L}_{\rm gauge\ field} = -\tfrac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \ .$$

The Higgs-field part of the Lagrangian is

$$\mathcal{L}_{\rm Higgs}=(D_\mu\phi)^\dagger D^\mu\phi+c^2(\phi^\dagger\phi)-\lambda(\phi^\dagger\phi)^2 \ .$$

The covariant derivative is

$$D_\mu=\partial_\mu+ig\frac{\tau^a}{2}A^a_\mu+ig'\tfrac{1}{2}B_\mu \ ,$$

where the matrices τ^a are the SU(2) Pauli matrices. The complex Higgs doublet

$$\phi=\frac{1}{\sqrt{2}}\begin{bmatrix}\phi_3+i\phi_4\\ \phi_1+i\phi_2\end{bmatrix}$$

has an electromagnetically neutral lower component and an upper component with electromagnetic charge $+1$. These charge assignments give an eigenstate of hypercharge and allow us to select a vacuum state that has U(1) electromagnetic symmetry by shifting the ϕ_1 field: $\phi_1=v+\chi$. The Higgs fields acquire masses: $m_1^2(v)=3\lambda v^2-c^2$ and $m_2^2(v)=m_3^2(v)=m_4^2(v)=\lambda v^2-c^2$, and the symmetry of the Lagrangian is broken: $SU(2)\times U(1)_Y\to U(1)_{EM}$. Note that at the classical minimum $\langle v\rangle_0=c\lambda^{1/2}$ the fields ϕ_2, ϕ_3, and ϕ_4 are Goldstone modes. The gauge bosons also acquire masses. The gauge-boson-mass term is of the form $\sim A^{\dagger A}_\mu M^2_{AB}(v)A^\mu_B$, where the mass matrix is nondiagonal:

$$M^2(v)=\begin{bmatrix}g^2v^2/4 & 0 & 0 & 0\\ 0 & g^2v^2/4 & 0 & 0\\ 0 & 0 & g^2v^2/4 & -gg'v^2/4\\ 0 & 0 & -gg'v^2/4 & g'^2v^2/4\end{bmatrix} . \tag{20}$$

In the usual way, we define the fields W^+_μ, W^-_μ, Z_μ, and A_μ as linear combinations of the fields A^a_μ and B_μ so that the new fields have masses $m_W^2(v)=g^2v^2/4$, $m_Z^2(v)=(g^2+g'^2)v^2/4$ and $m_A^2(v)=0$.

The fermion part of the Lagrangian is given by

$$\mathcal{L}_{\rm fermions}=\bar\psi_R i\gamma^\mu\left[\partial_\mu+i\frac{g'}{2}YB_\mu\right]\psi_R+\bar\psi_L i\gamma_\mu\left[\partial_\mu+i\frac{g'}{2}YB_\mu+i\frac{g}{2}A^a_\mu\tau_a\right]\psi_L \ .$$

There are three generations of fermions, which are grouped into left-handed SU(2) doublets and right-handed SU(2) singlets. For example,

$$\psi_R=\tfrac{1}{2}(1+\gamma_5)e,\ \ \tfrac{1}{2}(1+\gamma_5)u,\ \ \tfrac{1}{2}(1+\gamma_5)d' \ ,$$

$$\psi_L=\tfrac{1}{2}(1-\gamma_5)\begin{bmatrix}\nu_e\\ e\end{bmatrix},\ \ \tfrac{1}{2}(1-\gamma_5)\begin{bmatrix}u\\ d'\end{bmatrix}_{3\ \rm colors} .$$

The primed quark states are electroweak eigenstates, which are constructed as follows. We define a vector of the mass eigenstates of the charge $-1/3$ quarks: $x=(d,s,t)$. The electroweak eigenstates are denoted x' and are given by $x'=Ux$ where U is the unitary 3×3 Kobayashi-Maskawa matrix (the specific form of U is not needed for this problem).

We work in the approximation that the only nonzero Yukawa coupling is the t-quark coupling denoted f. When the Higgs field is shifted, only the t quark will acquire mass and thus, this approximation is equivalent to setting $m_\psi/T\sim0$ for $m_\psi\neq m_t$. In this approximation, the usual Yukawa term becomes

$$\mathcal{L}_{\rm Yukawa}=f\bar q'_L\tilde\phi t_R+\text{H.c.}$$

The Yukawa term is invariant under a U(1) hypercharge transformation. The t-quark wave function is represented by t and q'_L is the isodoublet:

$$\begin{bmatrix}t\\ b'\end{bmatrix}_L=\tfrac{1}{2}(1-\gamma_5)\begin{bmatrix}t\\ b'\end{bmatrix} .$$

The field $\tilde\phi=i\sigma_2\phi^*$ is the Higgs doublet with hypercharge -1. After shifting the Higgs field the Yukawa term becomes

$$\begin{aligned}\mathcal{L}_{\rm Yukawa}=&f\frac{v}{\sqrt{2}}\bar tt+\frac{f}{\sqrt{2}}(\bar t\phi_1 t)-i\frac{f}{\sqrt{2}}(\bar t\gamma_5\phi_2 t)\\ &+\frac{f}{2\sqrt{2}}[\,\bar b'(1+\gamma_5)(-\phi_3+i\phi_4)t\\ &-\bar t(1-\gamma_5)(\phi_3+i\phi_4)b'\,] \ ,\end{aligned}$$

and the t quark acquires a mass $m_t^2(v)=f^2v^2/2$.

We work in the R_ξ gauge, where the gauge-fixing part of the Lagrangian is

$$\begin{aligned}\mathcal{L}_{\rm gauge\ fixing}=&-\frac{1}{2\xi}(\partial^\mu A^a_\mu-\tfrac{1}{2}\xi gv\chi^a)^2\\ &-\frac{1}{2\xi}(\partial^\mu B_\mu-\tfrac{1}{2}\xi g'v\chi^2)^2 \ ,\end{aligned} \tag{21}$$

where $\chi_a=\phi_2,\phi_3,\phi_4$. We choose the Landau value of the gauge-fixing parameter $\xi\to0$. The cross terms in Eq. (21) combine with the cross terms from $(D_\mu\phi)^\dagger(D^\mu\phi)$ to produce total divergences, which integrate to zero. In the Landau gauge, the ghost fields are massless and do not contribute to the v-dependent part of the one-loop effective potential. Note that the effective potential itself is gauge dependent, as noted in Ref. [7(a)]. However, physical quantities obtained from the effective potential, such as the critical temperature, are gauge independent [14].

In the original basis, where the relevant fields are the gauge bosons $A_A^\mu=(A_a^\mu,B^\mu)$, the vector propagator is

$$iD_{\mu\nu}^{AB}(q)=-i\left[\frac{g_{\mu\nu}-q_\mu q_\nu/q^2}{q^2-M^2(v)+i\epsilon}\right]^{AB},$$

where $M^2(v)$ is the nondiagonal mass matrix given in Eq. (20). After diagonalization, we obtain the propagators for the W_μ and Z_μ fields,

$$iD_{\mu\nu}(q)=-i\frac{g_{\mu\nu}-q_\mu q_\nu/q^2}{q^2-m^2(v)+i\epsilon},$$
$$m^2(v)=m_W^2(v) \text{ or } m_Z^2(v),$$

and the A_μ field,

$$iD_{\mu\nu}(q)=-i\frac{g_{\mu\nu}-q_\mu q_\nu/q^2}{q^2+i\epsilon}.$$

The ghost, Higgs-boson and quark propagators, and the vertices are standard.

We can calculate the effective potential for the standard model in the same way that we did for the scalar model in Sec. II. We work in the loop expansion. To zeroth order, we have the tree potential

$$V(v)_{\text{tree}}=-\tfrac{1}{2}c^2v^2+\tfrac{1}{4}\lambda v^4.$$

To go beyond zeroth order consistently we calculate the one-loop and ring diagram contributions separately.

B. One-loop contributions

We will write separately the zero- and finite-temperature contributions to the one-loop result, and the contributions from the Higgs-boson, gauge-boson, and t-quark loops. We have

$$V_1(v)=V_1^{(0)}(v)+V_1^{(T)}(v),$$
$$V_1^{(0)}(v)=V_{1,\phi}^{(0)}(v)+V_{1,\text{gb}}^{(0)}(v)+V_{1,\psi}^{(0)}(v),$$
$$V_1^{(T)}(v)=V_{1,\phi}^{(T)}(v)+V_{1,\text{gb}}^{(T)}(v)+V_{1,\psi}^{(T)}(v).$$

We will rewrite each of these contributions in terms of the three functions $f(m_x(v))$, $g(m_x(v))$, and $h(m_x(v))$, which are defined below. The argument $m_x^2(v)$ represents one of the masses in the problem: $m_1^2(v)$, $m_2^2(v)$, $m_W^2(v)$, $m_Z^2(v)$, or $m_t^2(v)$. The function $f(m_x(v))$ is equal to the one-loop zero-temperature potential for the simple scalar model [Eq. (4)]. Regularizing as in Sec. II, $f(m_x(v))$ depends on the cutoff parameter Λ:

$$f(m_x(v))=\frac{\Lambda^2}{32\pi^2}m_x^2(v)+\frac{m_x^4(v)}{64\pi^2}\left[\ln\left[\frac{m_x^2(v)}{\Lambda^2}\right]-\frac{1}{2}\right]. \tag{22}$$

The function $g(m_x(v))$ is equal to the one-loop finite-temperature potential for the simple scalar model [Eq. (7)]:

$$g(m_x(v))=\frac{T}{2\pi^2}\int dk\,k^2\times\ln(1-\exp\{-\beta[k^2+m_x^2(v)]^{1/2}\}). \tag{23}$$

The t quarks are fermions and their contribution to the one-loop potential is proportional to

$$h(m_t(v))=\frac{T}{2\pi^2}\int dk\,k^2\ln(1+\exp\{-\beta[k^2+m_t^2(v)]^{1/2}\}). \tag{24}$$

With these definitions we have

$$V_{1,\phi}^{(0)}(v)=f(m_1(v))+3f(m_2(v)), \tag{25a}$$

$$V_{1,\text{gb}}^{(0)}(v)=3f(m_Z(v))+6f(m_W(v)), \tag{25b}$$

$$V_{1,\psi}^{(0)}(v)=-12f(m_t(v)), \tag{25c}$$

and

$$V_{1,\phi}^{(T)}(v)=g(m_1(v))+3g(m_2(v)), \tag{26a}$$

$$V_{1,\text{gb}}^{(T)}(v)=3g(m_Z(v))+6g(m_W(v)), \tag{26b}$$

$$V_{1,\psi}^{(T)}(v)=-12h(m_t(v)). \tag{26c}$$

The numerical factors in Eqs. (25) and (26) arise as follows. There is a factor 3 for the three Higgs bosons that have equal mass, a factor 3 for the three polarizations of the Z boson, a factor 6 for the three polarizations times two W bosons, and a factor 12 for the t quarks from two spins times two particle-antiparticle degrees of freedom times three colors.

We consider $V_1^{(0)}(v)$ first. As in the case of the scalar theory, this term contains all of the ultraviolet divergences and we can renormalize by adding counterterms [Eq. (5)] to remove the cutoff dependence. To determine A and B we write $A=a(\Lambda)-\delta c^2$ and $B=b(\Lambda)+\delta\lambda$, and choose $a(\Lambda)$ and $b(\Lambda)$ to cancel the infinite coefficient of the v^2 and v^4 terms, respectively. Then we impose the conventional renormalization condition

$$\left[\frac{dV^{(0)}(v)}{dv^2}\right]_{v=\langle v\rangle_0}=0,$$

where $V^{(0)}(v)=V(v)_{\text{tree}}+V_1^{(0)}(v)$. This condition gives $\delta c^2=\delta\lambda c^2/\lambda$. We use $\delta\lambda\sim\lambda^2\sim 0$, since corrections to the tree potential will not be significant in the high-temperature region. We have

$$V^{(0)}(v) = v^2\left[-\frac{1}{2}c^2 + \frac{3\lambda}{32\pi^2}c^2\right] + v^4\left[\frac{1}{4}\lambda - \frac{1}{64\pi^2}\left[6\lambda^2 + \frac{3g^4}{16} + \frac{3(g^2+g'^2)^2}{32} - \frac{3f^2}{2}\right]\right]$$
$$+\frac{1}{64\pi^2}\left[6m_W^4(v)\ln\left[\frac{\lambda v^2}{c^2}\right] + 3_Z^4(v)\ln\left[\frac{\lambda v^2}{c^2}\right] - 12m_t^4(v)\ln\left[\frac{\lambda v^2}{c^2}\right]\right.$$
$$\left. + m_1^4(v)\ln\left[\frac{m_1^2(v)}{2c^2}\right] + 3m_2^4(v)\ln\left[\frac{m_2^2(v)}{2c^2}\right]\right] . \tag{27}$$

Next we consider $V_1^{(T)}(v)$. We do not want to restrict ourselves to the ultrarelativistic regime $m(v)/T \ll 1$ and therefore we do the integrations in Eqs. (23) and (24) numerically. There is no difficulty with this procedure for the gauge-boson sector. For the Higgs sector, however, a calculational problem arises as a result of the fact that the Higgs-boson masses squared are negative for finite values of v. Specifically, $m_1^2(v) = 3\lambda v^2 - c^2$ is negative for $v^2 < c^2/3\lambda$, and $m_2^2(v) = m_3^2(v) = m_4^2(v) = \lambda v^2 - c^2$ is negative for $v^2 < c^2/\lambda$. In these regions, the square root in Eq. (23) is imaginary. To avoid this difficulty, we need to combine the ring diagrams and one-loop contributions. As discussed in Sec. II, the sum is the one-loop expression with the masses replaced by the shifted masses, $m_i^2(v) + \pi_i(0)$, which are positive for T large enough. The calculation of the ring diagrams is described in the next section and the condition on T is discussed. The sum $V_{1,\phi}^{(T)}(v) + V(v)_{\rm ring}^{\phi}$ is given in Eq. (31).

C. Ring diagrams

There are contributions to the ring diagrams from gauge-boson and Higgs-boson loops. We need to calculate the gauge-boson and Higgs-boson polarization tensors in the infrared limit. As in Sec. II, we work to leading order in $m(v)/T$, even when $m(v)/T \sim 1$, since $V(v)_{\rm ring} \sim 0$ there.

First we consider the Higgs polarization tensors. The diagrams that contribute are shown in Fig. 8. The notation is as follows: $\pi_i(0)$ is the polarization tensor for the ith Higgs field, and $\pi^{\phi}_{(A_\mu^a)}(0)$, $\pi^{\phi}_{(B_\mu)}(0)$, $\pi^{\phi}_{(\phi)}(0)$, and $\pi^{\phi}_{(\psi)}(0)$ indicate contributions to the Higgs polarization tensor from the SU(2), U(1), Higgs, and t-quark fields, respectively. The results are

$$\begin{aligned}\pi_i(0) &= \pi_\phi^{(A_\mu^a)}(0) + \pi_\phi^{(B_\mu)}(0) \\ &\quad + \pi_\phi^{(\phi)}(0) + \pi_\phi^{(\psi)}(0) , \\ \pi_\phi^{(A_\mu)^a}(0) &= \tfrac{1}{8}g^2T^2 , \\ \pi_\phi^{(B_\mu)}(0) &= \tfrac{1}{16}(g^2+g'^2)T^2 , \\ \pi_\phi^{(\phi)}(0) &= \tfrac{1}{2}\lambda T^2 , \\ \pi_\phi^{(\psi)}(0) &= \tfrac{1}{4}f^2T^2 .\end{aligned} \tag{28}$$

The Higgs-boson ring diagram contributions are given by [compare with Eq. (15)]

$$V(v)_{\rm ring}^{\phi_i} = -\tfrac{1}{2}T\sum_n \int \frac{d^3\mathbf{q}}{(2\pi)^3}\sum_{N=1}^{\infty}\frac{1}{N}\left[-\frac{1}{\omega_n + \mathbf{q}^2 + m_i(v)}\pi_i(0)\right]^N . \tag{29}$$

As discussed previously, we combine Eqs. (26a) and (29) to obtain functions of the shifted mass:

$$V_\phi^{(T)}(v) = V_{1,\phi}^{(T)}(v) + V(v)_{\rm ring}^{\phi} \tag{30}$$
$$= g(m_1^2(v)) + 3g(m_2^2(v)) + V(v)_{\rm ring}^{\phi_1} + 3V(v)_{\rm ring}^{\phi_2} \tag{31}$$
$$= g(m_1^2(v) + \pi_1(0)) + 3g(m_2^2(v) + \pi_2(0)) .$$

The functions $g(m_x^2(v) + \pi_x(0))$, for $x = 1,2$ are given by

$$g(m_x^2(v) + \pi_x(0)) = \frac{T}{2\pi^2}\int dk\, k^2 \ln(1 - \exp\{-\beta[k^2 + m_x^2(v) + \pi_x(0)]^{1/2}\}) . \tag{32}$$

The argument of the square root in Eq. (32) is real for all values of k when $m_x^2(v) + \pi_x(0) \geq 0$, and for all values of v when $\pi_x(0) \geq c^2$. From Eq. (28) this condition becomes

$$T^2 \geq T_1^2 = \frac{16c^2}{8\lambda + 3g^2 + g'^2 + 4f^2} . \tag{33}$$

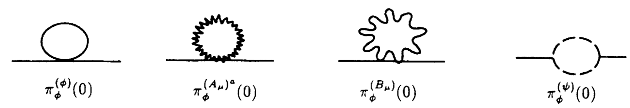

FIG. 8. Contributions to Higgs polarization tensors.

As in the case of the scalar model, this condition is equivalent to the requirement that the effective mass [Eq. (18)] is positive. To lowest order, we obtain the effective mass from the quadratic term in the effective potential. The quadratic term in the finite-temperature one-loop potential is obtained by expanding Eq. (26) in $m(v)/T$. Combining with the quadratic part of the zero-temperature term [Eq. (27)] and dropping the first-order corrections to the tree potential gives

$$V(v)_{\rm eff}^{\rm quad}=\frac{-c^2v^2}{2}+\frac{T^2}{24}[m_1^2(v)+3m_2^2(v)+6m_W^2(v)+3m_Z^2(v)+6m_t^2(v)] .$$

Differentiating we obtain

$$m_{\rm eff}^2\approx\left[\frac{\partial^2 V(v)_{\rm eff}^{\rm quad}}{\partial v^2}\right]_{v=0}=\pi_x(0)-c^2 ,$$

which is positive for $\pi_x(0)\geq c^2$. Thus, both the Higgs contributions to the effective potential and the Higgs-boson effective masses are real when $T\geq T_1$. Note that, for any values of the parameters, the phase transition will be first order as long as there is some temperature T_1 for which $m_{\rm eff}^2=0$. Increasing the temperature infinitesimally above T_1 will produce infinitesimal positive curvature at the origin, which indicates a local minimum at the origin and a first-order phase transition.

Next we calculate the gauge-boson polarization tensors. We work in the original basis, where the relevant fields are A_μ^a, and B_μ. We will write both the mass term and the polarization tensor in matrix form. The mass matrix $M_{AB}^2(v)$ is given in Eq. (20). The polarization tensor in the infrared limit is denoted $\pi_{AB}(0)$. We obtain an expression for this quantity as follows. We define the projection operators $T_{\mu\nu}$ and $L_{\mu\nu}$ by

$$T_{00}=0,\quad T_{0i}=T_{i0}=0,\quad T_{ij}=\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2} ,$$
$$L_{\mu\nu}=\frac{k_\mu k_\nu}{k^2}-g_{\mu\nu}-T_{\mu\nu} . \tag{34}$$

Expanding the polarization tensor we can write $\pi_{\mu\nu}^{AB}(0)=\pi_T^{AB}(0)T_{\mu\nu}+\pi_L^{AB}(0)L_{\mu\nu}$. In the infrared limit, $T_{00}=0$, $L_{00}=-1$, $T_i{}^i=-2$, and $L_i{}^i=0$. This gives $-\pi_L^{AB}(0)=\pi_{00}^{AB}(0)$, $2\pi_T^{AB}(0)=\pi_{ii}^{AB}(0)=0$ and, therefore,

$$\pi_{\mu\nu}^{AB}(0)=-L_{\mu\nu}\pi_{00}^{AB}(0) . \tag{35}$$

In the limit $m_W(v)/T$ and $m_Z(v)/T\ll 1$, $\pi_{00}^{AB}(0)$ is diagonal:

$$\pi_{00}(0)=\begin{bmatrix}\pi_{00}^{(2)}(0) & 0 & 0 & 0\\ 0 & \pi_{00}^{(2)}(0) & 0 & 0\\ 0 & 0 & \pi_{00}^{(2)}(0) & 0\\ 0 & 0 & 0 & \pi_{00}^{(1)}(0)\end{bmatrix} , \tag{36}$$

where the superscripts (2) and (1) indicate the polarization tensors for SU(2) and U(1) bosons, respectively.

The diagrams that contribute to $\pi_{00}^{(2)}(0)$ and $\pi_{00}^{(1)}(0)$ are shown in Fig. 9. We define $\pi_{\rm gb}^{(2)}(0)$, $\pi_\phi^{(2)}(0)$, and $\pi_\psi^{(2)}(0)$ as the contributions to the SU(2) gauge-boson polarization tensor from the gauge-boson, Higgs-boson, and t-quark sectors, respectively. Similarly, $\pi_\phi^{(1)}(0)$ and $\pi_\psi^{(1)}(0)$ are the contributions to the U(1) gauge-boson polarization tensor from the Higgs-boson and t-quark sectors, respectively. The results are

$$\begin{aligned}&\pi_{00}^{(2)}(0)=\pi_{\rm gb}^{(2)}(0)+\pi_\phi^{(2)}(0)+\pi_\psi^{(2)}(0) ,\\ &\pi_{00}^{(1)}(0)=\pi_\phi^{(1)}(0)+\pi_\psi^{(1)}(0) ,\\ &\pi_{\rm gb}^{(2)}(0)=\tfrac{2}{3}g^2T^2 ,\\ &\pi_\phi^{(2)}(0)=\tfrac{1}{6}g^2T^2 ,\\ &\pi_\psi^{(2)}(0)=g^2T^2 ,\\ &\pi_\phi^{(1)}(0)=\tfrac{1}{6}g'^2T^2 ,\\ &\pi_\psi^{(1)}(0)=\tfrac{5}{3}g'^2T^2 .\end{aligned} \tag{37}$$

The gauge-boson ring diagram contribution is given by

$$V(v)_{\rm ring}^{\rm gb}=-\frac{i}{2}\int\frac{d^4q}{(2\pi)^4}{\rm Tr}\sum_{N=1}^{\infty}\frac{1}{N}[-i\pi_{AB}^{\mu\nu}(0)iD_{\nu\lambda}^{BC}(q)]^N . \tag{38}$$

In the Landau gauge the gauge-boson propagator is

$$iD_{\mu\nu}^{AB}(q)=i(T_{\mu\nu}+L_{\mu\nu})\left[\frac{1}{q^2+M^2(v)+i\epsilon}\right]^{AB} . \tag{39}$$

Since $T_{\mu\nu}T^\nu{}_\lambda=-T_{\mu\lambda}$, $L_{\mu\nu}L^\nu{}_\lambda=-L_{\mu\lambda}$, and $L_{\mu\nu}T_\lambda^\nu=0$, we have, from Eqs. (35), (38), and (39),

$$V(v)_{\rm ring}^{\rm gb}=\frac{i}{2}(-L_\mu{}^\mu)\int\frac{d^4q}{(2\pi)^4}{\rm Tr}\sum_{N=1}^{\infty}\frac{1}{N}\left[\left[\frac{1}{q^2+M(v)+i\epsilon}\right]^{AB}\pi_{00}^{BC}(0)\right]^N ,$$

where the trace now operates only on the indices A,B. At finite temperature we have [compare Eqs. (15) and (29)]

$$V(v)_{\rm ring}^{\rm gb}=-\tfrac{1}{2}T\sum_n\int\frac{d^3\mathbf{q}}{(2\pi)^3}{\rm Tr}\sum_{N=1}^{\infty}\frac{1}{N}\left[\left[-\frac{1}{\omega_n^2+\mathbf{q}^2+M(v)}\right]^{AB}\pi_{00}^{BC}(0)\right]^N .$$

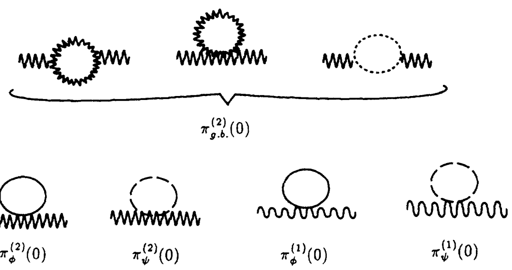

FIG. 9. Contributions to gauge-boson polarization tensor.

Using Eqs. (36) and (37) we can do the integration. We obtain

$$V(v)^{gb}_{\rm ring}=-\frac{T}{12\pi}{\rm Tr}\{[M^2(v)+\pi_{00}(0)]^{3/2}-M^3(v)\}\ . \quad (40)$$

We note that an expansion of the gauge-boson contributions to the one-loop finite-temperature part of the effective potential [Eq. (26b)] would show that the terms proportional to $m^3(v)$ do not cancel between $V^{(T)}_{1,gb}(v)$ and $V(v)^{gb}_{\rm ring}$ as they do for the corresponding terms in the Higgs sector. Thus, this cancellation is not a general feature of a consistent calculation of the effective potential. However, the squares of the gauge-boson masses are positive for all v, and therefore both the gauge-boson masses and gauge-boson contributions to the effective potential are always real.

The final expression for the effective potential is obtained by combining Eqs. (26b), (26c), (27), (31), and (40). The resulting expression is real for

$$T^2\geq T_1^2=\frac{16c^2}{8\lambda+3g^2+g'^2+4f^2}\ .$$

IV. RESULTS

We have plotted the effective potential for the following values of the parameters. For the SU(2) and U(1) couplings we use $g=0.637$ and $g'=0.344$. For the zero-temperature condensate we use $\langle v\rangle_0=246$ GeV. These values reproduce the correct values for the W- and Z-boson vacuum masses. We do not know the values of the t-quark Yukawa coupling (f) and the scalar coupling (λ) or equivalently, the vacuum values of the t-quark mass and the Higgs-boson mass. However, vacuum stability places limits on these parameters [15]. We have chosen three sets of values for the vacuum Higgs-boson mass (m_ϕ) and t-quark mass (m_t). We note that electroweak perturbation theory is reliable when $\alpha_W T/m(v)\ll 1$, where the electroweak fine-structure constant is $\alpha_W=g^2/4\pi$ and $m(v)=m_W(v)=gv/2$. The condition is $v/T\gg g/2\pi=0.10$. The effective potential cannot be calculated perturbatively in the range that does not satisfy this condition, and these regions should be ignored in the graphs of the effective potential, which are described below.

Figure 10 shows the effective potential when the vacuum Higgs-boson and t-quark masses are 55 and 110 GeV, respectively. These masses correspond to a scalar self-coupling of $\lambda=0.025$ and a t-quark Yukawa coupling of $f=0.632$. The dotted line indicates the real part of potential that is obtained when the ring diagram contributions are not included.

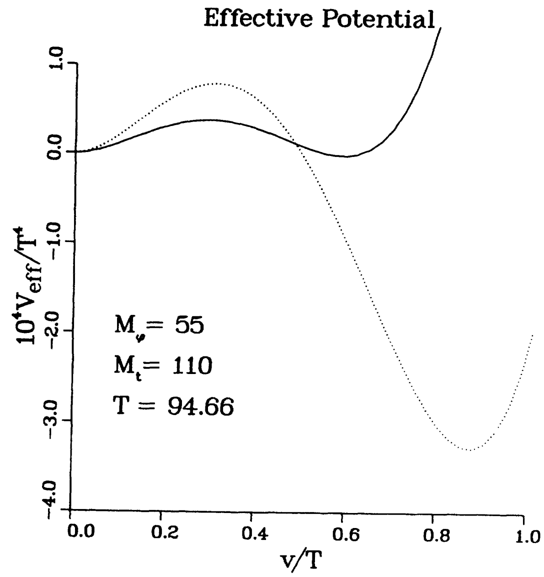

FIG. 10. The effective potential without ring diagram contributions (dotted line) and with ring diagram contributions (solid line). Masses and temperatures are measured in GeV.

Figure 11 shows the effective potential when $T=94.56$ GeV (solid line) and $T=94.69$ GeV (dotted line). Both curves show two distinct minima, one at zero v/T and one at nonzero v/T. The critical temperature is the tem-

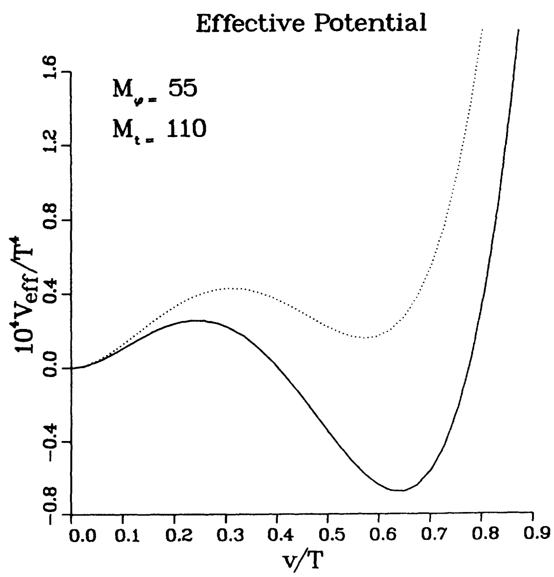

FIG. 11. The effective potential at $T=94.69$ GeV (dotted line) and $T=94.56$ GeV (solid line). Masses and temperatures are measured in GeV.

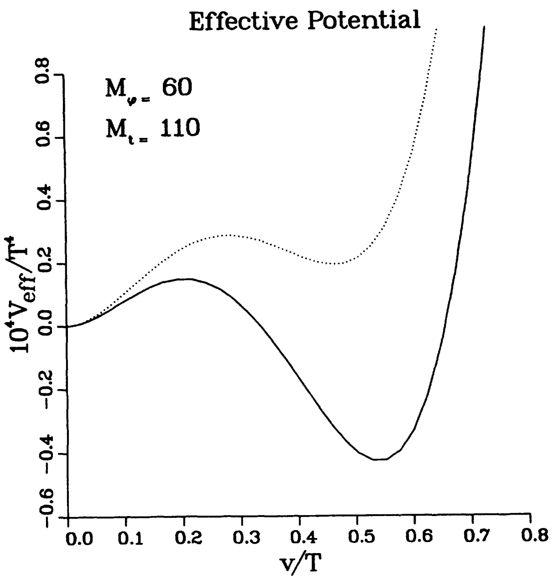

FIG. 12. The effective potential at $T=102.52$ GeV (dotted line) and $T=102.37$ GeV (solid line). Masses and temperatures are measured in GeV.

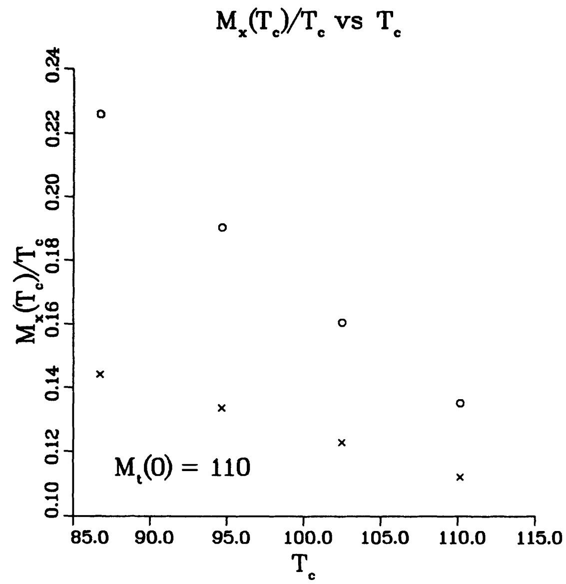

FIG. 14. The W-boson mass (open circles) and Higgs-boson mass (crosses) as a function of critical temperature.

perature at which the two minima are degenerate. For this choice of parameters, we obtain $T_c=94.66$ GeV. Figure 12 shows the effective potential when the vacuum Higgs-boson mass is 60 GeV ($\lambda=0.030$) and the vacuum t-quark mass is 110 GeV ($f=0.632$). The solid line corresponds to a temperature of 102.37 GeV and the dotted line to $T=102.52$ GeV. The critical temperature is 102.47 GeV. In Fig. 13, the vacuum Higgs-boson mass is 60 GeV ($\lambda=0.030$) and the t-quark mass is 115 GeV ($f=0.661$). The solid line corresponds to $T=99.69$ GeV and the dotted line to $T=99.87$ GeV. The critical temperature is $T=99.82$ GeV.

Figures 14 and 15 show plots of the W-boson mass and Higgs-boson mass as a function of critical temperature (T_c) and the temperature T_1, which is defined in Eq. (33). In both figures, the open circles indicate the W-boson mass and the crosses indicate the Higgs-boson mass.

There has been a lot of work done with models that include more than one Higgs doublet. It would be interesting to see if this phase transition persists in such models.

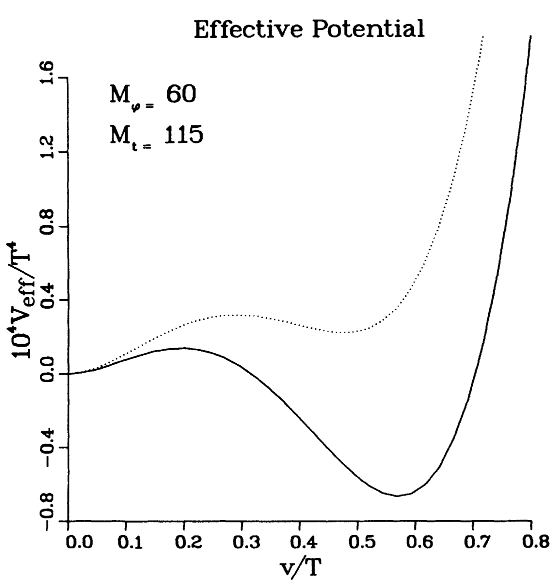

FIG. 13. The effective potential at $T=99.87$ GeV (dotted line) and $T=99.69$ GeV (solid line). Masses and temperatures are measured in GeV.

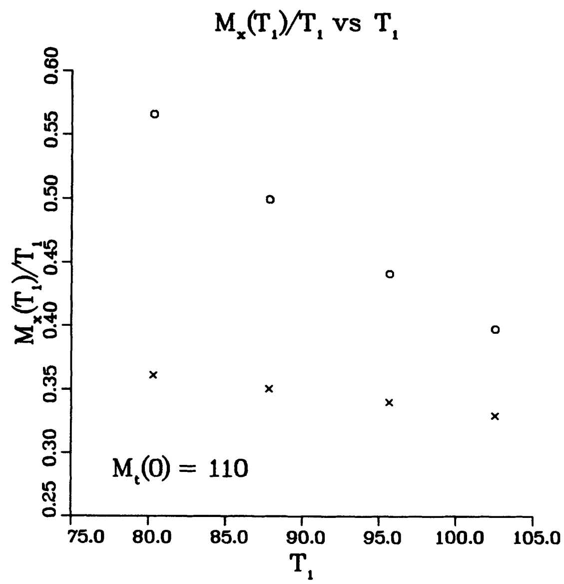

FIG. 15. The W-boson mass (open circles) and Higgs-boson mass (crosses) as a function of the temperature T_1 as defined in Eq. (33).

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