Proton decay modes in SU(15) grand unification

Palash B. Pal
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403
(Received 22 August 1991)

In the SU(15) grand unification model, proton decay is mediated by Higgs bosons only. They can be $B+L$ or $B-L$ conserving, depending on the choice of the Higgs sector. The dominant decay modes involve different generations of fermions. Thus, kaonic final states are preferred over pionic ones. If some discrete symmetry is imposed on the model, the proton can even be absolutely stable, unlike the situation in any other grand unified model.

PACS number(s): 13.30.Eg, 12.10.Dm, 14.20.Dh

The discovery of proton decay would be the most spectacular consequence of grand unification. Thus, whenever a new grand unified model is proposed, one rushes to check its predictions about the proton lifetime and decay modes. This issue assumed a lot of importance in the recent discussions of a grand unified model based on the gauge group SU(15). In this model, the fifteen known left-chiral spin-$\frac{1}{2}$ fields of the same generation are put in the fundamental representation of the gauge group:

$$\Psi = \left(u_r u_b u_y d_r d_b d_y \ \hat{u}_r \hat{u}_b \hat{u}_y \hat{d}_r \hat{d}_b \hat{d}_y \ \nu_e e^- e^+\right)_L , \quad (1)$$

where the caret denotes antiparticles, and the subscripts r, b, y represent color indices. The representations of the other generations are exactly the same.

In the early papers, Adler [1] as well as Frampton and Lee [2] noted that each gauge boson carries a well-defined baryon number and therefore cannot mediate baryon-number violating processes. They also considered a restricted Higgs boson sector which does not violate baryon number. But it was then pointed out by me [3] and others [4] that the baryon number is part of the gauge symmetry of this model, and so it must be broken in order to avoid a massless gauge boson coupling to baryon number. The Higgs boson sector therefore had to be enlarged. Subsequently, Frampton and Kephart (FK) [5] discussed various scenarios for proton decay, and deduced bounds on the parameters of the Higgs boson sector from the known bounds on proton lifetime.

In this article we discuss the decay modes of the proton, and show that they can be very different from the ones predicted from SU(5) or SO(10) grand unified models. In fact, with the choice of the Higgs-boson content of FK [5], the lowest-dimensional operators for proton decay obey the selection rule $\Delta B + \Delta L = 0$, as opposed to the $\Delta B - \Delta L = 0$ modes predicted by the SU(5) grand unification model. One can, however, choose the Higgs sector differently so as to obtain $(B-L)$-conserving proton decay modes even in SU(15), which we discuss as well. Even in this case, we show that the final states are quite different from the SU(5) prediction. We reanalyze the scenarios advocated by FK [5] and show that some of their diagrams, though baryon-number violating, do not contribute to proton decay. Finally, we point out that it is also possible to have an absolutely stable proton in SU(15).

We begin with a summary of the symmetry-breaking chain and the Higgs multiplets which have some component whose vacuum expectation value (VEV) can perform the breakings. To break SU(15) down to the standard-model gauge group, we need the following multiplets:

$$\Phi^{[klm]} : \mathbf{455}, \quad T^k{}_l : \mathbf{224}, \quad H^{[kl]}{}_{[pq]} : \mathbf{10800}, \quad (2)$$

where the square brackets denote antisymmetrization of the indices. To break the standard-model gauge group down to $SU(3)_c \times U(1)_Q$ as well as to give masses to quarks and charged leptons, one needs the multiplets

$$S^{\{kl\}} : \mathbf{120} \quad \text{or} \quad A^{[kl]} : \mathbf{105}. \quad (3)$$

A possible symmetry-breaking chain has been shown in Fig. 1. One can consider other possibilities, as we will discuss later.

Notice that in Fig. 1 there is only one VEV which violates baryon number, viz., the VEV of **455** having the quantum numbers of $\langle \hat{u}\hat{d}\hat{d} \rangle$. Since baryon number is part of the gauge symmetry, this VEV is the only source of baryon-number violation in the model. Thus, proton-decay diagrams must involve this VEV.

It is convenient to discuss the issue of proton decay by using the method of effective operators. In this method, we write down effective operators involving scalars and fermions which are invariant under the full SU(15) group. When the scalars develop VEV's, one obtains an effective operator involving fermions only, which is baryon-number violating when the scalar VEV's are.

Since the quarks carry one-third units of baryon number, we need at least three quark operators to obtain a $\Delta B = 1$ effective operator, as is required for proton decay. Angular momentum conservation would then demand that the lowest-order effective operators for proton decay involve four fermionic fields. As we indicated before, it must also involve at least one Φ field to accommodate B violation.

And it is easy to see that in fact one needs two occurrences of the Φ field in the effective operator. Since Φ has an odd number of indices of the gauge group and all other Higgs multiplets have an even number of them, it

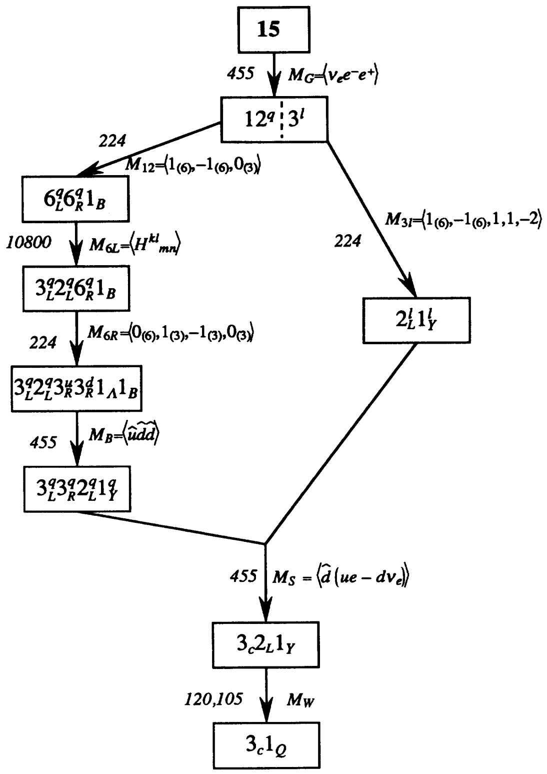

FIG. 1. A possible chain of symmetry breaking. See Eqs. (2) and (3) for identifying the representations. The numbers n denote a factor SU(n) in the gauge group if $n > 1$, and a U(1) factor if $n = 1$. The superscripts q or l indicate whether only quarks (and antiquarks) or only leptons (and antileptons) are nonsinglets under that part of the gauge group. If one considers the **224** as a traceless matrix, its VEV's are diagonal and the notation $1_{(6)}$, e.g., stands for six consecutive entries of unity. In the **455**, the symbol $\langle \hat{d}ue \rangle$, e.g., stands for the VEV of the color-singlet combination of the components with one index having the quantum numbers of $\hat{d}$, another of u and another of e. One can contemplate chains with fewer steps by equating two or more energy scales.

is impossible to contract the gauge indices of Φ unless it occurs an even number of times. Moreover, it was noted [6] that in the full gauge-invariant Lagrangian, there is a global symmetry

$$\Phi^{klm} \longrightarrow \exp(i\theta)\, \Phi^{klm} \,, \tag{4}$$

with the Higgs content specified in Eqs. (2) and (3). This implies that, if one uses one Φ with upper indices, one must use the other Φ with lower indices in the effective operator. From these considerations, let us write down one SU(15)-invariant effective operator,

$$\mathcal{O}_1 = \left\{ \Psi^k \Psi^l \right\} \left\{ \Psi^m \Psi^n \right\} \Phi_{klm} \Phi^{pqr} S_{np} A_{qr} \,, \tag{5}$$

and see whether it can induce proton decay. The notation $\left\{ \Psi^k \Psi^l \right\}$ is shorthand for $(\Psi^k)^T C \Psi_l$, where C is the conjugation matrix for fermions.

Baryon-number violation can occur if we replace Φ_{klm} by its VEV, and if the indices are such that this VEV corresponds to the B-violating VEV in the model. In fact, since the indices here are lower, this VEV has the quantum numbers of $\langle \hat{u}\hat{d}\hat{d} \rangle^*$, i.e., has $\Delta B = 1$ as is necessary for proton decay. However, proton decay requires not only baryon-number violation, but lepton-number violation as well. Lepton number is not part of the gauge symmetry, but is well defined for all components of Ψ. This can be used to assign lepton number to the gauge bosons and Higgs bosons. In other words, for any multiplet $\phi^{ij\cdots}$, we will count a lepton number $+1$ for each occurrence of the indices 13 or 14, and -1 for each occurrence for the index 15. Since lower indices are complex conjugates, they will have just the opposite assignments. It is then easy to see that lepton-number conservation is assured in the Lagrangian by gauge invariance, and therefore must be violated spontaneously.

Let us see how this can occur through $\mathcal{O}_1$. We assume that the only components of the multiplets S and A that have nonzero VEV's are the ones which give masses to the quarks and the charged leptons; i.e., they do not carry any lepton number. Then, the only source of lepton-number violation in Fig. 1 are the two VEV's at scales M_G and M_S. Note that both VEV's have $\Delta L = 1$. Since $\Delta B = 1$ as stated before, the 4-fermion operator obtained by replacing the scalar fields with their VEV's is $B - L$ conserving.

But this shows that the operator cannot contribute to proton decay. The reason is that the first three fermionic fields in $\mathcal{O}_1$ have to be $\hat{u}$, $\hat{d}$, and $\hat{d}$ because these indices contract with the B-violating VEV. Electric charge conservation then demands that the other fermionic field must be uncharged, and $B-L$ conservation demands that it has $L = -1$. No such field is present in the theory.

However, from Eq. (5) we can obtain another operator by performing the contractions in a different way:

$$\mathcal{O}_2 = \left\{ \Psi^k \Psi^l \right\} \left\{ \Psi^m \Psi^n \right\} \Phi_{klr} \Phi^{pqr} S_{mp} S_{nq} \,. \tag{6}$$

This one can indeed induce proton decay. To see what sort of decay products it leads to, we first note that this operator vanishes if all the fermion fields belong to the same generation. To see this, consider the first two fermion-field operators. They are antisymmetric under the interchange of the gauge indices k and l because Φ_{klr} is. They are also antisymmetric under the exchange of spinorial indices (not shown) since, as explained after Eq. (5), they are contracted by the conjugation matrix C which is antisymmetric. Fermi statistics then demands that that they must be antisymmetric under the exchange of generation indices, which means that Ψ^k and Ψ^l must belong to different families. One can see that the same comment applies for Ψ^m and Ψ^n. Thus, at least two different generations have to be involved in the operator $\mathcal{O}_2$. The quark level operator for proton decay arising from Eq. (6) would then be

$$\hat{u}\hat{s}\; \hat{u}\mu^+ \tag{7}$$

since it cannot involve any charm quark. This would give rise to the proton decay mode

$$p \longrightarrow \mu^+ K^0 \,, \tag{8}$$

which is a very different mode than the one favored by SU(5) grand unification. One can get other processes, e.g., $p \to \mu^+\pi^0$ or $p \to e^+\pi^0$ through mixing, but these will obviously be more suppressed.

Let us now see whether these decay modes can be consistent with known bounds on proton stability despite the possibility [1–3] of low unification scale of order 10^7 to 10^{11} GeV. A typical diagram generating $\mathcal{O}_2$ has been shown in Fig. 2. An order-of-magnitude estimate of this diagram yields the coefficient of the 4-fermion operator to be

$$\mathcal{K}_2 \sim \left(\frac{m_f}{M_W}\right)^2 \frac{\lambda_{SS}\lambda_{S\Phi}M_S M_B M_W^2}{M_G^6}\,. \tag{9}$$

Here, the quantity m_f is the mass of a typical fermion, and comes from the Yukawa couplings. The quartic scalar couplings are denoted by λ_{SS} and $\lambda_{S\Phi}$ in an obvious notation, and we have assumed that all the virtual colored scalars in this diagram have masses of order M_G, the largest scale in the model. The quantities M_S, M_B, etc. are explained in Fig. 1. The mass scales in the numerator correspond to different VEV's, neglecting factors of gauge coupling constants for a rough estimate. Known bounds on proton lifetime imply

$$\mathcal{K} \lesssim 10^{-30}\,\mathrm{GeV}^{-2} \tag{10}$$

for the 4-fermion operators [7]. In $\mathcal{O}_2$, since the second generation fermions are involved, let us use $m_f \simeq$ 100 MeV for a rough estimate. One then obtains from Eq. (9) the following bound on the scales and couplings of the model:

$$\frac{\lambda_{SS}\lambda_{S\Phi}M_S M_B}{M_G^6} \lesssim 10^{-28}\,\mathrm{GeV}^{-4}\,. \tag{11}$$

Even if, e.g., $M_S \sim M_B \sim M_G$, the above inequality can be satisfied by any $M_G \gtrsim 10^7$ GeV for $\lambda_{SS}, \lambda_{S\Phi} \sim 1$. In general M_B and M_S are smaller than M_G so that the bounds on M_G are less stringent. In any case, this bound allows roughly all values of M_G allowed by renormalization-group (RG) analysis performed on the symmetry-breaking scales [2,3], assuming $M_{6L} = M_{6R} = M_B = M_{3l}$ in Fig. 1.

Frampton and Kephart [5] considered an alternative scenario where the $\mathrm{U}(1)_B$ part of the gauge group is broken by the VEV of a **3003** dimensional multiplet which transforms like a completely antisymmetric 5-rank tensor of the gauge group. In the notation of Eq. (1), the VEV required to break $\mathrm{U}(1)_B$ lies in the components 10, 11, 12, 13, 14; i.e., it has the gauge transformation properties of $\hat{d}\hat{d}\hat{d}\nu_e e^-$. Therefore, this VEV has $B = -1$, $L = 2$. If this is the only B-violating VEV, the selection rules for proton decay are very different, as we will now see.

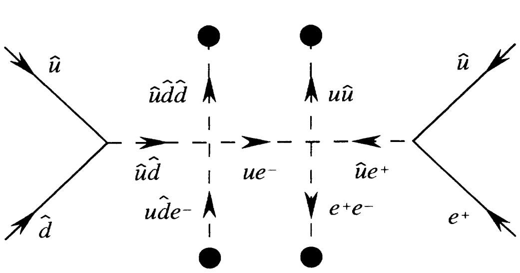

FIG. 2. Tree diagram generating $\mathcal{O}_2$. The labels on the Higgs-boson lines represent, via Eq. (1), the transformation properties under SU(15). The indices should all be considered as upper indices.

The gauge-invariant effective operator must involve the multiplet h now. Since it has an odd number of indices, we need another multiplet with an odd number of indices so that all indices can be contracted. For this, we cannot use h again if we want to obtain a $\Delta B = 1$ operator after putting the VEV's. Thus, the 3-index multiplet Φ must be used. We write down the operator

$$\mathcal{O}_3 = \left\{\Psi^k\Psi^l\right\}\left\{\Psi^m\Psi^n\right\} h_{klmpq} A_{nr} \Phi^{pqr}\,. \tag{12}$$

FK [5] showed that there is a tree diagram which can give rise to this operator, which we show in Fig. 3. From this, one notices that Φ^{pqr} obtains the VEV which has $L = 1$. Combining with the VEV of h having $B = 1$ and $L = -2$, the operator $\mathcal{O}_3$ has $B = 1$, $L = -1$; i.e., it conserves $B+L$. This is very different from the prediction of popular grand unified models.

The decay modes are also quite novel. Because of Fermi statistics, two different generations are involved here, just as for the operator $\mathcal{O}_2$ discussed before. This means that at the 4-fermion level, the operators that can contribute to proton decay are

$$\hat{d}\hat{s}\;\hat{d}\mu^-\,, \quad \hat{u}\hat{s}\;\hat{d}\nu_\mu\,. \tag{13}$$

The leading proton-decay modes corresponding to these operators will be, respectively,

$$p \to \pi^+K^+\mu^-\,, \quad p \to K^+\nu_\mu\,. \tag{14}$$

As in the case with $\mathcal{O}_2$, other processes will arise but they will be suppressed by intergenerational mixing angles.

The $B + L$ conservation obeyed in these processes is expected on general grounds from Weinberg's arguments [8] using F parity. His essential results can be rephrased as follows. Suppose one writes down all $\mathrm{SU}(3)_c\times\mathrm{U}(1)_Q$-invariant baryon-number-violating operators using only the known fermions, restricting oneself to four fermionic fields only. The operators conserve $B - L$ if they violate weak isospin by integral amounts including zero. On the other hand, if they violate weak isospin by half-integral amounts, the operators are $B + L$ conserving. In $\mathcal{O}_3$, the VEV of the A field violates weak isospin by 1/2 and, hence, the $(B + L)$-conserving operator.

From Fig. 3, we now estimate the 4-fermion operator generated from $\mathcal{O}_3$, getting

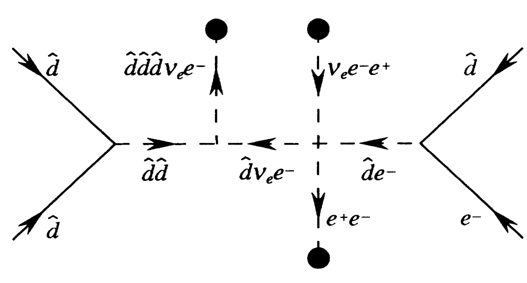

FIG. 3. Tree diagram generating $\mathcal{O}_3$.

$$\mathcal{K}_3 \sim \left(\frac{m_f}{M_W}\right)^2 \frac{\lambda_{A\Phi}\mathcal{M}_{A\Phi h} M_G M_B M_W}{M_G^6}, \qquad (15)$$

where $\mathcal{M}_{A\Phi h}$ is the trilinear Higgs-boson coupling having the dimension of mass. Equation (10) then implies

$$\frac{\lambda_{A\Phi}\mathcal{M}_{A\Phi h} M_B}{M_G^5} \lesssim 10^{-26}\,\mathrm{GeV}^{-3}, \qquad (16)$$

taking $m_f \approx 100\,\mathrm{MeV}$ as before. If both M_B and the trilinear coupling are close to M_G, one needs $M_G \gtrsim 10^9$ GeV if $\lambda_{A\Phi} \approx 1$. This is an important constraint, since it can rule out a good part of the range of values allowed for M_G from RG analysis [2, 3]. Of course, if $\lambda_{A\Phi}$ or $\mathcal{M}_{A\Phi h}$ is much smaller, constraints on M_G can be easier.

We now go back to the other proton-decay diagrams discussed by FK [5]. They considered a situation where the multiplet A, though present in the theory, has no VEV. In this case, they draw a 1-loop diagram for proton decay. In the language of effective operators, their diagram is equivalent to the operator

$$\mathcal{O}_4 = \left\{\Psi^k\Psi^l\right\}\left\{\Psi^m\Psi_n\right\} h_{klmpq}\Phi^{npq}. \qquad (17)$$

Notice that one of the Ψ field operators appears with a lower index here. This means that it represents a $\mathbf{15}^*$ multiplet, i.e., the mirror fermions which must be present in this model in order to cancel gauge anomalies. However, all mirror fermions must be heavier than the proton or else they would had been detected long ago. Thus, $\mathcal{O}_4$ cannot give rise to proton decay, contrary to the statement by FK [5]. It can, however, give rise to other baryon-number-violating processes involving the mirror fermions.

The final scenario considered by FK [5] assumes the multiplet A to be altogether absent in the model, and only the symmetric multiplet S couples to fermions. They claim that in this case, proton decay is induced by a 2-loop operator. However, one should note that the diagram they give has $\Delta B = 3$ from three VEV's of h, and therefore cannot contribute to proton decay. Instead, one can try to construct effective $\Delta B = 1$ operators involving the 5-index Higgs boson multiplet h. The simplest such operators are

$$\mathcal{O}_5 = \left\{\Psi^k\Psi^l\right\}\left\{\Psi^m\Psi^n\right\} h_{klmpq}S_{nr}\Phi^{pqr},$$
$$\mathcal{O}_6 = \left\{\Psi^k\Psi^l\right\}\left\{\Psi^m\Psi^n\right\} h_{klmpq}S_{nj}\Phi^{pqr}T^j{}_r. \qquad (18)$$

Once again, both of these can accommodate only $\frac{1}{2}$ unit of weak isospin violation and therefore give rise to $(B+L)$-conserving decay processes such as those in Eq. (14).

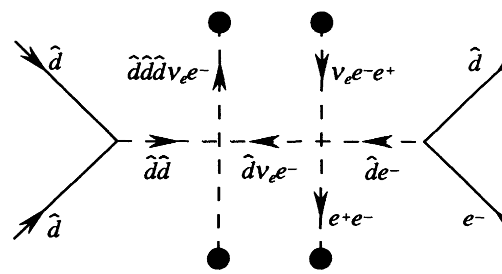

FIG. 4. Tree diagram generating $\mathcal{O}_6$. The line without any arrow represents the adjoint Higgs field.

The operator $\mathcal{O}_6$ can be generated by the tree-level graph shown in Fig. 4. An order-of-magnitude estimate here gives for the coefficient of the 4-fermion operator

$$\mathcal{K}_6 \sim \left(\frac{m_f}{M_W}\right)^2 \frac{\lambda_{S\Phi}\lambda'_{S\Phi hT} M_G M_B M_W M_{12}}{M_G^6}, \qquad (19)$$

where we take the largest possible VEV of the adjoint **224**, which appears at the scale M_{12}. If the quartic couplings are all of order unity, the bounds from this diagram will be similar to those from Eq. (16).

If, in Fig. 4, the adjoint Higgs line does not go to the vacuum but to any other line, we obtain a 1-loop diagram for $\mathcal{O}_5$. This will be slightly more suppressed than $\mathcal{O}_6$ because of loop integration factors of order $1/(8\pi^2)$.

The bounds derived above make it clear that for some choice of parameters in the model, the proton-decay rate might be close to the present experimental limit. If, for example, $(B+L)$-conserving decay modes are discovered, it will be a strong evidence in favor of the SU(15) unification model. However, as we discussed, depending on the way baryon number is violated, one can also obtain $(B-L)$-conserving decays.

It should also be pointed out that the proton can even be absolutely stable in SU(15) model. This happens if B violation occurs only through the VEV of h and the Lagrangian has a discrete symmetry

$$h \to -h. \qquad (20)$$

In this case, all gauge-invariant operators will be even in h and therefore will not be able to accommodate baryon-number violation by any odd number. This is a dramatic characteristic, not shared by any known grand unification model.

I thank N. G. Deshpande for comments on the manuscript. The research was supported by the Department of Energy Grant No. DE-FG06-85ER40224.

[1] S. L. Adler, Phys. Lett. B **225**, 143 (1989).
[2] P. H. Frampton and B-H. Lee, Phys. Rev. Lett. **64**, 619 (1990).
[3] P. B. Pal, Phys. Rev. D **43**, 236 (1991).
[4] U. Sarkar, A. Mann, and T. G. Steele, Report No. PRL-TH-90-18, 1990 (unpublished).
[5] P. H. Frampton and T. W. Kephart, Phys. Rev. D **42**, 3892 (1990).
[6] N. G. Deshpande, P. B. Pal, and H. C. Yang, Phys. Rev. D **44**, 3702 (1991).
[7] The precise bounds depend on the specific decay mode. For rough estimates, we use the same bound for all modes.
[8] S. Weinberg, Phys. Rev. D **22**, 1694 (1980).