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Late-phase-transition-induced fluctuations in the cosmic neutrino distribution and the formation of structure in the Universe

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We show that late-time ($T_c \leq 1$ eV) cosmic vacuum phase transitions which induce separation of phases and which involve a discontinuity in neutrino properties (e.g., mass) across phase boundaries could give rise to spatial inhomogeneities in the distribution of neutrinos. The nucleation physics of the phase transition would determine the spatial scale of the fluctuations. These density perturbations would be born in the nonlinear regime and could have masses in the range of $10^6 M_\odot - 10^{13} M_\odot$. If the fluctuations are shells, as expected, and there is gravitational modification of the original phase-transition nucleation scale then the upper limit on their masses could be considerably larger ($\approx 10^{18} M_\odot$), possibly encompassing the largest structures in the Universe. The motivation for this work stems, in part, from the possibility that future experiments (i.e., solar neutrino experiments) may suggest new, low-energy-scale weak-interaction phenomena, such as neutrino flavor mixing.

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I. INTRODUCTION

In this paper we discuss how late-time vacuum phase transitions involving neutrino properties could produce fluctuations in the cosmic distribution of neutrinos. These fluctuations would have spatial scales determined for the most part by the internal nucleation dynamics of the phase transition, not by gravitational clumping. The key ingredients of this mechanism are as follows: (1) a first-order phase transition to effect separation of phases; and (2) a discontinuity in neutrino properties (e.g., mass or mean free path) across the phase boundary. This paper deduces the astrophysical consequences of such a phase transition. We emphasize that most of our conclusions follow from generic features of first-order phase transitions (e.g., nucleation scale and duration) occurring at a specific temperature once the above two conditions are met and do not depend sensitively on the detailed dynamics of the particular weak-interaction model employed.

Recent studies have explored how late-time (after photon decoupling) phase transitions may account for some aspects of large-scale structure formation in the Universe [1–3]. These models represent an alternative to the standard picture. The standard galaxy formation ideas involving Gaussian primordial fluctuations from the end of the inflation epoch with some variety of cold dark matter may have difficulty reconciling the existence of highly evolved structures at redshifts $z \geq 4.5$ and the existence of ubiquitous large structures such as the "great wall" or large-scale velocity flows with the high degree of isotropy observed in the cosmic microwave background radiation [4]. Although the current data are not yet unequivocal in this regard it is worthwhile to explore alternatives to this standard picture. An alternative model is to have the generation of fluctuations occur after recombination. Such late-phase-transition models can usually circumvent background radiation anisotropy bounds, and may even explain the largest observed structures in the Universe, such as the possible "bubbles" associated with the recently reported (though as yet unconfirmed) redshift quasiperiodicity [5,6].

In Sec. II we discuss the essential features which nonstandard weak-interaction models must possess in order to produce fluctuations with our mechanism. The nucleation and phase-separation physics of generic small-supercooling-limit phase transitions as applied to the early Universe is discussed in Sec. III, while bubble wall motion, fluctuation generation, and cosmic microwave background distortions are discussed in Sec. IV. Conclusions are given in Sec. V.

II. NONSTANDARD WEAK INTERACTIONS

Astrophysical considerations have fueled speculation on massive neutrinos, though it remains to be seen if neutrinos actually have mass and, if so, how they acquire it. Among the models crafted to give neutrinos mass are those which use spontaneous symmetry breaking at low energy scales [7]. These are of particular interest because they may imply a cosmic vacuum phase transition associated with the symmetry-breaking event. As outlined above, our fluctuation generation mechanism requires that if such a phase transition occurs it must be first order to effect phase separation, and it must provide for differing neutrino properties, such as mass, across phase boundaries. As far as we know there are as yet no weak-interaction neutrino-mass theories which unambiguously possess these properties and at the same time are compatible with experimental and astrophysical constraints. Nevertheless it is interesting that several well known models in this genre are likely to possess these properties. If neutrino masses *are* acquired at low energy scales then a successful theory which explains them may share some of the features of these models. For this reason we will discuss some of these models, though we emphasize that our fluctuation generation mechanism is not specifically dependent on them.

The triplet Majoron model [7] for the weak interaction has the interesting property that neutrino-neutrino scattering cross sections can be very much larger than in the standard model. Raffelt and Silk [8] have discussed how the large neutrino scattering cross sections, and concomitant short mean free paths, in such a model might allow light neutrinos in the early Universe to mimick cold dark matter. We now know, however, from the recent Z^0 width experiments [9] that the triplet version of the Majoron model must be incorrect. The triplet of Higgs fields inherent in this model gives a coupling to the Z^0 that counts as two extra neutrino species and since the experimental result is that the equivalent number of neutrinos [9] in the Z^0 decay is $N_\nu = 2.98 \pm 0.06$ (in close correspondence to the known number of neutrino families), we must rule out the specific couplings in this model. The same conclusion would apply to any model of the weak interaction in which neutrinos and the intermediate bosons couple to light scalars.

The singlet Majoron model is not ruled out by these experiments, but this model lacks the enhanced neutrino-neutrino scattering cross sections of the triplet model. Nevertheless, there are suggestions of weak-interaction models which might retain some aspects of the triplet Majoron model (especially the enhanced $\nu\nu$ interactions) yet evade direct coupling to the Z^0. Some of these models avoid strong Z^0 couplings by invoking right-handed neutrinos [10], while others [7] resort to a hybrid singlet-triplet Majoron model. This latter case has been specifically discussed in the context of 17-keV neutrinos and, so far as we know, it remains an open question whether or not this model could be adapted to lower energy scales.

Since the triplet Majoron model has the essential points of weak-interaction physics we wish to explore, and is particularly simple to calculate with, in what follows we will sometimes refer to it for illustrative or demonstration purposes. However, our arguments merely presume the existence of massive neutrinos and some extra $\nu\nu$ coupling consistent with known constraints [11,12]. Only in this regard will we assume that the actual weak interaction shares some general properties of the triplet Majoron model.

We require that neutrinos have zero mass at high temperatures and acquire masses when the temperature drops through a critical temperature T_c. This might be engineered in a manner similar to the Majoron models by invoking a symmetry-breaking transition. In these models a U(1) charge symmetry associated with lepton number or family symmetry is spontaneously broken at low temperature. The masses of neutrinos are related to the vacuum expectation value of the part of the Higgs field ($\langle V \rangle$) corresponding to lepton number or family symmetry by

$$m_{\nu_i} = g_{ii} \langle V \rangle \, , \tag{1}$$

where i runs over e, μ, and τ, and g_{ii} is a small number characterizing the strength of the coupling. In the case of electron neutrinos in the triplet Majoron model, laboratory double β decay and supernova considerations yield an upper limit $g_{ee} \le 10^{-4}$ [12,13]. The coupling constants for the other neutrinos are not known but must be $g_{ii} \le 1$ in this model. Any model of the weak interaction which builds in a nonstandard $\nu\nu$ interaction strength must not exceed the experimental bounds on these properties [11–13].

We will assume that there is a first-order phase transition associated with whatever mechanism generates neutrino mass and extra interactions. In this case the stable, low-temperature, or broken phase nucleates via thermal perturbations or quantum tunnelling in the manner described by Coleman [14–16]. Bubbles of the broken phase nucleated in this manner expand until they coalesce. In the unbroken phase the decoupled neutrinos are massless, aside from thermal effects, and free-stream to the horizon; whereas, in the broken phase, the neutrinos may form a tightly coupled fluid, with short mean free paths, though they remain decoupled from the rest of the matter and radiation since they only interact weakly with these particles. We note that our fluctuation generation mechanism only requires some *discontinuity* in neutrino properties at the phase boundary. We will not require that the mass of the neutrinos be rigorously zero in the symmetric phase as, in fact, thermal effects would be expected to produce effective masses for neutrinos there.

In the triplet Majoron model $\nu\nu$ scattering can be mediated by the Nambu-Goldstone boson associated with the U(1) symmetry breaking (the "Majoron"). The $\nu\nu$ scattering cross section in this case is roughly

$$\sigma \approx \frac{g^4}{16\pi} T^{-2} \approx (2.5 \times 10^{-32}\ \mathrm{cm}^2) \left[\frac{g}{10^{-5}} \right]^4 \left[\frac{T}{\mathrm{eV}} \right]^{-2} , \tag{2}$$

where g is one of the g_{ii}. This cross section can be large enough that the neutrino mean free path would be very short compared to the nucleation scale of the phase transition. In the broken phase the neutrinos will be equilibrated among themselves when $T_c \approx 1$ eV, and one expects the neutrinos to feed into the lightest species [8], because in the triplet Majoron model lepton number is not conserved in $\nu\nu$ interactions. Lepton number violation is, of course, specific to the Majoron model. In the triplet Majoron model the lightest particle would probably be the Majoron particle itself. For temperatures $T_c \le 1$ eV the neutrinos may not come into equilibrium, given bounds on extra or "secret" neutrino interactions [11–13], though the mean free paths in the broken phase may still be small compared to the horizon.

We note that there are well known constraints on nonstandard neutrino interactions such as the neutrino-Majoron or Majoron-mediated neutrino-neutrino cross sections. These comprise the so-called "secret" neutrino interactions, the best limits on which come from the neutrino detection for Supernova 1987A [11–13]. The upper limit on the neutrino cross section from this source is $\sigma_\nu < 10^{-25}$ cm^2, which is some three orders of magnitude *larger* than the largest cross section suggested in the triplet Majoron model [Eq. (2)], which is taken as our fiducial example.

In the triplet Majoron model the mean free path for neutrinos in the broken phase is approximately

$$\lambda \approx (n\sigma)^{-1} \approx 16\pi^2 g^{-4} T^{-1} \approx (3\times 10^{17}\ \text{cm}) \left[\frac{g}{10^{-5}}\right]^{-4} \left[\frac{T}{\text{eV}}\right]^{-1}, \tag{3a}$$

where we assume that the number of target neutrinos and "Majorons" is very roughly $n \approx T^3$. The diffusion length in some fraction of a Hubble time, δH^{-1}, is then

$$\lambda_{\text{dif}} \approx (\delta H^{-1}\lambda)^{1/2} . \tag{3b}$$

We note that the Hubble time at this epoch is of order $H^{-1} \approx m_{\text{Pl}} T^{-2}$, where m_{Pl} is the Planck mass, even though we are close to the matter-dominated epoch if $T \approx 1$ eV.

The ratio of the diffusion length in time δH^{-1} to the free-streaming length (δH^{-1}) is

$$r = \frac{\lambda_{\text{dif}}}{\delta H^{-1}} = \left[\frac{\lambda}{\delta H^{-1}}\right]^{1/2} \approx \frac{4\pi}{g^2}\left[\frac{T}{\delta m_{\text{Pl}}}\right]^{1/2}, \tag{4}$$

where the last equality is only for the triplet Majoron model. We will assume that the actual weak interaction produces a large enough difference between neutrino transport cross sections in the broken and unbroken phases to give a value for r which is greater than unity. Any such weak interaction model will certainly meet the limits on "secret" interactions.

III. LATE PHASE TRANSITIONS AND PHASE SEPARATION

A first-order phase transition associated with the change in neutrino properties is required in order that phase separation take place. Hogan has given a simple model for homogeneous nucleation of phase in the small supercooling limit [16]. In this model the nucleation rate per unit volume is assumed to be of the form

$$p(T) = CT^4 e^{-S(T)} , \tag{5a}$$

where $S(T) = a[T_c/(T_c - T)]$ is the nucleating action, C is an unimportant scale factor of order unity, and a is a monotonically increasing function of temperature. Integrating the nucleation rate through the epoch of bubble coalescence (the end of the phase transition) and assuming that the bubble walls move at the speed of light yields an estimate of the time requires for bubble coalescence, expressed here as a fraction δ of the Hubble time H^{-1},

$$\delta \approx \left[4B \ln\left[\frac{m_{\text{Pl}}}{T_c}\right]\right]^{-1}, \tag{5b}$$

where B is the logarithmic derivative of the nucleating action S, in units of the Hubble time at the epoch of the phase transition, and has been argued to be of order unity [16]. The scale δ results from the comparison of a rapid nucleation rate and the very slow gravitational expansion of the Universe. In this limit most bubbles will be of size δH^{-1} at coalescence. This is because larger bubbles would have to have been nucleated early, near T_c, where the nucleation rate is exponentially small, and smaller bubbles would have to be nucleated near the end of the phase transition where the effective nucleation rate is again small since very little unbroken phase remains.

We wish to emphasize that the above expression for the size scale of the bubbles or equivalently, the duration of phase separation epoch, is insensitive to the detailed dynamics of the underlying weak-interaction theory. In fact this is because the bubble size scale is set by the comparison of the very slow expansion time scale to the relatively much faster weak nucleation time scale in the limit where this ratio is assumed to be small. The main assumptions employed in deriving this bubble size are that we can treat the phase transition in the small supercooling limit and that the nucleating action has the indicated dependence on the degree of supercooling. These conditions are expected to be met for many if not most first-order phase transitions in nature [16]. Furthermore, this bubble size is the single most important determinant of the characteristics of fluctuations produced by our mechanism.

Subsequent to the phase transition, gravitational interactions may modify the effective fluctuation scales by effecting mergers or fragmentation of bubble walls [1,2,3,17]. The distribution of bubble sizes at coalescence and the extent to which the resulting structure resembles a lattice or tessalation will be discussed elsewhere [17].

IV. BUBBLE WALL MOTION AND NEUTRINO DENSITY FLUCTUATIONS

The Coleman [14] picture in which the bubble walls rapidly accelerate to the speed of light is strictly true only in the $T = 0$ limit. In the late phase transitions we consider here the neutrinos are massless (or nearly so) on one side of the phase boundary and massive on the other, at least up to thermal effects. Energy and momentum conservation then require that the speed of the bubble wall associated with the phase boundary be less than the speed of light. This does not greatly effect the analysis of the mean bubble size discussed above [16], so long as the wall moves near the sound speed. Were the wall to move considerably more slowly than this the mean bubble size will differ from that given in Eq. (5b). The fluid velocities on either side of the wall can, in principle, be found from analyses of relativistic shocks and detonation waves [18–20] in the extreme limit where the neutrinos constitute an equilibrated fluid. We can, however, identify two relevant regimes: the first where neutrinos in the broken phase are nonrelativistic and could dominate the Universe; and the second where neutrinos may be relativistic in both phases.

Where neutrinos in the broken phase are nonrelativistic the bubble wall will resemble a detonation front moving into a collisionless-relativistic fluid, and leaving behind a nonrelativistic fluid of neutrinos which may dominate the mass-energy. By "collisionless" here we mean that the neutrino mean free path in the symmetric phase is large compared to the bubble size (certainly true), *not* that the effective mass of neutrinos in this phase from forward scattering interactions or thermal effects is exactly zero. The mean free path is determined by the transport cross section, not the forward-scattering amplitude. We would need a detailed weak-interaction neutrino-mass model in order to calculate the effects of, for example, thermal excitations of scalar fields on neutrino properties in each phase. If this is done and it turns out that neutrino properties are identical across the phase boundaries then there will be no neutrino fluctuations generated by our mechanism. In most laboratory phase transitions which exhibit phase separation or spinodal decomposition, rarely are all the properties of particles or quasiparticles identical in the two phases [16].

If the weak interaction had the lepton-number-violating character of the triplet Majoron model then, as discussed above, the broken phase would consist of the lightest neutrino or coupling boson. In this case, discussed in Ref. [8], neutrino domination could occur only in the unlikely situation where the ν_e is the lightest neutrino, with a closure mass of order 15 eV to 40 eV. Not only is this mass range for the ν_e potentially subject to experimental elimination [11], but the triplet Majoron model would be incapable of producing a neutrino with closure mass if the temperature is $T_c \leq 1$ eV [see Eq. (1)]. We emphasize that this unlikely scenario is peculiar to the triplet Majoron model alone and will not characterize the extensions of the standard model of the weak interaction which we require in this paper. For instance, a weak interaction model which retains the "strong" $\nu\nu$-scattering cross sections but where flavor changing reactions are either absent or of normal weak-interaction strength, i.e., insignificant, could allow the ν_τ to dominate with a mass in the range required for closure. This is obviously a more viable scenario from the standpoint of schematic models of neutrino mass hierarchies [11].

Although there is no direct experimental evidence for massive neutrinos, there are suggestions that neutrino mass could play a role in the solution of several problems in astrophysics. Notable among these are the solar neutrino problem [21] and the missing-mass/dark-matter problem. The Z^0-width experiment has greatly narrowed the field of dark-matter particle candidates [9], underscoring the importance of neutrinos. Particle physics models and Mikheyev-Smirnov-Wolfenstein (MSW) mixing schemes for the solution of the solar neutrino problem suggest that the ν_μ or ν_τ are likely the most massive neutrinos, and therefore the best candidates for a closure mass neutrino.

In the limit where the mass of neutrinos change discontinuously at the phase boundary there are two effects which may act to concentrate neutrinos and, hence, mass. First, the increase in mass at the phase boundary means that neutrinos striking the wall from the unbroken medium may have an appreciable probability to bounce back into the unbroken medium [22]. The wall will then tend to push neutrinos ahead of it, and this enhanced density will be preserved when the walls collide. Furthermore, since the neutrinos will interact strongly with the wall, and the broken medium behind it, they will decelerate when they finally are overtaken by, or otherwise cross, the wall. This implies that the neutrino density will be enhanced immediately behind the wall in the broken phase. The astrophysical effects of our fluctuations are interesting so long as r is greater than unity.

In the limit where the wall moves sufficiently slowly an estimate of the neutrino density jump across the wall can be made from detailed balance: Equating the fluxes across the wall in both directions yields a rough neutrino concentration factor at least of order of the neutrino velocity ratio, $r^{-1} \approx (2T/m_\nu)^{-1/2}$. In fact, the concentration factor will be larger than this when proper account is taken of energy and momentum conservation at the phase boundary. In the hydrodynamic limit for the neutrino gases, the wall would resemble a detonation front where relativistic neutrinos are converted to a nonrelativistic fluid. The Chapman-Jouget condition would apply in this case, so that the velocity of the fluid behind the front would be the sound speed. The neutrino density in such a model would be enhanced in a thin layer behind the front by a factor that depends on the ratio of the upstream and downstream fluid velocities, and which could be large. The strict hydrodynamic limit is unlikely to apply here, however, because the neutrinos upstream of the wall are collisionless, and the mean free paths of neutrinos in the broken medium, though small, may still be large compared to the thin enhanced-density zone length scale (or maybe even the rarefaction zone scale) one would calculate in the hydrodynamic limit.

One can get an idea of the "hydrodynamic" concentration factor by taking account of the diffusion of neutrinos away from the front into the broken medium and the resultant lower limit on the length scale of the density-enhanced region. The concentration factor r^{-1} will be larger than the ratio of the bubble size to the neutrino diffusion length in a coalescence time: $r^{-1} \geq (\delta H^{-1}/\lambda)^{1/2}$. This will be adequate for our subsequent analysis because all we really need to know is that the fluctuations will be in the nonlinear regime, corresponding to $r^{-1} \geq 1$.

If the mass in the horizon (neutrino dominated) is M_H^ν then the mass in the shells produced at the end of the phase transition will be $M_{\rm bubble}^\nu$, where

$$\frac{M^{\nu}_{\rm bubble}}{M^{\nu}_{H}} \approx 3\delta^2 \left[\frac{\omega}{H^{-1}}\right] r^{-1} , \tag{6a}$$

where ω is the width of the enhanced density region when the walls collide. If most of the wall's width is due to diffusion, as argued above, then

$$\frac{\omega}{H^{-1}} \geq \delta^{1/2} \left[\frac{\lambda}{H^{-1}}\right]^{1/2} . \tag{6b}$$

If neutrinos or baryons are subsequently accreted on these structures, then the relevant mass scale of the fluctuations, for the purpose of comparison with structures at the present epoch, would be just the total mass enclosed in radius δH^{-1}, so

$$\frac{M^{\nu}_{\rm bubble}}{M^{\nu}_{H}} \approx \delta^3 r^{-1} . \tag{6c}$$

We caution that gravity may significantly alter the bubble geometries between the end of the phase transition and the present epoch, as previously explained, so that it is not clear what value of δ to employ in Eq. (6c). A reasonable range for δ would be $10^{-6} \leq \delta \leq 10^{-3}$ so that $10^2 M_{\odot} \leq M^{\nu}_{\rm bubble} \leq 10^{13} M_{\odot}$, where we have assumed a value of r^{-1} consistent with the triplet Majoron model value in Eq. (4). The lower mass limit in this range comes from the demand that the neutrino diffusion length in a coalescence time is much less than H^{-1}, so that we are safely in the diffusive limit in the broken phase during the phase transition. In the case of the triplet Majoron model, this constraint would mean that $\delta \gg 10^{-26} g^{-4} (T/\text{eV})$.

There is a minimum mass for fluctuation growth corresponding to the "neutrino Jeans mass" when neutrino interactions are important. This Jeans mass scale (actually the Jeans length) is obtained by equating the dynamical time to the sound crossing time. The neutrino Jeans mass is then

$$M^{\nu}_{J} \approx \frac{m^3_{\rm Pl}}{2T^2} v_s^3 , \tag{7}$$

where $m_{\rm Pl}$ is the Planck mass, T is the phase-transition temperature, and v_s is the sound speed. Where neutrino masses or effective masses (from neutrino-neutrino interactions) are large enough to render the neutrinos nonrelativistic, or nearly so, then the Jeans mass will be roughly the same as for a conventional matter dominated Universe when the temperature is $T \approx 1$ eV, that is, $M^{\nu}_{J} \approx 10^6 M_{\odot}$. If we are not dealing with a neutrino-dominated Universe then it may be that the neutrinos would still be relativistic at the end of the phase transition, so that the sound speed would be $c/\sqrt{2}$ and the Jeans mass would be roughly the same as the horizon mass. Fluctuations in this regime would be damped by neutrino diffusion until the epoch where the Jeans mass fell below the fluctuation scale.

The upper limit on the bubble mass range is set by the demand that the fluctuations not perturb the temperature of the cosmic microwave background radiation by more than $\Delta T/T < 10^{-5}$. An upper bound on the induced anisotropy for a *spherical* fluctuation due to differential redshift/blueshift is [1]

$$\frac{\Delta T}{T} \leq G\rho l^2 \approx \left[\frac{m_{\nu}}{m^2_{\rm Pl}}\right] T^3 (\delta H^{-1})^2 , \tag{8}$$

where G is the gravitational constant, ρ the mass density, and l a typical size scale for the fluctuation. This expression is based on a simple "Sachs-Wolfe" argument. Equation (8) predicts that fluctuations generated by a phase transition at a temperature of $T \approx 1$ eV should have $\Delta T/T < 10^{-6}$–10^{-5} on an angular scale of about one-half degree. This result is marginally safe from observational elimination at this point (see the discussion in Ref. [1], but we emphasize that the actual microwave background distortions from these fluctuations could be considerably smaller than this if, for example, the fluctuations are on shells [1] as expected. If the fluctuations are shells then the upper limit on the mass of the bubbles is considerably relaxed so that, in principle, $M^{\nu}_{\rm bubble}$ could encompass the largest structures in the Universe, $M \approx 10^{18} M_{\odot}$, corresponding to 100 megaparsecs at the present epoch. We note, however, that this would require a significant increase in the effective δ over the nucleation scale of Eq. (5b). This increase would have to be effected by gravitational processes [23].

We could extend this discussion to phase transitions with $T_c > 1$ eV, but we would need to make several modifications in the calculation of the expected cosmic background radiation perturbations. These perturbations would be considerably larger than in the post-photon-decoupling case. Additionally, the phase transition treatment would have to be modified to account for relativistic neutrinos in each phase, though the magnitude of the neutrino concentration effect would be comparable.

V. CONCLUSIONS

In conclusion, we have discussed how a first-order phase transition involving neutrino properties and occurring after photon decoupling may produce fluctuations in the cosmic distribution of neutrinos. The gross characteristics of these fluctuations are determined for the most part by the temperature at which the phase transition takes place. In order for our fluctuation generation mechanism to work we require two features of an underlying weak-interaction neutrino-mass theory: first, that there be a first-order phase transition associated with neutrino properties in order that we have separation of phases for a time; and, second, that there be some difference in neutrino properties like mass or mean free path across the phase boundaries.

We have extended the work of Raffelt and Silk [8] to include the effects of phase separation induced by a first-order phase transition associated with the epoch when neutrinos acquire masses and, possibly, extra interactions. Of course, the paradigm model for the weak interaction used in previous studies, the triplet Majoron model, is incorrect. Nevertheless, we have demonstrated that extensions of the standard model of the weak interaction in which neutrinos have mass and additional interactions may lead to nonlinear perturbations in the spatial distribution of neutrinos. If particle physicists find such a model then it may have important implications for the production of structure in the Universe, because if the phase transition which generates the fluctuations occurs after the photons decouple then induced cosmic microwave background perturbations will be below present observational bounds over a wide range of mass scales. Finally, if the current Ga-solar-neutrino experiments do suggest an MSW neutrino oscillation solution to the solar neutrino problem, then new physics involving neutrino flavor-mixing will be indicated. Perhaps such new physics will involve enhanced $\nu\nu$ interactions or late phase transitions.

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