Electroweak phase transition and baryogenesis

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We give an analytic treatment of the one-Higgs-doublet, electroweak phase transition which demonstrates that the phase transition is first order. The phase transition occurs by the nucleation of thin-walled bubbles and completes as a temperature where the order parameter $\langle\phi\rangle_T$ is significantly smaller than it is when the origin becomes absolutely unstable. The rate of anomalous baryon-number violation is an exponentially sensitive function of $\langle\phi\rangle_T$. In very minimal extensions of the standard model it is quite easy to increase $\langle\phi\rangle_T$ so that anomalous baryon-number violation is suppressed after the completion of the phase transition. Hence, baryogenesis at the electroweak phase transition is tenable in minimal extensions of the standard model with one Higgs doublet.

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I. INTRODUCTION

In the minimal standard model, electroweak symmetry breaking is induced by the ground state of a single doublet scalar field. We can write the potential for the real scalar component of the doublet which acquires a vacuum expectation value as

$$U(\phi)=\tfrac{1}{4}\lambda(\phi^2-\sigma^2)^2 \ . \tag{1.1}$$

In a cold and empty (or relatively empty) universe, the Higgs field ϕ can minimize its energy and hence the free energy of the system by choosing a vacuum expectation value $\langle\phi\rangle=\sigma$. However, the early Universe was neither cold nor empty, and the presence of an ambient, thermal distribution of particles changes this picture. Although the vacuum energy of the system is still reduced by shifting the classical value of the Higgs field away from $\langle\phi\rangle=0$, we now pay the price of adding free energy to the particles in the surrounding plasma as they acquire a mass (see Fig. 2). When the temperature is high enough, the free energy required to give mass to a thermal distribution of particles exceeds the vacuum energy liberated by displacing the Higgs-field vacuum expectation value from the origin. This occurs for temperatures above a critical temperature we call T_1, where the configuration of the Higgs field that minimizes the free energy of the system is $\langle\phi\rangle=0$. Thus, at temperatures large compared to the scale of electroweak physics, the minimum of the effective Higgs potential is at the origin. This is the origin of symmetry restoration at high temperature [1–3]. As the Universe cools, when the temperature drops to the critical temperature T_1, a new minimum appears, separated from the origin by a hump. When the free-energy barrier separating the two extrema is small enough, bubbles of true vacuum are nucleated and grow. The thermal decay rate from the unstable state carries a suppression [4,6]

$$\Gamma/\mathcal{V}\propto e^{-F_c(T)/T} \ , \tag{1.2}$$

where $F_c(T)$ is the surplus free energy needed to create a bubble of true vacuum large enough to grow indefinitely. Bubbles of true vacuum smaller than this critical size collapse under surface tension. For a bubble larger than this critical size, as the radius of the bubble increases the free energy liberated by the expanding volume of true vacuum exceeds the free energy required to increase the bubble's surface area. Such a bubble will grow and convert space to a true vacuum. A static bubble which is exactly the critical size is in unstable equilibrium; it is a saddle-point solution of the free-energy functional.

At temperature T_2, where the second derivative of the potential at the origin vanishes, fluctuations can classically roll towards the global minimum without surmounting an energy barrier. If the phase transition has not yet completed by the time the temperature drops to T_2, the transition no longer occurs through bubble nucleation. We call the transition first order if it proceeds by bubble nucleation. So a necessary condition for a first-order phase transition is that bubbles occupy most of space before the temperature drops to T_2. We will satisfy a slightly more stringent condition. At temperatures very close to T_2 the loop expansion parameter becomes large [2]. So in addition, a reliable analytical determination of the phase transition requires that it complete while the effective loop expansion parameter is small.

In the next section we write the finite-temperature effective potential for the one-Higgs-doublet standard model in a form which is useful for the analytic understanding of the nature of the phase transition. We take the Higgs boson to be lighter than about 150 GeV, and use the high-temperature approximation, which we will show is highly accurate for all aspects of the phase transi-

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tion. In Sec. III we discuss the nature of bubbles which could be nucleated to trigger the phase transition. The scaling arguments of Sec. III suggest that an examination of whether the phase transition occurs via nucleation of thin-walled bubbles is warranted. In Sec. IV, the free energy of these thin-walled bubbles is calculated, and in Sec. V it is shown that the thin-walled bubble free energy is small enough that the phase transition does complete by the rapid nucleation of exclusively thin-walled bubbles. Our analysis is purely analytic, and we obtain formulas for such quantities as the temperature of the Universe at the completion of the phase transition and the number of bubbles nucleated per horizon volume.

Section VI provides an important application of our results to the question of depletion of the baryon asymmetry after the phase transition. The standard model contains an anomaly which is baryon-number violating [5]. At high temperatures the rate of anomalous baryon-number violation can be quite large [6–10]. This has stimulated a great deal of interest in the possibility of creating the baryon asymmetry at the electroweak phase transition (EWPT) [11–17]. A successful scenario must explain why the baryon excess created at the electroweak phase transition is not washed out after the phase transition completes. We show that extremely simple additions to the standard model avoid washout for any Higgs-boson mass up to 150 GeV.

II. EVOLUTION OF THE POTENTIAL

The tree-level potential for the physical Higgs scalar is

$$U(\phi)=\frac{\lambda_0}{4}(\phi^2-\sigma^2)^2 , \tag{2.1}$$

where λ_0 is related to the Higgs-boson mass by $m_H^2=2\lambda_0\sigma^2$. To reliably analyze the dynamics of this field, we need to include the interactions of the Higgs field with virtual particles and with the heat bath.

The one-loop, zero-temperature potential $V(\phi)$ can be written as the sum of the classical potential and a one-loop correction: $V(\phi)=U(\phi)+\bar{V}_1(\phi)$. If we adopt the renormalization prescriptions (i) $V''(\sigma)m_H^2$ and (ii) $V'(\sigma)=0$, for each degree of freedom to which the Higgs boson is coupled, the zero-temperature one-loop correction to the effective potential is (see Appendix A)

$$\bar{V}_1(\phi)=\pm\frac{1}{64\pi^2}\{m^4(\phi)\ln[m^2(\phi)/m^2(\sigma)] -\tfrac{3}{2}m^4(\phi)+2m^2(\phi)m^2(\sigma)-\tfrac{1}{2}m^4(\sigma)\} , \tag{2.2}$$

where the $\pm$ is for bosons (fermions) and $m(\phi)$ is the mass of the particle in the presence of a background field ϕ. Equation (2.2) is valid for particles which have a mass of the form $m^2=a+b\phi^2$ in the mass-eigenstate basis. In addition to these quantum corrections, we must also include the interaction between the Higgs field and the hot electroweak plasma. Taking the Higgs boson sufficiently light we can ignore the contribution of scalar loops. Appendix B contains a critical review of the standard calculation of the temperature-dependent effective potential. From Appendix B, at high temperature, the effective potential for the standard model [16,18] can be reliably written

$$V(\phi,T)=D(T^2-T_2^2)\phi^2-ET\phi^3+\tfrac{1}{4}\lambda_T\phi^4 , \tag{2.3}$$

where

$$D=\tfrac{1}{24}[6(m_W/\sigma)^2+3(m_Z/\sigma)^2+6(m_t/\sigma)^2] ,$$

and the coefficient of the term linear in temperature is

$$E=\frac{1}{12\pi}[6(m_W/\sigma)^3+3(m_Z/\sigma)^3]\simeq 10^{-2} .$$

The temperature-dependent ϕ^4 coupling is

$$\lambda_T=\lambda-\sum_B g_B\frac{m_B^4}{16\sigma^4\pi^2}\ln(m_B^2/c_BT^2) +\sum_F g_F\frac{m_F^4}{16\sigma^4\pi^2}\ln(m_F^2/c_FT^2) , \tag{2.4}$$

where the $B(F)$ denotes bosons (fermions), $g_{B(F)}$ is the number of degrees of freedom, c_F and c_B are constants which can be found in Appendix B, and the masses in (2.4) are evaluated at $\langle\phi\rangle=\sigma$. The physical-Higgs-boson mass is related to λ by

$$m_H^2=(2\lambda+12B)\sigma^2 , \tag{2.5}$$

where

$$B=\frac{1}{64\pi^2\sigma^4}(6m_W^4+3m_Z^4-12m_t^4) .$$

We define T_2 as the temperature where $V''(\phi=0)=0$. From Appendix B,

$$T_2^2=\frac{m_H^2-8B\sigma^2}{4D}\equiv\chi^2(m_t,m_H)m_H^2 . \tag{2.6}$$

Because this result was obtained at the origin, it is valid to all orders in m/T. The quantity χ is plotted against m_H for various values of m_t in Fig. 1.

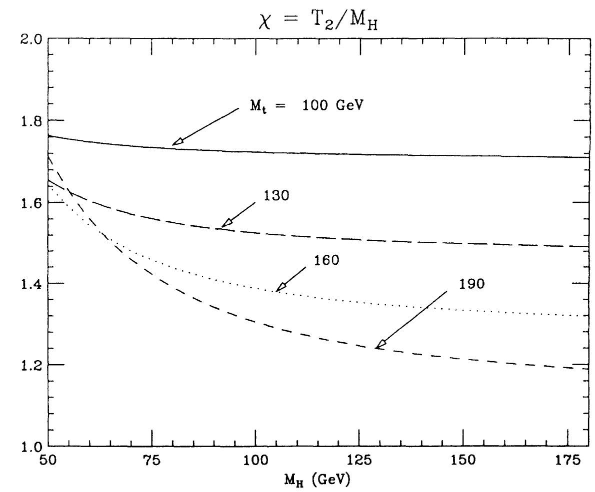

FIG. 1. The ratio $\chi=T_2/m_H$ plotted against the Higgs-boson mass for three different top-quark masses.

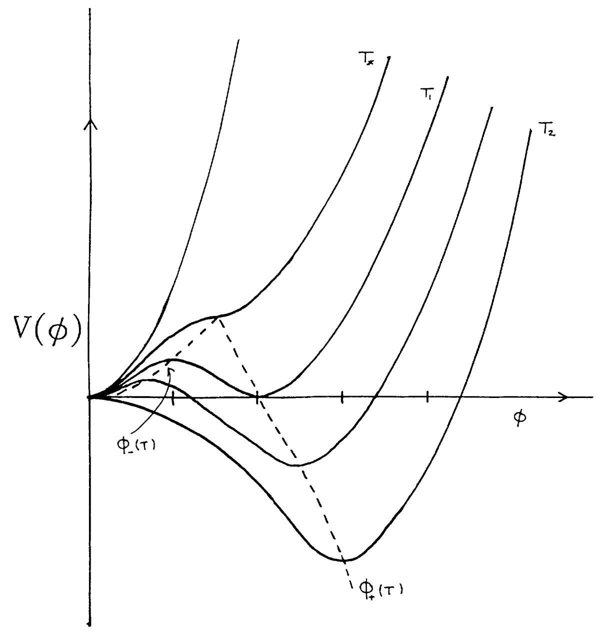

FIG. 2. A schematic picture of the effective potential for temperatures T_*, T_1, $T_1>T>T_2$ and T_2.

At temperatures well above T_2, the only minimum of the potential is $\langle\phi\rangle=0$. As the early Universe cools down from this high temperature, a second local minimum of the potential first appears (as an inflection point) when the temperature reaches

$$T_*^2=T_2^2\frac{1}{1-9E^2/8\lambda_T D} \tag{2.7}$$

at a value of the field $\phi_*=3ET_*/2\lambda_T$. At lower temperatures, this point splits into a barrier ϕ_- and a local minimum ϕ_+ which subsequently evolve as

$$\phi_\pm=\frac{3ET}{2\lambda_T}\pm\frac{1}{2\lambda_T}\sqrt{9E^2T^2-8\lambda_T D(T^2-T_2^2)}\ . \tag{2.8}$$

The evolution of $\phi_\pm$ is shown in Fig. 2. We define the temperature T_1 to be the temperature at which the second minimum becomes degenerate with the origin, $V(\phi_+(T_1))=0$. Hence, if we divide Eq. (2.3) by ϕ^2, T_1 occurs where the resulting quadratic equations has two real equal roots. This gives the relation

$$T_1^2=\frac{1}{1-\dfrac{E^2}{\lambda_T D}}T_2^2\ . \tag{2.9}$$

For Higgs-boson masses above the current experimental limit, the difference in temperature between T_1 and T_2 is small compared to the temperature. Writing $T_1=T_2+\tau$, we find $\tau\ll T_2$ provided $m_H\gtrsim 10$ GeV, where

$$\tau=\left[\frac{E^2}{2\lambda_T D}\right]T_2\ . \tag{2.10}$$

From Eqs. (2.8) and (2.9) we see that

$$\phi_\pm(T_1)=\frac{3ET_1}{2\lambda_T}\pm\frac{ET_1}{2\lambda_T},\quad \phi_\pm(T_2)=\frac{3ET_2}{2\lambda_T}\pm\frac{3ET_2}{2\lambda_T}\ . \tag{2.11}$$

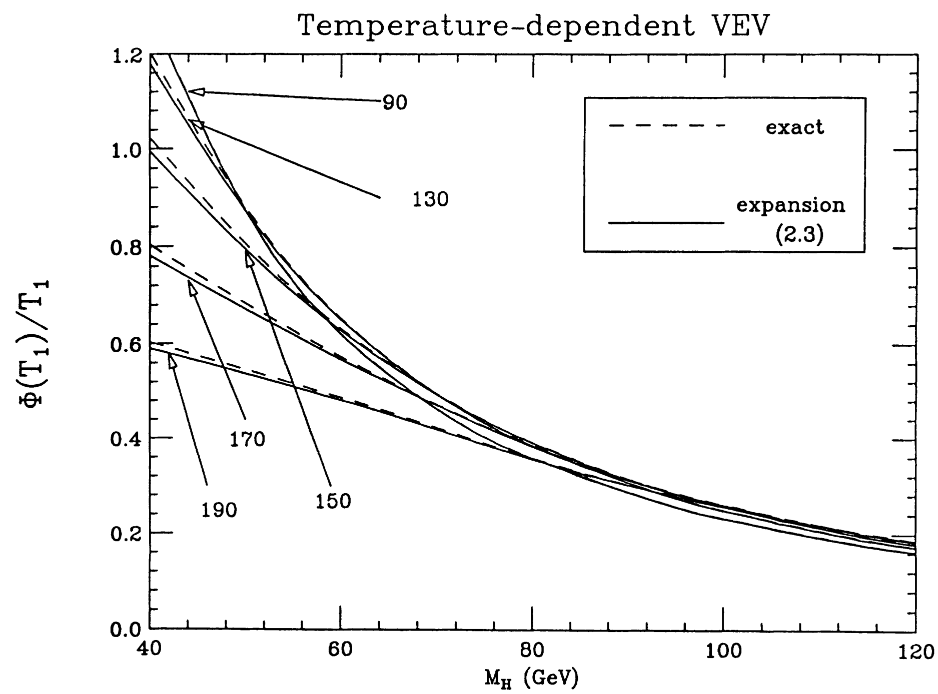

FIG 3. The temperature-dependent vacuum expectation value of the Higgs field vs the Higgs-boson mass for five different top-quark masses. The dashed line is the exact result, while the solid curve represents the approximation of Eq. (2.3).

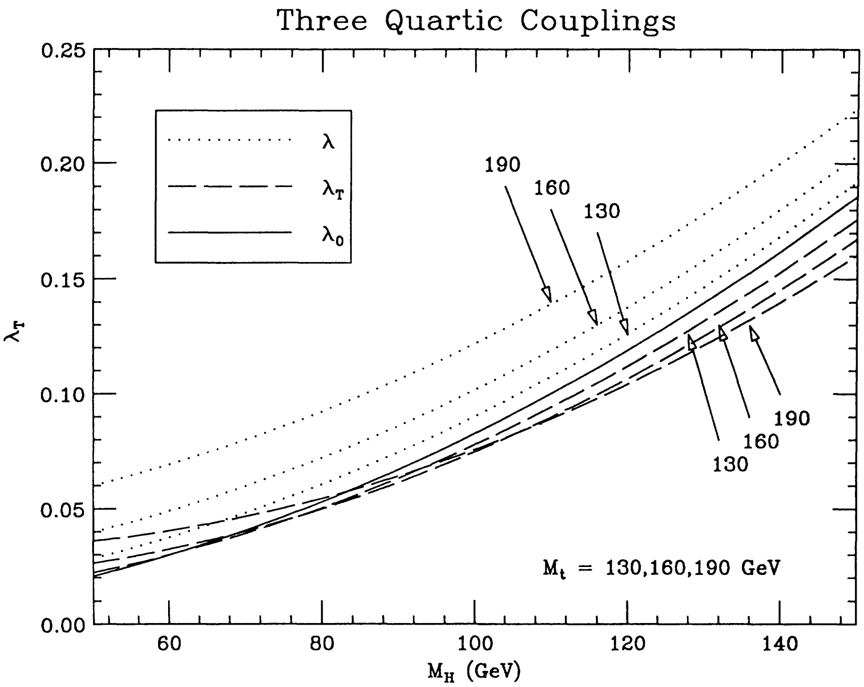

FIG. 4. The tree-level, one-loop, and temperature-dependent quartic scalar couplings vs the Higgs-boson mass for three different top-quark masses.

It will prove convenient to write the potential in terms of the scaled field ϕ', where $\phi=(ET/\lambda_T)\phi'$, for which

$$V(\phi)=\lambda_T\left[\frac{ET}{\lambda_T}\right]^4\left[\frac{T_1^2(T^2-T_2^2)}{T^2(T_1^2-T_2^2)}\phi'^2-\phi'^3+\frac{1}{4}\phi'^4\right] . \tag{2.12}$$

If $T_1-T_2 \ll T_1+T_2$, then for any T such that $|T_1-T| \ll T_1$, the potential reduces to the simple form

$$V(\phi)=\lambda_T\left[\frac{ET}{\lambda_T}\right]^4[(1-\epsilon)\phi'^2-\phi'^3+\tfrac{1}{4}\phi'^4] , \tag{2.13}$$

where $\epsilon=(T_1-T)/(T_1-T_2)$. In terms of the scaled field,

$$\phi'_{\pm}=\tfrac{1}{2}(3\pm\sqrt{1+8\epsilon}) . \tag{2.14}$$

The critical temperatures T_*, T_1, and T_2 correspond to the following values of ϵ:

$$\epsilon_*=-3/8, \quad \epsilon_1=0, \quad \epsilon_2=1 . \tag{2.15}$$

The largest values of m/T which are of importance in this paper correspond to $T\simeq T_1$ and $\phi\simeq\phi_+(T_1)$. Is the high-temperature expansion a good approximation in this case? If the high-temperature approximation is valid for the top quark, it will be valid for all other particles as well. In the temperature region of interest, for the top quark we have

$$\frac{m_t(T)}{T}\simeq\lambda_t\frac{\phi_+(T_1)}{T_1}\simeq\lambda_t\frac{2E}{\lambda_T} . \tag{2.16}$$

In Appendix B we show that the high-temperature approximation is valid to better than 5% provided this quantity is less than 1.6. An inspection of Figs. 3 and 4 demonstrates that our use of the high-temperature approximation is well justified.

III. A HEURISTIC DISCUSSION OF THE SADDLE POINT

After the Universe cools down to a temperature below T_1, the previously global minimum $\langle\phi\rangle=0$ becomes metastable. The subsequent conversion of the Universe to the true-vacuum state $\langle\phi\rangle=\phi_+(T)$ takes place by the nucleation of true-vacuum bubbles. Accordingly, we need to determine the free-energy barrier such bubbles must surmount in order to grow. Consider a true-vacuum bubble in a sea of false vacuum $\langle\phi\rangle=0$. Let $\langle\phi\rangle=\phi'$ at the center of the bubble (see Fig. 5).

By convention we choose the state $\langle\phi\rangle=0$ to have free-energy zero, $V(0)=0$. Then the surplus free energy of a true-vacuum bubble is

$$F=\int d^3x[\tfrac{1}{2}(\nabla\phi)^2+V(\phi,T)] . \tag{3.1}$$

The free energy of a true-vacuum bubble has two contributions: a surface free energy F_S, coming from the derivative terms in (3.1), and a volume term F_V, which arises from the difference in free-energy density inside and outside the bubble. These two contributions scale like

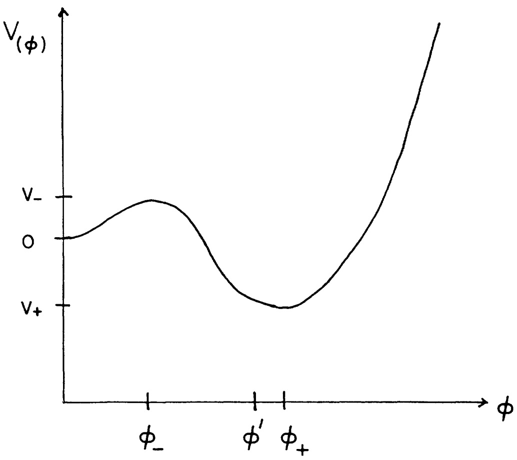

FIG. 5. The effective potential.

$$F_S \sim 2\pi R^2 \left[\frac{\delta\phi}{\delta R} \right]^2 \delta R, \quad F_V \sim -\frac{4\pi}{3} R^2 \bar{V} , \tag{3.2}$$

where R is the radius of the bubble, δR is the thickness of the bubble wall, $\delta\phi = \phi'$,[1] and $\bar{V}$ is minus the average value of the potential inside the bubble. When $V(\phi_-)$ is large compared to $-V(\phi')$, it is important to minimize the contribution to F_V coming from regions near $\phi \simeq \phi_-$. More precisely, when the height and the width of the barrier near ϕ_- are not small compared to the depth and width of the well at ϕ_+, for the optimal solution, ϕ will change quickly between 0 and ϕ', and δR will be small. This is the situation for temperatures just below T_1. Hence, the first bubbles which could be formed are thin-wall bubbles.

As the temperature subsequently drops towards T_2, the barrier in ϕ space tends to zero, and the difference in free-energy density between the states $\langle \phi \rangle = 0$ and $\langle \phi \rangle = \phi_+$ increases. When the size of the hump in the potential at ϕ_- becomes small compared to the depth of the well at ϕ_+, it is favorable to make δR as large as possible so as to minimize the "surface" term F_S. Hence, $\delta R \sim R$, and we should work in a thick-wall approximation. So, whether the EWPT proceeds by the nucleation of thick-walled or thin-walled bubbles depends on how large the rate of bubble nucleation becomes before thick-walled bubbles are energetically preferred. Our purpose, throughout this section, is to gain a qualitative understanding of the dependence of bubble free energies on the shape of the effective potential. Accordingly, we shall not be too concerned with the precise value of the numerical prefactors which occur in estimates of bubble parameters. Simple qualitative and semiquantitative estimates of these two cases will tell us what kind of bubble to examine with closer scrutiny. Let us first consider the simpler case of thick-walled bubbles.

[1]For thin-walled bubbles ϕ' will lie at the absolute minimum, while for thick-walled bubbles we must allow for the possibility that ϕ' is somewhat less than ϕ_+.

Thick-walled bubbles

For thick-walled bubbles $\delta R \sim R$, and the surface energy of the bubble grows like R. In contrast, the negative volume term increases in magnitude like R^3. Thus, a thermal fluctuation producing a bubble of true vacuum, which starts from a radius of zero and expands in radius to envelope the system, must have a free energy greater than or equal to some critical value. The critical radius of the bubble, R_c, is the bubble radius where the total free energy of the bubble reaches a maximum. Differentiating (3.2), $R_c \sim \delta\phi/\sqrt{2\bar{V}}$, and the thick-wall bubble free energy is

$$F_c \simeq 2\pi R_c (\delta\phi)^2 - \frac{4\pi}{3} R_c^3 \bar{V} \sim \frac{(\delta\phi)^3}{\sqrt{\bar{V}}} . \tag{3.3}$$

Note that for thick-walled bubbles the magnitude of F_c depends on the relative sizes of the shift in ϕ and the potential difference between the center and the outside of the bubble and not the height of the barrier in π space.[2] As a quick estimate for the potential (2.3), defining $r = 1 - \epsilon$, as the temperature approaches T_2 the critical free energy scales like

$$F_c \sim \frac{ET}{\lambda_T^{3/2}} r^{3/2} . \tag{3.4}$$

Thin-walled bubbles

For thin-walled bubbles, in addition to the contribution of the derivative term to the bubble wall free energy, inside the bubble walls there is also a positive contribution to the free energy from the barrier in ϕ-space, $V_b \sim V_-$. As a function of the bubble radius and thickness, the thin-wall bubble free energy is

$$F(R) \sim 2\pi R^2 \delta R \left[\left[\frac{\delta\phi}{\delta R} \right]^2 + 2V_b \right] - \frac{4\pi}{3} R^3 \bar{V} . \tag{3.5}$$

The saddle point corresponds to $\delta R \simeq \delta\phi/\sqrt{2V_b}$ and $R_c \sim \sqrt{V_b/\bar{V}}\,(\delta\phi/\sqrt{\bar{V}})$. So the critical free energy is

$$F_c \sim \frac{(\delta\phi)^3}{\sqrt{\bar{V}}} \left[\frac{V_b}{\bar{V}} \right]^{3/2} . \tag{3.6}$$

So the thin-wall bubble radius is found by scaling the

[2]While it is true that a large barrier in $V(\phi)$ vs ϕ space makes it hard to fluctuate a bubble of true vacuum, and a smaller barrier makes it easier, we should remember that the real barrier is in configuration space, and we should be wary of intuition based on the one-dimensional mechanics.

thick-wall radius by $\sim\sqrt{V_b/\bar{V}}$ and the thin-wall free energy increases by a factor of $\sim(V_b/\bar{V})^{3/2}$ relative to the thick-wall case. For the potential of (2.3), $\bar{V}\simeq -V(\phi_+)$, $V_b\simeq V(\phi_-)$, and $\delta\phi=\phi_+$. Since the first bubbles to form as the Universe cools to a temperature below T_1 will be thin-walled bubbles, substituting $\phi_\pm$ into Eq. (3.6), recalling $\epsilon=(T_1-T)/(T_1-T_2)$, and expanding for small ϵ, the first bubbles to form must surmount a free-energy barrier which scales like

$$F_c(\epsilon)\sim\frac{ET_1}{\epsilon^2(2\lambda_T)^{3/2}} . \tag{3.7}$$

As we will discuss in Sec. V, a free-energy barrier $O(100T)$ is small enough to allow bubbles to nucleate. Accordingly, the estimate (3.7) tells us we should make a careful study of bubble free energies in the thin-wall approximation the instant after the temperature drops below T_1.

IV. SADDLE-POINT FREE ENERGIES IN THE THIN-WALL APPROXIMATION

We can adapt Coleman's thin-wall approximation [19] to derive an analytic formula for the critical free energy valid in the limit that the temperature approaches T_1 from below.[3] The thin-wall analysis has been applied to the general case of thermal vacuum stability by Linde [20]. However we find his conclusions regarding the applicability of the thin-wall approximation to realistic gauge theories overly pessimistic. For the potential of Eq. (2.3), if we shift our field $\phi\to\phi'=\phi+(ET/\lambda_T)$, we can cast the potential in a form where the validity of Coleman's thin-wall approximations will be transparent. Recall $\delta=E^2/\lambda_T D$, $\epsilon=(T_1-T)/(T_1-T_2)$, and define $v=ET/\lambda_T$. In terms of the shifted field the potential becomes

$$V(\phi)=\left[\frac{\lambda_T}{4}[\phi^2-v^2(1+2\epsilon)]^2-\epsilon(2\lambda_T v^3)(\phi+v)\right]\times[1+O(\delta\epsilon)] \tag{4.1}$$

plus terms independent of ϕ. For a universe filled with false vacuum $\langle\phi\rangle=-v$, the true-vacuum bubble of minimum free energy, which is just large enough to grow, is a static O(3)-invariant solution to the equations of motion. Hence it satisfies

$$\frac{d^2\phi}{dr^2}+\frac{2}{r}\frac{d\phi}{dr}=V'(\phi,T) , \tag{4.2}$$

where the prime denotes differentiation with respect to ϕ. Integrating the appropriate solution to (4.2) gives the critical free energy

$$F_c=4\pi\int r^2dr\left[\frac{1}{2}\left[\frac{d\phi}{dr}\right]^2+V(\phi,T)\right] . \tag{4.3}$$

In the limit of small ϵ, the bubble wall thickness is negligible when compared to the bubble radius. So for small ϵ, inside the bubble walls we can neglect the term in (4.2) linear in spatial derivatives. This, together with the boundary conditions that the derivative of ϕ and the free-energy density vanish outside of the bubble, implies

$$\frac{d\phi}{dr}=\sqrt{2V} \tag{4.4}$$

inside the bubble walls. We have defined the zero of $V(\phi)$ in (4.1) so that $V(-v)=0$. From (4.3) and (4.4) we can write the resulting bubble free energy as

$$F\simeq 4\pi R^2\int_{\phi(R-\delta R)}^{\phi(R+\delta R)}\sqrt{2V(\phi)}d\phi+4\pi\int_0^R dr\, r^2V(\phi_+,T) . \tag{4.5}$$

As a function of the bubble radius, the free energy is given by[4]

$$F(R)=\frac{8\pi}{3}\sqrt{2\lambda_T}v^3R^2-\epsilon\frac{16\pi}{3}\lambda_T v^4R^3 . \tag{4.6}$$

Varying with respect to R, we find that $R_c=2/[\sqrt{2\lambda_T}(3\epsilon v)]$. Hence, in the limit of small ϵ, the critical free energy to temperature ratio is[5]

$$F_c/T_1=\left[\frac{64\pi}{81}\right]\frac{E}{\epsilon^2(2\lambda_T)^{3/2}}\simeq(2.85)\left[\frac{50\text{ GeV}}{m_H}\right]^3\epsilon^{-2} . \tag{4.7}$$

As we will discuss in Sec. V, the phase transition completes when the free energy to temperature ratio is on the order of 100. This is achieved for $\epsilon\simeq\frac{1}{6}(50\text{ GeV}/m_H)^{3/2}$. For completeness, we note that to lowest order in ϵ, inside the bubble walls ϕ behaves like a domain wall and the well-known solution is

$$\phi(r)=-v\tanh\left[\left[\frac{\lambda_T}{2}\right]^{1/2}v(r-R)\right] . \tag{4.8}$$

V. BUBBLE PRODUCTION, EVOLUTION, AND NUMBER

Having determined the free energy of a bubble large enough to grow indefinitely, we examine the rate for pro-

[3]The result we obtain here will be exact in the limit ϵ goes to zero, for the potential of (2.3).

[4]The reader is cautioned that formulas (4.12) and (4.21) of Ref. [19] are off by a factor of 2.

[5]By rescaling the spatial coordinate r in Eq. (4.3), with the potential given by (4.1), the free energy can be written $F_c(T)/T=(64\pi/81)[E/(2\lambda_T)^{3/2}]f(\epsilon)$, where f is a function which depends only on ϵ. The thin-wall approximation gives $f=\epsilon^{-2}$.

ducing such bubbles. As the early Universe cooled from temperature T_1 to temperature T_2, a point was reached where classical thermal fluctuations were large enough to nucleate bubbles of true vacuum. These thermal fluctuations produced bubbles of true vacuum at a rate per unit volume

$$\Gamma/\mathcal{V}=\Lambda^4(T)e^{-F_c(T)/T}, \tag{5.1}$$

where $F_c(T)$ is the free energy of a fluctuation large enough to pass over the energy barrier separating the two vacua, and Λ is a characteristic scale in the theory. For definiteness we take $\Lambda^4=\omega T^4$. As we shall see, the temperatures we are interested in are on the order of the particle masses. Moreover, because the nucleation rate is dominated by the exponential, the exact value of the prefactor is not very important, so the effect of $\omega\neq 1$ will be negligible.

The onset of nucleation

Let us begin by determining when the onset of bubble nucleation occurs. In the radiation-dominated era, the time-temperature relationship is $t=\xi M_{\rm Pl}/T^2$, where t is the age of the Universe, $M_{\rm Pl}=1.22\times10^{19}$ GeV is the Planck mass, and $\xi\simeq 1/34$ near the electroweak phase transition.[6] Because the horizon size scales like $d_H=2t$, the size of a causal volume at a temperature T is

$$V_H(T)\sim 8\xi^3M_{\rm Pl}^3/T^6\ . \tag{5.2}$$

Without being overly precise about the numerical prefactor, we take (5.2) as our definition of a causal volume. For probabilities small compared to one, the probability that a bubble was nucleated inside a causal volume during a temperature interval dT is given by

$$dP\sim 16\omega\xi^4\frac{M_{\rm Pl}^4}{T^4}e^{-F_c(T)/T}\frac{dT}{T}\ . \tag{5.3}$$

In this section, it will be convenient to make use of the fact that the critical bubble free energy to temperature ratio, (4.7), has a Taylor expansion about a temperature T_0 given by

$$F_c(T)/T=[F_c(T_0)/T_0](1+2x+3x^2+\cdots)\ , \tag{5.4}$$

where $x=(T-T_0)/(T_1-T_0)$. Between the temperatures T_1 and T_2 the free energy is a quickly changing function of temperature but the change in temperature itself between T_1 and T_2 is insignificant. Define T_N as the temperature reached when the first bubble is nucleated in a typical horizon. Using the expansion (5.4), we integrate Eq. (5.3) and note that the integrand is sharply peaked about $x=0$. The first bubble nucleates in a typical horizon when $P\sim1$ giving

$$F_c(T_N)/T_N\sim 134\ . \tag{5.5}$$

This ratio varies by about 2% as the Higgs-boson mass increases from 50 to 100 GeV.

The phase transition

In order to claim that the phase transition really proceeds by bubble nucleation, it is necessary but not sufficient to require that T_2 is small enough relative to T_N to ensure that nucleated bubbles grow enough to convert the Universe to true vacuum before the temperature drops to T_2, where the free-energy barrier disappears. A reliable determination of the phase transition requires that it completes before the loop expansion parameter in the SU(2)×U(1) model becomes large [2]. Moreover, because the anamalous baryon-number-violating processes which persist after the EWPT completes are very sensitive to the value of $\phi_+(T_\Omega)$,[7] a careful determination of the temperature at which the phase transition completes is required.

Define $dN(T)$ as the number of bubbles per unit volume nucleated between temperatures T and $T+dT$. From (5.1),

$$dN(T)=2\omega\xi M_{\rm Pl}Te^{-F_c(T)/T}dT\ . \tag{5.6}$$

Consider a true-vacuum bubble expanding with a constant terminal velocity β with respect to the plasma. There are two contributions to the bubble wall expansion. One comes from the propagation of the bubble wall through the fluid and the other is from the expansion of the Universe. Hence,

$$dR=\beta dt-a(T)\frac{dT}{T}\ , \tag{5.7}$$

where $a(T)$ is the Robertson-Walker scale factor. In the radiation-dominated era, this leads to the differential equation

$$\frac{dR}{dT}=-2\beta\xi\frac{M_{\rm Pl}}{T^3}-\frac{a(T)}{T}\ . \tag{5.8}$$

During the electoweak phase transition a bubble nucleated at a temperature T', and expanding with a velocity β, has a radius

$$R_B(T,T')=R_N(T')\frac{T'}{T}+2\beta\xi\frac{M_{\rm Pl}}{T}\left[\frac{1}{T}-\frac{1}{T'}\right]\ , \tag{5.9}$$

where $R_N(T')$ is the radius of a true-vacuum bubble when it is produced at a temperature T'.

We can imagine two qualitatively different types of first-order phase transitions. In a bubble expansion dom-

[6] $\xi=\left[\left[\frac{90}{32\pi^3}\right]\frac{1}{\sum_B g_B+\frac{7}{8}\sum_F g_F}\right]^{1/2}$.

[7] ϕ_+ increases by a factor of 3/2 as the temperatures drops from T_1 to T_2.

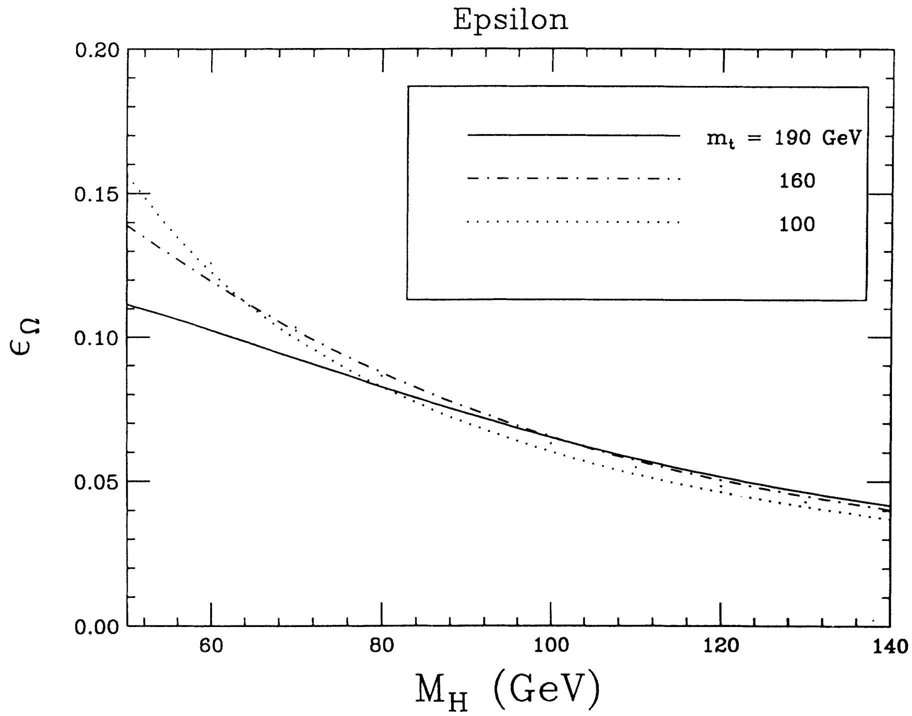

FIG. 6. Epsilon at the end of the phase transition vs the Higgs-boson mass for three different top-quark masses.

inated first-order phase transition, a typical observer in the fluid sees the surrounding volume converted to true vacuum as a nearby bubble of true vacuum expands and a bubble wall passes over the volume. A second type of first-order phase transition occurs if the nucleation rate turns on very suddenly and is so rapid that a typical volume element in the fluid is converted to true vacuum by a nucleation event. We call this second type of first-order phase transition nucleation dominated. The nucleation rate increases on a time scale set by the expansion rate of the Universe. At a time when the bubble nucleation rate is large enough to produce bubbles copiously, but small enough that the conversion of space to true vacuum by the nucleation event itself is still negligible, previously nucleated bubbles will grow appreciably in size before there is a substantial increase in the nucleation rate. This is a result of the smallness of the temperature relative to the Planck mass. As a result, the vast majority of space is converted to true vacuum by the expansion of bubbles as opposed to being on the site of a nucleation event. So, the volume of a bubble produced at temperature T' can be safely written

$$V(T,T')=\frac{32\pi}{3}\beta^3\xi^3\frac{M_{\rm Pl}^3}{T^3}\left[\frac{1}{T}-\frac{1}{T'}\right]^3 . \quad (5.10)$$

Integrating over the duration of bubble production, the fraction of a typical horizon that is true vacuum at time t is[8]

$$f\simeq\int dt'V(t',t)\Gamma/\mathcal{V}$$
$$\simeq\frac{64\pi}{3}\omega\beta^3\xi^4(M_{\rm Pl}/T)^4$$
$$\times\int_T^{T_1}\frac{T\,dT'}{T'^2}\left[1-\frac{T}{T'}\right]^3 e^{-F_c(T')/T'} . \quad (5.11)$$

End of the phase transition

Define T_Ω as the temperature reached when the phase transition has completed. In the region of interest, the temperature change is insignificant compared to the change in $F_c(T)$. Defining $x=(T-T_\Omega)/T_\Omega$, using the Taylor expansion (5.4) about T_Ω, we see that the integrand of (5.11) is sharply peaked at temperatures very near T_Ω. The phase transition completes when the fraction of true vacuum approaches unity, giving

[8]This expression for the fraction of space containing a true vacuum overcounts the volume where bubbles overlap. The exact [21] formula is $1-f=\exp[-\int\Gamma/\mathcal{V}V(t',t)dt']$. Hence, our definition of the end of the phase transition corresponds to the era where only $1/e$ of space is still a false vacuum.

$$F_c(T_\Omega)/T_\Omega \simeq 96 - 7\ln\left[\frac{2\lambda_T\sigma^2}{(50\text{ GeV})^2}\right] - 4\ln\left[\frac{100\text{ GeV}}{T_1}\right] - \ln\beta^3 . \quad (5.12)$$

The expanding bubbles are not extremely nonrelativistic, giving $F_c(T_\Omega)/T_\Omega \simeq 100$. Hence $\epsilon_\Omega \simeq \frac{1}{7}(50\text{ GeV}/m_H)^{3/2}$, and the phase transition completes the instant after the Universe cools to temperature T_1. The precise value of ϵ_Ω is shown as a function of the Higgs-boson mass for several top-quark masses in Fig. 6.

The number of bubbles

Define T_n as the temperature reached by the time n bubbles have been nucleated inside the comoving volume coincident with the horizon volume at the end of the phase transition:

$$n \simeq 16\omega\xi^4\left[\frac{M_{\rm Pl}}{T}\right]^4 \int_{T_n}^{T_1} e^{-\beta F_c(T)/T}\frac{dT}{T} . \quad (5.13)$$

Equating T_n with T_Ω, the number of bubbles produced per horizon by the end of the phase transition satisfies the simple relation

$$n \simeq \frac{1}{16\pi}\beta^{-3}\left[\frac{dF_c}{dT}\right]_\Omega^3 = \frac{1}{2\pi}\beta^{-3}\left[\frac{T_\Omega}{T_1 - T_\Omega}\right]^3\left[\frac{F_c(T_\Omega)}{T_\Omega}\right]^3 . \quad (5.14)$$

It is easy to understand why the number of bubbles per horizon increases as the derivative of the saddle-point free energy increases. At the end of the phase transition, the faster the nucleation rate is changing, the shorter the duration of bubble nucleation. A larger number of bubbles needs to be produced for the phase transition to complete in a shorter time interval.

VI. AVOIDING WASHOUT OF THE BARYON ASYMMETRY

Can baryogenesis occur at the one-Higgs-doublet EWPT? The picture of the phase transition which we have given, valid for Higgs-boson masses from the experimental limit of 46 GeV up to about 150 GeV, certainly shows that the Universe is far from thermal equilibrium after the temperature drops below T_N. At high temperature, anomalous baryon-number violation can be quite rapid [6–10]. Hence the possibility of successful baryogenesis rests on two issues: sufficient CP violation; avoiding washout of the $B+L$ asymmetry, after T_Ω.

The numerical size of CP violation in the standard model is insufficient for baryogenesis. This is true even if the strong CP parameter $\bar{\theta}$ were of order unity at high temperatures, because the physical effects of $\bar{\theta}$ are repressed by light-quark Yukawa couplings. Nevertheless, it is simple to add new physics to the standard model which yields sufficient CP violation without changing the behavior of the EWPT. One possibility is that this new physics yields operators of the form $(1/M^2)\phi^*\phi F\tilde{F}$ in the low-energy theory, where F is the electroweak field strength [11]. Hence it may be possible to create a significant baryon asymmetry at the one-Higgs-doublet EWPT.

We are left with the problem of how to avoid the $B+L$ asymmetry being depleted just after T_Ω as soon as thermal equilibrium is reestablished. Several authors [12,16] have argued that anomalous baryon-number violation will washout any baryon asymmetry for Higgs-boson masses larger than some critical value, m_{H_c}. Although there are uncertainties in the calculation of m_{H_c}, it is in the vicinity of 50 GeV, perilously close to the experimental lower bound. Hence, the one-Higgs-doublet EWPT of the minimal standard model, extended only to include CP-violating operators of the form $(1/M^2)\phi^*\phi F\tilde{F}$, does not yield an acceptable baryon asymmetry, with the possible exception that the Higgs-boson mass is very close to 46 GeV. (There is also the possibility that m_H is much larger than 100 GeV, in which case we do not know the nature of the phase transition.)

In this section we show that the problem of baryon washout in one-Higgs-doublet models can be solved in a way similar to the solution of the problem of sufficient CP violation. Particles can be added to the standard model such that our analysis of the one-Higgs-doublet EWPT persists, but baryon washout is avoided for Higgs-boson masses all the way up to 150 GeV. We find that these additional particles affect the EWPT indirectly, by changing the numerical values of the parameters of the effective potential (B,D,E,λ_T). However, our formulas for quantities of interest $[T_{1,2}\phi_\pm(T_{1,2}),\epsilon_\Omega$, etc.] are still correct when written in terms of these parameters B,D,E,λ_T.

As we go beyond the minimal standard model, why not go to the two-Higgs-doublet model? We do not do this because this greatly complicates the EWPT. In general, one cannot just define a single linear combination of the Higgs bosons as the one which gets a vacuum expectation value (VEV), because this combination is T dependent. No complete analysis of the phase transition exists. For example, we do not know the quantities $\phi_{1+}(T_\Omega)$ and $\phi_{2+}(T_\Omega)$ which are relevant for baryogenesis. The advantage of the one-Higgs-doublet EWPT is that we know essentially everything about the phase transition, so that we can use the requirement of avoiding baryon washout as a guide to what new physics should exist.

The analysis of this section will be valid for a whole class of models. This class of models has a single Higgs doublet and has the EWPT proceed by the nucleation of thin-wall bubbles. More precisely, the class is defined by three criteria.

(i) The EWPT is induced by a single Higgs doublet, and the coefficient of the tree-level ϕ^4 term is sufficiently small that the Higgs contribution to $V(\phi,T)$ can be neglected.

(ii) All particle masses are such that at temperatures near T_1 the high T approximation, (B6), for $V(\phi,T)$ is valid, with the exception of particles so massive that their thermal contributions to the effective potential are Boltzmann suppressed to the point that they can be ignored.

(iii) The EWPT proceeds by the thermal fluctuation of thin-wall bubbles. This means that[9]

$$\epsilon_\Omega \simeq \frac{1}{12}\frac{E^{1/2}}{\lambda_T^{3/4}} \ll 1 . \tag{6.1}$$

These one-Higgs-doublet models all have the EWPT proceed as we have described, except now the parameters B, D, E, λ_T can differ significantly from their standard-model values. We now calculate how the baryon washout rate depends on these parameters. We will not attempt to calculate the numerical value for the washout rate, however.

At $T < T_\Omega$ the rate for anomalous baryon number violation is proportional to $\exp[-F_{sp}(T)/T]$, where $F_{sp}(T)$ is the sphaleron free energy at temperature T. In the region of interest to us $T \simeq T_\Omega$ (which is just below T_1), and we do not know what $F_{sp}(T)$ is. This is because the usual approximation of keeping only the $T^2\phi^2$ terms in the high T expansion of $\Delta V_1(\phi, T)$ [22] is not good at temperatures near T_Ω. In particular it is clear that the term $-ET\phi^3$ cannot be neglected. Although the $\phi^2 T^2$ terms are the largest ϕ-dependent terms in the m/T expansion, they combine with the zero-temperature ϕ^2 terms to cancel when the temperature is T_2. One possibility is to try to find some lower temperature where the ϕ^3 term can be dropped, but where the high T expansion is still good. However, since the baryon washout rate decreases as T is lowered, this will only yield a lower bound on the amount of depletion. Alternatively one can do a numerical analysis for the sphaleron energy at temperatures just below T_1 [12]. We will assume that the sphaleron free energy is linear in $\phi(T)/T$, a form motivated by the inclusion of the $T^2\phi^2$ terms. Hence we take

$$\frac{dn_B}{n_B} = \frac{\Gamma}{HT} dT , \tag{6.2a}$$

where

$$\frac{\Gamma}{HT} = C_1 \exp\left[-\left[C_2 \frac{\phi_+(T)}{T}\right]\right] \tag{6.2b}$$

and $H = H(T)$ is the Hubble parameter. C_2 is a large dimensionless constant which we assume has a more mild dependence on (B, D, E, λ_T) than does $\phi_+(T)$ in the region of $T = T_\Omega$. We take C_1 to be a constant, although our results are unchanged if it has a large power dependence on T.

The most likely T region for baryon washout is immediately below T_Ω. If the washout is to be limited, the washout rate must have frozen out well before T_2: $(\Gamma/H)_{T_2} \ll 1$. Taking $\phi_+(T) = (ET/\lambda_T)\phi'_+(T)$ where ϕ'_+ is given in Eq. (2.14), and integrating (6.2a) gives a logarithmic depletion of

$$\ln\left[\frac{n_B(T_\Omega)}{n_B(0)}\right] \simeq \frac{\lambda_T}{2C_2 E}\left[\frac{T_1 - T_2}{T_1}\right]\left[\frac{\Gamma}{H}\right]_{T_\Omega} , \tag{6.3}$$

where

$$\left[\frac{\Gamma}{H}\right]_{T_\Omega} \simeq C_1 T_1 \exp\left[-C_2 \frac{E}{2\lambda_T}(3 + \sqrt{1 + 8\epsilon_\Omega})\right] . \tag{6.4}$$

When $\epsilon_\Omega \ll 1$, it can be dropped from Eq. (6.4). Whether depletion is significant is largely a question of whether the baryon washout rate freezes out before or after T_Ω, i.e., of whether $(\Gamma/H)_{T_\Omega}$ is greater or less than unity. This entire class of one-Higgs-doublet models has a baryon washout rate which is exponentially sensitive to the ratio E/λ_T, but is relatively insensitive to B and D, and therefore to T_1 and T_2. There is significant sensitivity to m_H because λ_T is related to m_H by

$$\lambda_T = \frac{m_H^2}{2\sigma^2} - (\Delta\lambda + 6B) , \tag{6.5}$$

where we have defined the logarithmic terms of Eq. (2.4) to be $\Delta\lambda$: $\lambda_T = \lambda - \Delta\lambda$.

Others have found a critical Higgs-boson mass of about 50 GeV [12,16] in the minimal standard model. This corresponds to

$$\left[\frac{E}{\lambda_T}\right]_c \simeq \frac{1}{2} \tag{6.6}$$

for low top-quark masses.

We can now see how simple it is to avoid baryon washout at the one-Higgs-doublet EWPT. Particles should be added to the standard model so that $E/\lambda_T > \frac{1}{2}$ for any desired value of m_H in the region of 45–150 GeV. This can be accomplished in two ways: add bosons with small $SU(2) \times U(1)$-preserving masses to increase E; add bosons so that $\Delta\lambda + 6B$ is increased. For a given Higgs-boson mass this decreases λ_T as can be seen from (6.5).

It is interesting that both possibilities involve additional bosons. Although fermions never contribute to E, they do contribute to $\Delta\lambda + 6B$. However, they tend to increase baryon depletion. This can be seen from the fact that a heavy top-quark mass in the standard model decreases the critical Higgs-boson mass:

$$m_{H_c}^2(m_t) = m_{H_c}^2(0) - \frac{3\sigma^2}{4\pi^2}\left[\frac{m_t}{\sigma}\right]^4 \left[4 \ln\left[\frac{m_t}{\sqrt{c_F} T_1}\right] + 3\right] . \tag{6.7}$$

We now give a specific simple extension of the standard model which avoids baryon washout even for Higgs-

[9]In order to use this formula for ϵ_Ω it is actually necessary to also require that $T_1 - T_2 \ll T_1 + T_2$. This is because our thin-wall analysis for ϵ_Ω was based on the effective potential of (2.13) which assumes $T_1 - T_2 \ll T_1 + T_2$. However if this constraint on $T_1 - T_2$ is not satisfied, Eq. (6.1) still holds provided ϵ is replaced by $F\epsilon$, where $F = (T_2/T_1)[2T_2/(T_1 + T_2)] \le 1$. In the majority of models where F is not close to 1, we find that F is not small either.

boson masses up to 150 GeV. We add a spin-0 multiplet S which is a singlet, so that the Lagrangian of the standard model is augmented by

$$\mathcal{L}_S = \partial^\mu S^* \partial_\mu S - M^2 S^* S - \lambda_S (S^* S)^2 - 2\zeta^2 S^* S H^* H \,, \tag{6.8}$$

where H is the Higgs-doublet field: $|H^0| = \phi/\sqrt{2}$. We take M^2 and ζ^2 to be positive so that $\langle S \rangle = 0$ at all temperatures, and the EWPT is that of the single Higgs doublet.

A simple possibility is to take M^2 sufficiently small that where $\phi \simeq \phi_+(T_1)$ the scalar mass $m_S^2 = M^2 + \zeta^2 \phi_+^2 \simeq \zeta^2 \phi_+^2$. In this case the S field contributes to E. If it gives the dominant contribution to E, then

$$\frac{E}{\lambda_T} = \frac{g_S}{\pi} \zeta^3 \left[\frac{100 \text{ GeV}}{m_H} \right]^2 , \tag{6.9}$$

where g_S is the degeneracy of the S multiplet, and we have chosen parameters such that $\Delta\lambda + 6B$ can be dropped in Eq. (6.5). The size of E/λ_T is restricted by requiring that the high T expansion be valid at T_1:

$$\frac{E}{\lambda_T} \leq \frac{1}{\zeta} \,. \tag{6.10}$$

Hence we must take $\zeta \leq 1$. Note that the criterion (6.1) for the thin-wall approximation is easily satisfied for E/λ_T much bigger than we need. There are a wide range of parameters that avoid baryon washout for all Higgs-boson masses up to 150 GeV.

As an example, consider $\zeta = 1$, $g_S = 1$, and $m_H = 100$ GeV. In this case $E/\lambda_T \simeq 2/\pi$, and all constraints and approximations are satisfied. Taking $m_t = 125$ GeV gives $T_2 \simeq 130$ GeV and $T_1 \simeq 148$ GeV. How small must the SU(2)-invariant S mass be in order that $m_S(T_1) \simeq \zeta \phi_+(T_1)$ is a valid approximation? We find that $M \leq \zeta (E/\lambda_T) T_1 \simeq 94$ GeV, which does not involve any more fine-tuning than for the Higgs doublet.

The simple extension of the standard model described by Eq. (6.8) can also avoid baryon washout, even if the S particle has a mass in the TeV range. At first sight this is surprising since S does not contribute to E. This is because $M \gg \zeta \phi_+(T_1)$, and because the S particles have an exponentially suppressed number density at T_1. The contribution of the S field to the effective potential $V_S(\phi)$ can then be obtained by expanding (A6) in a power series in $\zeta\phi/M$:

$$V_S(\phi) = \frac{g_S \zeta^4}{64\pi^2} \left[-\phi^2(\phi^2 - \sigma^2) + \frac{1}{3} \frac{\phi^6}{\sigma^2} \right] \frac{\zeta^2 \sigma^2}{M^2} + O\left[\frac{\zeta^2 \sigma^4}{M^4} \right] . \tag{6.11}$$

The decoupling behavior as $M \to \infty$ is manifest. Since $V_S'(\sigma) = V_S''(\sigma) = 0$, this contribution does not alter the minimum of the potential or the relation for the Higgs-boson mass: $m_H^2 = 2(\lambda_{T_{\rm SM}} + 6B_{\rm SM})\sigma^2$. For our purposes the most important consequence of V_S is to correct the coefficient of ϕ^4:

$$\lambda_T = \lambda_{T_{\rm SM}} - \frac{g_S \zeta^4}{16\pi^2} \frac{\zeta^2 \sigma^2}{M^2} \,. \tag{6.12}$$

This is a cruical correction since it changes the m_H/λ_T relation, increasing the critical Higgs-boson mass:

$$m_{H_c}^2(\zeta) = m_{H_c}^2(\zeta=0) + \frac{g_S \zeta^4}{8\pi^2} \sigma^2 \frac{\zeta^2 \sigma^2}{M^2} \,. \tag{6.13}$$

Hence if $(g_S \zeta^4)(\zeta^2 \sigma^2/M^2) = 3$, the critical Higgs-boson mass squared is increased by $(50 \text{ GeV})^2$. For example taking $g_S \zeta^4 = 15$ and $\zeta^2 \sigma^2/M^2 = \frac{1}{5}$ would give a bare S mass of $M = 0.56\zeta$ TeV. Baryon depletion can be avoided even if the additions to the standard model have masses in the TeV region.

We conclude that successful baryogenesis at the one-Higgs-doublet EWPT can occur provided the Higgs doublet is given two new interactions: one to violate CP and the other to enhance E/λ_T to avoid baryon workout.

VII. CONCLUSION

We have presented a completely analytic treatment of the electroweak phase transition (EWPT), valid for all Higgs-boson masses from the experimental limit of 46 GeV up to about 150 GeV. The electroweak phase transition is first order and proceeds by the nucleation of thin-walled bubbles. We give the precise value of the temperature at which the phase transition completes as a function of the top-quark and Higgs-boson masses. In addition to characterizing the electroweak phase transition, we determine the value of the Higgs field VEV after the phase transition completes, the number of bubbles nucleated per horizon, and many other quantities. Our formulas also apply to many extensions of the standard model which have the EWPT occur through just one field acquiring VEV. Additional particles can significantly alter quantities such as the temperature at which the phase transition completes, through their virtual effects on parameters in the Higgs potential. An important result of our analysis is determination of $\langle \phi \rangle_T$ at the end of the phase transition. The temperature-dependent VEV $\langle \phi \rangle_T$ increases by a factor of 3/2 as the temperature drops from T_1 to T_2. Accordingly, determination of when the phase transition completes is essential because the rate of anomalous baryon-number violation is an exponentially sensitive function of $\langle \phi \rangle_T$. If the rate of anomalous baryon-number violation is large after thermal equilibrium is reestablished, any $B+L$ asymmetry generated during the EWPT will be at best ephemeral.

Much attention has recently been paid to the exciting possibility that the cosmological baryon excess may be produced at the EWPT. The possibility that this occurs in a model with a single Higgs doublet has largely been ignored. This is because it has been shown that even if sufficient baryon asymmetry could be generated, immediately after the EWPT it would be destroyed by anomalous baryon-number violation, at least for Higgs-boson masses above about 50 GeV. As a demonstration of the utility of the analysis of the EWPT presented in this paper, we have shown that this baryon washout is very easily avoided in simple extensions of the standard model. Perhaps the simplest is the addition of a gauge-singlet scalar boson that receives a contribution to its mass from the Higgs boson VEV. In a subsequent paper we will show that the baryon asymmetry of the Universe could be generated at the one-Higgs-doublet EWPT.

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APPENDIX A: THE ZERO-TEMPERATURE, ONE-LOOP EFFECTIVE POTENTIAL

Consider an ensemble of particles i, which receive a contribution to their mass from the vacuum expectation value of a scalar field ϕ. In the mass-eigenstate basis, the unrenormalized one-loop contribution to the effective potential is

$$\Delta V_1=\pm\left[-\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\ln[-k^2+m^2(\phi)-i\epsilon]\right], \quad \text{(A1)}$$

where the $\pm$ is for bosons (fermions), respectively. Going to Euclidean space, introducing a cutoff Λ, and integrating we find

$$\Delta V_1=\pm\frac{1}{32\pi^2}\left[\frac{1}{2}m^4(\phi)\ln\left[\frac{m^2(\phi)}{\Lambda^2}\right]-\frac{1}{4}m^4(\phi)+m^2(\phi)\Lambda^2\right]. \quad \text{(A2)}$$

We write the renormalized potential V_1 as the sum of the tree-level potential $U(\phi)$, the one-loop correction ΔV_1, and a counterterm potential $V_{\rm ct}$:

$$V_1=U(\phi)+\Delta V_1(\phi)+V_{\rm ct}(\phi)\ . \quad \text{(A3)}$$

We denote the renormalized one-loop correction by $\bar V_1=\Delta V_1(\phi)+V_{\rm ct}(\phi)$. At $\phi=\sigma$, we choose to impose renormalization conditions that preserves the tree-level values of m_H, and σ:

$$\text{(i)}\ \frac{d}{d\phi}\bar V_1=0\ ,\qquad \text{(ii)}\ \frac{d^2}{d\phi^2}\bar V_1=0\ . \quad \text{(A4)}$$

For a collection of particles with masses of the form $m^2(\phi)=\mu^2+g\phi^2$, imposing the renormalization conditions above, we find[10]

$$V_1(\phi)=\sum_i\pm\frac{g_i}{64\pi^2}\left[m_i^4(\phi)\ln\frac{m_i^2(\phi)}{m_i^2(\sigma)}-\frac{3}{2}m_i^4(\phi)+2m_i^2(\phi)m_i^2(\sigma)\right] \quad \text{(A5)}$$

plus terms independent of ϕ. We will have occasion to use two special cases of this formula. For a particle with mass $m(\phi)=\lambda\phi$, we recover the well-known result [18]

$$\bar V_1(\phi)=\pm\frac{\lambda^4}{64\pi^2}[\phi^4\ln(\phi^2/\sigma^2)-\tfrac{3}{2}\phi^4+2\phi^2\sigma^2]\ . \quad \text{(A6)}$$

Finally, for a particle with a large SU(2)-conserving mass $m^2(\phi)=M^2+g\phi^2$ where $M^2\gg g\sigma^2$, the one-loop contribution decouples as

$$V(\phi)=\pm\frac{g^3}{64\pi^2}(\sigma^4\phi^2-\sigma^2\phi^4+\tfrac{1}{3}\phi^6)M^{-2}+O(M^{-4})\ . \quad \text{(A7)}$$

APPENDIX B: THE EFFECTIVE POTENTIAL OF THE STANDARD MODEL

For the tree-level potential

$$U(\phi)=\frac{\lambda_0}{4}(\phi^2-\sigma^2)^2\ , \quad \text{(B1)}$$

λ_0 is related to the Higgs-boson mass by $m_H^2=2\lambda_0\sigma^2$. The one-loop potential is the sum of the classical potential and a one-loop correction $V=U+\bar V_1$. If we adopt the renormalization prescriptions (i) $V''(\sigma)=m_H^2$ and (ii) $V'(\sigma)=0$, the relation between m_H and λ_0 will be preserved. In this case, for each degree of freedom, the one-loop corrections to the effective potential is given by (A5):[11]

$$\bar V_1(\phi)=\pm\frac{1}{64\pi^2}\{m^4(\phi)\ln[m^2(\phi)/m^2(\sigma)]-\tfrac{3}{2}m^4(\phi)+2m^2(\phi)m^2(\sigma)-\tfrac{1}{2}m^4(\sigma)\}, \quad \text{(B2)}$$

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[10]Strictly speaking, the tree-level Higgs-boson mass preserved by (A4) is the Higgs-boson mass at zero Euclidean momentum. The formulas we obtain here will have corrections coming from running the mass from zero Euclidean momentum up to m_H [23]. The divergence in (A5) for Goldstone bosons is an artifact of this running. This is because the Goldstone-boson contribution gives an infinite running of the Higgs-boson mass between $p^2=m_H^2$ and zero Euclidean momentum. Alternatively one could impose renormalization conditions at different value of ϕ. In that case, a singularity in V'' would still exist for the reasons mentioned above, and parameters of effective potential would be related to measurable quantities by a running in both ϕ space and momentum space. In this paper we work with Higgs-boson self-couplings small enough that we can neglect scalar loops which include the Goldstone bosons.

[11]We have added a constant so that the one-loop contribution to the cosmological constant vanishes at $\langle\phi\rangle=\sigma$.

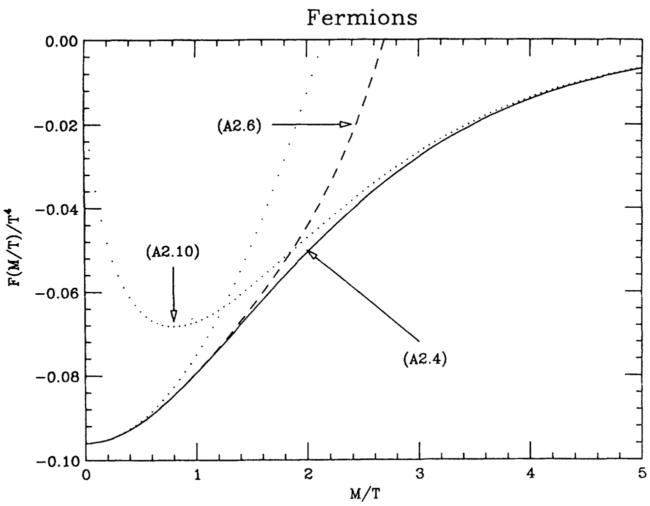

FIG. 7. A fermion's contribution to the free energy as a function of m/T. The solid curve is the exact contribution to one loop. The dashed [dotted] curve represents the approximation (B6) [(B10)].

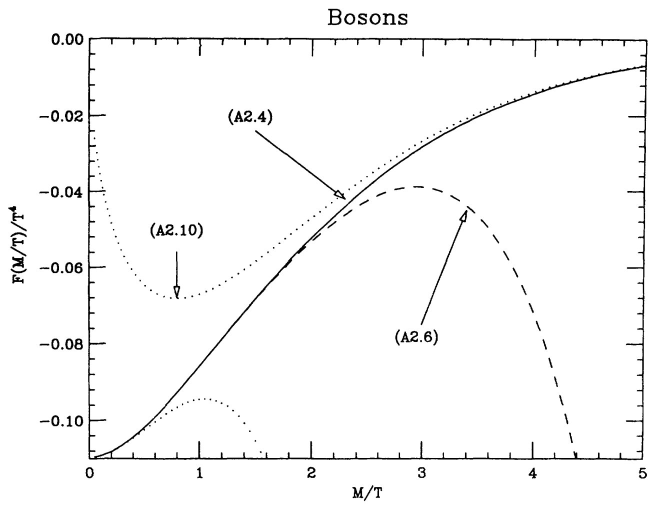

FIG. 8. A boson's contribution to the free energy as a function of m/T. The solid curve is the exact contribution to one loop. The dashed [dotted] curve represents the approximation (B6) [(B10)].

where the $\pm$ is for bosons (fermions) and $m(\phi)$ is the mass of the particle in the presence of a background field. Neglecting the Higgs-doublet contribution to $\bar{V}_1$, the one-loop, zero-temperature, effective potential for the physical Higgs scalar is given by the well-known expression [18]

$$V(\phi)=-\tfrac{1}{2}(\lambda+2B)\sigma^2\phi^2+\tfrac{1}{4}\lambda\phi^4+B\phi^4\ln(\phi^2/\sigma^2) \qquad \text{(B3)}$$

plus terms independent of ϕ. Here ϕ, the physical Higgs scalar, has a mass $m_H^2=2\lambda_0\sigma^2=(2\lambda+12B)\sigma^2$, $\langle\phi\rangle=\sigma$, and

$$B=\frac{1}{64\pi^2\sigma^4}(6m_W^4+3m_z^4-12m_t^4) \; .$$

When the system is in contact with a hot thermal reservoir, such as in the early Universe, the effective potential for the Higgs boson must be modified to include the interactions between the Higgs field and the hot ambient plasma. The thermal one-loop corrections to the effective potential for the Higgs boson is just the free energy of the Bose-Einstein and Fermi-Dirac distributions of particles getting a mass from ϕ:

$$\Delta V_1(\phi,T)=-\sum_F \frac{g_F T^4}{2\pi^2}\int_0^\infty dx\, x^2\ln(1+e^{-\sqrt{x^2+\beta^2 m_F^2}}) + \sum_B \frac{g_B T^4}{2\pi^2}\int_0^\infty dx\, x^2\ln(1-e^{-\sqrt{x^2+\beta^2 m_B^2}}) \; , \qquad \text{(B4)}$$

where $m_{B(F)}$ is the mass of a boson (fermion) in the presence of a background field ϕ, $g_{B(F)}$ is the number of degrees of freedom, $\beta=1/T$ and $(F)B$ denotes a sum over (fermions) bosons, respectively. Expanding the argument of the logarithm and integrating, the integral equation for $\Delta V_1(\phi,T)$, can be written in terms of a sum:

$$\Delta V_1(\phi,T)=\sum_F \frac{g_F T^4}{2\pi^2}\left[\sum_{n=1}^\infty \frac{(-1)^n}{n^2}(\beta m_F)^2 K_2(\beta m_F n)\right] - \sum_B \frac{g_B T^4}{2\pi^2}\left[\sum_{n=1}^\infty \frac{1}{n^2}(\beta m_B)^2 K_2(\beta m_B n)\right] . \qquad \text{(B5)}$$

Since the modified Bessel function K_2 falls off exponentially for large values of its argument, the expression (B5) is well suited to numerical computation when m/T is large. In the high-temperature limit, when $m(\phi)/T$ is small, Eq. (2.6) can be expanded in powers of $m(\phi)/T$ [3]. Excluding the terms independent of ϕ,

$$\Delta V_1(\phi,T)=\sum_B g_B\left[\frac{1}{24}m_B^2T^2-\frac{1}{12\pi}m_B^3T-\frac{m_B^4}{64\pi^2}\ln(m_B^2/c_BT^2)\right] + \sum_F g_F\left[\frac{1}{48}m_F^2T^2+\frac{m_B^4}{64\pi^2}\ln(m_F^2/c_FT^2)\right]+O(M^6/T^2)+O(M^6/T^2\ln(m/T)) \; , \qquad \text{(B6)}$$

where $\ln(c_B)=\frac{3}{2}+2\ln 4\pi-2\gamma\simeq 5.41$ and $\ln(c_F)=\frac{3}{2}+2\ln\pi-2\gamma\simeq 5.41$ and $\ln(c_F)=\frac{3}{2}+2\ln\pi-2\gamma\ \simeq 264.$[12]

Expanding to order T^0 in m/T and neglecting all one-loop Higgs-boson self-interactions, we add (B3) and (B6) to obtain the one-loop, temperature-dependent potential [16,18]

$$V(\phi,T)=[DT^2-(\tfrac{1}{2}\lambda+B\sigma^2)\sigma^2]\phi^2-E\phi^3T+\tfrac{1}{4}\lambda_T\phi^4$$
$$=D(T^2-T_2^2)\phi^2-ET\phi^3+\tfrac{1}{4}\lambda_T\phi^4\ , \qquad \text{(B7)}$$

where the constants $D,E,$ and λ_T are given in Sec. II. The absolute instability of the origin occurs when the temperature reaches T_2:

$$T_2^2=\frac{(2\lambda+4B)\sigma^2}{4D}=\frac{m_H^2-8B\sigma^2}{4D}\equiv\chi^2(m_t,m_H)m_H^2\ . \qquad \text{(B8)}$$

As well as providing good qualitative behavior, the standard m/T expansion (B6) gives reliable quantitative results up to surprisingly large values of m/T. For what values of m/T is the high-temperature approximation accurate? From Figs. 7 and 8 we see that the high-temperature approximation agrees with exact potential to better than 5% for $m/T<1.6$ (2.2) for fermions (bosons). It is interesting to note that unless great accuracy is required, for any value of m/T, either the high-temperature or a low-temperature approximation can be used to obtain a simple formula for the free energy. For large values of x, the modified Bessel function K_2 has the asymptotic behavior:

$$K_2(x)=\left[\frac{\pi}{2x}\right]^{1/2}e^{-x}\left[1+\frac{15}{8x}+\cdots\right]\ . \qquad \text{(B9)}$$

So for large values of m/T the temperature-dependent contribution to the free energy is given by

$$\Delta V_1(\phi)=\left[\frac{m(\phi)}{2\pi T}\right]^{3/2}e^{-m(\phi)/T}\left[1+\frac{15T}{8m(\phi)}+\cdots\right]\ . \qquad \text{(B10)}$$

For any value of m/T, the better of (B6) or (B10) will give a value for $\Delta V_1(\phi)$ which is good to better than 10%.

[12]This comes from Doland and Jackiw [3] where the 2.64 was misprinted as 2.84, we thank David Brahm for bringing this to our attention.

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