Scattering in soliton models and boson-exchange descriptions

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(Received 23 April 1991)

We analyze the relationship between the conventional one-boson-exchange model (OBE) for nuclear forces with the description of the nucleon as a soliton within the context of the two-dimensional sine-Gordon model. We find that the soliton-soliton S matrix contains poles and residues compatible with the ones used in the OBE. Implications of this result for the four-dimensional case are briefly discussed.

PACS number(s): 11.10.Lm, 11.20.Dj, 11.80.$-$m

I. INTRODUCTION

The idea that the nucleon is a chiral soliton has attracted considerable attention in the past few years [1]. While the exact effective meson dynamics of which the nucleon is a soliton is not known, the generic character and properties of the low-lying baryons can be understood from simple chiral models of the Skyrme type [1]. In the semiclassical approximation, the models provide a natural setting for analyzing systematically dynamical issues related to meson-nucleon and nucleon-nucleon scattering [2]. This is a distinct advantage over relativistic bag models and nonrelativistic constituent-quark models.

Overall, semiquantitative success has been obtained for the bulk properties of the nucleon and pion-nucleon scattering. If some early failures of the model predictions can be overcome [3,4], there are still persisting fundamental deficiencies that are understood if not cured [5].

The nucleon-nucleon scattering amplitude has been more of a challenge since the classical two-soliton solution is not known. A product ansatz yields a nucleon-nucleon potential which compares favorably with the empirically motivated potentials at short and long distances, but fails to reproduce the intermediate-range attraction in the scalar channel [6]. A recent reanalysis of the nucleon-nucleon potential beyond the product approximation yields, however, a more favorable result in the intermediate range [18].

Recently two of us [7], following on previous attempts [8], have argued that the nucleon-nucleon problem in the context of Skyrme models can be systematically analyzed using a double expansion in both the range and $\hbar$. The ambiguities related to the ansatz dependence can be lifted through the pion fluctuations. As a result, an attractive component is seen in the central potential at the two-pion range.

Undoubtedly, this is a step forward in the process of describing unambiguously the two-nucleon problem in the context of soliton models. What is unclear, however, is to what extent this soliton description is compatible with the conventional one-boson-exchange (OBE) approach. Put differently, can we deduce that the nucleon is or is not a soliton directly from its S matrix?

To address some of these issues, we have turned to the two-dimensional sine-Gordon model, where exact form factors [9] as well as S-matrix elements [10] are now known. It should be stressed, however, that the two-dimensional case is particular, since bosonic theories such as the sine-Gordon model can be exactly fermionized and vice versa. This is not the case in four dimensions. This is an important difference to be kept in mind throughout.

The paper is organized as follows. In Sec. II, we analyze the meson-soliton form factor. It exhibits a pole at zero momentum transfer even though the mesons are massive, in contrast with conventional models of the Yukawa type. We also extract the meson-soliton coupling, and check the semiclassical expansion against the exact result to one loop. In Sec. III, we analyze the soliton-soliton S matrix. We find that the S matrix contains poles and residues similar to the ones expected from OBE. Implications of these results to the original nucleon-nucleon problem are discussed in Sec. IV. In the Appendix, we discuss the nonrelativistic reduction of the exact meson-soliton scattering amplitude for the sine-Gordon model.

II. MESON-SOLITON FORM FACTOR

Consider the sine-Gordon Lagrangian

$$L=\int dx\left[\frac{1}{2}(\partial_\mu\phi)^2+\frac{m^2}{g^2}\cos(g\phi)\right] \quad (1)$$

with the classical one-soliton solution

$$\phi_s(x)=\frac{4}{g}\arctan(e^{+mx}) . \quad (2)$$

Besides the soliton, there are (composite) meson states with masses in the range $g^0\rightarrow g^{-2}$

$$m_n=2M\sin\left[\frac{n\gamma}{16}\right] \quad n=1,2,\ldots \quad \left[n<\frac{8\pi}{g^2}\right] , \quad (3)$$

where M is the renormalized soliton mass and $\gamma=g^2(1-g^2/8\pi)^{-1}$. The semiclassical expansion is justified for small g.

The relevant quantity in our case is the meson-soliton form factor

$$F(Q)=\langle p_2|\phi(0)|p_1\rangle , \quad (4)$$

where $|p\rangle$ refers to a momentum eigenstate of the soliton, and $Q=p_1-p_2$ (Breit frame). An explicit evaluation of (4) can be obtained using either the Kerman-Klein method [11] or the collective-quantization approach [12]. The form factor to lowest order is simply the Fourier transform of the soliton profile (2)

$$F(q)=\int_{-\infty}^{+\infty} e^{iqx}\phi_s(-x)dx=-\frac{2i\pi}{g}\frac{1}{q}\text{sech}(q\pi/2m)\ . \tag{5}$$

Note that to leading order, the form factor is strong (order g^{-1}).

In the complex q plane, the right-hand side exhibits poles at $q=0$ and $q_n=im(2n+1)$ with $n=0,1,\ldots$. The pole at $q=0$ does not arise in models with a conventional Yukawa coupling with massive mesons, and is characteristic of solitons. This may be readily understood, if we note that

$$\lim_{q\to 0} iqF(q)=\int dx\ \phi_s'(x)=\phi_s(+\infty)-\phi_s(-\infty)\neq 0\ . \tag{6}$$

The poles at $q=q_n$ correspond to poles in $t=Q^2=-q^2$ and arise in conventional Yukawa models with derivative couplings as well. In particular, the positions of the poles are in agreement with (3) for small g. Hence, we may define the meson-soliton coupling constant to lowest order [13] as follows:

$$f_1=\lim_{q\to q_1}(q^2-q_1^2)[-iqF(q)]=-\frac{8}{g}\ . \tag{7}$$

The occurrence of only odd poles suggests that the original meson field is odd under charge conjugation.

To estimate the role of the quantum effects on the structure of the meson-soliton form factor we will evaluate the one-loop correction to (5). For that, we will use the collective-quantization method discussed by Tomboulis [12]. For the sine-Gordon model, the renormalized Hamiltonian to order g^2 reads

$$H=M_s+H_0+H_1+O(g^2)\ , \tag{8}$$

where

$$H_0=\int dx\left[\frac{\pi^2}{2}+\frac{\xi'^2}{2}+\frac{m^2\xi^2}{2}[2\tanh^2(mx)-1]\right],$$
$$H_1=\frac{1}{3}gm^2\int dx\ \xi\left[\xi^2-\frac{3}{4\pi}\int\frac{dk}{\sqrt{k^2+1}}\right]\frac{\sinh(mx)}{\cosh^2(mx)}\ . \tag{9}$$

In H_1 the term linear in the meson coupling is the mass counterterm following normal ordering in the meson sector. To this order, the soliton does not recoil. The meson field is quantized in a box of length L,

$$\xi(x,t)=\sum_n\frac{1}{2\omega_n}[b_n\psi_n(x)e^{-i\omega_n t}+b_n^\dagger\psi_n^*(x)e^{i\omega_n t}] \tag{10}$$

with $\omega_n^2=m^2+(mk_n)^2$, and

$$\psi_n(x)=\frac{[\tanh(mx)-ik_n]e^{imk_nx}}{[(1+k_n)^2L-(2/m)\tanh(mL/2)]^{1/2}} \tag{11}$$

for a discrete set of momenta mk_n.

The one-loop correction to the semiclassical expression (5) for the form factor can be obtained using time-dependent Rayleigh-Schrödinger perturbation theory. Generically

$$\langle p_2|\phi(x)|p_1\rangle_g=\sum_m\left[\frac{\langle p_2|H_1|m\rangle\langle m|\phi(x)|p_1\rangle}{E_0-E_m}+\frac{\langle p_2|\phi(x)|m\rangle\langle m|H_1|p_1\rangle}{E_0-E_m}\right]. \tag{12}$$

Using the usual decomposition for the full quantum field together with the quantized meson fluctuations (10) and (11), we obtain

$$\langle p_2|\phi(x)|p_1\rangle_g=\lim_{L\to\infty}\int dz\,dy\ e^{iqz}\sin(g\phi_s(y))\times A_L(x,y,z)B_L(y)\ , \tag{13}$$

where the integrands are given by

$$A_L=\sum_n\frac{1}{2\omega_n^2}[\psi_n(x-z)\psi_n^*(y)+\psi_n(y)\psi_n^*(x-z)]\ , \tag{14}$$

$$B_L=\frac{m^2g}{4}\left[\sum_n\frac{1}{\omega_n}\psi_n(y)\psi_n^*(y)-\frac{1}{2\pi}\int\frac{dk}{\sqrt{k^2+1}}\right] \tag{15}$$

and ϕ_s is the soliton profile (2). In the continuum limit ($L\to\infty$) the discrete sums can be turned into continuous integrals with the proper weight for the meson density of states around a nonmoving soliton, i.e.,

$$\rho(k_n)=\frac{mL}{2\pi}-\frac{1}{2\pi}\delta'(k_n)\ , \tag{16}$$

where $\delta(k_n)=-2\arctan(k_n)$, denotes the phase shift of the box eigenfunctions (11). As a result

$$B_\infty=\frac{m^2g}{8\pi}\int\frac{dk}{(k^2+1)^{3/2}}[\tanh^2(my)-1]\ . \tag{17}$$

Note that the result is finite (as it should be) following the cancellation between the divergence in the mode sum and the mass counterterm in (9). This provides an additional check that mass renormalization in the trivial vacuum sector is sufficient to renormalize the model in the nontrivial topological sector as well. Similarly,

$$A_\infty=\frac{1}{4\pi m}\int dk\ \frac{e^{imk(x-y-z)}}{(k^2+1)^2}[k+i\tanh(m(x-z))]\times[k-i\tanh(my)]+\text{c.c.} \tag{18}$$

Inserting (17) and (18) in (13) and performing some algebra, we obtain

$$\langle p_2|\phi(x)|p_1\rangle_g=\frac{mg}{4\pi^2}\int_{-\infty}^{\infty}dz\ e^{iqz}\int_{-\infty}^{\infty}\frac{dk}{(k^2+1)^2}\int_{-\infty}^{\infty}dy\ \frac{\sinh(my)}{\cosh^4(my)}G_1(x,z,k,y)\ , \tag{19}$$

where

$$G_1(x,z,k,y)=[\tanh(m(x-z))\tanh(my)+k^2]\cos(mk(x-z-y)) - k[\tanh(m(x-z))-\tanh(my)]\sin(mk(x-z-y)) \, . \tag{20}$$

The integrations are performed in the order indicated, so that no spurious divergence is generated. Using the result

$$\int_0^\infty dt \, \frac{\cos(at)}{\cosh^\nu(bt)} = \frac{2^{\nu-2}}{b\Gamma(\nu)} \Gamma\left[\frac{\nu}{2}+\frac{ai}{2b}\right]\Gamma\left[\frac{\nu}{2}-\frac{ai}{2b}\right] \tag{21}$$

and integrating by parts, gives

$$\langle p_2|\phi(x)|p_1\rangle_g = \left[\frac{-ig}{8q}\right] e^{iqx} \operatorname{sech}\left[\frac{q\pi}{2m}\right]\left[\frac{q^2}{m^2}\right] . \tag{22}$$

Combining (5) with (22) yields the form factor to order g^2

$$\langle p_2|\phi(0)|p_1\rangle = -\frac{i2\pi}{q}\operatorname{sech}\left[\frac{q\pi}{2m}\right] \times \left[\frac{1}{g}+\frac{gq^2}{16\pi m^2}+O(g^2)\right] . \tag{23}$$

While the pole position has not changed, the residue has been modified. The quantum effects are expected to renormalize the bare position of the pole (here meson mass) and affect the strength of the residues. In fact we expect this result to hold true to higher orders in perturbation theory around the soliton background and even in higher dimensions.

The semiclassical description of the meson-soliton form factor and the one-loop correction are consistent with the exact result derived in [9,10], in the form

$$F(Q) = -\frac{i2\pi}{g}\frac{e^{J(\theta_-)}}{q}\left[\frac{\cosh\theta_+/2}{\sqrt{\cosh\theta_1\cosh\theta_2}}\right] \times \frac{\cosh(\theta_-/2)}{\cosh(4\pi\theta_-/\gamma)} , \tag{24}$$

where

$$J(\theta_-) = \int_0^\infty \frac{dx}{x}\frac{\sin^2(x\theta_-/2)\sinh(x(\pi-\xi)/2)}{\cosh(\pi x/2)\sinh(\pi x)\sinh(3x/2)} . \tag{25}$$

Here $\theta_\pm$ are the rapidities

$$\theta_\pm = \theta_1 \pm \theta_2, \quad p_{1,2} = M(\cosh\theta_{1,2}, \sinh\theta_{1,2}) . \tag{26}$$

The large parentheses in (24) involve a relativistic kinematic factor (numerator) and energy normalization factors (denominator). The combination of these factors and $1/q$ is relativistically invariant. In the region $0<\mathrm{Im}\theta_-<2\pi$, the function $J(\theta_-)$ is regular, so that the exact poles of the form factor follow from the position of the poles in $\operatorname{arccosh}(4\pi\theta_-/\gamma)$, i.e.,

$$\theta_n = \frac{i\gamma}{8}(2n+1), \quad n=0,1,\ldots \tag{27}$$

in agreement with the poles derived using the semiclassical approximation.

Using the substitution $\theta\to v=p/M$ in the semiclassical limit, we have

$$J(\theta_-) = \frac{g^2}{16\pi}\left[\frac{q^2}{m^2}\right]+O(g^4) , \tag{28}$$

$$\cosh\left[\frac{\pi\theta_-}{2\xi}\right] = \cosh\left[\frac{\pi q}{2m}\right]+O(g^4) . \tag{29}$$

Inserting (28) and (29) into (24) we obtain after a little algebra

$$\langle p_2|\phi(0)|p_1\rangle = -\frac{i2\pi}{q}\operatorname{sech}\left[\frac{q\pi}{2m}\right] \times \left[\frac{1}{g}+\frac{gq^2}{16\pi m^2}+O(g^2)\right] \tag{30}$$

in agreement with (23). This result shows agreement between perturbation theory in the presence of a single-soliton background and the exact result to one-loop level, extending the lowest-order check first done in [14].

III. SOLITON-SOLITON SCATTERING

The exact relativistic S matrix for the sine-Gordon model has been derived by Zamolodchikov and Zamolodchikov [10]. The form of the S matrix follows from factorization (absence of pair creation), unitarity and crossing. It depends only on the relative rapidities $\theta=\theta_-$, and is a meromorphic function of θ in the strip $0<\mathrm{Im}\theta<\pi$, following the mapping $s=4M^2\cosh^2(\theta/2)$. The edges of the strip correspond to the cuts in the s plane. Crossing symmetry follows from $\theta\to i\pi-\theta$. The one-boson-exchange (OBE) pole at $t=m^2$ corresponds to $\theta=i\xi=i\gamma/8$.

The explicit form of the S matrix may be taken as $S(\theta)=S_3(\theta)+S_2(\theta)$ where [10]

$$S_3(\theta) = i\coth\left[\frac{4\pi\delta}{\gamma}\right]\coth\left[\frac{4\pi\theta}{\gamma}\right]S_2(\theta) . \tag{31}$$

Here δ is a parameter equal to π when crossing is enforced. $S_2(\theta)$ has a simple pole at $\theta=i\xi$ as shown below, and the hyperbolic cotangent has a simple zero, so $S_3(\theta)$ will not contribute to the residue. $S_2(\theta)$ may be written as [10]

$$S_2(\theta) = \frac{2}{\pi}\sin\left[\frac{4\pi^2}{\gamma}\right]\sinh\left[\frac{4\pi\theta}{\gamma}\right] \times \sinh\left[\frac{4\pi(i\pi-\theta)}{\gamma}\right]U(\theta) , \tag{32}$$

where

$$U(\theta)=\Gamma\left[\frac{8\pi}{\gamma}\right]\Gamma\left[1+i\frac{8\theta}{\gamma}\right]\Gamma\left[1-\frac{8\pi}{\gamma}-i\frac{8\theta}{\gamma}\right]\prod_{n=1}^{\infty}\left[\frac{R_n(\theta)R_n(i\pi-\theta)}{R_n(0)R_n(i\pi)}\right],$$

$$R_n(\theta)=\frac{\Gamma(2n(8\pi/\gamma)+i(8\theta/\gamma))\Gamma(1+2n(8\pi/\gamma)+i(8\theta/\gamma))}{\Gamma((2n+1)(8\pi/\gamma)+i(8\theta/\gamma))\Gamma(1+(2n-1)(8\pi/\gamma)+i(8\theta/\gamma))}\ . \tag{33}$$

For $\theta=i\zeta$ the string of products

$$\prod_{n=1}^{\infty}\frac{(\zeta/\pi)+2n-1}{(\zeta/\pi)+2n-2}\left[\frac{2n-1}{2n}\right]^2\frac{2n+1-(\zeta/\pi)}{2n-(\zeta/\pi)} \tag{34}$$

is finite. At weak coupling $\gamma\rightarrow 0$

$$\frac{3\pi}{\gamma}\prod_{n=2}^{\infty}\left[\frac{2n-1}{2n}\right]^3\frac{2n+1}{2n-2}=\frac{3\pi}{\gamma}\left[\frac{32}{3\pi^2}\right]. \tag{35}$$

The prefactor which converts this to $U(\theta)$ has a simple pole

$$\Gamma(\pi/\zeta)\frac{1}{1+i\theta/\zeta}\Gamma(2-\pi/\zeta)=\frac{1-\pi/\zeta}{1+i\theta/\zeta}\frac{\pi}{\sin(\pi^2/\zeta)} \tag{36}$$

and the prefactor that converts $U(\theta)$ to $S_2(\theta)$ gives

$$\frac{1}{\pi}\sin\frac{\pi^2}{\zeta}\ . \tag{37}$$

Collecting all the pieces, going to the weak-coupling limit, and converting to the Mandelstam variables yield the pole

$$\frac{8M^2}{t-m^2}\ . \tag{38}$$

On the other hand, the Yukawa Lagrangian consistent with the coupling (7),

$$\mathcal{L}=-i\frac{8}{g}\bar{\psi}\gamma_\mu\gamma_5\psi\partial^\mu\phi \tag{39}$$

gives the S matrix

$$\langle p_1'p_2'|(S-1)|p_1p_2\rangle=(2\pi)^2\frac{\delta^2(p_1'+p_2'-p_1-p_2)}{\sqrt{2E_12E_22E_1'2E_2'}}T(s,t,u)\ , \tag{40}$$

where the T-matrix amplitude follows from the Born diagrams. The restricted character of the kinematics in two dimensions and the mass-shell condition yield in the t channel

$$\langle p_1'p_2'|(S-1)|p_1p_2\rangle=(2\pi)^2[\delta(p_1'-p_1)\delta(p_2'-p_2)-\delta(p_1'-p_2)\delta(p_2'-p_1)]\frac{2p/m}{\sqrt{1-t/4M^2}}\frac{8M^2}{t-m^2} \tag{41}$$

with $p_1=-p_2=p$, $E_1=E_2=\sqrt{M^2+p^2}$ and $t=-(p_1-p_2)^2$ (center-of-mass frame). There is a pole at $t=m^2$ (one-meson exchange) with a residue that matches the residue in (38) in the semiclassical limit.

In a way, this is a nice surprise. Indeed, in soliton models the meson field ϕ and the soliton field ψ obey a nonlocal commutation relation,

$$[\phi(x,t),\psi(y,t)]=-\frac{2\pi}{g}\theta(x-y)\psi(y,t) \tag{42}$$

following from the fact that the creation of a soliton must change the vacuum at infinity. One would have thought that since locality properties are altered, so will analyticity properties. Our explicit calculation shows that this is not the case.

We note, that the soliton-(anti)soliton S matrix does obey crossing, since ψ is local with respect to itself as well as $\bar{\psi}$. So, if the meson is considered as a soliton-antisoliton bound state, this will generate an s-channel pole and, by crossing, a t-channel pole. So the appearance of poles in the exact S matrix is rather natural.

This simple argument also implies the existence of poles for $t=(nm)^2\ (n\geq 2)$. To exhibit these higher-order poles, and as a further check on our preceding derivation, we will use an alternative analysis borrowing on some results for the exact S matrix established by Smirnov [9]. For that, consider $\Omega(\theta)$ to be a regular function in the strip $0<\mathrm{Im}\theta<2\pi$ and such that

$$S(-\theta)=\Omega(\theta)S(\theta)=\Omega(\theta-i2\pi)\ , \tag{43}$$

$$\Omega(\theta)\Omega(\theta-i\pi)=\eta^{-1}(\theta+i\pi/2)\ , \tag{44}$$

where $\eta(\theta)$ is an even function with simple poles at $\theta_n=i\pi/2+in\zeta$ with $n\geq 0$ and zeros at $\theta_n=i3\pi/2+in\zeta$ with $n\geq 1$. The auxiliary function $\eta(\theta)$ satisfies the property [9]

$$\eta(\theta-i\zeta)=\eta(\theta)\frac{\cosh((\theta+i\pi/2)/2)}{\cosh((\theta-i\pi/2-i\zeta)/2)}\ . \tag{45}$$

In terms of (43)–(45) we can rewrite the scattering amplitude in the following form:

$$S(\theta)=\frac{\eta(\theta+i\pi/2)}{\eta(\theta-i\pi/2)}\ . \tag{46}$$

Using (45) we obtain

$$S(\theta)=S(\theta-i\zeta)\coth(\theta/2)\coth((\theta-i\zeta)/2)\ . \tag{47}$$

Iterating (47) we easily find

$$S(\theta)=S(\theta-in\zeta)\coth(\theta/2)H_n^2(\theta)\coth((\theta-in\zeta)/2)\ , \tag{48}$$

where we have defined

$$H_n(\theta)=\prod_{k=1}^{n-1}\coth([\theta-i(n-k)\zeta]/2)\ . \tag{49}$$

Expression (48) for the scattering amplitude displays a string of simple poles at $\theta_n=in\zeta=in\gamma/8$ with residues given by

$$R_n=-2\coth(\theta_n/2)H_n^2(\theta_n)\ . \tag{50}$$

In the semiclassical limit the S matrix near the nth pole reads (t channel)

$$S=\frac{S_n}{t-m_n} \tag{51}$$

with the location of the poles and residues given, respectively, by

$$m_n=nm,\quad \frac{S_n}{8M^2}=\frac{(-)^{n+1}}{[(n-1)!]^2}\left[\frac{16}{g^2}\right]^{2(n-1)}\ . \tag{52}$$

For $n=1$ this result is in agreement with (38).[1]

It is not hard to show that additional Yukawa couplings corresponding to the breather modes (bound-meson states) with couplings commensurate with their intrinsic charge conjugation, reproduce the scattering amplitude (51) in the semiclassical limit. This description is consistent with the OBE approach using higher-mass resonances.

IV. DISCUSSION

We have seen that in two dimensions, form factors and scattering amplitudes exhibit the same analyticity properties as those derived with conventional Yukawa couplings. In other words, the OBE description is compatible with the soliton-soliton S-matrix element in the semiclassical limit. The question then becomes: How much of this carries to four dimensions? Unfortunately, analytic expressions are not known for Skyrme models, even in the semiclassical limit. Moreover, the exact integrability of the sine-Gordon model together with the fact that it can be fermionized exactly, are important features of two dimensions that may limit considerably the relevance of the preceeding arguments to four dimensions.

However, there are reasons to believe that the meson-soliton form factor behaves more similarly to those of conventional Yukawa models. For definiteness, we may take the classical partially conserved axial-vector current (PCAC) pion field to be

$$\Pi^a(\mathbf{x})=f_\pi R^{ab}\frac{x^b}{r}\sin\Theta(r)\ , \tag{53}$$

where f_π is the pion decay constant, and R is a spin-isospin rotation matrix that is unimportant for the remainder of the discussion. Since the chiral angle obeys the boundary conditions

$$\Theta(0)=\pi,\quad \Theta(r)=O(e^{-mr})\quad (r\to\infty) \tag{54}$$

it follows that the form factor

$$F^a(\mathbf{q})=\int d^3x\, e^{-i\mathbf{q}\cdot\mathbf{x}}\Pi^a(\mathbf{x}) \tag{55}$$

has a singularity at $\mathbf{q}^2=-m^2$, but no pole at $\mathbf{q}=0$. The latter is due to the fact that $\partial_\mu\Pi^a$ is related to the axial-vector current rather than the topological current as in two dimensions.

For the soliton-soliton S matrix, we expect a pole at $t=m^2$ from the general dimension-independent argument of Sec. III. This is consistent with the fact that the classical interaction energy of two solitons falls off exponentially with their distance. Moreover, the above two-dimensional results encourage us to believe that the residue relates naturally to the pion-nucleon coupling. We note that in nature, the pion-nucleon coupling can be obtained from pion-nucleon dispersion relations, and OBE can be tested in the higher partial waves of the nucleon-nucleon scattering amplitude. Experimentally, the two are consistent.

The assumption that analyticity properties hold true for the chiral soliton is actually nontrivial, if we recall that in four dimensions the meson and soliton fields obey nonlocal commutation relations, while in QCD $\bar{q}q$ (meson) and qqq (baryons) fields obey local ones. We note, however, that the pion (meson) field in Skyrme models is a coarse-grained version of $\bar{q}q$, so nonlocality is in fact expected on a length scale of the order of $\hbar^0$. Unfortunately this argument breaks down in the chiral limit, where the nonlocality extends over an infinite range.

Finally, we would like to add that besides the positive result of the two-dimensional analysis presented above with its suggestive implications for four dimensions, there are also strong reasons to favor the idea that nucleons are chiral solitons, and this independently of the phenomenology. Indeed, chiral symmetry dictates that the effective low-energy theory of pions is given by the nonlinear σ model [15], and once we add vector mesons, especially the ω, it is difficult not to wind up with solitons. Moreover, it is hard to believe that the relationship between the chiral anomaly and the spin statistics of the soliton [16] is just a mere coincidence.

ACKNOWLEDGMENTS

We express our warm gratitude to V. E. Korepin and F. A. Smirnov for clarifying discussions. R.P. thanks M. Rocek for guidance and C.C. thanks the Theory Group at the Department of Physics University of Lecce, Sezione I.N.F.N. at Lecce, Italy. This work was supported by the Department of Energy under Grant No. DE-FG02-88ER40388 with the State University of New York.

APPENDIX: MESON-SOLITON SCATTERING AMPLITUDE

The exact meson-soliton scattering amplitude is given by [10]

[1]We have checked that a rerun of the first argument yields also the same higher-order poles.

$$S_1(\theta_-)=\frac{\sinh\theta_- + i\cos(\gamma/16)}{\sinh\theta_- - i\cos(\gamma/16)} . \quad \text{(A1)}$$

In the meson-soliton center-of-mass frame $p+k=0$, the scattering amplitude (A1) can be rewritten in momentum space

$$S_1(k)=\frac{k(\sqrt{m^2+k^2}-\sqrt{M^2+k^2})-imM\cos(\gamma/16)}{k(\sqrt{m^2+k^2}-\sqrt{M^2+k^2})+imM\cos(\gamma/16)} . \quad \text{(A2)}$$

In the semiclassical (nonrelativistic) limit $\gamma\sim g^2$ and (A2) reduces to

$$S_1=1+\frac{2imk-2m^2}{k^2+m^2}+O(g^2) . \quad \text{(A3)}$$

On the other hand, the process of a meson scattering off a soliton in the semiclassical description corresponds to background field scattering off a static soliton. If $\psi_k(x)$ designates the continuum scattering wave function of the meson, then from (11) we have

$$\psi_k(x)=\frac{1}{\sqrt{2\pi}}\frac{k}{\omega_k}e^{ikx}\left[1+\frac{im}{k}\tanh(mx)\right] , \quad \text{(A4)}$$

where $\omega_k=\sqrt{m^2+k^2}$, $-\infty<k<\infty$. The S matrix to order g^2 follows from (A4) through the identification (modulo the plane-wave phase factors)

$$S_p\sim\frac{\psi_k(+\infty)}{\psi_k(-\infty)} . \quad \text{(A5)}$$

A little algebra shows that this agrees with (A3). Here, we point out that with the Yukawa coupling defined as in (39), it was shown in [17] that the Born diagrams for meson-soliton scattering give the same result as potential scattering (A5). Our result, while in agreement with [17], shows that (A5) follows directly from the exact scattering amplitude in the semiclassical limit, as it should.

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