# Vector polarization at high momentum transfers in QCD

Carl E. Carlson
*Physics Department, College of William and Mary, Williamsburg, Virginia 23187*

M. Chachkhunashvili
*High Energy Physics Institute, Tbilisi State University, Georgia 380086*

(Received 7 October 1991)

We show in the context of perturbative QCD that all single transverse vector polarizations or analyzing powers are zero in the high-momentum-transfer limit. The proof depends on hadron helicity conservation, which applies to exclusive processes that are dominated by a short-distance reaction mechanism. Some comments are made about the available data.

PACS number(s): 13.88.+e, 12.38.Cy, 13.60.Le, 25.20.−x

## I. INTRODUCTION

At sufficiently high momentum transfers, exclusive as well as inclusive processes can be calculated in quantum chromodynamics (QCD) using perturbation theory. Simple consequences of perturbative QCD, requiring no detailed calculation, are the laws which state how differential cross sections scale with energy at a fixed c.m. scattering angle [1], and the prediction of hadron helicity conservation [2]. The latter states that the sum of the helicities of the outgoing hadrons in a given process is equal to the corresponding sum from initial hadrons. Hadron helicity conservation applies to ordinary hadrons whose smallest significant Fock component consists of quarks and/or antiquarks and requires that the process in question be dominated by short-distance mechanisms.

A corollary of hadron helicity conservation is that transverse vector polarization quantities (polarizations of outgoing particles or analyzing powers) will be zero at high momentum transfers. For two-body-to-two-body processes, $A+B\rightarrow C+D$, parity invariance forbids vector polarization in the scattering plane. Hence, for strong or electromagnetic two-body-to-two-body processes at high momentum transfers, one can say that there will be no vector polarization in any direction. The nonpolarization result is known for proton-proton elastic scattering [3] and for the outgoing proton in deuteron photodisintegration [4], and is perhaps believed in other cases. We shall show how the general case can be proved. We will start by showing how the theorem works for spin-$\frac{1}{2}$ and spin-1 particles using the examples of polarization of the outgoing proton in deuteron photodisintegration and of the vector analyzing power of the deuteron in the same process. Then we prove the general case. We will close with some comments upon the available data and with some general conclusions.

## II. DEMONSTRATION OF ZERO LINEAR POLARIZATION

The proof, following from the hadron helicity conservation theorem, that there is no vector polarization in high momentum transfer, can be illustrated for spin-$\frac{1}{2}$ and spin-1 particles using an example interesting in its own right, namely, deuteron photodisintegration, $\gamma+d\rightarrow p+n$.

For the discussion of the polarization of the outgoing proton, we choose coordinates with $\hat{z}$ along the proton momentum direction, $\hat{y}$ normal to the scattering plane, and $\hat{x}$ in the scattering plane and perpendicular to the proton momentum. Then for the proton, the helicity is the same as the spin along the $\hat{z}$ direction. Since this is a two-body-to-two-body reaction, polarization in the scattering plane is forbidden by parity conservation and we only need further prove that polarization in the $\hat{y}$ direction is zero.

Let $\mathcal{M}(\lambda_\gamma,\lambda_d;\lambda_p,\lambda_n)$ be the helicity amplitude with $\lambda_i$ being the helicity of particle $i$. Often it will be useful to have an amplitude where one of the particles has a definite value of spin projection in some other direction, and we shall use notation such as $\mathcal{M}(\lambda_\gamma,\lambda_d;s_y=\pm\frac{1}{2},\lambda_n)$ to indicate the amplitude where the proton is polarized along the $\pm\hat{y}$ direction. Then the polarization of the proton is given by

$$P_y=\frac{\sigma(\uparrow)-\sigma(\downarrow)}{\sigma(\uparrow)+\sigma(\downarrow)}=\frac{1}{D}\sum_{\lambda_\gamma,\lambda_d,\lambda_n}[|\mathcal{M}(\lambda_\gamma,\lambda_d;s_y=\tfrac{1}{2},\lambda_n)|^2-|\mathcal{M}(\lambda_\gamma,\lambda_d;s_y=-\tfrac{1}{2},\lambda_n)|^2]\ , \tag{1}$$

where the denominator $D$ is a sum over all helicity amplitudes squared. For spin-$\frac{1}{2}$ particles, eigenstates of the $y$ component of spin are related to eigenstates of the $z$ component of spin by

$$|\pm\tfrac{1}{2}\rangle_y=(1/\sqrt{2})(|+\tfrac{1}{2}\rangle_z\pm i|-\tfrac{1}{2}\rangle_z)\ . \tag{2}$$

Thus, one may write the polarization in terms of helicity amplitudes:

$$P_y=\frac{1}{D}\sum_{\lambda_\gamma,\lambda_d,\lambda_n}i\sqrt{2}\mathcal{M}^*(\lambda_\gamma,\lambda_d;\lambda_p=\tfrac{1}{2},\lambda_n)\times\mathcal{M}(\lambda_\gamma,\lambda_d;\lambda_p=-\tfrac{1}{2},\lambda_n)+\text{c.c.} \tag{3}$$

 

Hadron helicity conservation states that

$$\lambda_d - \lambda_n = \lambda_p \ . \tag{4}$$

Since a given term in the expression for the polarization has two factors with the same $\lambda_d$ and $\lambda_n$ but different $\lambda_p$, at least one of the factors is not conserving hadron helicity, and so is tending to zero at high momentum transfer compared to its helicity-conserving counterpart. Hence, the numerator in the expression for $p_y$ must be tending to zero compared with the denominator.

The argument is similar in the case of spin 1. Consider the vector analyzing power in the same reaction, $\gamma + d \to p + n$. Now the coordinate system should have the $z$ axis along the direction of the incoming deuteron (one can define this direction in the c.m. frame) with the $y$ axis still normal to the scattering plane. In terms of cross sections with spin-up and spin-down target deuterons, the vector analyzing power is

$$\begin{aligned} A_y &= \frac{\sigma(\uparrow)-\sigma(\downarrow)}{\sigma(\uparrow)+\sigma(\downarrow)} \\ &= \frac{1}{D}\sum_{\lambda_\gamma,\lambda_p,\lambda_n} [\,|\mathcal{M}(\lambda_\gamma,s_y=1;\lambda_p,\lambda_n)|^2 \\ &\qquad - |\mathcal{M}(\lambda_\gamma,s_y=-1;\lambda_p,\lambda_n)|^2\,] \ . \end{aligned} \tag{5}$$

Eigenstates of $y$ direction spin for spin-1 particles are given by

$$|\pm 1\rangle_y = \tfrac{1}{2}(|1\rangle_z \pm i\sqrt{2}|0\rangle_z - |-1\rangle_z) \ , \tag{6}$$

and so

$$\begin{aligned} A_y = \frac{1}{D}\sum_{\lambda_\gamma,\lambda_p,\lambda_n} & i[\mathcal{M}^*(\lambda_\gamma,\lambda_d=1;\lambda_p,\lambda_n) \\ & -\mathcal{M}^*(\lambda_\gamma,\lambda_d=-1;\lambda_p,\lambda_n)] \\ & \times\mathcal{M}(\lambda_\gamma,\lambda_d=0;\lambda_p,\lambda_n)+\text{c.c.} \end{aligned} \tag{7}$$

At momentum transfers sufficiently high for hadron helicity conservation to be valid, two of the three helicity amplitudes involved in each term of the expression for the analyzing power must tend toward zero compared to a helicity-conserving amplitude, and so the analyzing power itself must tend to zero.

The proof of nonpolarization for the general case $A+B \to C+D \cdots$ is essentially the same, no matter which particle polarization is studied. We consider particle $C$, assuming that $C$ is a hadron and that it has spin $j$. We will give the proof for vector polarization in the $y$ direction. Direction $\hat{z}$ is along the momentum of $C$, and $\hat{y}$ is some direction normal to $\hat{z}$. Consider polarization as the difference of cross sections for spin up— along $\hat{y}$— with some value $m_y = m$ (it does not have to be the maximum possible $m_y$) and spin down with the same magnitude $m$:

$$\begin{aligned} p_y &= \frac{\sigma(m)-\sigma(m)}{\sigma(m)+\sigma(-m)} \\ &= \frac{1}{D}\sum_{\lambda_A,\lambda_B,\lambda_D} [\,|\mathcal{M}(\lambda_A,\lambda_B;s_y=m,\lambda_D,\ldots)|^2 \\ &\qquad - |\mathcal{M}(\lambda_A,\lambda_B;s_y=-m,\lambda_D,\ldots)|^2\,] \ . \end{aligned} \tag{8}$$

The states with definite spin projection in the $y$ direction are obtained from states with definite spin in the $z$ direction by

$$\begin{aligned} |m\rangle_y &= R_z(\pi/2)R_y(\pi/2)|m\rangle_z \\ &= |m'\rangle_z e^{-im'\pi/2} d_{m'm}(\pi/2) \ , \end{aligned} \tag{9}$$

where the $R_i$ are rotation operators and $d_{m'm}$ (or $d^{(j)}_{m'm}$) are representations of the operator for rotations about the $y$ axis for spin-$j$ particles. This induces the relation

$$\mathcal{M}(\ldots,s_y=m) = \mathcal{M}(\ldots,s_z=m')e^{-im'\pi/2}d_{m'm}(\pi/2) \ , \tag{10}$$

where we have only explicitly indicated the polarization of particle $C$. Thus,

$$\begin{aligned} p_y = \frac{1}{D}\sum_{m'} & |\mathcal{M}(\ldots,m')|^2(d^2_{m'm} - d^2_{m',-m}) \\ & + \sum_{m'\neq m''} \mathcal{M}^*(\ldots,m')\mathcal{M}(\ldots,m'')e^{i\pi(m'-m'')/2} \\ & \times(d_{m'm}d_{m''m} - d_{m',-m}d_{m'',-m})+\text{c.c.} \end{aligned} \tag{11}$$

We have not indicated the sum over polarizations of $A$, $B$, $D,\ldots$; all amplitudes are helicity amplitudes and the argument of each $d$ function is $\pi/2$. The second sum is zero compared to the denominator by the usual argument, which is that if the amplitude with $m'$ satisfies hadron helicity conservation, then the amplitude with $m''$ cannot, and vice versa. The first sum is zero because of the theorem [5]

$$d_{m',m}(\pi/2) = (-1)^{j-m'}d_{m',-m}(\pi/2) \ . \tag{12}$$

The proof applies to any direction perpendicular to the momentum of the hadron in question, so there will be no vector polarization in any transverse direction at energies or situations sufficient for hadron helicity conservation to work.

## III. COMMENTS AND CONCLUSION

While perturbative QCD prohibits vector polarization, there are other polarizations that are allowed at any momentum transfer. For example, we have the perturbative QCD prediction of the tensor polarization in high-momentum-transfer $e$-$d$ elastic scattering [6], $t_{20} = -\sqrt{2}$, and the general theorem for double quantities in $p$-$p$ elastic scattering at 90° in the c.m. [7,8]:

$$A_{zz} - A_{xx} - A_{yy} = 1 \ . \tag{13}$$

Here $A_{zz}$ is an asymmetry between beam and target polarizations parallel and antiparallel along the $\hat{z}$ direction, and similarly for $A_{xx}$ and $A_{yy}$; $\hat{y}$ is normal to the scattering plane and $\hat{z}$ parallels the incoming proton momentum. The theorem requires just rotation, parity, and time-reversal invariance, and Fermi statistics, so it applies to any strong-interaction theory, and shows that the $A_{ii}$ cannot all be small [9].

Returning to vector polarization, the data do not presently support the prediction of a zero result. Perhaps best known [3] are the measurements of the analyzing power in proton-proton elastic scattering; when at $s=47$ GeV$^2$ and $-t=9.7$ GeV$^2$ (i.e., $p_{\mathrm{lab}}=24$ GeV and $p_\perp=2.7$ GeV) one finds $A_y=(20.4\pm3.9)\%$. However, as emphasized by Ralston and Pire [10], the possibility that long-distance, or Landshoff [11], processes are important in purely hadronic reactions can vitiate the hadron helicity conservation arguments. Hadron helicity conservation requires quark helicity conservation, plus neglect of the orbital angular momentum of a hadron's constituents projected along the momentum direction of that hadron. This is not justified if the interaction region is large. To the extent Landshoff process are important one may see polarization in purely hadronic reactions even at high energy. Thus, electromagnetic processes become of greater interest.

There are polarization data for the proton in deuteron photodisintegration, $\gamma+d\to p+n$, and for target and outgoing baryons in meson photoproduction, $\gamma+N\to m+B$. In deuteron photodisintegration, the available proton polarization data [12] are for $E_\gamma$ up to 1.1 GeV at a c.m. scattering angle of 120° and show polarization that is large and negative. (For the perturbative QCD scaling rule falloff of $d\sigma/dt$, deuteron photodisintegration stands out as a case that appears to work at a remarkably low energy. The quantity $s^{11}d\sigma/dt$ for the 90° c.m. data looks constant [13] in the higher-energy region of $E_\gamma$, beginning at $E_\gamma$ just 1.1 GeV, the same as the top energy for the polarization data, and a momentum transfer $-t=1.2$ GeV$^2$, smaller than in any other known case.)

In kaon photoproduction, $\gamma+p\to K+\Lambda$, data are available for the polarization of $\Lambda$ [14,15] for $E_\gamma$ up to 1.3 GeV and c.m. scattering angle 90°. The polarization of $\Lambda$ is of the order of $-\frac{1}{3}$ for $E_\gamma$ in the range 1.0 to 1.2 GeV and may be shrinking in magnitude as one approaches 1.3 GeV. In pion photoproduction there are data for both target and outgoing nucleon polarization at 90° c.m. for $E_\gamma$ up to 1.6 GeV, and for $E_\gamma$ as high as 16 GeV at smaller angles [16,17]. There is no systematic trend toward small polarizations with increasing energy.

However, none of the photoproduction experiments are at a high momentum transfer from the viewpoint of perturbative QCD. A sufficient momentum transfer for hadron helicity conservation to work is not known, but one can get an estimate from the momentum transfer necessary for the scaling predictions of perturbative QCD to work. Best developed are the data on baryon electroproduction, where the proton elastic form factor $G_M$ and the leading electromagnetic transition form factors for the 1520 and 1688 resonance bumps appear to follow to the perturbative QCD scaling rules starting at momentum transfers $Q^2$ of about 5 GeV$^2$ [18]. The scaling of

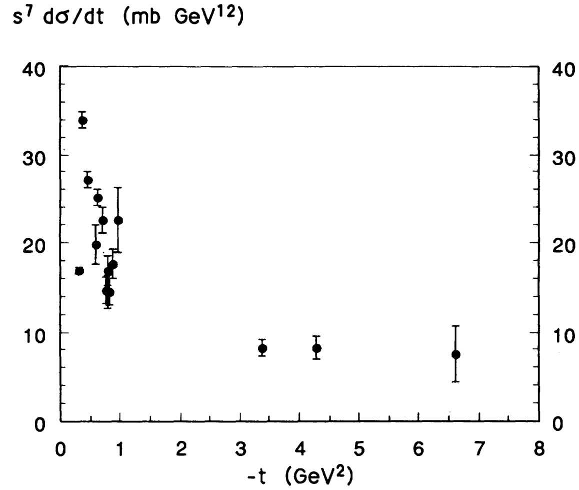


FIG. 1. A plot of $s^7 d\sigma/dt$ for pion photoproduction, $\gamma+p\to\pi^++n$ at 90° in the c.m. Perturbative QCD predicts this quantity should be constant at high momentum transfers. The data are from [16], and not all data with $-t$ below 0.75 GeV$^2$ are shown. (This figure is similar to one given as a log-log plot in [19].)

the differential cross section at 90° c.m. for pion photoproduction seems to work for $-t$ above 3 GeV$^2$, when the data are presented on a linear plot as in Fig. 1. Now, for the highest-energy polarization data in kaon photoproduction, $-t$ and $-u$ are 0.61 GeV$^2$ and 0.34 GeV$^2$, respectively. Similarly, none of the pion photoproduction polarization data have both $-t$ and $-u$ much greater than 1 GeV$^2$. Thus, in these cases we are well below the benchmark momentum transfers where perturbative QCD may be working in other situations.

We shall close by emphasizing the value of polarization measurements at higher momentum transfer, momentum transfers at least as large as the threshold for the perturbative QCD scaling law data to appear in the baryon electroproduction or meson photoproduction data. Pion photoproduction at 90° in the c.m. with an $E_\gamma=4$ or 6 GeV beam would give $-t$ and $-u$ both in excess of 3 or 5 GeV$^2$, respectively, and the relevant measurements should be possible at a laboratory such as CEBAF.

## ACKNOWLEDGMENTS

We wish to thank V. Burkert, R. Gilman, and J. Napolitano for useful conversations. C. E. C. wishes to thank the National Science Foundation for support under Grant No. PHY-9112173, and M.C. wishes to thank Nathan Isgur and the CEBAF theory group for its hospitality.

[1] S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973); Phys. Rev. D **11**, 1309 (1975); V. A. Matveev, R. M. Muradyan, and A. V. Tavhkhelidze, Lett. Nuovo Cimento 7, 719 (1973).
[2] S. J. Brodsky and G. P. Lepage, Phys. Rev. D **22**, 2848 (1980).
[3] D. G. Crabb *et al.*, Phys. Rev. Lett. **65**, 3241 (1990).
[4] R. Gilman (private communication).
[5] A. R. Edmonds, *Angular Momentum in Quantum Mechanics*, 2nd ed. (Princeton University Press, Princeton,

NJ, 1960).

[6] C. E. Carlson and F. Gross, Phys. Rev. Lett. **53**, 127 (1984).

[7] G. Farrar, S. Gottlieb, D. Sivers, and G. Thomas, Phys. Rev. D **20**, 202 (1979).

[8] S. J. Brodsky, C. E. Carlson, and H. Lipkin, Phys. Rev. D **20**, 2278 (1979). This and the preceding reference give the direction conventions that fix the signs of the terms in Eq. (13).

[9] F. Z. Khiari *et al.*, Phys. Rev. D **39**, 45 (1989).

[10] J. P. Ralston and B. Pire, Phys. Rev. Lett. **65**, 2343 (1990); in *Electromagnetic Physics With Internal Targets,* Proceedings of the Topical Conference, Stanford, CA, 1989, edited by R. G. Arnold (World Scientific, Singapore, 1989), p. 229.

[11] P. V. Landshoff, Phys. Rev. D **10**, 1024 (1974); P. Cvitanovic, *ibid.* **10**, 338 (1974).

[12] A. S. Bratashevskii *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 174 (1982) [JETP Lett. **36**, 216 (1982)].

[13] J. Napolitano *et al.*, Phys. Rev. Lett. **61**, 2530 (1988).

[14] D. E. Groom and J. H. Marshall, Phys. Rev. **159**, 1213 (1967).

[15] R. A. Adelseck and B. Saghai, Phys. Rev. C **42**, 108 (1990).

[16] D. Menze, W. Pfeil, and R. Wilcke, *Physics Data: Compilation of Pion Photoproduction Data* (Zentralstelle für Atomkernenergie-Dokumentation, Eggenstein-Leopoldshafen, 1977).

[17] C. C. Morehouse *et al.*, Phys. Rev. Lett. **25**, 835 (1970).

[18] P. Stoler, Phys. Rev. D **44**, 73 (1991).

[19] R. L. Anderson *et al.*, Phys. Rev. D **14**, 679 (1977).