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Variant of the $S_3 \times Z_3$ model for the 17-keV neutrino

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In extending the recently proposed $S_3 \times Z_3$ model of quark mass matrices to include leptons to account for a possible 17-keV neutrino, a solution was already obtained with a certain choice for the Majorana neutrino mass matrix. We show here that there is another choice which results in a qualitatively new solution. There will be three pseudo Dirac neutrinos, one of which is formed mostly out of ν_e and $\bar{\nu}_\mu$. The 17-keV neutrino decays into $\nu_{e,\mu}$ and a Majoron with a lifetime of about 3×10^{-3} s.

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In the context of the standard SU(2)×U(1) electroweak gauge model with only scalar doublets, it has recently been shown [1] that an $S_3 \times Z_3$ discrete symmetry can be used to derive two successful empirical relationships: $|V_{us}| \simeq (m_d/m_s)^{1/2}(1+m_s^2/m_b^2|V_{cb}|^2)^{-1/4}$ and $m_s^2/m_b^2 \simeq -V_{ub}V_{cb}/V_{us}$. It has also been shown [2] that, with the addition of scalar singlets, this model can be extended in parallel to include leptons so that the electron neutrino may have a massive component with $\sin\theta < 0.06$. This is below the current experimental indication [3–5] of $\sin\theta \simeq 0.09$ for the existence of a 17-keV neutrino. (The experiment of Hime and Jelley [5] measured $\sin^2\theta = 0.0085 \pm 0.0006 \pm 0.0005$.) Whereas further experimentation is clearly needed to confirm or refute this observation, it is theoretically interesting to know whether such a large mixing can be accommodated at all in the $S_3 \times Z_3$ model. As it turns out, a less obvious extension of Ref. [1] does exist which is qualitatively different than that of Ref. [2]. The resulting model has three pseudo Dirac neutrinos, one of which is formed mostly out of ν_e and $\bar{\nu}_\mu$. The 17-keV neutrino decays into $\nu_{e,\mu}$ and a Majoron with a lifetime of about 3×10^{-3} s.

As in Refs. [1,2], the usual six quarks and six leptons of the standard model are organized into doublets and singlets under S_3, and the scalar sector consists of four SU(2) doublets and three SU(2) singlets. The $S_3 \times Z_3$ assignments of the scalar SU(2) doublets remain the same, but those of the scalar SU(2) singlets are now changed. In Table I we list all the leptons and scalars together with their $S_3 \times Z_3$ and lepton-number (L) assignments. As in Ref. [2], the charged-lepton mass matrix is of the form

$$M_l = \begin{bmatrix} 0 & h_1\langle\phi_2^0\rangle & h_2\langle\eta_1^0\rangle \\ h_1\langle\phi_2^0\rangle & 0 & h_2\langle\eta_2^0\rangle \\ h_3\langle\eta_1^0\rangle & h_3\langle\eta_2^0\rangle & h_4\langle\phi_1^0\rangle \end{bmatrix}, \tag{1}$$

in exact analogy with M_d in Ref. [1], and the 3×3 mass matrix linking $(\nu_1,\nu_2,\nu_3)_L$ to $(\nu_1,\nu_2,\nu_3)_R$ is again given by

$$M_\nu = \begin{bmatrix} h_5\langle\bar{\eta}_1^0\rangle & 0 & 0 \\ 0 & h_5\langle\bar{\eta}_2^0\rangle & 0 \\ 0 & 0 & h_6\langle\bar{\phi}_1^0\rangle \end{bmatrix}, \tag{2}$$

in exact analogy with M_u in Ref. [1]. However, the Majorana mass matrix linking $(\nu_1,\nu_2,\nu_3)_R$ to itself is now different than that of Ref. [2], namely,

$$M'_\nu = \begin{bmatrix} 0 & 0 & h_7\langle\chi_1^0\rangle \\ 0 & 0 & h_7\langle\chi_2^0\rangle \\ h_7\langle\chi_1^0\rangle & h_7\langle\chi_2^0\rangle & h_8\langle\chi_3^0\rangle \end{bmatrix}. \tag{3}$$

In fact, it can easily be shown that within the context of $S_3 \times Z_3$, there are only two forms of M'_ν which are of rank 2: either the one given above or that of Ref. [2]

TABLE I. Assignment of leptons and scalars under $S_3 \times Z_3$ and L. The elements of Z_3 are 1, ω, and ω^2, with $\omega^3=1$.

	S_3	Z_3	L
$\begin{pmatrix}\nu_1\\ l_1\end{pmatrix}_L, \begin{pmatrix}\nu_2\\ l_2\end{pmatrix}_L$	2	ω	1
$(\nu_2,\nu_1)_R, (l_2,l_1)_R$	2	ω^2	1
$\begin{pmatrix}\nu_3\\ l_3\end{pmatrix}_L, \nu_{3R}, l_{3R}$	1	1	1
$\begin{pmatrix}\phi_1^+\\ \phi_1^0\end{pmatrix}$	1	1	0
$\begin{pmatrix}\phi_2^+\\ \phi_2^0\end{pmatrix}$	1	ω^2	0
$\begin{pmatrix}\eta_1^+\\ \eta_1^0\end{pmatrix}, \begin{pmatrix}\eta_2^+\\ \eta_2^0\end{pmatrix}$	2	ω	0
(χ_1^0,χ_2^0)	2	ω	−2
χ_3^0	1	1	−2

where nonzero entries occur only in the $(\nu_1,\nu_2)_R$ submatrix.

Consider now the entire 6×6 neutrino mass matrix spanned by $\bar{\nu}_{1L}$, $\bar{\nu}_{2L}$, $\bar{\nu}_{3L}$, ν_{1R}, ν_{2R}, and ν_{3R}:

$$M=\begin{bmatrix} 0 & 0 & 0 & \xi A & 0 & 0 \\ 0 & 0 & 0 & 0 & A & 0 \\ 0 & 0 & 0 & 0 & 0 & B \\ \xi A & 0 & 0 & 0 & 0 & C \\ 0 & A & 0 & 0 & 0 & D \\ 0 & 0 & B & C & D & E \end{bmatrix}, \tag{4}$$

where $\xi=\langle\eta_1^0\rangle/\langle\eta_2^0\rangle=m_u/m_c\simeq 4\times 10^{-3}$ as in Refs. [1,2]. Note first that if $E=0$, then M can be rotated into the form

$$M'=\begin{bmatrix} 0 & m \\ m^T & 0 \end{bmatrix}, \tag{5}$$

where m is a 3×3 matrix. Hence M' is the mass matrix for three Dirac neutrinos. In the case with $E\neq 0$, but A^2, $B^2 \ll C^2$, D^2, E^2, it turns out that M contains two pseudo Dirac neutrinos; if $E\ll\sqrt{C^2+D^2}$ as well, there are three pseudo Dirac neutrinos. The characteristic equation for M is given by

$$\begin{aligned}\lambda^2[\lambda^4-E\lambda^3-(A^2+B^2+C^2+D^2)\lambda^2 \\ +A^2E\lambda+A^2(B^2+C^2)] \\ -\xi^2A^2[\lambda^4-E\lambda^3-(A^2+B^2+D^2)\lambda^2 \\ +A^2E\lambda+A^2B^2]=0\ .\end{aligned} \tag{6}$$

Let $A^2,B^2\ll C^2,D^2,E^2$; then the eigenvalues are approximately given by

$$\lambda\simeq\tfrac{1}{2}[E\pm\sqrt{E^2+4(C^2+D^2)}]\ , \tag{7}$$

$$\lambda\simeq\pm\frac{AC}{\sqrt{C^2+D^2}}+\frac{A^2D^2E}{2(C^2+D^2)^2}\ , \tag{8}$$

and

$$\lambda\simeq\pm\frac{\xi AB}{C}+\frac{\xi^2A^2E}{2C^2}\ . \tag{9}$$

Presumably, the 17-keV neutrino is to be identified with the eigenvlaues of Eq. (8), which have eigenstates given by

$$\begin{aligned}n_{3,4}\simeq\frac{1}{\sqrt{2}}\Bigg[-\frac{\xi D}{C},1,\mp\frac{BD}{C\sqrt{C^2+D^2}},\mp\frac{D}{\sqrt{C^2+D^2}}, \\ \pm\frac{C}{\sqrt{C^2+D^2}},-\frac{AD}{C^2+D^2}\Bigg]\ ,\end{aligned} \tag{10}$$

and the much lighter neutrino of Eq. (9) is associated with the eigenstates

$$n_{1,2}\simeq\frac{1}{\sqrt{2}}\left[1,\frac{\xi D}{C},\mp 1,\pm\frac{B}{C},\pm\frac{\xi^2BD}{C^2},-\frac{\xi A}{C}\right]\ . \tag{11}$$

This means that the two pseudo Dirac neutrinos are formed by combining

$$\nu_{2L}-(\xi D/C)\nu_{1L}-[AD/(C^2+D^2)]\bar{\nu}_{3R}$$

with

$$[D\nu_{1R}-C\nu_{2R}+(BD/C)\bar{\nu}_{3L}]/\sqrt{C^2+D^2}\ ,$$

and by combining

$$\nu_{1L}+(\xi D/C)\nu_{2L}-(\xi A/C)\bar{\nu}_{3R}$$

with

$$\bar{\nu}_{3L}-(B/C)\nu_{1R}-(\xi^2BD/C^2)\nu_{2R}\ .$$

Hence, we should take ν_{2L} to be mostly ν_τ, and thus l_{2L} should be mostly τ_L. In other words, in contrast with the model of Ref. [2], where $(l_{1,2,3})_L$ correspond roughly to $(e,\mu,\tau)_L$, we must examine M_l of Eq. (1) anew for a solution with $l_{2L}\simeq\tau_L$.

As in Ref. [2], we write

$$M_l=\begin{bmatrix} 0 & a & \xi b \\ a & 0 & b \\ \xi c & c & d \end{bmatrix}, \tag{12}$$

but now we assume $b^2\gg a^2,c^2,d^2$. Then the diagonalization of M_l according to

$$M_lM_l^\dagger=V\begin{bmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{bmatrix}V^\dagger \tag{13}$$

results in

$$m_\tau\simeq b\ ,\quad m_\mu\simeq\sqrt{c^2+a^2}\ ,$$

$$m_e\simeq|a^2d-2\xi abc|/b\sqrt{c^2+a^2}\ ,$$

and

$$V\simeq\begin{bmatrix} \cos\theta & \sin\theta & \xi \\ (d/b)\sin\theta & -(d/b)\cos\theta & 1 \\ -\sin\theta & \cos\theta & d/b \end{bmatrix}, \tag{14}$$

where $\tan\theta\equiv a/c$. Since $(\nu_e,\nu_\mu,\nu_\tau)_L$ are defined to be the SU(2)-doublet partners of $(e,\mu,\tau)_L$, Eq. (14) implies that

$$\nu_{eL}\simeq\cos\theta\nu_{1L}-\sin\theta\nu_{3L}+(d/b)\sin\theta\nu_{2L}\ , \tag{15}$$

$$\nu_{\mu L}\simeq\cos\theta\nu_{3L}+\sin\theta\nu_{1L}-(d/b)\cos\theta\nu_{2L}\ , \tag{16}$$

and

$$\nu_{\tau L}\simeq\nu_{2L}+\xi^*\nu_{1L}+(d/b)\nu_{3L}\ . \tag{17}$$

Using Eqs. (10) and (11), we can relate $(\nu_{1,2,3})_L$ to the mass eigenstates $n_{1,2,3,4}$:

$$\nu_{1L}\simeq\frac{1}{\sqrt{2}}\left[n_1+n_2-\frac{\xi D}{C}(n_3+n_4)\right]\ , \tag{18}$$

$$\nu_{3L}\simeq\frac{-1}{\sqrt{2}}\left[n_1-n_2+\frac{BD}{C\sqrt{C^2+D^2}}(n_3-n_4)\right]\ , \tag{19}$$

and

$$\nu_{2L} \simeq \frac{1}{\sqrt{2}} \left[n_3 + n_4 + \frac{\xi D}{C}(n_1 + n_2) \right] . \tag{20}$$

Hence ν_{eL} has an overlap with $(n_3+n_4)/\sqrt{2}$ given by $-(\xi D/C)\cos\theta = (d/b)\sin\theta$, and an overlap with $(n_3-n_4)/\sqrt{2}$ given by $(BD/C\sqrt{C^2+D^2})\sin\theta$, which is negligibly small. Note that $(n_3+n_4)/\sqrt{2}$ is mostly $\nu_{\tau L}$, which interacts with τ_L, but $(n_3-n_4)/\sqrt{2}$ is mostly its Dirac partner which is inert. Note also that from Eqs. (15), (16), (18), and (19), ν_{eL} and $\nu_{\mu L}$ are dominated by $(n_1 \pm n_2)/\sqrt{2}$, which are Dirac partners. In other words, a linear combination of ν_e and ν_μ combines with the orthogonal combination of $\bar{\nu}_e$ and $\bar{\nu}_\mu$ to form a pseudo Dirac neutrino.

Returning to the (e,μ,τ) mass matrix, we see that

$$\frac{m_e}{m_\mu} \simeq \sin\theta \left| \frac{d}{b}\sin\theta - 2\xi\cos\theta \right| . \tag{21}$$

Because of this constraint and the assumption that $d^2 \ll b^2$, $(d/b)\sin\theta$ is always much smaller than 0.09. For example, if $\sin\theta \simeq 1$, then $d \simeq m_e m_\tau / m_\mu \simeq 8.6$ MeV and $(d/b)\sin\theta \simeq 0.005$. Consider now the massive component of ν_{eL}, which is mostly $(n_3+n_4)/\sqrt{2}$. Its overlap is given by $-(\xi D/C)\cos\theta + (d/b)\sin\theta$ and should be about 0.09 in magnitude. On the other hand, the overlap with $\nu_{\mu L}$ is $(\xi D/C)\sin\theta + (d/b)\cos\theta$, which should be less than 0.03 from $\nu_\mu \to \nu_\tau$ oscillation data [6]. Hence, it is desirable to have $\tan\theta \simeq -(d/b)/(\xi D/C)$ so that ν_μ-ν_τ mixing can be ignored. In that case, Eq. (21) implies that $\sin\theta \simeq 0.3$ for $\xi = +m_u/m_c$ and $\cos\theta > 0$. Also we have $\xi D/(C\cos\theta) \simeq 0.092$, or $C/D \simeq 0.043$, and $AC/\sqrt{C^2+D^2} \simeq 17$ keV, or $A \simeq 400$ keV. Since B, C, and E are not yet constrained, we can adjust their values for the $\nu_e(\bar{\nu}_\mu)$ pseudo Dirac mass and look for a solution which can explain the solar-neutrino deficit in terms of the Mikheyev-Smirnov-Wolfenstein (MSW) effect [7]. It has recently been shown [8] that a value of about 4×10^{-8} eV^2 for the product of Δm_{12}^2 and $\sin^2(2\theta_{12})$ is desirable for this purpose. In this model, $\sin^2(2\theta_{12}) \simeq \cos^2(2\theta) \simeq 0.67$ for $\sin\theta \simeq 0.3$ and

$$\Delta m_{12}^2 \simeq 2 \left| \frac{\xi AB}{C} \right| \left| \frac{\xi^2 A^2 E}{C^2} \right| . \tag{22}$$

Hence, if $B \simeq 10$ keV, $C \simeq 10$ GeV, and $E \simeq 0.85$ GeV, we find the $\nu_e(\bar{\nu}_\mu)$ pseudo Dirac mass to be 1.5×10^{-3} eV and $\Delta m_{12}^2 \simeq 6\times10^{-8}$ eV^2. Therefore, going back to Eq. (7), we see that the two heavy Majorana neutrinos are most likely also pseudo Dirac neutrinos with a mass given by $\sqrt{C^2+D^2}$, as long as an MSW solution is imposed.

So far we have shown that with Eq. (3), the $S_3 \times Z_3$ model has a solution with three pseudo Dirac neutrinos: a very massive one perhaps of order 10^2 GeV, a not-so-massive one of 17 keV, and a light one of 10^{-3} eV. The electron neutrino has an overlap of 0.092 with the 17-keV neutrino, the muon neutrino has a negligible overlap with the 17-keV neutrino, and the τ neutrino is mostly the active component of the 17-keV neutrino. A linear combination of ν_e and ν_μ has a very small mass difference with its orthogonal combination; hence, ν_e-ν_μ oscillations do occur and $\sin^2(2\theta_{12})\Delta m_{12}^2 \simeq 4\times10^{-8}$ eV^2 can be realized for an optimal explanation of all solar-neutrino data in terms of the MSW effect. Another kind of oscillation occurs because ν_τ is mostly $(n_3+n_4)/\sqrt{2}$ and there is a mass different between n_3 and n_4 given by Eq. (8) corresponding to

$$\Delta m_{34}^2 \simeq 2 \left| \frac{AC}{\sqrt{C^2+D^2}} \right| \frac{A^2 D^2 E}{(C^2+D^2)^2} , \tag{23}$$

which ranges from about 1 to 10^2 eV^2 in this model. The effective ν_e mass for double-β decay is less than 3×10^{-5} eV, much below the present experimental upper limit of about 1 eV.

We now consider the decay of the 17-keV neutrino. Since lepton number is spontaneously broken by $\langle \chi_i^0 \rangle$ in this model, there exists a massless Goldstone boson called the Majoron which is a linear combination of $\chi_{1,2,3}^0$ and $\bar{\chi}_{1,2,3}^0$. Being all SU(2) singlets [9], they do not contribute to the invisible width of the Z boson and are thus consistent with the observation of the CERN e^+e^- collider LEP that the effective number of light neutrinos is just 3 [10]. Using Eqs. (3), (10), and (11), we find that $n_{3,4}$ do couple to $n_{1,2}$ through χ_i^0; hence, the decay $n_{3,4} \to n_{1,2}$ + Majoron is allowed. For h_7 of order unity, the effective coupling is approximately given by $ABD/C(C^2+D^2)$ for $(n_3+n_4)/\sqrt{2}$ and $\xi A/C$ for $(n_3-n_4)/\sqrt{2}$. We estimate their contribution to be of order 10^{-8} to 10^{-7}; hence, the decay lifetime of the 17-keV neutrino is roughly in the range 3×10^{-2} to 3×10^{-4} s.

An almost Dirac 17-keV neutrino may also contribute an extra degree of freedom to the effective number of neutrinos in the expansion of the early Universe. This may affect nucleosynthesis and violate the present astrophysical upper bound [11] of 3.3. However, it has been pointed out recently by Babu and Rothstein [12] that if the mass difference squared between the two mass eigenstates which make up the 17-keV pseudo Dirac neutrino is less than about 10 eV^2 (which is possible here), then the effective number of neutrinos remains at 3. On the other hand, there is also an upper bound of the order 10 keV on the mass of a Dirac or pseudo Dirac neutrino with an inert partner from supernova dynamics [13]. This is a serious problem for our model as well as any other which postulates an inert partner for the 17-keV neutrino.

Recently, there has been a deluge of papers [14–31] on the theoretical interpretation of a possible 17-keV neutrino. The model we have presented here is distinguished by its connection to an existing model of quark mass matrices [1], allowing it to offer a unified description of both quarks and leptons, and by its prediction of three pseudo Dirac neutrinos.

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