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First moments and the polarized-gluon contribution to g_1^p

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It is demonstrated that the gluonic contribution $\int_x^1 (dz/z)\Delta C_G(z)\Delta G(x/z,Q^2)$ to the polarized structure function $g_1^p(x,Q^2)$ in the experimentally accessible region $x \geq 0.01$ is unaffected by the debate concerning the first moment of g_1^p. Agreement with present measurements can be obtained in terms of $\Delta G(x,Q^2)$, without involving a polarized-strange-quark contribution, even for $\int_0^1 \Delta C_G(z)dz=0$. Present data allow for a negative polarized-gluon distribution $\Delta G(x,Q^2)$ as well.

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The polarized deep-inelastic structure function of the proton, $g_1^p(x,Q^2)$, is related to the polarized parton distributions for $f=3$ light flavors via

$$g_1^p(x,Q^2)=\frac{1}{12}\Delta C_q * \Delta q_3+\frac{5}{36}\Delta C_q * \Delta q_8 +\frac{1}{3}\left[\Delta C_q * \Delta s+\frac{\alpha_s(Q^2)}{2\pi}\Delta C_G * \Delta G\right], \tag{1}$$

with $\Delta q_3=\Delta u-\Delta d$ and $\Delta q_8=\Delta u+\Delta d-2\Delta s$, where $\Delta q \equiv q_+ - q_- + \bar{q}_+ - \bar{q}_-$, and $\Delta G \equiv G_+ - G_-$ with $q_\pm$ denoting the parton densities aligned $(+)$ and antialigned $(-)$ with the proton spin. Equation (1) has been supplemented by the, formally higher-order, gluonic contribution [1,2] and the α_s/π correction to the fermionic Wilson coefficient [3] $\Delta C_q(z)=\delta(1-z)+O(\alpha_s/\pi)$, insignificant for our purposes, has been included for consistency, with the convolutions being defined by

$$\Delta C_f * \Delta f \equiv \int_x^1 \frac{dz}{z}\Delta C_f(z)\Delta f\left[\frac{x}{z},Q^2\right].$$

The European Muon Collaboration (EMC) [4] has measured $g_1^p(x,Q^2)$ for $x \geq 0.01$, which, when integrated, resulted in

$$\int_{0.01}^1 g_1^p(x,Q^2)dx=0.123\pm0.010\pm0.015 \tag{2}$$

at $\langle Q^2\rangle = 10.7\ \mathrm{GeV}^2$. Extrapolating into the unmeasured region $x<0.01$ gives [4] $\int_0^1 g_1^p(x,Q^2)dx=0.126$, which shows that almost all of the relevant contribution of $g_1^p(x,Q^2)$ to the first moment in (2) stems from the region $x \geq 0.01$. This is important for theoretical attempts to explain the surprising result in (2) which lies significantly below the "naive" ($\Delta s=\Delta G=0$) expectation [5] $\int_{0.01}^1 g_1^p(x,Q^2)dx \simeq 0.2$, derived from Eq. (1) using the hyperon β-decay constraints [4]

$$\int_0^1 \Delta q_3(x,Q^2)dx=g_A=1.254\pm0.006, \qquad \int_0^1 \Delta q_8(x,Q^2)dx=0.68\pm0.04 \tag{3}$$

where for the flavor-nonsinglet octet densities $\Delta q_{3,8}$ the range of integration can, with sufficient accuracy [6], be extended down to $x=0$. In general, a finite strange Δs as well as a finite gluonic ΔG contribution in Eq. (1) can be responsible for bringing this "naive" expectation into agreement with the EMC result (2), but for definiteness we assume a small strange component, $\Delta s(x,Q^2)\simeq 0$, and concentrate on the gluonic term in (1) and on its contribution to g_1^p in the experimentally measured region $0.01 \leq x < 1$.

The debate concerning the ΔG contribution to $g_1^p(x,Q^2)$ in Eq. (1) results from the well-known factorization scheme dependence of the higher-order Wilson coefficient $\Delta C_G(z)$. In particular, the factorization scheme dependence of the first moment $\Delta C_G^{(1)} \equiv \int_0^1 \Delta C_G(z)dz$ has attracted a lot of attention due to its relevance for the first moment of $g_1^p(x,Q^2)$. Many arguments have been presented in favor of some particular choice of $\Delta C_G^{(1)}$ but one may doubt whether matters of mere convention, such as the choice of a factorization scheme, are really relevant [7]. To study this question we choose two seemingly different $\Delta C_G(z)$ corresponding to two different values of $\Delta C_G^{(1)}$ frequently considered in the literature, i.e. [1,3,8–10], $\Delta C_G^{a(1)}=-1$ and [11,12] $\Delta C_G^{b(1)}=0$, where

$$\Delta C_G^a(z)=(2z-1)\ln\frac{1-z}{z}-a(2z-1), \tag{4a}$$

$$\Delta C_G^b(z)=(2z-1)\ln\frac{1-z}{z}-b(2z-1)+2(1-z), \tag{4b}$$

with Eq. (4a) taken from Refs. [1,3,9, and 10], and Eq. (4b) adopted from Refs. [11] and [12]. Moreover, the forms presented in Eqs. (4a) and (4b) are only illustrative since different regularization schemes will yield different coefficients $\Delta C_G(z)$, even if $\Delta C_G^{(1)}$ takes the same value as, e.g., in Ref. [8] for $\Delta C_G^{(1)}=-1$. It should be noted that the free constants a and b in Eqs. (4a) and (4b) reflect the freedom in the factorization [11,13] of the mass regulated contribution [1,3,8–12] $(2z-1)\ln Q^2/m^2$, with m being an entirely *arbitrary* mass parameter, and are a crucial element in the *definition* of $\Delta G^{a,b}(z,Q^2)$: changing these constants a and b amounts to linear transformations among the members $\Delta G(z,Q^2)$ and $\Delta\Sigma(z,Q^2)=\Delta u+\Delta d+\Delta s$ of the flavor-singlet sector.

In order to compare the gluonic contributions to $g_1^p(x,Q^2)$ in Eq. (1) corresponding to the two different ΔC_G in Eqs. (4a) and (4b), one obviously needs some independent information concerning $\Delta G(z,Q^2)$ which is, unfortunately, missing, apart from the positivity constraint,

$$|\Delta G(z,Q^2)| \le G(z,Q^2) , \tag{5}$$

with $G=G_+ + G_-$ being the usual unpolarized-gluon distribution. A possible ansatz for ΔG proposed [9] to account for the measured [4] $g_1^p(x,Q^2)$ in terms of a negative gluonic contribution in Eq. (1), i.e., corresponding to $\Delta C_G^{a(1)}=-1$ which results from Eq. (4a), is

$$\Delta G^a(z)=16.3z^{-0.3}(1-z)^7 , \tag{6}$$

chosen so as to satisfy Eq. (5) at $Q^2\simeq 4\ \mathrm{GeV}^2$, relevant for the low-$x$ EMC data responsible for the result in (2), with G taken from Duke-Owens (set 1) [14]. This choice for ΔG^a together with $a\simeq -1$ to 3 in Eq. (4a) yields a satisfactory agreement with experiment as shown [15] in Fig. 1 where $a=2$ has been taken for definiteness. Using the same procedure in connection with Eq. (4b), corresponding to $\Delta C_G^{b(1)}=0$, it is simply a matter of searching for an appropriate $\Delta G^b(z,Q^2)$ and values for b in Eq. (4b) satisfying

$$\int_x^1 \frac{dz}{z}\Delta C_G^b(z)\Delta G^b\left[\frac{x}{z},Q^2\right] \simeq \int_x^1 \frac{dz}{z}\Delta C_G^a(z)\Delta G^a\left[\frac{x}{z},Q^2\right] . \tag{7}$$

For illustration we have chosen two drastically different polarized-gluon densities at $Q^2\simeq 4\ \mathrm{GeV}^2$:

$$\Delta G^{b_-}(z)=-13.2z^{0.7}(1-z)^{10}G(z) , \tag{8$'$}$$

$$\Delta G^{b_+}(z)=+12.2z^{0.6}(1-z)^{14}G(z) , \tag{8$''$}$$

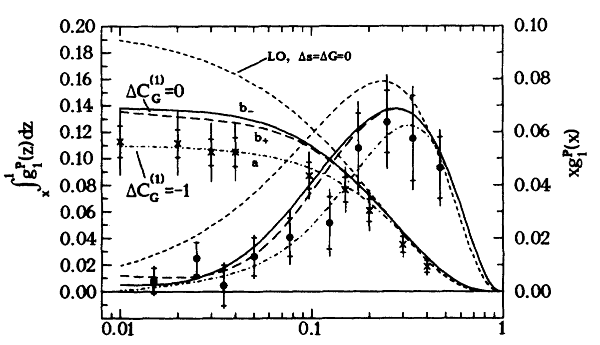

FIG. 1. Predictions for $g_1^p(x,Q^2)$ in Eq. (1), assuming $\Delta s\simeq 0$, and its areas for $x\ge 0.01$ using the gluonic Wilson coefficients of Eqs. (4a) and (4b) together with the polarized-gluon distributions of Eqs. (6) and (8), respectively. Cases a and b refer to the first moments $\Delta C_G^{a(1)}=-1$ and $\Delta C_G^{b(1)}=0$. The short dashed curves refer to the $\Delta q_{3,8}$ leading-order (LO) contributions in Eq. (1). The data are taken from Ref. [4].

which are of course only to be trusted for $z\ge 0.01$ where data exist. With the Kwiecinski-Martin-Stirling-Roberts set B [KMSR(B_-)] parametrization [16] taken for G, these densities satisfy Eq. (5). Taking $b_-\simeq -3$ and $b_+\simeq 2$ in Eq. (4b) for the two choices ΔG^{b_-} and ΔG^{b_+}, respectively, the resulting contributions to $g_1^p(x,Q^2)$ in Eq. (1) are, as shown in Fig. 1, indistinguishable from that of $\Delta G^a(z)$ when folded with $\Delta C_G^a(z)$, keeping in mind the large uncertainties attached to the data as well as to the choice of $\Delta G(x,Q^2)$ and of the parameters a and b in Eqs. (4a) and (4b). It should be recalled that, despite the radically different first moments $\Delta C_G^{a(1)}=-1$ and $\Delta C_G^{b(1)}=0$, both Wilson coefficients $\Delta C_G^a(z)$ and $\Delta C_G^b(z)$ in Eqs. (4a) and (4b) result in indistinguishable contributions to $g_1^p(x,Q^2)$ in the experimentally accessible region $x\ge 0.01$.

The polarized-gluon distributions in Eqs. (6) and (8) are compared for illustration in Fig. 2 which integrate to

$$\begin{aligned}
&\int_{0.01}^1 \Delta G^a(x)dx = 4.1 ,\\
&\int_{0.01}^1 \Delta G^{b_-}(x)dx = -4.4 ,\\
&\int_{0.01}^1 \Delta G^{b_+}(x)dx = 4.6 .
\end{aligned} \tag{9}$$

It should be emphasized that presently available measurements [4] of $g_1^p(x,Q^2)$ do not even constrain the sign of ΔG and allow for a *negative* ΔG as well.

These results clearly demonstrate that as long as we do not possess some independent [17] reliable information on $\Delta G(x,Q^2)$, the debate concerning its contribution to $g_1^p(x,Q^2)$ is unresolvable and, moreover, unaffected by our attitude towards the recommendable values of the first moments of $\Delta G(x)$ as well as of $\Delta C_G(x)$.

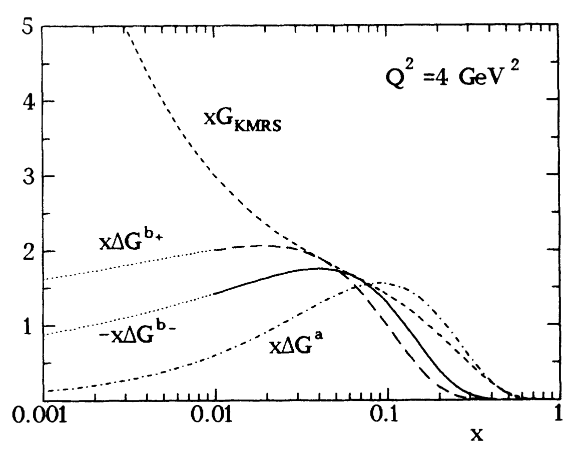

FIG. 2. Comparison of the polarized-gluon distributions of Eqs. (6) and (8) used for our predictions in Fig. 1. The unpolarized KMSR(B_-) gluon distribution, relevant for Eqs. (8′) and (8″), is taken from Ref. [16]. The polarized-gluon distribution of Altarelli and Stirling [9] in Eq. (6), shown by the dashed-dotted curve, satisfies Eq. (5) with $G(z,Q^2)$ at $Q^2=4\ \mathrm{GeV}^2$ taken from Ref. [14]. These distributions should obviously be trusted only for $x\ge 0.01$ where data exist.

[1] G. Altarelli and G. G. Ross, Phys. Lett. B **212**, 391 (1988).
[2] A. V. Efremov and O. V. Teryaev, Dubna Report No. E2-88-287, 1988 (unpublished).
[3] P. Ratcliffe, Nucl. Phys. **B223**, 45 (1983).
[4] EMC, J. Ashman *et al.*, Nucl. Phys. **B328**, 1 (1989).
[5] M. Gourdin, Nucl. Phys. **B38**, 418 (1972); J. Ellis and R. L. Jaffe, Phys. Rev. D **9**, 1444 (1974); **10**, 1669(E) (1974).
[6] G. G. Ross, in *Proceedings of the XIVth International Symposium on Lepton and Photon Interactions,* Stanford, California, 1989, edited by M. Riordan (World Scientific, Singapore, 1990), p. 41.
[7] It should be noted that ΔC_q is subject to the *same* ambiguities as ΔC_G. For definiteness and simplicity we have adopted ΔC_q from Ref. [3]. We have checked, however, that our results and conclusions are essentially unaffected by this restriction.
[8] R. D. Carlitz, J. C. Collins, and A. H. Mueller, Phys. Lett. B **214**, 229 (1988).
[9] G. Altarelli and W. J. Stirling, Particle World **1**, 40 (1989).
[10] G. G. Ross and R. G. Roberts, Rutherford Report No. RAL-90-062 (unpublished).
[11] G. T. Bodwin and J. Qiu, Phys. Rev. D **41**, 2755 (1990).
[12] A. V. Manohar, Phys. Rev. Lett. **66**, 289 (1991); U. Ellwanger, Phys. Lett. B **259**, 469 (1991).
[13] L. Mankiewicz, Phys. Rev. D **43**, 64 (1991); W. Vogelsang, Z. Phys. C **50**, 275 (1991).
[14] D. W. Duke and J. F. Owens, Phys. Rev. D **30**, 49 (1984).
[15] The parametrizations for $\Delta u(x,Q^2)$ and $\Delta d(x,Q^2)$ at $Q^2 \simeq 4$ GeV^2 are taken, for definiteness, from Ref. [10] according to the "constrained $\Delta s=0$, $\Delta G \neq 0$ fit". Similar results can be obtained for the parametrizations of Ref. [9], for example.
[16] J. Kwiecinski, A. D. Martin, W. J. Stirling, and R. G. Roberts, Phys. Rev. D **42**, 3645 (1990).
[17] Only processes where $\Delta G(x,Q^2)$ occurs *directly* already in the lowest order, as for example in $\gamma g \rightarrow c\bar{c}$ for open charm or J/ψ production, appear to be a promising source for extracting some information about ΔG: M. Glück and E. Reya, Z. Phys. C **39**, 569 (1988); M. Glück, E. Reya, and W. Vogelsang, Nucl. Phys. **B351**, 579 (1991); J. P. Guillet, Z. Phys. C **39**, 75 (1988); R. M. Godbole, S. Gupta, and K. Sridhar, Phys. Lett. B **255**, 120 (1991). A theoretically consistent determination of $\Delta G(x,Q^2)$ from scaling violations of $g_1(x,Q^2)$ will not be feasible as long as the polarized two-loop splitting functions $\Delta P_{ij}^{(2)}(x)$ are not available.