Constraints on additional Z bosons

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(Received 19 July 1991)

Z-pole, W-mass, and weak neutral-current (WNC) data as well as direct collider limits are used to constrain the mass and mixing of possible heavy Z_2 bosons with couplings expected in grand unified theories and $SU(2)_L \times SU(2)_R \times U(1)$ models. The data now enable the top-quark mass and Z_2 properties to be limited simultaneously. The indirect (Z-pole, M_W, WNC) data stringently constrain the Z_1^0-Z_2^0 mixing angle θ ($|\theta|<0.01$ in most cases). For models with arbitrary Higgs representations, the limits on the Z_2 mass M_2 from indirect and direct constraints are rather weak (typically 160–400 GeV), but in specific models in which M_2 and θ are correlated, the constraints are much stronger ($M_2 > 500$–1000 GeV). The weak angle in the modified minimal subtraction scheme is well determined even allowing for extra Z's and an arbitrary Higgs structure: $\sin^2\hat{\theta}_W(M_Z)=0.2334^{+0.0008}_{-0.0011}$, for the models considered, where the uncertainties include m_t, compared to the SU(2)×U(1) model value 0.2333±0.0008. The 95%-C.L. upper limits on m_t ($m_t < 182$ GeV for Higgs doublets and singlets only, and $m_t < 310$ GeV for arbitrary Higgs representations) continue to hold in the presence of the extra Z's considered. The implications of these results for ordinary and supersymmetric grand unification, atomic parity violation, charged-current universality, $R(e^+e^- \to \mu^+\mu^-$, hadrons) below the Z pole, and superstring theories are discussed. A nongauge model in which the Z_2 has the same couplings as the ordinary Z is included for comparison.

PACS number(s): 13.10.+q, 12.15.Ji, 12.15.Mm, 14.80.Er

I. INTRODUCTION

One of the simplest extensions of the standard model is the possible existence of heavy neutral Z_2 bosons associated with extra U(1) gauge symmetries. Such bosons could occur with masses in the range (100 GeV–10 TeV) accessible to direct searches at high-energy colliders [1,2] or indirect manifestations involving high-precision Z-pole and weak neutral-current (WNC) experiments [3–7]. If observed, such extended gauge structures could provide an important probe of underlying physics at much higher mass scales.

For some time, the most stringent constraints on extra Z's came from indirect WNC data [3–5]. Recently, however, many precise new results have been obtained in Z-pole experiments at the CERN e^+e^- collider LEP [8,9], on M_W [10,11], in atomic-parity violation [12,13], and in other neutral-current phenomena [14,15]. In addition, the Collider Detector at Fermilab (CDF) group has obtained the preliminary 95%-C.L. lower limit [16]

$$\sigma(\bar{p}p \to Z_2)B(Z_2 \to e^+e^-) < 1.6 \text{ pb} \tag{1}$$

on the production cross sections times branching ratio into e^+e^- at $\sqrt{s}=1800$ GeV.

We have previously studied the implications of the high-precision experiments for the SU(2)×U(1) model [15]. We found that in the minimal model (with a single Higgs doublet)

$$\begin{aligned} &\sin^2\hat{\theta}_W(M_Z)=0.2334\pm0.0008 , \\ &m_t = 124^{+28+20}_{-34-15} \text{ GeV} , \\ &m_t < 182 \text{ GeV, } 95\% \text{ C.L.} , \end{aligned} \tag{2}$$

where the uncertainty in the [modified minimal subtraction scheme ($\overline{\text{MS}}$)] weak angle includes the effects of m_t and of a Higgs-boson mass M_H in the range 50–1000 GeV. The central m_t value is for $M_H=250$ GeV, with the second error due to M_H, while the upper limit on m_t is for $M_H=1$ TeV. In SU(2)×U(1) models with extended Higgs structures, nonleading effects in m_t and the $b\bar{b}$ contribution to the Z width allow a separation of the effects of m_t from those of higher-dimensional Higgs representations, yielding [15]

$$\begin{aligned} &\sin^2\hat{\theta}_W(M_Z)=0.2333\pm0.0008 , \\ &\rho_0=0.992\pm0.011 , \\ &m_t < 310 \text{ GeV, } 95\% \text{ C.L.} , \end{aligned} \tag{3}$$

where

$$\rho_0 = \frac{\sum_i (t_i^2 - t_{3i}^2 + t_i)|\langle \varphi_i \rangle|^2}{\sum_i 2t_{3i}^2 |\langle \varphi_i \rangle|^2} , \tag{4}$$

where t_i and t_{3i} are the total and third component of weak isospin of φ_i. In Eq. (3), the errors in $\sin^2\hat{\theta}_W$ and ρ_0 include m_t, and the limit on m_t allows arbitrary ρ_0. This strong constraint on ρ_0 for arbitrary m_t is especially important because most viable superstring theories predict $\rho_0=1$ [17]. The results in (3) still hold in the presence of nondegenerate SU(2) multiplets of heavy fermions and bosons, which affect the W and Z self-energies at the loop level [18–21], except that the ρ_0 value applies to $\rho_0/(1-\alpha T)$ where T is defined in [19].

In this paper we extend the analysis to models involving an extra Z_2 boson and include the direct constraint in

Eq. (1). There have been other recent studies of the implications of LEP data [22], atomic parity violation [23], or all data [24,25] for Z_2 models. The present analysis is useful, however, because it is part of an ongoing project [4,14,15] to systematically incorporate and utilize all existing data, and because it makes no *a priori* assumptions about m_t except the direct CDF limit $m_t > 89$ GeV [26]. Other recent studies consider astrophysical and cosmological constraints [27]. These achieve stringent limits ($M \gtrsim 1$ TeV), but only apply if the Z_2 couples to new neutrinos which are relatively light ($m_\nu \lesssim 1$ MeV).

It is impractical to deal with a totally general extended neutral sector, because, in addition to their masses and mixings, one must specify the couplings of each extra Z to all of the relevant chiral fermions. We will therefore concentrate on the specific types of Z_2 bosons expected in SO(10) and E_6 grand unified theories and $SU(2)_L \times SU(2)_R \times U(1)$ theories. These are particularly well motivated theoretically, especially in view of the successful predictions of supersymmetric grand unified theories for low-energy coupling constants [15,28]. For comparison, we will also discuss limits on a heavy Z' boson with the same couplings as the ordinary Z, although such a boson is not expected in gauge theories. For simplicity, we assume that only one new boson is light enough to be relevant. All of the cases we consider will be family universal and have no flavor-changing vertices.

For each type of Z_2 boson (i.e., for each set of gauge couplings) we consider three models, which differ in the assumptions concerning the quantum numbers of the Higgs fields which generate the Z-boson mass matrix. In each case, there is a relation [29]

$$\tan^2\theta = \frac{M_0^2 - M_1^2}{M_2^2 - M_0^2} \tag{5}$$

between Z_1^0-Z_2^0 mixing angle θ and the two mass eigenvalues M_1 and M_2, where M_0 is the prediction for the Z mass in the absence of mixing. The least constrained (ρ_0-free) model makes no assumption concerning the Higgs sector. It allows arbitrary SU(2) representations for the Higgs fields, and is the analog of allowing $\rho_0 \neq 1$ in the SU(2)×U(1) model. In this case at tree level $M_0 = M_W/\sqrt{\rho_0}\cos\theta_W$ with ρ_0 arbitrary, so that M_0 is not known independently. Then M_1, M_2, and θ are all free parameters, along with $\sin^2\hat{\theta}_W$ and m_t. If one assume that all SU(2) breaking is due to Higgs doublets ($\rho_0 = 1$ models), then, at the tree level, $M_0 = M_W/\cos\theta_W$ (in practice, radiative corrections to the formula for M_0 must be included). Since M_W is a known function of $\sin^2\hat{\theta}_W$ and m_t, M_1 can be eliminated and there are only four parameters, M_2, θ, $\sin^2\hat{\theta}_W$, and m_t. Finally, in specific models one specifies not only the SU(2) assignments but the U_1' assignments of the Higgs fields. Since the same Higgs multiplets generate both M_1 and θ, one has an additional constraint

$$\theta = C\frac{g_2}{g_1}\frac{M_1^2}{M_2^2} , \tag{6}$$

where g_1 and g_2 are the coupling constants of the two U(1)'s, so that θ and M_2 are not independent. In some cases C is a definite number and in others it spans a finite range, depending on the ratios of Higgs vacuum expectation values (VEV's).

In Sec. II we will detail the Z pole, weak neutral current, W mass, and other experimental inputs, the models considered, and the formalism used for describing the effects of an extra Z.

The results are given in Sec. III. The indirect high-precision data strongly restrict the mixing angle θ, with $|\theta| < 0.01$ in most cases. For the ρ_0-free and $\rho_0 = 1$ models, the indirect and direct constraints on the mass M_2 are generally comparable but rather weak, typically $M_2 > 160$–400 GeV. However, in specific models in which there is a relation between θ and M_2, Eq. (6), the limits on M_2 are driven by those on θ and are usually much stronger ($M_2 > 500$–1000 GeV). These results should be compared with the rather weak limit on the extra charged W in general $SU(2)_L \times SU(2)_R \times U(1)$ models, $M_{W_2} > 300$ GeV [30], and the much stronger limit, $M_{W_2} > 1.4$ TeV [31] in specific models with left-right symmetry. It is also found that the weak angle is determined in the presence of the extra Z_2's considered almost as well as in the standard model,

$$\sin^2\hat{\theta}_W(M_Z) = 0.2334^{+0.0008}_{-0.0011} , \tag{7}$$

where the uncertainty is dominated by m_t. Similarly, the 95%-C.L. limits $m_t < 182$ GeV (standard model) and $m_t < 310$ GeV [SU(2)×U(1) with $\rho_0 \neq 1$] continue to hold in the SU(2)×U(1)×U′(1) models considered here.

In Sec. IV we discuss the implications of these results. Since the value of $\sin^2\hat{\theta}_W(M_Z)$ is essentially the same as in the standard model, the result of [15,28] that the three running gauge couplings meet in a supersymmetric extension of the standard model, but not in the standard model itself, continues to hold in many models with an extra Z_2. This supports the notion of supersymmetric grand unification, provided the unified group breaks directly to SU(3)×SU(2)×U(1)×U′(1). Unlike in SU(2)×U(1), however, one cannot obtain a useful lower bound on the ρ_0 parameter Eq. (4) if one allows for the presence of a heavy Z_2 of arbitrary mass (such limits *could* be obtained if the mass of the Z_2 were known). An extra Z_2 within the allowed $M_2 - \theta$ range could significantly shift the value of the weak charge measured in atomic parity violation, as is hinted at by the present data [12] (though the effect is not statistically significant at present). However, an extra Z_2 in the allowed range makes no significant contribution to the possible anomalies observed in charged-current universality tests [32–35] or in the e^+e^- cross section below the Z pole [36].

In Sec. V we summarize our results.

II. INPUTS AND FORMALISM

The weak neutral current, Z pole, and M_W data used in this analysis are the same as in [15], which in turn was part of an ongoing project, begun in [4], to collect and utilize all relevant precision data. All significant low-energy data on ν-hadron, νe, and l-hadron ($l = e, \mu$) interactions are included, as well as $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ below the Z pole [36]. (We view $e^+e^- \to$ hadrons below the Z pole as more a test of QCD than of the electroweak sector. However, the implications of extra Z_2's will be discussed in Sec. IV). The M_W results used are from CDF [10] and UA2 [11]. For the LEP Z-pole experiments we use a recent average of the ALEPH, DELPHI, L3, and OPAL results [9] which correctly takes common systematic errors into account. (A more recent average [37] gives almost identical results. We continue to use those in [9] for consistency with the SU(2)×U(1) analysis in [15].) The LEP and other recent results are summarized in Table I, which is repeated from [15] for convenience. It is seen that the data are in excellent agreement with standard-model expectations.

In the presence of extra U(1)'s, the neutral-current gauge interactions are [3–7]

$$-L_{\rm NC} = eJ^\mu_{\rm em}A_\mu + \sum_{\alpha=1}^{n} g_\alpha J^\mu_\alpha Z^0_{\alpha\mu} \ , \qquad (8)$$

where Z^0_1 is the SU(2)×U(1) boson and Z^0_α, $\alpha \ge 2$, are additional bosons in the weak-eigenstate basis. The g_i are the gauge couplings, with $g_1^2 \equiv g^2 + g'^2 = g^2/\cos^2\theta_W$. For the grand-unified-theory- (GUT-) motivated cases considered here the couplings of the extra Z's are

$$g_2 = \left(\tfrac{5}{3}\right)^{1/2} \sin\theta_W g_1 \lambda_g^{1/2} \ , \qquad (9)$$

where λ_g depends on the symmetry-breaking pattern, but is of order unity [39]. For the case in which the underlying group breaks directly to SU(3)×SU(2)×U(1)×U′(1), one expects $\lambda_g = 1$. The currents in Eq. (8) are

$$\begin{aligned} J^\mu_\alpha &= \sum_i \bar\psi_i \gamma^\mu [\epsilon_L^{(\alpha)}(i) P_L + \epsilon_R^{(\alpha)}(i) P_R] \psi_i \\ &= \tfrac{1}{2} \sum_i \bar\psi_i \gamma^\mu (g_V^{i(\alpha)} - g_A^{i(\alpha)} \gamma_5) \psi_i \ , \end{aligned} \qquad (10)$$

where the sum extends over fermions ψ_i, $P_{L,R} \equiv (1 \mp \gamma^5)/2$, $\epsilon^{(\alpha)}_{L,R}$ are the chiral couplings to Z^0_α, and

$$g^{i(\alpha)}_{V,A} = \epsilon_L^{(\alpha)}(i) \pm \epsilon_R^{(\alpha)}(i) \qquad (11)$$

are the vector and axial-vector couplings. In particular, the SU(2)×U(1) couplings are

$$\begin{aligned} \epsilon_L^{(1)}(i) &= t^i_{3L} - \sin^2\theta_W q_i \ , \\ \epsilon_R^{(1)}(i) &= -\sin^2\theta_W q_i \ , \end{aligned} \qquad (12)$$

where t^i_{3L} and q_i are, respectively, the third component of weak isospin and electric charge of fermion i.

For simplicity, we will restrict ourselves to the case of one additional boson Z^0_2. We consider the following GUT-inspired cases.

(a) Z^0_χ, occurring in SO(10)→SU(5)×U(1)$_\chi$.

(b) Z^0_ψ, occurring in E_6→SO(10)×U(1)$_\chi$.

(c) $Z^0_\eta \equiv \sqrt{3/8}\, Z^0_\chi - \sqrt{5/8}\, Z^0_\psi$, occurring in many superstring-inspired models in which E_6 breaks directly to a rank-5 group.

TABLE I. Experimental values for LEP observables [9], M_W/M_Z [11], M_W [10], the weak charge in cesium Q_W [12,13], and the tree-level $\sin^2\theta^0$ from $\overset{(-)}{\nu}_\mu e \to \overset{(-)}{\nu}_\mu e$ [38], compared with the standard-model predictions for $M_Z = 91.174 \pm 0.021$ GeV, $m_t = 124^{+28}_{-34}$ GeV, and 50 GeV $< M_H <$ 1 TeV. Only the first five LEP observables are independent. The two errors for Q_W(Cs) are experimental and theoretical (in square brackets). The first uncertainty in the predictions is from the uncertainties in M_Z and Δr, the second is from m_t and M_H, and the third (in brackets) is the theoretical QCD uncertainty. The older neutral-current quantities described in [4,15] are also used in the analysis.

Quantity	Value	Standard model
M_Z (GeV)	91.174±0.021	
Γ_Z (GeV)	2.487±0.009	2.485±0.0021±0.008±[0.011]
$\Gamma_{l\bar l}$ (MeV)	83.3±0.4	83.5±0.1±0.2
$R = \Gamma_{\rm had}/\Gamma_{l\bar l}$	20.94±0.12	20.78±0.003±0.016±[0.13]
$A_{\rm FB}(\mu)$	0.0154±0.0048	0.0142±0.0002±0.0015
$\Gamma_{\rm had}$ (MeV)	1744±10	1735±1.6±5.6±[11]
$\Gamma_{\rm inv}$ (MeV)	493±10	499.4±0.3±1.3
N_ν	2.97±0.04±0.04	3
σ^h_p (nb)	41.44±0.28	41.44±0.02±0.02±[0.21]
$\bar g_A^2$	0.250±0.001	0.251±0±0.001
$\bar g_V^2$	0.0013±0.0004	0.0012±0±0.0001
M_W (GeV)	79.91±0.39	80.05±0.03±0.19
M_w/M_Z	0.8831±0.0055	0.8780±0.0001±0.0022
Q_W(Cs)	−71.04±1.58±[0.88]	−73.12±0.08±0.05
$\sin^2\theta^0$	0.240±0.009±0.008	0.232±0.0003±0.001

(d) The general E_6 boson [40]

$$Z^0(\beta)=\cos\beta Z_\chi^0+\sin\beta Z_\psi^0 , \tag{13}$$

where β is a mixing angle. We will take $0\le\beta<\pi$. The χ, ψ, and η are special cases with $\beta=0,\pi/2$, and $-Z_\eta^0=Z^0(\beta=\pi-\arctan\sqrt{5/3})$, respectively.

The chiral couplings are

$$\epsilon_L^{(\alpha)}(i)=Q_\alpha(i),\quad \epsilon_R^{\alpha}(i)=-Q_\alpha(\bar{i}) , \tag{14}$$

where the charges Q are listed in Table II. In these cases λ_g in Eq. (9) are typically in the range $\frac{2}{3}-1$ [39].

We will also consider the $SU(2)_L\times SU(2)_R\times U(1)$ (LR) model [41], for which the boson Z_{LR}^0 orthogonal to the Z_1^0 couples to the current

$$J_{LR}^\mu=\left[\frac{3}{5}\right]^{1/2}\left[\alpha J_{3R}^\mu-\frac{1}{2\alpha}J_{B-L}^\mu\right] , \tag{15}$$

where J_{3R} is the third component of $SU(2)_R$ and B (L) is baryon (lepton) number. J_{3R} is constructed so that all of the right-handed fermions are doublets, and all the left-handed fermions are singlets. Thus,

$$\epsilon_L^{(LR)}(i)=\left[\frac{3}{5}\right]^{1/2}\left[-\frac{1}{2\alpha}\right](B-L)_i , \tag{16}$$

$$\epsilon_R^{(LR)}(i)=\left[\frac{3}{5}\right]^{1/2}\left[\alpha t_{3R}^i-\frac{1}{2\alpha}(B-L)_i\right] . \tag{17}$$

In Eqs. (15) and (17),

$$\alpha=\left[\frac{1-(1+g_L^2/g_R^2)\sin^2\theta_W}{g_L^2/g_R^2\sin^2\theta_W}\right]^{1/2} , \tag{18}$$

where $g_{L,R}$ are the $SU(2)_{L,R}$ gauge couplings. In this case, Eq. (9) holds with $\lambda_g=1$ by construction.

In the special case of left-right symmetry ($g_L=g_R$),

$$\alpha=\left[\frac{1-2\sin^2\theta_W}{\sin^2\theta_W}\right]^{1/2}\simeq 1.53 . \tag{19}$$

TABLE II. Couplings of the Z_χ^0, Z_ψ^0, and Z_η^0 to a 27-plet of E_6. The SO(10) and SU(5) representations are also indicated. The couplings are shown for the left-handed (L) particles and antiparticles. The couplings of the right-handed particles are minus those of the corresponding L antiparticles. The D is an exotic SU(2)-singlet quark with charge $-\frac{1}{3}$. $(E^0,E^-)_{L,R}$ is an exotic lepton doublet with vector SU(2) couplings. N and S are new Weyl neutrinos which may have large Majorana masses.

SO(10)	SU(5)	$2\sqrt{10}Q_\chi$	$\sqrt{24}Q_\psi$	$2\sqrt{15}Q_\eta$
16	$10(u,d,\bar{u},e^+)_L$	-1	1	-2
	$5^*(\bar{d},\nu,e^-)_L$	3	1	1
	$1\bar{N}_L$	-5	1	-5
10	$5(D,\bar{E}^0,E^+)_L$	2	-2	4
	$5^*(\bar{D},E^0,E^-)_L$	-2	-2	1
1	$1S_L^0$	0	4	-5

The LR model is similar to the Z_χ. In fact, the couplings coincide for $\alpha=\sqrt{2/3}\simeq 0.82$ (i.e., $\sin^2\theta_W=\frac{3}{8}$). It is well known that $SU(2)_L\times SU(2)_R\times U(1)$ can be embedded in SO(10). However, in that case, a low (~ 1 TeV) $SU(2)_R$-breaking scale is not realistic, because it predicts much too high a value for $\sin^2\theta_W$ ($\simeq 0.28$) [42]. The simplest realistic versions involve a large $SU(2)_R$-breaking scale, around $10^{10}-10^{11}$ GeV. However, in most versions, the $Z_{L,R}^0$ acquires this same large mass scale and is not relevant to low-energy physics [17]. Nevertheless, the $SU(2)_L\times SU(2)_R\times U(1)$ model is interesting because it can occur outside of the SO(10) context, or in SO(10) models with more complicated symmetry-breaking patterns [43].

For completeness we also consider a heavy Z' with the same couplings as the ordinary Z. Such couplings are not expected in extended gauge theories, but are a useful reference point for comparing the sensitivities of experiments.

After spontaneous symmetry breaking, the n weak-eigenstate bosons Z_α^0 in Eq. (8) are related to the mass-eigenstate bosons Z_α of definite mass M_α by

$$Z_\alpha=\sum_{\beta=1}^{n}U_{\alpha\beta}Z_\beta^0 , \tag{20}$$

where U is an orthogonal matrix. For one extra Z ($n=2$), U is just

$$U=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} . \tag{21}$$

θ, M_1^2, and M_2^2 are obtained by diagonalizing the Z_1^0-Z_2^0 mass matrix:

$$M^2=\begin{bmatrix}2g_1^2\sum_i t_{3i}^2|\langle\phi_i\rangle|^2 & 2g_1g_2\sum_i t_{3i}Q_i|\langle\phi_i\rangle|^2\\ 2g_1g_2\sum_i t_{3i}Q_i|\langle\phi_i\rangle|^2 & 2g_2^2\sum_i Q_i^2|\langle\phi_i\rangle|^2\end{bmatrix}$$

$$\equiv\begin{bmatrix}M_0^2 & c\\ c & M_S^2\end{bmatrix} , \tag{22}$$

where $\langle\phi_i\rangle$ is the VEV of a Higgs field ϕ_i with three-component of weak isospin t_{3i} and J_2^0 charge Q_i. Hence, $M_0^2=M_W^2/\rho_0\cos^2\theta_W$ is the squared Z_1^0 mass in the absence of mixing, where ρ_0 is defined in Eq. (4). For arbitrary M^2, one has the relation Eq. (5) between θ, M_1, M_2, and M_0 (i.e., ρ_0). If there are more than one extra Z, then the right-hand side (RHS) of Eq. (5) becomes an upper bound on the analogue of $\tan^2\theta$ [29].

If one allows completely arbitrary Higgs representations ("ρ_0-free" models), including SU(2) multiplets higher than doublets, then ρ_0 and M_0 are arbitrary, and there is no relation between θ, M_1, and M_2. These, as well as $\sin^2\hat{\theta}_W(M_Z)$ and m_t, are free parameters. (We also consider the effect of varying M_H from 50 GeV to 1 TeV.)

In the "$\rho_0=1$" models we assume that SU(2)-breaking is due to Higgs doublets, so that $\rho_0=1$ and $M_0=M_W/\cos\theta_W$ is a known function of $\sin^2\hat{\theta}_W$, m_t, and

M_H. (One must, of course, incorporate the SM radiative corrections, as is discussed below.) Then, M_1 can be eliminated and the free parameters are θ, M_2, $\sin^2\hat{\theta}_W$, and m_t.

Finally, in specific "minimal-Higgs" models, one specifies not only the SU(2) quantum numbers t_i, t_{3i}, but also the U′(1) charges Q_i of the Higgs fields. In that case, there is an additional relation between θ and the masses. For simplicity, we assume that $M_2 \gg M_1$. For small mixing that implies $M_S^2 \gg M_0^2, |c|$, i.e., that there is an SU(2)-singlet Higgs field with a large VEV. Equation (6) then follows easily, where the coefficient is given by

$$C = -\frac{g_1}{g_2}\frac{c}{M_0^2} = -\frac{\sum_i t_{3i} Q_i |\langle \phi_i \rangle|^2}{\sum_i t_{3i}^2 |\langle \phi_i \rangle|^2} . \tag{23}$$

It is convenient to rewrite this as

$$\hat{\theta} \equiv C \hat{\rho}_2 , \tag{24}$$

where

$$\hat{\theta} \equiv \frac{g_2}{g_1}\theta, \quad \hat{\rho}_2 \equiv \frac{g_2^2}{g_1^2}\rho_2 , \tag{25}$$

with

$$\rho_2 \equiv \frac{M_W^2}{M_2^2 \cos^2\theta_W} \simeq \frac{M_1^2}{M_2^2} . \tag{26}$$

For the E_6 bosons χ, ψ, η, and β, when considering minimal Higgs models, we will assume that the Higgs fields are restricted to 27-plets, as is motivated by most superstring models. These have the same quantum numbers as the fermions in Table II. The neutral color-singlet members of the 27-plet have the same U′(1) charges as the ν, E^0, or $\bar{E}^0$. The expressions for $C = \hat{\theta}/\hat{\rho}_2$ in terms of $x \equiv \langle \phi_\nu \rangle$, $\bar{v} \equiv \langle \phi_{\bar{E}^0} \rangle$, and $v \equiv \langle \phi_{E^0} \rangle$ are listed for the χ, ψ, and η models in Table III. It is usually assumed that $x=0$, because $\langle \phi_\nu \rangle \neq 0$ would cause severe problems with charged-current universality if ϕ_ν were the scalar partner of the ν_e, ν_μ, or ν_τ [$x \neq 0$ could occur for scalar 27-plets that are not the supersymmetric (SUSY) partners of the ordinary fermions]. It is also usually assumed that $\sigma \equiv |\bar{v}/v|^2 > 1$ for the η and ψ models, since the t and b masses are proportional to $\bar{v}$ and v, respectively.

The Z_{LR} must be treated separately. There are two popular choices. In the first [$LR(1)$] one introduces Higgs fields which transform as

$$\Phi = (2,\bar{2},0), \quad \delta_L = (2,1,1/2), \quad \delta_R = (1,2,1/2) \tag{27}$$

with respect to the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ group [41]. These can be accommodated in a 27-plet of E_6, with the neutral components identified as $(\Phi \leftrightarrow E^0, \bar{E}^0)$, $(\delta_L \leftrightarrow \nu)$, and $(\delta_R \leftrightarrow N)$. Φ, δ_L, and δ_R develop VEV's after spontaneous symmetry breaking, leading to the expressions for C in Table III. The LR model is usually considered without supersymmetry, i.e., δ_L is not the SUSY partner of the leptons. In that case there is no restriction on x, and C can vary from $-\sqrt{3/5}/\alpha$ to $\sqrt{3/6}\,\alpha$. The supersymmetric version with $x=0$ has $C = \sqrt{3/5}\,\alpha$.

Another popular model [$LR(2)$] introduces Higgs fields:

$$\Phi = (2,\bar{2},0), \quad \Delta_L = (3,1,1), \quad \Delta_R = (1,3,1) . \tag{28}$$

Δ_L and Δ_R are not in the 27-plet of E_6. Δ_L and Δ_R are popular in nonstring models [44], since they can generate Majorana masses for the neutrinos. We assume that the neutral field in Δ_L has a negligible VEV due to the stringent constraint from the ρ_0 parameter discussed below. Then $C = \sqrt{3/5}\,\alpha$. In the special case

TABLE III. Expressions for $C \equiv \hat{\theta}/\hat{\rho}_2$ in the χ, ψ, and η E_6 models as a function of $x = \langle \phi_\nu \rangle$, $\bar{v} = \langle \phi_{\bar{E}^0} \rangle$, and $v = \langle \phi_{E^0} \rangle$. The values or ranges for $x=0$ and for ($x=0$, $|\bar{v}/v| > 1$) are also shown. The $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model $LR(1)$ can be accommodated in a 27-plet of E_6. The range allows $x \neq 0$, as is relevant for nonsupersymmetric versions. $LR(2)$ invokes Higgs triplets and cannot be accommodated in a 27-plet. $\langle \Delta_L^0 \rangle = 0$ is assumed.

Model	C	$C(x=0)$	$C(x=0, \bar{v}/v>1)$
χ	$\frac{2}{\sqrt{10}}\frac{\lvert\bar{v}\rvert^2+\lvert v\rvert^2-3/2\lvert x\rvert^2}{\lvert\bar{v}\rvert^2+\lvert v\rvert^2+\lvert x\rvert^2}$	$\frac{2}{\sqrt{10}}$	$\frac{2}{\sqrt{10}}$
ψ	$-\sqrt{2/3}\frac{\lvert\bar{v}\rvert^2-\lvert v\rvert^2+\lvert x\rvert^2/2}{\lvert\bar{v}\rvert^2+\lvert v\rvert^2+\lvert x\rvert^2}$	$\sqrt{2/3}[-1,1]$	$\sqrt{2/3}[-1,0]$
η	$\frac{4}{\sqrt{15}}\frac{\lvert\bar{v}\rvert^2-\lvert v\rvert^2/4-\lvert x\rvert^2/4}{\lvert\bar{v}\rvert^2+\lvert v\rvert^2+\lvert x\rvert^2}$	$\frac{4}{\sqrt{15}}\left[-\frac{1}{4},1\right]$	$\frac{4}{\sqrt{15}}\left[\frac{3}{8},1\right]$

Model	C	Range
$LR(1)$	$\sqrt{3/5}\,\alpha\frac{\lvert\bar{v}\rvert^2+\lvert v\rvert^2-\lvert x\rvert^2/\alpha^2}{\lvert\bar{v}\rvert^2+\lvert v\rvert^2+\lvert x\rvert^2}$	$\sqrt{3/5}[-1/\alpha,\alpha] \underset{\text{SUSY}}{\rightarrow} \sqrt{3/5}\,\alpha$
$LR(2)$	$\sqrt{3/5}\,\alpha$	$\sqrt{3/5}\,\alpha \underset{g_L=g_R}{\rightarrow} \sqrt{3/5}\frac{\sqrt{\cos 2\theta_W}}{\sin\theta_W}$

of left-right symmetry ($g_L = g_R$), this becomes $\sqrt{3/5}\sqrt{\cos 2\theta_W}/\sin\theta_W$, so that $\Theta/\rho_2 = g_2 C/g_1 = \sqrt{\cos 2\theta_W} \simeq 0.735$.

The presence of extra Z_2 bosons changes the standard-model expressions for observables in a number of ways. In particular, the relation between the Z_1 mass M_1 and $\sin^2\hat{\theta}_W$ is modified by mixing, as in Eq. (5). Low-energy neutral amplitudes are modified by (a) the downward shift in M_1, (b) the change in the Z_1 couplings due to mixing (this is especially important for the vector coupling to charged leptons, which is small in the SM), and (c) by Z_2 exchange. Similarly, Z-pole observables are modified by the changes in the $M_1 - \sin^2\hat{\theta}_W$ relation and in the Z_1 couplings. It should be emphasized that, except for the $M_1 - \sin^2\hat{\theta}_W$ relation, the effects of the extra Z cannot be parametrized by the $S-T$ formalism of [19], which describes types of heavy physics which only affect boson self-energies.

It was shown in [3] that the effective four-Fermi interaction

$$-L_{\text{eff}}^{\text{SM}} = \frac{4G_F}{\sqrt{2}} J_1^2 \tag{29}$$

for low-energy neutral-current interactions in the standard model is replaced by

$$-L_{\text{eff}} = \frac{4G_F}{\sqrt{2}} (\rho_{\text{eff}} J_1^2 + 2w J_1 J_2 + y J_2^2) \tag{30}$$

in the presence of an extra Z_2. The coefficients are defined by

$$\rho_{\text{eff}} = \rho_1 \cos^2\theta + \rho_2 \sin^2\theta \;, \tag{31}$$

$$w = \frac{g_2}{g_1} \cos\theta \sin\theta (\rho_1 - \rho_2) \;, \tag{32}$$

$$y = \left[\frac{g_2}{g_1} \right]^2 (\rho_1 \sin^2\theta + \rho_2 \cos^2\theta) \;, \tag{33}$$

where

$$\rho_1 \equiv \frac{M_W^2}{M_1^2 \cos^2\theta_W} \tag{34}$$

and ρ_2 is defined in Eq. (26).

For small ρ_2 and θ, these reduce to

$$\rho_{\text{eff}} \sim \rho_1, \quad w \sim \hat{\theta}, \quad y \simeq \hat{\rho}_2 \;, \tag{35}$$

where $\hat{\theta}$ and $\hat{\rho}_2$ are defined in Eq. (25). From Eq. (5), we have

$$\rho_1 \simeq \rho_0 \left[1 + \frac{\rho_0 \theta^2}{\rho_2} \right] . \tag{36}$$

(Including SU(2)-breaking loop effects [19], $\rho_0 \to \rho_0/(1-\alpha T)$.) For $\rho_0 = 1$ this reduces to

$$\rho_1 \simeq 1 + \theta^2/\rho_2 \; . \tag{37}$$

$\rho_1 - 1$ is first order in small quantities, because $|\theta|/\rho_2$ is bounded by a number of order unity in all sensible models [cf. Eqs. (23)–(25)]. In the minimal Higgs models, these expressions can be rewritten

$$\rho_{\text{eff}} \simeq \rho_1 \simeq 1 + C^2 \hat{\rho}_2, \quad w \simeq C\hat{\rho}_2, \quad y \simeq \hat{\rho}_2 \; . \tag{38}$$

For example, the effective Lagrangian for neutrino-quark or neutrino-electron interactions is

$$-L_{\text{eff}} = \frac{4G_f}{\sqrt{2}} \bar{\nu} \gamma^\mu P_L \nu J_\mu \;, \tag{39}$$

where

$$J_\mu = \sum_i \epsilon_L(i) \bar{\psi}_i \gamma_\mu P_L \psi_i + \epsilon_R(i) \bar{\psi}_i \gamma_\mu P_R \psi_i \; . \tag{40}$$

In the standard model, the $\epsilon_{L,R}(i)$ are equal to $\epsilon_{L,R}^{(1)}(i)$ defined in Eq. (12), while, including a Z_2,

$$\begin{aligned} \epsilon_{L,R}(i) = {} & \rho_{\text{eff}} \epsilon_{L,R}^{(1)}(i) + w[\epsilon_{L,R}^{(2)}(i) + 2\epsilon_L^{(2)}(\nu)\epsilon_{L,R}^{(1)}(i)] \\ & + 2y \epsilon_L^{(2)}(\nu) \epsilon_{L,R}^{(2)}(i) \; . \end{aligned} \tag{41}$$

Similarly, the effective parity-violating interaction between leptons and quarks is

$$-L^{eH} = -\frac{G_f}{\sqrt{2}} \sum_i (C_{1i} \bar{e} \gamma_\mu \gamma^5 e \bar{q}_i \gamma^\mu q_i + C_{2i} \bar{e} \gamma_\mu e \bar{q}_i \gamma^\mu \gamma^5 q_i) \; . \tag{42}$$

In the standard model,

$$\begin{aligned} C_{1i} &= 2 g_A^{e(1)} g_V^{i(1)} \;, \\ C_{2i} &= 2 g_V^{e(1)} g_A^{i(1)} \;, \end{aligned} \tag{43}$$

for $i = u, d$. Including a Z_2,

$$\begin{aligned} C_{1i} = {} & 2\rho_{\text{eff}} g_A^{e(1)} g_V^{i(1)} + 2w(g_A^{e(1)} g_V^{i(2)} + g_A^{e(2)} g_V^{i(1)}) \\ & + 2y g_A^{e(2)} g_V^{i(2)} \;, \end{aligned} \tag{44}$$

$$\begin{aligned} C_{2i} = {} & 2\rho_{\text{eff}} g_V^{e(1)} g_A^{i(1)} + 2w(g_V^{e(1)} g_A^{i(2)} + g_V^{e(2)} g_A^{i(1)}) \\ & + 2y g_V^{e(2)} g_A^{i(2)} \; . \end{aligned} \tag{45}$$

Expression for e^+e^- observables below the Z pole are given in [6].

The tree-level expression for the partial width for $Z \to f_i \bar{f}_i$, where $f_i = \nu_e, e^-, u, d, \ldots$ is given in SU(2)×U(1) models by

$$\Gamma_{Z \to f_i \bar{f}_i} = \frac{\rho_0 C_i G_F M_Z^3}{6\pi\sqrt{2}} (V_i^2 + A_i^2) \; . \tag{46}$$

In Eq. (46), $V_i \equiv g_V^{i(1)}$, $A_i \equiv g_A^{i(1)}$, $C_i = 1$ (3) for leptons (quarks) is the color factor, and the f_i mass has been ignored. (In practice, the masses of the heavier fermions must be included.) The coefficient comes about by rewriting

$$\frac{g^2 M_Z}{8\cos^2\theta_W} = \rho_0 \frac{G_F M_Z^3}{\sqrt{2}} \tag{47}$$

at the tree level. The second form incorporates the bulk of the radiative corrections, except for the m_t dependence. Including Z_1^0-Z_2^0 mixing, Eq. (46) is replaced by

$$\Gamma_{Z\to f_i\bar{f}_i}=\frac{\rho_1 C_i G_F M_1^3}{6\pi\sqrt{2}}(V_i^2+A_i^2)\ , \tag{48}$$

where now

$$\begin{aligned} V_i&=\cos\theta g_V^{i(1)}+\frac{g_2}{g_1}\sin\theta g_V^{i(2)}\simeq g_V^{i(1)}+\hat{\theta}g_V^{i(2)}\ ,\\ A_i&=\cos\theta g_A^{i(1)}+\frac{g_2}{g_1}\sin\theta g_A^{i(2)}\simeq g_A^{i(1)}+\hat{\theta}g_A^{i(2)}\ . \end{aligned} \tag{49}$$

Hence, mixing is manifested by the change in the coefficient ($\rho_0 M_Z^3\to\rho_1 M_1^3$), the explicit θ dependence in the vector and axial-vector couplings, and in the M_1-$\sin^2\hat{\theta}_W$ relation.

The tree-level expressions for asymmetries on the Z pole depend on the vector and axial-vector couplings V_i and A_i [45]. For example, neglecting small interference terms,

$$A_{\rm FB}(i)=\frac{3}{4}\frac{2V_eA_e}{(V_e)^2+(A_e)^2}\frac{2V_iA_i}{(V_i)^2+(A_i)^2}\ , \tag{50}$$

$$A_{\rm pol}(i)=\frac{2V_iA_i}{(V_i)^2+(A_i)^2} \tag{51}$$

are, respectively, the forward-backward asymmetry and final polarization in $e^+e^-\to f_i\bar{f}_i$, while the initial-state polarization asymmetry is

$$A_{LR}=\frac{2V_eA_e}{(V_e)^2+(A_e)^2}\ . \tag{52}$$

These formulas are unchanged by the existence of a Z_2, provided one uses the expressions in Eq. (49) for V_i and A_i and the correct M_1-$\sin^2\hat{\theta}_W$ relation. (The contributions of Z_2 exchange are negligibly small on or near the Z_1 pole [45,6].)

The equations above are valid at tree level only. It is necessary to include the full standard-model radiative corrections to all observables. For example, M_0 in Eq. (5) must be identified as the full SU(2)×U(1) expression for the Z mass, including radiative corrections, rather than just the tree-level formula $M_W/\sqrt{\rho_0}\cos\theta_W$ [46]. Similarly, we include the standard-model radiative corrections to WNC amplitudes, partial Z widths, and Z-pole asymmetries as is described in [15]. The top-quark mass is left as a free parameter except for the direct CDF constraint $m_t>89$ GeV [26]. We assume a Higgs-boson mass 50 GeV $<M_H<1$ TeV with a central value of 250 GeV and ignore the possible effects of additional Higgs fields on the radiative corrections.

We neglect any new radiative corrections due to the extra Z_2 boson. This is plausible, since we are searching for small tree-level effects of order θ or M_1^2/M_2^2, and changes in the radiative corrections are suppressed by an additional factor of α. This point has been quantified by Degrassi and Sirlin [47], who show that the dominant Z_2 loop effects are box diagrams in muon decay. These modify slightly the relation between the Fermi constant, extracted from muon decay, and the W and Z masses. However, for the allowed values of M_2 that we will obtain in Sec. III, these effects are negligible.

TABLE IV. Branching ratios (%) into e^+e^- for χ, ψ, η, LR, and Z' (heavy Z), assuming that three full 15-plet, 16-plet, or 27-plet channels are open. Fermion masses are ignored for open channels. The numbers in parentheses assume $M_2<2m_t$.

	Z_χ	Z_ψ	Z_η	Z_{LR}	Z'
15-plet	6.1(6.3)	4.4(5.1)	3.7(4.5)	2.5(2.8)	3.0(3.3)
16-plet	4.2(4.3)	4.2(4.8)	2.4(2.7)	1.9(2.1)	
27-plet	2.8(2.8)	0.93(0.95)	0.93(0.97)		

For the gauge coupling of the Z_2, we will use Eq. (9) with $\lambda_g=1$. From Eqs. (32), (33), (49), it is apparent that to leading order all observables depend on the combination $\hat{\theta}=g_2\theta/g_1$, and on g_1M_2/g_2. Hence, limits quoted on θ and M_2 for $\lambda_g=1$ apply to $\lambda_g^{1/2}\theta$ and $M_2/\lambda_g^{1/2}$ for other values of λ_g.

To compare with the direct CDF limit on $\bar{p}p\to Z_2\to e^+e^-$ in Eq. (1), we calculate the production cross section using the expressions in [1], with the appropriate Z_2 couplings to u and d quarks in each case. The branching ratio into e^+e^- depends on how many decay channels are open. As is shown in Table II, there are a number of exotic fermions in each 27-plet of E_6, and these could be light enough to be produced in Z_2 decays. In Table IV, we show $B(Z_2\to e^+e^-)$ for three representative cases: (a) "15-plet" model, in which it is assumed that only the three standard-model family channels (three 15-plets) are open; (b) "16-plet," in which three families of SO(10) 16-plets (i.e., the 15-plet plus the $\bar{N}_L$) are light; (c) "27-plet," in which three full E_6 27-plets are assumed

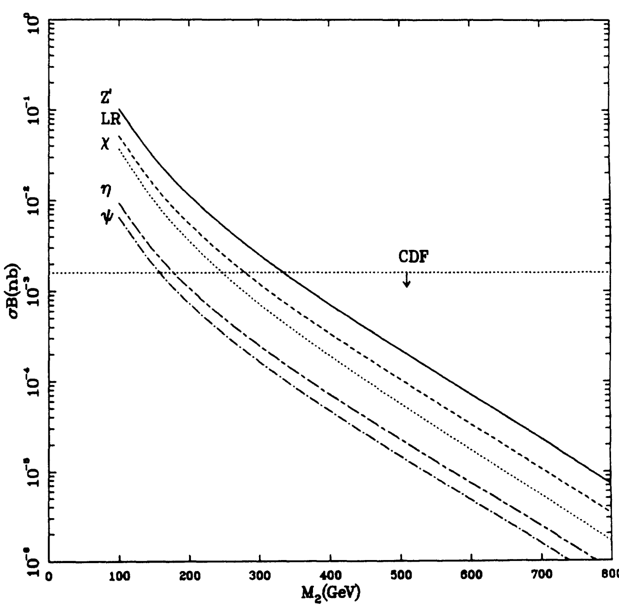

FIG. 1 Predicted values of $\sigma(\bar{p}p\to Z_2)B(Z_2\to e^+e^-)$ as a function of M_2 for the χ, ψ, η, LR, and Z' bosons at $\sqrt{s}=1800$ GeV. The preliminary CDF limit $\sigma B<1.6$ pb (95% C.L.) is also shown. The branching ratios assume that three 27-plet channels are open for Z_ψ and Z_η, three 16-plets for Z_χ, Z_{LR}, and three 15-plets for the Z'.

TABLE V. Lower limits on the mass M_2 of an extra Z boson for the Z_χ, Z_ψ, Z_η, Z_{LR}, and Z' models. The indirect limits are for the ρ_0=free, $\rho_0=1$, and for minimal Higgs models with $\sigma=0,1,5,\infty$. Both the 95%-C.L. and 90%-C.L. (in parentheses) limits are given. In all cases, $m_t>89$ GeV and $\sin^2\hat{\theta}_W$ are free parameters. The direct limits are 95% C.L. based on the CDF result in Eq. (1), assuming $B(Z_2\to e^+e^-)$ for models with three 15-plets, 16-plets, and 27-plets. The 27-plet limits for the Z_ψ and Z_η assume that the $Z_2\to t\bar{t}$ channel is closed.

	Z_χ	Z_ψ	Z_η	Z_{LR}	Z'
ρ_0=free	322(343)	158(172)	181(194)	389(419)	756(824)
$\rho_0=1$	321(340)	160(174)	182(195)	389(421)	779(855)
$\sigma=0$	553(592)	585(624)	211(221)	857(914)	
$\sigma=1$	553(592)	163(178)	483(531)	857(914)	
$\sigma=5$	553(592)	716(785)	773(842)	857(914)	
$\sigma=\infty$	553(592)	912(1000)	923(1004)	857(914)	
15-plet	273	250	266	299	335
16-plet	248	246	236	281	
27-plet	224	160	182		

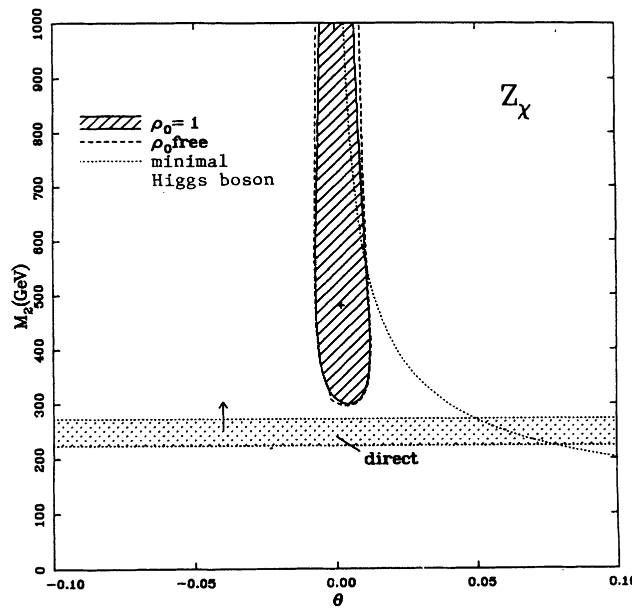

FIG. 2. 90%-C.L. ($\Delta\chi^2=4.6$) allowed region in M_2 and θ for the SO(10) boson Z_χ using indirect (WNC, $M_{W,Z}$, Z-pole) data for the cases ρ_0 free and $\rho_0=1$. m_t and $\sin^2\hat{\theta}_W$ are free parameters and $\lambda_g=1$ is assumed. Also shown are the additional constraint in the minimal Higgs case, best fit point, and the 95%-C.L. lower limit on M_2 from the direct CDF search, for $B(Z_2\to e^+e^-)$ corresponding to three 15-plets (upper end of range) and three 27-plets (lower end).

to be much lighter than the Z_2. The branching ratios for each case are shown in Table IV, assuming for simplicity that the fermion masses for all open channels (including $Z_2\to t\bar{t}$) are negligible. The branching ratios for a heavy $m_t>M_2/2$ are also shown. The predicted values of $\sigma(\bar{p}p\to Z_2)B(Z_2\to e^+e^-)$ for the χ, ψ, η, LR, and Z' for typical branching ratios are shown as a function of M_2 in Fig. 1, along with the preliminary CDF limit. From such curves one can read off the direct production limits on M_2.

III. RESULTS

Our results are summarized in Figs. 2–9 and Tables V–VIII. Figures 2–6 show the 90%-C.L. allowed contours in θ-M_2 for the χ, ψ, η, LR, and Z' bosons for two classes of assumptions concerning the Higgs structure (ρ_0 free, and $\rho_0=1$) described in Sec. II, assuming $\lambda_g=1$ in Eq. (9). The additional constraints in the minimal Higgs models are also shown, for $x=0$ and various values of $\sigma\equiv|\bar{v}|^2/|v|^2$ (Table III). $\sin^2\hat{\theta}_W$ and $m_t>89$ GeV (as well as M_1 in the ρ_0=free case) are left as free parameters. It is seen that the limits on θ are quite stringent, except the Z_η. In all cases, θ is constrained far better than in [4,5], mainly due to the LEP and atomic parity violation experiments. The M_2 limits are also improved somewhat compared to [4,5], but the differences are smaller. The limits are fairly strong for the χ and LR models, with the indirect constraints (WNC, $M_{W,Z}$, Z pole) stronger than the direct production limits. The ψ and η limits are much weaker, with the direct and indirect constraints comparable. The Z' (heavy Z) constraints are considerably stronger, due to the fact that the Z' couplings to ordinary fermions are larger than those of the other bosons. In all cases the best fit (indicated in the figures) occurs for finite M_2, but the standard model ($\theta=0$, $M_2=\infty$) is well within the 90%-C.L. contours. It

TABLE VI. Best fit values and 95%-C.L. upper ($\theta_{\max}$) and lower ($\theta_{\min}$) limits on the mixing angle θ for the ρ_0=free and $\rho_0=1$ models. The numbers in parentheses are the uncertainties in the best-fit values.

	Z_χ	Z_ψ	Z_η	Z_{LR}	Z'
ρ_0=free, θ	0.0012(50)	0.0040(58)	−0.021(12)	0.0018(41)	−0.0038(29)
$\theta_{\min}$	−0.0070	−0.0060	−0.038	−0.0048	−0.0087
$\theta_{\max}$	+0.0094	+0.012	+0.002	+0.0079	+0.0020
$\rho_0=1,\theta$	0.0019(44)	0.0047(47)	−0.021(11)	0.0024(32)	−0.0036(32)
$\theta_{\min}$	−0.0048	−0.0025	−0.038	−0.0025	−0.0086
$\theta_{\max}$	+0.0097	+0.013	−0.002	+0.0083	+0.0005

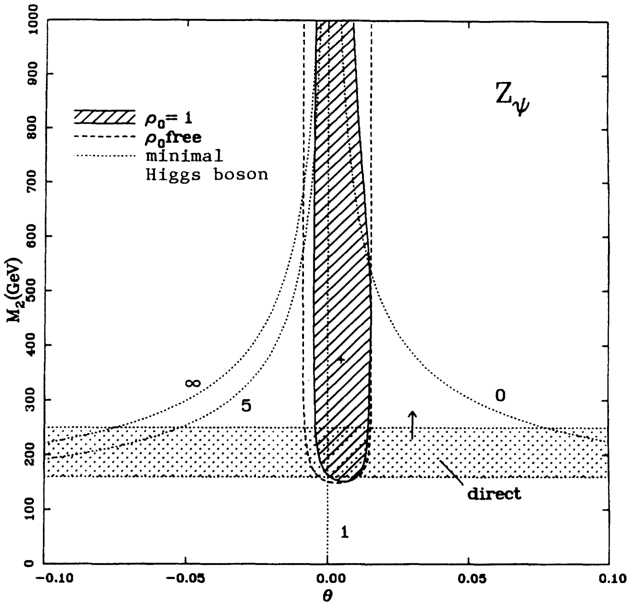

FIG. 3. Same as Fig. 2, only for the E_6 boson Z_ψ. In the miminal Higgs case the constraints are for $\sigma=0,1,5,\infty$.

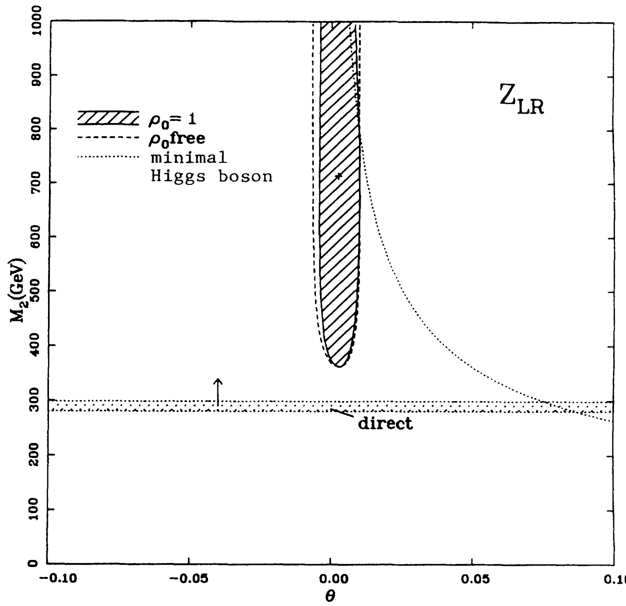

FIG. 5. Same as Fig. 2, only for the $SU(2)_L \times SU(2)_R \times U(1)$ boson Z_{LR}. The direct limits are for three 15-plets and three 16-plets.

is also apparent from Figs. 2–5 that in most cases the limits on M_2 in the minimal Higgs models are much stronger than in the more general cases because small masses imply large mixing angles. The C parameter in Eq. (23) is independent of σ for the Z_χ and Z_{LR}, but spans a finite range for the Z_ψ and Z_η. In the latter cases, one expects $\sigma > 1-2$, but the full range from $0-\infty$ is shown for completeness. All of the mass and mixing angle limits are summarized in Tables V and VI, respectively. Results are shown for the general E_6 boson $Z(\beta)$ in Figs. 7 and 8 as a function of $\cos\beta$.

The standard-model value of the $(\overline{\mathrm{MS}})$ weak-angle $\sin^2\hat{\theta}_W(M_Z)=0.2334\pm0.0008$ [15], where the uncertainty is dominated by m_t, is hardly affected by extended $SU(2)\times U(1)$ models with $\rho_0\neq 1$ [Eq. (3)]. In fact, even allowing extra Z_2 bosons the central value and uncertainties in $\sin^2\hat{\theta}_W$ are almost the same as in the standard model, as can be seen in Table VII; there are now so many precision experiments, each of which has different dependences on $\sin^2\hat{\theta}_W$, θ, M_2, etc., that there is little room for compensation between the effects of θ, for ex-

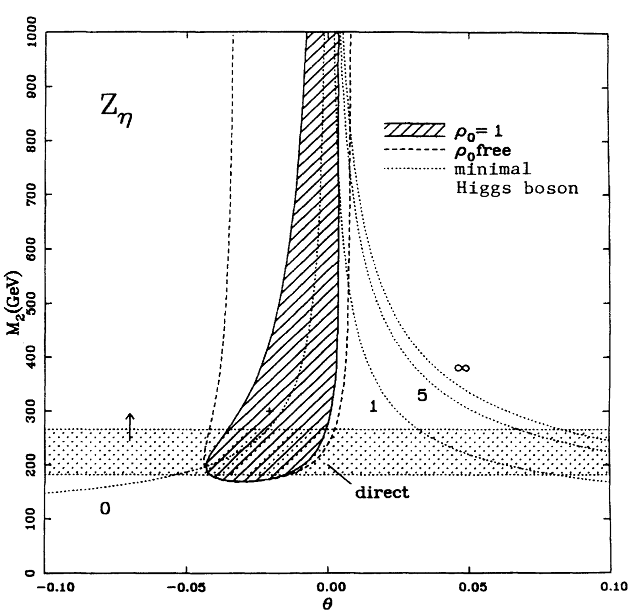

FIG. 4. Same as Fig. 3, only for Z_η.

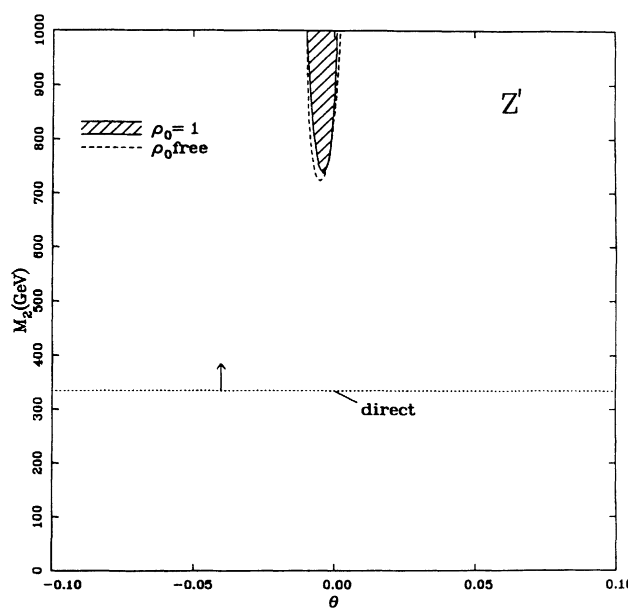

FIG. 6. Same as Fig. 2, only for the heavy Z' with the same couplings as the Z. The direct constraint assumes three 15-plets. There is no analogue of the minimal Higgs case.

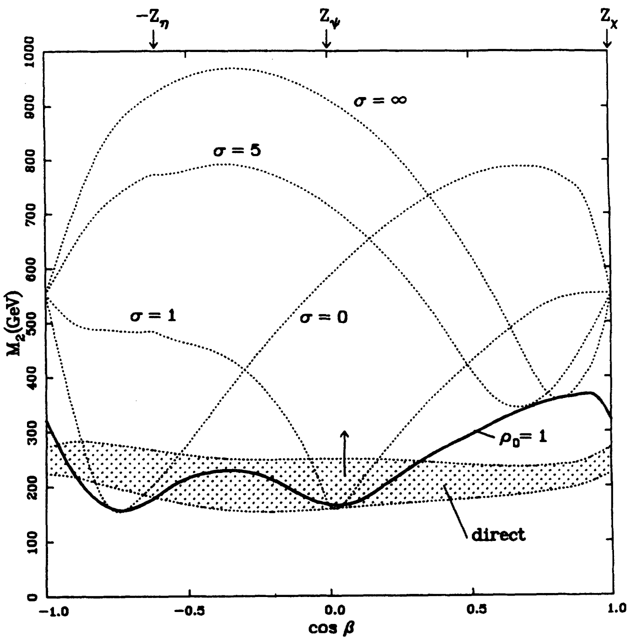

FIG. 7. 95%-C.L. lower limits of M_2 for the general E_6 boson $Z(\beta)$ defined in Eq. (13) as a function of $\cos\beta$. The solid line is the indirect limit in the $\rho_0=1$ case. (The ρ_0=free limit is almost identical.) The dotted lines are the limits in minimal Higgs models for various values of σ. The band is the direct limit for branching ratios ranging from three 15-plets (upper edge) to three 27-plets (lower edge). The special cases Z_χ, Z_ψ, and $-Z_\eta$ are indicated.

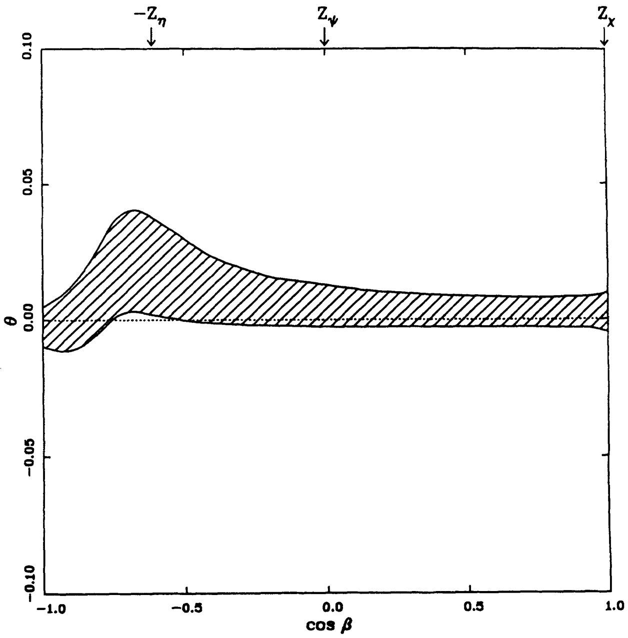

FIG. 8. 95%-C.L. upper and lower limits on the mixing angle θ as a function of $\cos\beta$.

ample, and $\sin^2\hat{\theta}_W$. Altogether, within the framework of SU(2)×U(1) and models involving the extra GUT and LR Z_2's considered here, and allowing an arbitrary Higgs sector (ρ_0), $\sin^2\hat{\theta}_W$ is determined to be

$$\sin^2\hat{\theta}_W(M_Z)=0.2334^{+0.0008}_{-0.0011} . \tag{53}$$

Again, most of the uncertainty is from m_t.

Similarly, the upper limits on m_t are the same as or slightly more stringent than the limits in SU(2)×U(1), for both the ρ_0=free and $\rho_0=1$ cases, as is shown in Table VIII. Both Z_1^0-Z_2^0 mixing and a large m_t lead to a smaller value of M_1 for a given $\sin^2\hat{\theta}_W$, so the effects cannot cancel each other. In particular, for $\rho_0=1$ mixing implies the presence of a factor $\rho_1\simeq 1+\theta^2/\rho_2\geq 1$ in neutral-current amplitudes and $\Gamma_Z\to f_i\bar{f}_i$, and ρ_1^{-1} in the formula for M_1^2. Including radiative corrections, ρ_1 is everywhere replaced by $\rho_1(1+\Delta\rho_t)$, where [48]

$$\Delta\rho_t=\frac{3G_F m_t^2}{8\sqrt{2}\pi^2}\simeq 0.0031\left[\frac{m_t}{100\text{ GeV}}\right]^2 \tag{54}$$

is positive. Clearly, the possible existence of $\rho_1>1$ strengthens the upper limit on $\Delta\rho_t$, while for large m_t the constraints on θ are more stringent. This is illustrated for the Z_χ model in Fig. 9. ρ_1 can be <1 for general ρ_0 and can compensate the quadratic m_t effects in $\rho_1(1+\Delta\rho_t)$. However, the logarithmic m_t dependence in the M_W and M_0 formulas and additional vertex corrections quadratic in m_t in $\Gamma_{Z\to b\bar{b}}$ allow a separation of ρ_1 from m_t, analogous to Eq. (3) in SU(2)×U(1), leading to the limits in Table VIII.

IV. IMPLICATIONS

It was shown in [15,28] that the three observed gauge couplings do not meet at a single unification scale if they are evolved to short distances using the standard-model particles in the β functions. However, they do meet within uncertainties in the minimal SUSY extension of the standard model, supporting the possibility of supersymmetric grand unification at $M_X\sim 10^{16}$ GeV. Equivalently, in the SUSY standard model one can use the observed α/α_s to predict $\sin^2\hat{\theta}_W(M_Z)=0.2324\pm0.0030\pm0.0010$, where the first error is mainly from $\alpha_s(M_Z)$, and the second from the masses of the new SUSY particles in the range M_Z-1 TeV. This is in striking agreement with the experimental value of 0.2334 ± 0.0008. There are other possibilities as well, such

TABLE VII. $\sin^2\hat{\theta}_W$ values determined from the data allowing for the existence of a Z_2, compared to the SU(2)×U(1) values. The uncertainties (in parentheses) include m_t and M_H.

	Z_χ	Z_ψ	Z_η	Z_{LR}	Z'	SU(2)×U(1)
ρ_0=free	0.2333(8)	0.2330(9)	0.2328(8)	0.2331(9)	0.2319(14)	0.2333(8)
$\rho_0=1$	0.2333(8)	0.2330(8)	0.2327(8)	0.2331(8)	0.2321(14)	0.2334(8)

TABLE VIII. 95%- (90%-) C.L. upper limits on m_t for the Z_χ, Z_ψ, Z_η, Z_{LR}, Z' models for the cases ρ_0=free and $\rho_0=1$, compared to the limits obtained assuming SU(2)×U(1). The direct CDF limit $m_t > 89$ GeV is incorporated in the analysis. The limits in the minimal Higgs models are similar to the $\rho_0=1$ case.

	Z_χ	Z_ψ	Z_η	Z_{LR}	Z'	$SU_2 \times U_1$
ρ_0=free	300(282)	307(290)	294(275)	309(292)	307(290)	310(294)
$\rho_0=1$	182(174)	182(172)	167(156)	181(172)	183(175)	182(174)

as an ordinary (non-SUSY) SO(10) breaking first to $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ at $M_X \sim 10^{16}-10^{17}$ GeV, and then to the standard model at an intermediate scale $M_R \sim 10^{10}-10^{11}$ GeV. However, M_R introduces an extra parameter in this case, so that $\sin^2\hat{\theta}_W$ is an input rather than a prediction.

As discussed in Sec. III, the experimental value of $\sin^2\hat{\theta}_W$ is hardly affected by the possible existence of the extra Z_2's considered here. However, one must consider whether the $\sin^2\hat{\theta}_W$ predictions of ordinary and SUSY GUT's are modified. The gauge-boson contributions to the SU(3)×SU(2)×U(1) β functions are not affected by the extra U′(1) in the χ, ψ, η, and β models, provided the grand unification (GUT) group breaks directly to (SUSY) SU(3)×SU(2)×U(1)×U′(1) at the unification scale. However, the extra fermions and scalars that may be present (Table II) may be problematic.

Within the ordinary or SUSY standard model the $\sin^2\hat{\theta}_W$ predictions are independent of the number of fermion families (except for small two-loop effects), but depend sensitively on the number of light Higgs doublets. For example, the successful SUSY standard-model prediction was for two doublets, while $\sin^2\hat{\theta}_W$ would be much too high ($\simeq 0.254$) for four doublets. This can be traced to the fact that a complete SU(5)-plet of fermions or scalars, such as a fermion 15-plet family (15=reducible $\mathbf{5}^* + \mathbf{10}$), contributes equally to the β function for each normalized α_i and therefore to the slope of $\alpha_i^{-1}(\ln\mu)$. This changes the value of α_i^{-1} at M_X, but not whether the three couplings meet or the $\sin^2\hat{\theta}_W$ prediction. On the other hand, a scalar doublet is not part of a complete SU(5)-plet. It contributes differently to the three β functions and therefore changes the predictions significantly.

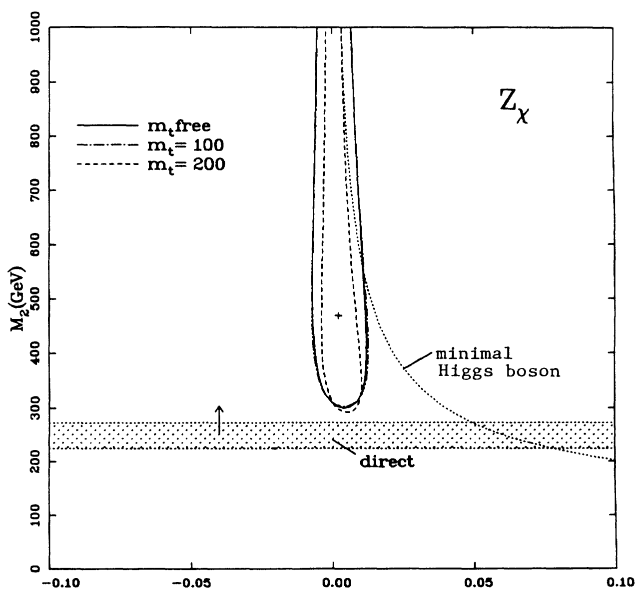

FIG. 9. 90%-C.L. allowed regions in M_2 and θ for the $\rho_0=1$ Z_χ model, for $m_t=100$ and 200 GeV, and for m_t a free parameter. The m_t free and $m_t=100$ GeV contours are almost indistinguishable, while the allowed θ range is much smaller for $m_t=200$ GeV.

In most SO(10) and E_6 models, each fermion family is assigned to a 16- or 27-plet (Table II). The extra SU(5) singlets have no effect on the SU(3)×SU(2)×U(1) couplings. As long as the members of each new SU(5) 5-plet are approximately degenerate [e.g., all light ($M_Z - 1$ TeV) or all superheavy ($\simeq M_X$)] they will not affect the $\sin^2\hat{\theta}_W$ predictions at one-loop level. There is no reason to expect such multiplet splittings, so the predictions for ordinary and SUSY unification are the same whether the light fermions fall into complete 15-plets, 16-plets, or 27-plets.

In realistic GUT's, however, the scalar multiplets must split into light sectors, including the Higgs doublets, and superheavy sectors, because the latter include colored scalars that can mediate proton decay. This apparently unnatural multiplet splitting is one aspect of the hierarchy problem that has plagued grand unified theories [49]. From Table II, each scalar 27-plet contains three colorless SU(2) doublets. The coupling constant predictions of the SUSY standard model will be preserved only if not only the colored scalars but also all but two of the color-singlet SU(2) doublets are superheavy.

Thus, the ordinary (SUSY) standard-model predictions of $\sin^2\hat{\theta}_W$ and unification continue to hold in the χ, ψ, η, and β models provided they break directly to SU(3)×SU(2)×U(1)×U′(1) and there are only one (two) light Higgs doublets.

The situation is somewhat different for the LR model, which assumes that the $SU(2)_R$-breaking scale is of order TeV. If $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is embedded in a non-SUSY SO(10), then in the simplest versions the experimental $\sin^2\hat{\theta}_W$ implies that $SU(2)_R$-breaking and the Z_{LR} mass are of order $M_R \sim 10^{10}-10^{11}$ GeV; much too high for the Z_{LR} to be experimentally relevant. Of course, modified models with more complicated Higgs structure could allow a much lower $SU(2)_R$-breaking scale [43] or a Z_{LR} much lighter than M_R [17].

It has been emphasized that the value of ρ_0 in Eq. (4) is a significant probe of new physics [17]. In SU(2)×U(1) it is possible to place strong constraints on m_t and ρ_0 [or

$\rho_0/(1-\alpha T)$] simultaneously [Eq. (3)]. In SU(2)×U(1)×U′(1) similar constraints can be obtained on ρ_1 and m_t, and a comparable *upper* limit on ρ_0 can be obtained. However, no lower limit on ρ_0 can be obtained. This is clear from Eq. (36). The data require $\rho_1 \sim 1$, but this can be accomplished for $\rho_0 \ll 1$ and small (allowed) values of θ for very small ρ_2 (i.e., very large M_2) [50]. Hence, one cannot at present place significant constraints on nonstandard Higgs representations if one allows for the possible compensating effects of an extra Z_2 of large mass [51]. On the other hand, if a Z_2 is actually discovered, it will eventually be possible to obtain ρ_2 and θ, allowing a determination of ρ_0 (assuming that there is only one extra Z light enough to be relevant).

In Table I it is seen that the experimental value [12] $Q_W^{\rm expt}=-71.04\pm1.58\pm0.88$ (the second error is theoretical [13]) of the weak charge in cesium is somewhat smaller in magnitude than the standard-model prediction:

$$Q_W^{\rm SM}=-376C_{1u}-422C_{1d}\simeq-73.1 \ , \tag{55}$$

where the coefficients $C_{1u,d}$ are defined in Eqs. (42) and (43). The prediction uses $\sin^2\hat\theta_W$ determined from M_Z, and turns out to be almost independent of m_t and M_H [19]. The discrepancy is not statistically significant ($\sim 1.1\sigma$) at present. However, the experimental uncertainty will soon be reduced considerably, and it is instructive to discuss how the theoretical prediction is modified by an extra Z_2.

We have studied numerically the value of Q_W predicted by the χ, ψ, η, and LR models for points on the 90%-C.L. contours in Figs. 2–5. For the χ and LR, $|Q_W|\le 71$ is predicted on the $\theta<0$ parts of the contours for $M_2 \lesssim 600$ GeV and also for small M_2 with $\theta\sim 0$. Similarly, $|Q_W|<72$ for the Z_η on the $\theta<0$ part of the contour for $250\lesssim M_2\lesssim 550$ GeV. For the other parts of the contours and for the Z_ψ the effects are small or yield $|Q_W|>73.1$. Clearly, a Z_2 boson would be a possible explanation of a small $|Q_W|$ if such were indicated by future experiments.

It is useful to show this analytically. From Eq. (44),

$$C_{1i}=C_{1i}^{\rm SM}+\delta C_{1i}^{\rm dir}+\delta C_{1i}^{\rm ind} \ , \tag{56}$$

where $C_{1i}^{\rm SM}$ is the standard-model prediction for C_{1i} using the weak angle $\sin^2\hat\theta_W^Z$, which is the value of $\sin^2\hat\theta_W$ obtained from the experimental M_1 assuming the validity of the standard model (one can use any convenient m_t, since $Q_W^{\rm SM}$ is almost independent of m_t). $\delta C_{1i}^{\rm dir}$ represents the direct effects on Q_W from the Z_2. From Eq. (44),

$$\delta C_{1i}^{\rm dir}\simeq(\rho_1-1)C_{1i}^{\rm SM}+\theta\frac{g_2}{g_1}(-g_V^{i(2)}+2g_V^{i(1)}g_A^{e(2)}) + 2\rho_2\left[\frac{g_2}{g_1}\right]^2 g_A^{e(2)}g_V^{i(2)} \ , \tag{57}$$

where the three terms are due to the shift in M_1, mixing in the Z_1 vertex, and Z_2 exchange, respectively. The indirect contribution $\delta C_{1i}^{\rm ind}$ is due to the fact that $\sin^2\hat\theta_W^Z$ obtained from M_1 (assuming the validity of the standard model) differs from the true $\sin^2\hat\theta_W$ because of the shift in M_1 from Z_1^0-Z_2^0 mixing. From Eqs. (43) and (12), $C_{1i}^{\rm SM}\sim -g_V^{i(1)}\simeq -t_{3L}^i+2q_i\sin^2\hat\theta_W$, so

$$\delta C_{1i}^{\rm ind}=\frac{-dC_{1i}^{\rm SM}}{d\hat s^2}(\hat s_Z^2-\hat s^2)\simeq-2q_i(\hat s_Z^2-\hat s^2) \ , \tag{58}$$

where $\hat s^2\equiv\sin^2\hat\theta_W$ and $\hat s_Z^2\equiv\sin^2\hat\theta_W^Z$.

Using Eq. (34) and the standard-model expression for M_Z, one has

$$M_1^2=\frac{A^2}{\rho_1\hat c^2\hat s^2}=\frac{A^2}{\hat c_Z^2\hat s_Z^2} \ , \tag{59}$$

where $\hat c^2\equiv1-\hat s^2$, $\hat c_Z^2\equiv1-\hat s_Z^2$, and $A^2=\pi\alpha/\sqrt{2}G_F(1-\Delta\hat r_Z)$, where $\Delta\hat r_Z$ is a radiative correction [52], so that

$$\hat s_Z^2-\hat s^2\simeq(\rho_1-1)\frac{\hat c^2\hat s^2}{\hat c^2-\hat s^2}\sim0.34(\rho_1-1) \ . \tag{60}$$

From Eqs. (55)–(60) one obtains $Q_W=Q_W^{\rm SM}+\Delta Q_W$, with

$$\Delta Q_W=(\rho_1-1)(Q_W^{\rm SM}+73.8)-\theta\frac{g_2}{g_1}(Q_W^{(2)}+2g_A^{e(2)}Q_W^{\rm SM}) + 2\rho_2\left[\frac{g_2}{g_1}\right]^2 g_A^{e(2)}Q_W^{(2)} \ , \tag{61}$$

where

$$Q_W^{(2)}\equiv-375g_V^{u(2)}-422g_V^{d(2)} \tag{62}$$

is the effective vector coupling of the Z_2 to cesium. The first term includes the ρ_1-1 part of $\delta C_{1i}^{\rm dir}$ (the $Q_W^{\rm SM}$ term) as well as $\delta C_{1i}^{\rm ind}$ (the 73.8). Since $Q^{\rm SM}\sim-73.1$ and $|\rho_1-1|\lesssim0.02$, the ρ_1-1 term is numerically irrelevant. (This is the same cancellation that makes $Q_W^{\rm SM}$ insensitive to m_t and Q_W almost independent of the T parameter [19].) Dropping this term,

$$\Delta Q_W\sim\gamma_1\theta+\gamma_2\rho_2 \ , \tag{63}$$

where the coefficients γ_1 and γ_2 can be read off from Eq. (61). The numerical values of $Q_W^{(2)}$, $g_A^{e(2)}$, γ_1, and γ_2 for the χ, ψ, η, and LR models are given in Table IX.

An important test of the standard model is whether the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix is unitary. One test concerns the prediction $(VV^\dagger)_{11}=1$. One recent analysis yields [34,33],

TABLE IX. Values of the parameters $Q_W^{(2)}$, $g_A^{e(2)}$, γ_1, and γ_2 relevant to atomic parity violation for the χ, ψ, η, and LR models, computed for $\sin^2\theta_W=0.2334$, $\lambda_g=1$ [Eq. (9)], and $\alpha=1.53$ [Eq. (18)]. The minimum values for $|Q_W|$ within the 90%-C.L. allowed regions in Figs. 2–5 are also listed.

	Z_χ	Z_ψ	Z_η	Z_{LR}
$Q_W^{(2)}$	267	0	163	162
$g_A^{e(2)}$	0.316	0.408	−0.129	0.593
γ_1	−138	37.2	−114	−46.9
γ_2	65.6	0	−16.4	74.7
$\lvert Q_W\rvert^{\min}$	67.5	72.5	71.4	68.6

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9970\pm0.0016\ , \qquad (64)$$

which is about two standard deviations from unity [53]. However, there are still some uncertainties concerning the nuclear mismatch correction. Some idea of the theoretical uncertainties can be obtained by comparing Eq. (64) with another recent analysis [35], which obtained 0.9989 ± 0.0012, which is in better agreement with unitarity. If there really is a discrepancy, it would indicate the presence of new physics that was not taken into account in the extraction of the V_{ij} from the experimental data, such as mixing of the ordinary fermions with a fourth family or with exotic fermions [54], or mixing between the $W^\pm$ and a heavy $W_R^\pm$ with $V+A$ couplings, as in $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models [55].

Marciano and Sirlin showed [32] that the presence of an extra Z_2 boson modifies the experimental values of V_{ij} determined from μ, β, and other decays: if the Z_2 couplings to e_L and d_L are different, then box diagrams involving W and Z_2 exchanges renormalize the coefficients of μ and β decay differently [56]. The result is that the right-hand side (RHS) of Eq. (64) is replaced by $0.9970\pm0.0016+\Delta$, where (in our notation)

$$\Delta=-\frac{5\alpha}{\pi\cos^2\theta_W}\epsilon_L^{(2)}(e)[\epsilon_L^{(2)}(e)-\epsilon_L^{(2)}(d)]\frac{\ln x}{x-1}\ , \qquad (65)$$

where $x=M_2^2/M_W^2$. In Eq. (65) we have ignored Z_1^0-Z_2^0 mixing effects, which are negligible for allowed θ. Using the smallest allowed M_2 for each ($\rho_0=1$) model, one obtains $0\geq\Delta\geq-0.0008$, 0, -0.0003, and -0.0002 for the χ, ψ, η, and LR models, respectively. Except for the Z_χ there are too small to be relevant, and in all of these cases Δ has the wrong sign to account for the discrepancy. In the $Z(\beta)$ model Δ is positive for $-0.395<\cos\beta<0$, but the maximum value (for $\cos\beta\sim-0.202$) is only 0.000 07. Hence, the extra Z_2's considered here are irrelevant to the possible anomaly in the unitarity relation Eq. (64).

Two possible anomalies have been observed in e^+e^- annihilation below the Z pole [36]. The ratio $R_{\mu\mu} \equiv\sigma(e^+e^-\to\mu^+\mu^-)/\sigma_{\rm QED}^{\mu\mu}$ of the cross section for $e^+e^-\to\mu^+\mu^-$ relative to the lowest-order QED result is expected to rise by several percent over the DESY e^+e^- storage ring PETRA–KEK TRISTAN energy range, due to the contribution from the virtual Z. (The pure weak term is larger than the weak-electromagnetic interference term due to the small value of $g_V^{e(1)}$.) For example, one expects $R_{\mu\mu}\sim1.07$ at $\sqrt{s}=60$ GeV. In fact, the PETRA and TRISTAN data do not show this rise and are consistent with $R_{\mu\mu}=1$, though no individual data point is below the standard-model prediction by more than $(1-1.5)\sigma$.

Similarly, the TRISTAN experiments [36] observe a gradual rise in $R_{\rm had}=\sigma(e^+e^-\to{\rm hadrons})/\sigma_{\rm QED}^{\mu\mu}$ above $\sqrt{s}=40$ GeV, due to the effects of the virtual Z. However, several of the measurements obtained $R_{\rm had}$ several percent higher than the standard value expectation, computed using the known values of M_Z and $\sin^2\hat{\theta}_W$, with a typical excess of $\simeq0.2$ in $R_{\rm had}$. Again, the statistical significance of the effect is not compelling [36].

One possible explanation of these effects, if real, would be the existence of a Z' boson [4,57,58,59]. If one uses the observed M_1 in the Z_1 propagator, the effects of the Z' on the standard-model predictions for $R_{\mu\mu}$ and $R_{\rm had}$ are (a) the shift in the value of $\sin^2\hat{\theta}_W$ obtained from M_1 from the true value (this effect is negligible for the $\sin^2\hat{\theta}_W$ allowed ranges in Table VII), (b) the change in the Z_1 couplings due to mixing, and (c) Z_2 exchange. The full expressions for these effects are given in [6]. Here we only give the results.

TABLE X. Shifts in $R_{\mu\mu}$ and $R_{\rm had}$ relative to the standard-model expectations for the χ, ψ, η, and LR models at $\sqrt{s}=60$ GeV, for the minimum values of M_2 at 95% C.L. for $\rho_0=1$ from Table V, and the 95%-C.L. $\theta_{\rm min}$ and $\theta_{\rm max}$ from Table VI. The shifts are small compared to the values $\Delta R_{\mu\mu}\sim-0.07$, $\Delta R_{\rm had}\sim+0.2$ suggested by the data.

		Z_χ	Z_ψ	Z_η	Z_{LR}
$M_2^{\rm min}$	$\Delta R_{\mu\mu}$	−0.014	+0.008	−0.020	+0.002
	$\Delta R_{\rm had}$	+0.050	+0.024	+0.061	+0.005
$\theta_{\rm min}$	$\Delta R_{\mu\mu}$	+0.0003	+0.0004	−0.003	+0.0005
	$\Delta R_{\rm had}$	0	+0.004	−0.050	+0.008
$\theta_{\rm max}$	$\Delta R_{\mu\mu}$	−0.0007	−0.002		−0.002
	$\Delta R_{\rm had}$	−0.0001	−0.022		−0.025

In Table X are listed the shifts $\Delta R_{\mu\mu}$ and $\Delta R_{\rm had}$ from the standard-model predictions for the χ, ψ, η, and LR models at the typical TRISTAN energy $\sqrt{s}=60$ GeV. For each model the shifts are listed for three cases, corresponding to the minimum M_2 for the $\rho_0=1$ model in Table V (with $\theta=0$), and for the $\theta_{\rm min}$ and $\theta_{\rm max}$ for $\rho_0=1$ in Table VI (with $M_2\to\infty$). It is seen that the only significant effects (of the right sign) are for the χ and η models with small M_2 (and $\theta\sim0$ for the η). Even in these cases the largest effects are $\Delta R_{\mu\mu}\sim-0.02$, $\Delta R_{\rm had}\sim0.06$, which are small compared to the values (-0.07 and $+0.2$, respectively) suggested by the data. We therefore conclude that if the e^+e^- anomalies are real, they cannot be accounted for by the types of Z_2 bosons considered in this paper.

V. SUMMARY

Many extensions of the standard-model gauge group allow the possibility of additional neutral Z_2 bosons light enough to be observable in precision experiments or at high-energy colliders. In this paper we have studied the existing constraints from Z-pole experiments, weak neutral-current experiments, M_W, and the direct CDF search at the Tevatron for typical flavor-diagonal and family-universal Z_2 bosons predicted by grand unified theories and $SU(2)_L\times SU(2)_R\times U(1)$ models. Particular attention is paid to the constraints on $\sin^2\hat{\theta}_W$ and m_t in the presence of the extra Z_2, and on the significance of the quantum numbers of the Higgs fields which break the SU(2) symmetry and lead to Z_1^0-Z_2^0 mixing.

The mixing angle θ is now well constrained, mainly by the LEP and atomic parity-violation data, with $|\theta|<0.01$ in most cases. The limits on M_2 from both indirect experiments and collider searches are still fairly weak, typically 160–400 GeV, if one allows an arbitrary Higgs structure, mainly because the LEP Z-pole observables are not sensitive to an extra Z_2 unless it mixes with the ordinary Z. In specific models for the Higgs structure, however, the limits are often much stronger ($M_2 > 500-1000$ GeV), because θ and M_2 are correlated so that a small M_2 would require a large (excluded) $|\theta|$.

The data are now good enough to simultaneously determine $\sin^2\hat{\theta}_W$ and to constrain m_t and the Z_2 parameters, even allowing arbitrary Higgs representations. For the models considered here, $\sin^2\hat{\theta}_W = 0.2334^{+0.0008}_{-0.0011}$, where the uncertainty includes m_t. This is to be compared with the SU(2)×U(1) value 0.2333±0.0008 for an arbitrary Higgs structure. The limits on $|\theta|$ and M_2 allow an arbitrary m_t, except for the direct CDF limit $m_t > 89$ GeV. Similarly, the SU(2)×U(1) limits on m_t ($m_t < 182$ GeV at 95% C.L. for Higgs doublets, and $m_t < 310$ GeV for arbitrary Higgs representations) continue to hold in the presence of the extra Z_2's considered. However, if one allows for the possible existence of a Z_2, one cannot obtain a significant constraint on the ρ_0 parameter, the value of which tests for the existence of SU(2) triplets or higher-dimensional representations of Higgs fields and which is an important constraint on superstring theories. Such constraints could be obtained if a Z_2 were observed and its mass and mixing determined.

The success of the prediction for $\sin^2\hat{\theta}_W$ (i.e., the meeting of the three coupling constants at a common unification scale) continues to hold in the presence of a Z_2 if there are only two light Higgs doublets. If future measurements of atomic parity violation observe a discrepancy from the standard-model predictions, as is hinted at by the present data, then a relatively light ($M_2 < 600$ GeV) Z_2 boson would be a serious candidate for the cause. However, the direct and indirect existing constraints essentially rule out the possibility that the types of Z_2 considered here contribute significantly to possible anomalies in charged current universality tests or in $R(e^+e^- \to \mu^+\mu^-$, hadrons) below the Z pole.

Future precision experiments are expected to increase the sensitivity of Z_2 searches up to ~ 1 TeV [6], while direct searches at the Superconducting Super Collider should be sensitive up to $\sim 5-10$ TeV [1].

ACKNOWLEDGMENTS

This work was supported by Department of Energy, Contract No. DE-AC02-76-ERO-3071.

[1] P. Langacker, R. Robinett, and J. Rosner, Phys. Rev. D **30**, 1470 (1984).

[2] V. Barger, N. G. Deshpande, J. L. Rosner, and K. Whisnant, Phys. Rev. D **35**, 2893 (1987).

[3] L. S. Durkin and P. Langacker, Phys. Lett. **166B**, 436 (1986).

[4] U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1987).

[5] G. Costa *et al.*, Nucl. Phys. **B297**, 244 (1988).

[6] P. Langacker, M. Luo, and A. K. Mann, Rev. Mod. Phys. (to be published).

[7] Early references are cited in [1]–[5].

[8] For a recent review, see F. Dydak, in *Proceedings of the 25th International Conference on High Energy Physics*, Singapore, 1990, edited by K. K. Phua and Y. Yamaguchi (World Scientific, Singapore, 1991).

[9] S. Lloyd, in the Aspen Winter Conference in Elementary Particle Physics, 1991 (unpublished).

[10] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **65**, 2243 (1990); Phys. Rev. D **43**, 2070 (1991).

[11] UA2 Collaboration, J. Alitti *et al.*, Phys. Lett. B **241**, 150 (1990).

[12] M. C. Noecker *et al.*, Phys. Rev. Lett. **61**, 310 (1988).

[13] S. A. Blundell, W. R. Johnson, and J. Sapirstein, Phys. Rev. Lett. **65**, 411 (1990); A. Dzuba *et al.*, Phys. Lett. **141A**, 147 (1989).

[14] Additional references are given in [15].

[15] P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991).

[16] CDF Collaboration, P. Derwent, in the Aspen Winter Conference in Elementary Particle Physics [9].

[17] M. Cvetic and P. Langacker, Phys. Rev. D **42**, 1797 (1990), and references therein.

[18] M. Veltman, Nucl. Phys. **B123**, 89 (1977).

[19] B. Lynn, M. Peskin, and R. Stuart, in *Physics at LEP*, LEP Jamboree, Geneva, Switzerland, 1985, edited by J. Ellis and R. Peccei (CERN Report No. 86-02, Geneva, 1986), Vol I, p. 90; M. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); M. Golden and L. Randall, Nucl. Phys. **B361**, 3 (1991); W. Marciano and J. Rosner, Phys. Rev. Lett. **65**, 2963 (1990); D. Kennedy and P. Langacker, *ibid.* **65**, 2967 (1990); Phys. Rev. D **44**, 1591 (1991); G. Altarelli and R. Barbieri, Phys. Lett. B **253**, 161 (1991); B. Holdom and J. Terning, *ibid.* **247**, 88 (1990).

[20] A. Denner *et al.*, Phys. Lett. B **240**, 438 (1990).

[21] S. Bertolini and A. Sirlin, Phys. Lett. B **257**, 179 (1991).

[22] G. Altarelli *et al.*, Phys. Lett. B **263**, 459 (1991); Report No. 5752-90-ADD (unpublished); Phys. Lett. B **261**, 146 (1991); **245**, 669 (1990); Nucl. Phys. **B342**, 15 (1990); Mod. Phys. Lett. A **5**, 495 (1990); J. Layssac, F. M. Renard, and C. Verzegnassi; Annecy Report No. LAPP-TH-290/90-REV (unpublished); F. M. Renard and C. Verzegnazzi, Phys. Lett. B **260**, 225 (1991).

[23] K. T. Mahanthappa and P. K. Mohapatra, Phys. Rev. D **43**, 3093 (1991); P. Langacker, Phys. Lett. B **256**, 277 (1991).

[24] F. del Aguila, J. M. Moreno, and M. Quiros, Nucl. Phys. **B361**, 45 (1991); Phys. Lett. B **254**, 479 (1991); Phys. Rev. D **40**, 2481 (1989); **41**, 134 (1990); **42**, 262(E) (1990).

[25] M. C. Gonzalez-Garcia and J. W. F. Valle, Nucl. Phys. **B345**, 312 (1990); Phys. Lett. B **236**, 360 (1990); **259**, 365 (1991).

[26] CDF Collaboration, F. Abe *et al.*, Phys. Rev. D **43**, 664 (1991).

[27] J. A. Grifols *et al.*, Phys. Rev. D **42**, 3293 (1990); M. C. Gonzalez-Garcia and J. W. F. Valle, Phys. Lett. B **240**, 163 (1990); J. L. Lopez and D. V. Nanopoulos, *ibid.* **241**, 392 (1990); A. E. Faraggi and D. V. Nanopoulos, Mod.

Phys. Lett. A **6**, 61 (1991).

[28] J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **249**, 441 (1990); U. Amaldi, W. de Boer, and H. Furstenan, *ibid.* **260**, 447 (1991); C. Giunti, C. W. Kim, and U. W. Lee, Johns Hopkins Report No. JHU-TIPAC-91003 (unpublished).

[29] P. Langacker, Phys. Rev. D **30**, 2008 (1984).

[30] P. Langacker and S. Uma Sankar, Phys. Rev. D **40**, 1569 (1989).

[31] G. Beall, M. Bander, and A. Soni, Phys. Rev. Lett. **48**, 848 (1982).

[32] W. J. Marciano and A. Sirlin, Phys. Rev. D **35**, 1672 (1987).

[33] W. Raus and G. Rasche, Phys. Rev. D **41**, 166 (1990).

[34] A. Sirlin, Columbia Report No. CU-TP-505 (unpublished).

[35] D. H. Wilkinson, Triumf Report No. TRI-PP-91-9 (unpublished).

[36] For a recent review, see K. Abe, in *Proceedings of the 25th International Conference on High Energy Physics* [8].

[37] D. Schaile, presented at the Ringberg Workshop on High Precision versus High Energies in e^+e^- Collisions, Ringberg Castle, Germany, 1991 (unpublished).

[38] CHARM II Collaboration, D. Geiregat *et al.*, Phys. Lett. B **232**, 539 (1989).

[39] R. Robinett and J. Rosner, Phys. Rev. D **25**, 3036 (1982); **26**, 2396 (1982); R. Robinett, *ibid.* **26**, 2388 (1982).

[40] This is the most general boson if E_6 breaks directly to the direct product of the SM and a group containing the extra U(1). For generalizations, see [24].

[41] For reviews, see R. N. Mohapatra, *Unifications and Supersymmetries* (Springer, New York, 1986); Langacker and Uma Sankar [30].

[42] For a recent discussion, see [15] and references therein.

[43] M. K. Parida, Phys. Lett. B **196**, 163 (1987); T. Hübsch *et al.*, Phys. Rev. D **31**, 2958 (1985), and references therein.

[44] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); Phys. Rev. D **23**, 165 (1981).

[45] See *Z Physics at LEP, 1,* Proceedings of the Workshop, Geneva, Switzerland, 1989, edited by G. Altarelli, R. Kleiss, and C. Verzegnassi (CERN Report No. 89-08, Geneva, 1989).

[46] This prescription eliminates a possible pathology in the limit $\theta \rightarrow 0$ [47].

[47] G. Degrassi and A. Sirlin, Phys. Rev. D **40**, 3066 (1989).

[48] M. Veltman, Nucl. Phys. **B123**, 89 (1977); M. Chanowitz, M. A. Furman, and I. Hinchliffe, Phys. Lett. **78B**, 285 (1978).

[49] For reviews, see P. Langacker, Phys. Rep. **72**, 185 (1981); in *Ninth Workshop on Grand Unification,* Proceedings, Aix-Les-Bains, France, 1988, edited by R. Barloutaud (World Scientific, Singapore, 1988), p. 3; G. G. Ross, *Grand Unified Theories* (Benjamin, Reading, MA, 1985).

[50] Actually, Eq. (36) is valid only for $\rho_2 \ll \rho_0$, but the same argument can be made using the exact equation derivable from Eq. (5).

[51] One cannot push this compensation too far, because large values of θ/ρ_2 would require large U′(1) charges for the Higgs fields responsible for the mixing.

[52] G. Degrassi, S. Fanchiotti, and A. Sirlin, Nucl. Phys. **B351**, 49 (1991).

[53] If one did not include radiative corrections to μ and β decay (which would be infinite in the four-Fermi or intermediate-vector-boson theories) one would obtain $(VV^\dagger)_{11} = 1.036$ [34].

[54] P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988); J. Maalampi and M. Roos, Phys. Rep. **186**, 53 (1990).

[55] L. Wolfenstein, Phys. Rev. D **29**, 2130 (1984). For recent developments, see [30].

[56] The μ decay diagrams also modify the relation between the Fermi constant and $M_{W,Z}$ [47], but the effects are negligible.

[57] A. A. Pankov and C. Verzegnassi, Phys. Lett. B **233**, 259 (1989).

[58] K. Hagiwara *et al.*, Phys. Rev. D **41**, 815 (1990).

[59] Limits on a Z_2 have also been studied by the VENUS Collaboration, K. Abe *et al.*, Phys. Lett. B **246**, 297 (1990).