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Approaching low-energy QCD with a gauged, nonlocal, constituent-quark model

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We study the minimal constituent-quark model with momentum-dependent quark mass in the presence of SU(3)×SU(3) external gauge fields $V_\mu(x)$, $A_\mu(x)$, $S(x)$, and $P(x)$. The model generates vertex functions, for any number of external fields and pseudo Goldstone bosons coupling to quarks, saturating the Ward-Takahashi identities of QCD. Parameters in the $O(p^4)$ chiral Lagrangian are expressed in terms of the quark mass function $\Sigma(p)$ and are surprisingly consistent with low-energy QCD. By use of the auxiliary-field method we discuss the relation of the model to the improved ladder approximation in QCD.

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I. INTRODUCTION

Much of what is known about low-energy QCD may be encoded into a finite number of parameters appearing in a chiral Lagrangian, expanded to some finite order in an energy expansion. There have been various attempts to model the underlying dynamics of QCD to understand the observed values of these parameters. This involves modeling the relevant degrees of freedom for QCD at intermediate energies, such as the low-lying resonances or constituent-mass quarks, and then integrating out these degrees of freedom to obtain the low-energy chiral Lagrangian. Clearly, if a low-energy derivative expansion of some model is to reproduce successfully the chiral Lagrangian of QCD, the model should be quite QCD-like at intermediate energies (0.2–2 GeV).

Our interest here is in the quark-based models [1], as opposed to resonance models, and their potential to make more direct contact with the underlying gauge-theory dynamics. The simplest constituent-quark models are based on linear or nonlinear σ models. A nice feature of such models is that they correctly incorporate the chiral symmetries of QCD. For example, they contain the non-Abelian chiral-anomaly structure of QCD and thus automatically generate the Wess-Zumino terms. And they have some hope of incorporating the explicit chiral-symmetry-breaking effects due to current quark masses and electroweak interactions.

But a major deficiency of any local σ model lies in its description of a constituent-quark mass. The latter is taken to be essentially momentum independent, whereas the dynamical quark mass in QCD will have significant momentum dependence in the range 0.2–2 GeV. The derivative expansion will be sensitive to this momentum dependence.

Closely related to this problem is that some quantities such as f_π turn out to be cutoff dependent because of the assumed high-momentum behavior of the constituent mass. Predictions are then limited to those quantities which are still convergent. Basically, it is rather difficult to model QCD dynamics in local models with a sharp cutoff.

A goal then is to build the dynamical mass into constituent-quark models at the Lagrangian level, by considering nonlocal effective Lagrangians [2–4]. Our main emphasis will be on maintaining the chiral symmetries of QCD. These chiral symmetries will be promoted to local symmetries by introducing external gauge fields, as is typical in the study of low-energy QCD. (The model described in our previous work [4] was not fully consistent with this symmetry structure.)

We will see that tree-level vertices derived from this Lagrangian, involving the couplings to quarks of any number of pseudo Goldstone bosons (PGB's) and external SU(3)×SU(3) gauge fields, will satisfy the infinite set of exact QCD Ward-Takahashi (WT) identities. Thus all implications of a momentum-dependent dynamical mass following solely from symmetry considerations are built into the model at the Lagrangian level.

Consider QCD with three light flavors $q=(u,d,s)$ in the presence of external vector, axial-vector, scalar, and pseudoscalar fields, $V_\mu(x)$, $A_\mu(x)$, $S(x)$, and $P(x)$: [$A_\mu(x)\equiv A_\mu^a(x)T^a$, etc., $\mathrm{Tr}T^aT^b=\frac{1}{2}\delta_{ab}$] [5]. The generating functional $\Gamma(V,A,S,P)$ for the Green functions of the corresponding currents is represented by

$$e^{i\Gamma(V,A,S,P)} = \frac{1}{N}\int DG_\mu\, Dq\, D\bar{q} \times \exp\left[i\int d^4x\, \mathcal{L}_{\mathrm{QCD}}(q,\bar{q},G;V,A,P,S)\right], \tag{1}$$

$$\mathcal{L}_{\mathrm{QCD}}=\mathcal{L}^0_{\mathrm{QCD}}+\bar{q}(\not{V}+\not{A}\gamma_5)q-\bar{q}(S-iP\gamma_5)q\ . \tag{2}$$

Under a local SU(3)×SU(3) transformation

$$q(x)\Longrightarrow \exp[i\alpha(x)+i\beta(x)\gamma_5]q(x)\ , \tag{3}$$

$\mathcal{L}_{\mathrm{QCD}}$ is invariant if, for infinitesimal $\alpha(x)$ and $\beta(x)$, the external fields transform as

$$\begin{aligned}
\delta V_\mu&=\partial_\mu\alpha+i[\alpha,V_\mu]+i[\beta,A_\mu]\ ,\\
\delta A_\mu&=\partial_\mu\beta+i[\alpha,A_\mu]+i[\beta,V_\mu]\ ,\\
\delta S&=i[\alpha,S]-\{\beta,P\}\ ,\\
\delta P&=i[\alpha,P]+\{\beta,S\}\ .
\end{aligned} \tag{4}$$

We postulate that another, more complicated representation of the QCD generating functional can be made in terms of constituent-mass quark fields. In this representation Goldstone fields must be introduced to maintain the chiral symmetries. With explicit symmetry-breaking effects, these fields are PGB's. But the gluons are assumed to be integrated out of the theory, so the generating functional has the form

$$e^{i\Gamma(V,A,S,P)} = \frac{1}{N'}\int DU\, D\psi\, D\bar{\psi} \times \exp\left[i\int d^4x\, \mathcal{L}_{\mathrm{QCD}'}(\psi,\bar{\psi},U;V,A,P,S)\right] . \tag{5}$$

ψ denotes a constituent-mass quark field. The PGB fields $\pi(x)\equiv\pi^a(x)T^a$ appear in

$$U(x)=e^{-2i\pi(x)\gamma_5/f_\pi} . \tag{6}$$

$\mathcal{L}_{\mathrm{QCD}'}$ is some complicated, nonlocal, nonrenormalizable Lagrangian. $\mathcal{L}_{\mathrm{QCD}'}$, for example, must include nonlocal four-fermion terms to account for the effects of gluon exchange at high energies. And the composite nature of the PGB's must also be evident at high energies.

Our hope is that there exist simpler constituent-quark Lagrangians involving the same fields and symmetries which capture some of the essential physics in the intermediate energy range and, by doing so, help serve as a bridge between high- and low-energy QCD.

We will first consider a minimally gauged nonlocal constituent- (GNC-)quark model and show how it generates vertex functions saturating the QCD WT identities (Sec. II). By studying the PGB propagator, we are led to choose a minimal model with $g_A=1$ and with no other free parameters other than the mass function $\Sigma(p)$. We then extract the parameters in the resulting effective low-energy chiral Lagrangian (Sec. III). Although confinement and the resonance structure of QCD is completely omitted in such a model, we end up with a surprisingly close similarity to low-energy QCD. The effects of a momentum-dependent quark mass will become clear.

In Sec. IV we make some effort to try to bridge the gap between the QCD Lagrangian $\mathcal{L}_{\mathrm{QCD}}$ and the constituent-quark Lagrangian $\mathcal{L}_{\mathrm{QCD}'}$ by use of the auxiliary-field method. We will find some motivation for the minimal GNC-quark model in the context of the improved ladder approximation in QCD. We end with some remarks in Sec. V.

II. MINIMAL GAUGED NONLOCAL CONSTITUENT-QUARK MODEL

We construct a GNC model with the following properties. Most importantly, it will involve the V_μ, A_μ, S, P external gauge fields and have the same local SU(3)$\times$SU(3) symmetry structure as QCD. The model will be nonlocal, but it will only be quadratic in the constituent-mass quark field, and it will have no kinetic terms for the PGB fields at tree order. The action is of the form

$$A_{\mathrm{GNC}}=\int d^4x\, d^4y\, \bar{\psi}(x)S^{-1}_{\pi,V,A,S,P}(x,y)\psi(y) . \tag{7}$$

$S^{-1}_{\pi,V,A,S,P}(x,y)$ is proportional to δ_{cd} in color space. A_{GNC} will reduce to the following when the PGB and external fields are set to zero:

$$\int d^4x\, d^4y \left[\frac{i}{2}\bar{\psi}(x)Z(x-y)\partial\!\!\!/\psi(y)-\frac{i}{2}\bar{\psi}(x)\overleftarrow{\partial\!\!\!/}Z(x-y)\psi(y) -\bar{\psi}(x)\Sigma(x-y)\psi(y)\right] . \tag{8}$$

In this way the large- (Euclidean) momentum behavior of the chiral-symmetry-breaking part of the tree-level quark propagator may be chosen to resemble that of the full quark propagator of QCD; namely, for the Fourier transform of $\Sigma(x-y)$, we could choose

$$\Sigma(p)\Longrightarrow\frac{c\,\ln(p^2)^a}{p^2}\quad \text{for } p^2\gg m^2 . \tag{9}$$

m is the constituent-quark mass scale and we may define it by $m\equiv\Sigma(m)$.

By integrating out the quarks and performing a derivative expansion, we obtain the chiral Lagrangian $\mathcal{L}_{\mathrm{eff}}$:

$$\int d^4x\, \mathcal{L}_{\mathrm{eff}}(\pi,V,A,S,P)=-\operatorname{tr}\ln S^{-1}_{\pi,V,A,S,P} . \tag{10}$$

We will discuss $\mathcal{L}_{\mathrm{eff}}$ in the next section. It is sensitive in particular to the momentum dependence expected in $\Sigma(p)$ for $p^2\approx m^2$.

The PGB propagator $\Delta(p)$ may also be extracted from the right-hand side (RHS) of (10). The final form of the model will yield the following behavior (for Euclidean p) in the chiral-symmetry limit:

$$\frac{\Delta^{-1}(p)}{p^2}\Longrightarrow\begin{cases}1 & \text{for } p^2\ll m^2 ,\\ \dfrac{c_\Delta}{p^2} & \text{for } p^2\gg m^2 .\end{cases} \tag{11}$$

This low-momentum behavior of $\Delta^{-1}(p)$ is required by the masslessness of the PGB's, while the soft high-momentum behavior is desirable in view of the compositeness of the PGB's.

We begin with the following action, which may be considered to be the minimally gauged version of the terms in (8) (we will constrain it further below):

$$A_{\mathrm{GNC}}=\int d^4x\, d^4y \left[\frac{i}{2}Z(x-y)[\bar{\psi}(x)\gamma^\mu Y(x,y)D_\mu\psi(y)-\bar{\psi}(x)\gamma^\mu\overleftarrow{D}_\mu Y(x,y)\psi(y)] -\Sigma(x-y)\bar{\psi}(x)\xi(x)X(x,y)\xi(y)\psi(y)-\delta(x-y)\bar{\psi}(x)[S(x)-i\gamma_5P(x)]\psi(y)\right] , \tag{12}$$

$$R_\mu = V_\mu + A_\mu\gamma_5,\quad L_\mu = V_\mu - A_\mu\gamma_5,\quad \xi^2(x) \equiv U(x)\ , \tag{13}$$

$$D_\mu = \partial_\mu - i\hat{\Gamma}_\mu\ ,\quad \overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + i\hat{\Gamma}_\mu\ , \tag{14}$$

$$X(x,y) = P\exp\left[-i\int_x^y \Gamma_\mu(z)dz^\mu\right]\ , \tag{15}$$

$$Y(x,y) = P\exp\left[-i\int_x^y \tilde{\Gamma}_\mu(z)dz^\mu\right]\ , \tag{16}$$

$$\begin{aligned}\Gamma_\mu &= \frac{i}{2}[\xi(\partial_\mu - iR_\mu)\xi^\dagger + \xi^\dagger(\partial_\mu - iL_\mu)\xi] \\ &= V_\mu + \frac{i}{2f_\pi^2}(\pi\partial_\mu\pi - (\partial_\mu\pi)\pi) - \frac{i}{f_\pi}[\pi, A_\mu] + \cdots\ ,\end{aligned} \tag{17}$$

$$\begin{aligned}\hat{\Gamma}_\mu &= \frac{1+g_A}{2}R_\mu + \frac{1-g_A}{2}iU^\dagger(\partial_\mu - iL_\mu)U \\ &= V_\mu + g_A A_\mu\gamma_5 + (1-g_A)F_\mu(\pi) + \cdots\ ,\end{aligned} \tag{18}$$

$$F_\mu(\pi) = \frac{1}{f_\pi}\partial_\mu\pi\gamma_5 + \frac{i}{f_\pi}[\pi, V_\mu]\gamma_5 - \frac{i}{f_\pi}[\pi, A_\mu] + \frac{i}{f_\pi^2}(\pi\partial_\mu\pi - (\partial_\mu\pi)\pi)\ , \tag{19}$$

$$\tilde{\Gamma}_\mu = \hat{\Gamma}_\mu|_{g_A \Rightarrow \tilde{g}_A}\ . \tag{20}$$

The constituent-quark field here transforms linearly under SU(3)×SU(3) transformations

$$\psi(x) \Longrightarrow \exp[i\alpha(x) + i\beta(x)\gamma_5]\psi(x) \equiv V(x)\psi(x)\ , \tag{21}$$

$$\begin{aligned}\xi &\Longrightarrow g(x,\alpha,\beta,\pi)\xi\exp(-i\alpha - i\beta\gamma_5) \\ &= \exp(i\alpha - i\beta\gamma_5)\xi g^\dagger(x,\alpha,\beta,\pi)\ ,\end{aligned} \tag{22}$$

$$\hat{\Gamma}_\mu \Longrightarrow V\hat{\Gamma}_\mu V^\dagger + iV\partial_\mu V^\dagger,\quad \tilde{\Gamma}_\mu \Longrightarrow V\tilde{\Gamma}_\mu V^\dagger + iV\partial_\mu V^\dagger\ , \tag{23}$$

$$\Gamma_\mu \Longrightarrow g\Gamma_\mu g^\dagger + ig\partial_\mu g^\dagger,\quad g(x,\alpha(x),0,\pi(x)) = e^{i\alpha(x)}\ . \tag{24}$$

$A_{\rm GNC}$ is also C and P invariant. We note that there are two parameters in the quark "kinetic term"; one is the standard axial-current coupling g_A and the other, $\tilde{g}_A$, appears in $Y(x,y)$. $\tilde{g}_A$ dependence drops out in the local limit: $Z(x-y) = \Sigma(x-y) = \delta(x-y)$. There is no analogous parameter in the quark "mass term" because of CP invariance.

The dependence of the action on the S and P external fields ensures that the quark condensate takes the standard form in terms of the quark propagator $S_0(p)$ implicit in (8):

$$\langle\bar{\psi}\psi\rangle \equiv \left.\frac{\delta\Gamma}{\delta S}\right|_{V,A,S,P=0} = -\int\frac{d^4p}{(2\pi)^4}{\rm Tr}S_0(p)\ . \tag{25}$$

(PGB loop corrections are ignored on the RHS.) A current quark mass matrix M is introduced by making the replacement

$$S(x) \Longrightarrow S(x) + M\ . \tag{26}$$

Note that this does not modify the PGB couplings to quarks. In the following we shall mostly be concerned with the chiral-symmetry limit $M=0$, unless noted otherwise.

First, we derive some of the tree-level vertices for external fields and PGB's coupling to quarks, to be used in the diagrammatic expansion of the RHS of (10). To expand the path-ordered exponentials, we use a technique partially utilized in [4] and described in detail in [6]. [But note that a locally SU(3)×SU(3) invariant model was not studied in those references. The vertices derived here will be different.] This method involves Fourier transforming and Taylor expanding the functions $\Sigma(p)$ and $Z(p)$, converting the powers of momenta into derivatives acting on the path-ordered exponentials and quark fields, Fourier transforming back, and then resumming. The property of the path integral used is

$$\frac{\partial}{\partial y^\nu}\int_x^y \Gamma_\mu(z)dz^\mu = \Gamma_\nu(y)\ . \tag{27}$$

The following are examples of a few of the vertices found in this way (in Minkowski space). We first give vertices for the fields Γ_μ, $\hat{\Gamma}_\mu$, and $\tilde{\Gamma}_\mu$, since by using the expansions of these quantities in (17) and (18) one finds contributions to vertices involving π's and the external fields (p_1 and p_2 are quark momenta):

$$\Lambda_\Gamma^{\mu a}(p_1,q,p_2=p_1+q) = G(p_2,p_1)(p_1+p_2)^\mu T^a\ , \tag{28}$$

$$\Lambda_{\hat{\Gamma}}^{\mu a}(p_1,q,p_2=p_1+q) = -\tfrac{1}{2}[Z(p_1)+Z(p_2)]\gamma^\mu T^a\ , \tag{29}$$

$$\Lambda_{\tilde{\Gamma}}^{\mu a}(p_1,q,p_2=p_1+q) = -\tfrac{1}{2}F(p_2,p_1)(\not{p}_1+\not{p}_2)(p_1+p_2)^\mu T^a\ , \tag{30}$$

$$G(p_2,p_1)=\frac{\Sigma(p_2)-\Sigma(p_1)}{p_2^2-p_1^2}\ ,$$
$$F(p_2,p_1)=\frac{Z(p_2)-Z(p_1)}{p_2^2-p_1^2}\ , \tag{31}$$

$$\begin{aligned}\Lambda^{\mu\nu ab}_{\hat\Gamma\hat\Gamma}(p_1,q_1,q_2,p_2=p_1+q_1+q_2)\\ &=G(p_2,p_1)g^{\mu\nu}T^bT^a+\left[\frac{G(p_2,p_1)-G(p_1+q_1,p_1)}{(p_2+p_1+q_1)\cdot q_2}\right](2p_1+q_1)^\mu(p_2+p_1+q_1)^\nu T^bT^a+[(q_1,\mu,a)\Longrightarrow(q_2,\nu,b)]\ ,\end{aligned} \tag{32}$$

$$\begin{aligned}\Lambda^{\mu\nu ab}_{\tilde\Gamma\tilde\Gamma}(p_1,q_1,q_2,p_2=p_1+q_1+q_2)\\ &=-\tfrac{1}{2}F(p_2,p_1)(\not p_1+\not p_2)g^{\mu\nu}T^bT^a\\ &\quad-\frac{1}{2}\left[\frac{F(p_2,p_1)-F(p_1+q_1,p_1)}{(p_2+p_1+q_1)\cdot q_2}\right](\not p_1+\not p_2)(2p_1+q_1)^\mu(p_2+p_1+q_1)^\nu T^bT^a+[(q_1,\mu,a)\Longrightarrow(q_2,\nu,b)]\ ,\end{aligned} \tag{33}$$

$$\Lambda_{\hat\Gamma\tilde\Gamma}(p_1,q_1,q_2,p_2=p_1+q_1+q_2)=-\tfrac{1}{2}F(p_2,p_1+q_1)\gamma^\mu(p_2+p_1+q_1)^\nu T^bT^a-\tfrac{1}{2}F(p_1+q_2,p_1)\gamma^\mu(2p_1+q_2)^\nu T^aT^b\ . \tag{34}$$

In the case of $\Lambda_{\hat\Gamma\tilde\Gamma}$, the arguments/indices (q_1,μ,a) are associated with the $\hat\Gamma$ field, etc. Vertices involving V_μ and/or A_μ only are completely determined by these vertices. For example,

$$\Lambda^{\mu a}_{\Lambda}=\Lambda^{\mu a}_{\Gamma}+\Lambda^{\mu a}_{\hat\Gamma}+\Lambda^{\mu a}_{\tilde\Gamma}\ , \tag{35}$$

$$\Lambda^{\mu a}_{A}=g_A\Lambda^{\mu a}_{\hat\Gamma}\gamma_5+\tilde g_A\Lambda^{\mu a}_{\tilde\Gamma}\gamma_5\ , \tag{36}$$

$$\Lambda^{\mu\nu ab}_{VV}(p_1,q_1,q_2,p_2)=\Lambda^{\mu\nu ab}_{\hat\Gamma\hat\Gamma}(p_1,q_1,q_2,p_2)+\Lambda^{\mu\nu ab}_{\tilde\Gamma\tilde\Gamma}(p_1,q_1,q_2,p_2)+\Lambda^{\mu\nu ab}_{\hat\Gamma\tilde\Gamma}(p_1,q_1,q_2,p_2)+\Lambda^{\nu\mu ba}_{\hat\Gamma\tilde\Gamma}(p_1,q_2,q_1,p_2)\ . \tag{37}$$

The vertices involving π's also receive contributions from the $\xi(x)$ fields appearing in the quark mass term in $A_{\rm GNC}$. For example, we find

$$\begin{aligned}&\Lambda^{a}_{\pi}(p_1,k,p_2=p_1+k)=-\frac{i\gamma_5}{f_\pi}[\Sigma(p_1)+\Sigma(p_2)]T^a\ ,\\ &\Lambda^{ab}_{\pi\pi}(p_1,k_1,k_2,p_2=p_1+k_1+k_2)=-\frac{1}{2f_\pi^2}[\Sigma(p_1)+\Sigma(p_2)+2\Sigma(p+k_1)]T^bT^a-\frac{1}{2f_\pi^2}G(p_2,p_1)(p_1+p_2)\cdot(k_2-k_1)T^bT^a\\ &\qquad+[(k_1,a)\Longrightarrow(k_2,b)]\ ,\\ &\Lambda^{\mu ab}_{V\pi}(p_1,q,k,p_2=p_1+q+k)=-\frac{i\gamma_5}{f_\pi}[T^b\Lambda^{\mu a}_{\tilde\Gamma}(p_1,q,p_1+q)+\Lambda^{\mu a}_{\tilde\Gamma}(p_1,q+k,p_2)T^b]\ ,\end{aligned} \tag{38}$$

for $g_A=1$ and $\tilde g_A=1$. The additional terms proportional to g_A-1 and $\tilde g_A-1$ may be easily found from the $\hat\Gamma$ and $\tilde\Gamma$ vertices.

All these vertices are free from kinematic singularities, and for precisely this reason the $\Lambda^{\mu a}_V$ vertex was advocated long ago by Ball and Chiu [7]. For example, $\Lambda^{\mu a}_V$ is nonsingular when $p_2=p_1$ and satisfies the original Ward identity in this limit.

Since the model has the same symmetry structure and quark propagator as QCD, the WT identities derived from the model will be identical in content to the WT identities of QCD. (Since the PGB appears explicitly in the model, there will be additional terms in the WT identities for one-particle-irreducible vertex functions which correspond to the PGB pole pieces of QCD vertex functions.) And in particular these identities will be satisfied in the model at zeroth loop order. In other words, the set of tree-order vertices from the model, involving any number of external gauge fields coupling to quarks, will saturate the full set of QCD WT identities.

The full vertex functions of QCD may be written as a sum of our singularity-free longitudinal vertex functions plus a remainder which includes transverse vertices. But we will find that our particular set of vertex functions by themselves account surprisingly well for certain characteristics of low-energy QCD.

Thus far we have a model with suitable high-energy behavior for the quark propagator; a reasonable PGB propagator imposes additional constraints. We consider contributions to $f_\pi^2\Delta^{-1}(p)$ from the diagrams in Fig. 1 and find that there are cutoff-dependent contributions in the present model. The first diagram has a Λ^2 dependence proportional to $(1-g_A)^2$ and independent of $Z(p)$. And both diagrams have cutoff-dependent terms proportional to $(1-\tilde g_A)^2$ and $(1-g_A)(1-\tilde g_A)$ involving derivatives of $Z(p)$. These also give Λ^2 dependence if $Z(p)$ behaves like a power of $\ln(p^2)$ at large p^2. Clearly, it is difficult to obtain a reasonable PGB propagator unless $g_A=\tilde g_A=1$.

Thus, for the final form of the model, we choose $g_A=\tilde g_A=1$. Phenomenologically, $g_A=1$ is not a bad

FIG. 1. Diagrams contributing to the PGB propagator and f_π. The vertices are momentum dependent.

choice for quark models, and there has been a recent theoretical argument [8] supporting the notion that constituent quarks should behave like bare Dirac fermions with $g_A=1$.

With this choice all the PGB couplings arise in the mass term depending on $\Sigma(x-y)$, and the high-energy behavior of $\Sigma(p)$ provides more suitable form-factor suppression of PGB vertices. The diagrams in Fig. 1 are now convergent, and the high- and low-energy behavior for $\Delta^{-1}(p)/p^2$ is given in (11) with

$$c_\Delta=\frac{N_c}{(2\pi)^2 f_\pi^2}\int dq^2\, q^2 \frac{\Sigma(q)^2}{Z(q)^2q^2+\Sigma(q)^2} \; . \tag{39}$$

The second diagram is responsible for the high-energy behavior in (11).

We have arrived at a minimal model with a few appealing high-energy properties; the quark propagator is QCD like, and the PGB propagator is at least reminiscent of a physical composite pion. In these respects the model clearly improves upon *local* constituent-quark models. As stated in the Introduction, we wish to model QCD in the intermediate energy regime, and thus it is appropriate to consider the model as an effective theory below some cutoff. The main point is that in comparison to local models, the model we have chosen with $g_A=\tilde{g}_A=1$ displays much less cutoff dependence. This is traced to the physically motivated momentum dependence of $\Sigma(p)$.

III. PARAMETERS IN THE CHIRAL LAGRANGIAN

We return to low energies and obtain the chiral Lagrangian $\mathcal{L}_{\rm eff}$ from (10). As a first application, we may calculate f_π by taking the small-momentum limit of the diagrams in Fig. 1. From the normalization $\Delta^{-1}(p)/p^2=1$ at small p^2, we obtain the following convergent expression:

$$f_\pi^2=\frac{N_c}{8\pi^2}\int dq^2\, q^2 \frac{([\Sigma\Sigma''-\Sigma'^2]q^2+2\Sigma\Sigma')\Sigma^2+Z^2([\Sigma'^2+\Sigma\Sigma'']q^4+2\Sigma^2)+2ZZ'q^2(\Sigma-2q^2\Sigma')\Sigma+2Z'^2q^4\Sigma^2}{(Z^2q^2+\Sigma^2)^2} \; . \tag{40}$$

This generalizes the following Pagels-Stokar formula [9]. When $Z(q)=1$, (40) is equivalent to, up to a vanishing total derivative,

$$f_\pi^2=\frac{N_c}{4\pi^2}\int dq^2\, q^2 \Sigma \left[\frac{\Sigma-\frac{1}{2}q^2\Sigma'}{(q^2+\Sigma^2)^2} \right] . \tag{41}$$

Of course, our knowledge of $\Sigma(q)$ and $Z(q)$ in QCD is limited, and it is usual to rely on the Schwinger-Dyson (SD) equation in improved ladder approximation to develop some feeling for $\Sigma(q)$ and $Z(q)$. It is common in this approximation, by choice of the Landau gauge, to set $Z(q)=1$. [In a different gauge, $\Sigma(q)$ and $Z(q)$ would presumably change in such a way as to keep the physics the same.] We adopt $Z(q)=1$ in the following.

We turn our attention to the standard coefficients L_1–L_{10} of ten terms in the chiral Lagrangian at order p^4. (See [5] for details.) In the model these L_i's are given by very lengthy integral expressions involving $\Sigma(q)$ and its derivatives. The important point is that they are all convergent, and we evaluate them without a cutoff. We will then compare these results to the renormalized values of L_i's in the QCD chiral Lagrangian. For the renormalization scale, we find it appropriate to choose $\mu=2m$, where m is the constituent mass.

As noted before [2,4], the Wess-Zumino terms are also directly determined and the coefficients turn out to be independent of $\Sigma(q)$ as required. But for the L_i's we must parametrize our ignorance of $\Sigma(q)$; as in [4], we do this in terms of one parameter A,

$$\Sigma(q)=\frac{(A+1)m^3}{q^2+Am^2} \; . \tag{42}$$

The normalization is such that $\Sigma(m)=m$. This simple function shares qualitative similarities with solutions of the improved ladder SD equation, those solutions being finite, positive, monotonically decreasing functions with $1/q^2$ behavior at large q^2 [ignoring the power of $\ln(q^2)$] and $\Sigma'(0)<0$.

We follow the procedure in [4]. We calculate the coefficients of the general set of $O(p^4)$ terms and then use lowest-order equations of motion to eliminate some terms in favor of the ten L_i's. The equations of motion introduce a nonzero L_7, but

$$L_1-\tfrac{1}{2}L_2=L_4=L_6=0 \; , \tag{43}$$

consistent with Zweig's rule [5]. Numerically, some of our results will be the same as found in [4] since the $\Sigma(q)$ dependence of the vertices in the two models is the same up to the first derivative $\Sigma'(q)$.

In particular, all results related to the current quark mass matrix M are the same. L_4–L_8 multiply $O(p^4)$ terms involving M. For the coefficient of the $O(p^2)$ M-dependent term, the model generates the usual $M\langle\bar{\psi}\psi\rangle$ combination with $\langle\bar{\psi}\psi\rangle$ as defined in (25). But $\langle\bar{\psi}\psi\rangle$ is sensitive to the high-energy behavior of $\Sigma(p)$, unlike the L_i's. Thus we replace $\langle\bar{\psi}\psi\rangle$ by a parameter $\langle\bar{\psi}\psi\rangle_\mu$, and we use this quantity to represent the physical condensate renormalized at scale μ. We also consider M to be renor-

TABLE I. Results for the coefficients $L_1, L_2, L_3, L_9, L_{10}$ appearing in the chiral Lagrangian at $O(p^4)$ in the chiral-symmetry limit are compared with the experimental values (first column). The model yields $L_1 = L_2/2$. The parameter A determines the form of $\Sigma(p)$ in Eq. (42). L_2, L_9, L_{10} are renormalization-scale dependent, and we give their values renormalized at m_η. We also give their raw values, corresponding to a renormalization scale at the matching scale.

$\times 10^3$	Expt	$A=1$	$A=2$	$A=3$	$A=4$
L_3	-3.6 ± 1.3	-6.15	-5.13	-4.70	-4.45
$L_2(m_\eta)$	1.6 ± 0.4	2.58	2.17	1.98	1.86
$L_9(m_\eta)$	7.4 ± 0.7	8.88	7.57	7.02	6.71
$L_{10}(m_\eta)$	-6.0 ± 0.7	-7.10	-5.72	-5.10	-4.73
L_2^0		2.31	2.00	1.88	1.81
L_9^0		8.54	7.34	6.88	6.65
L_{10}^0		-6.76	-5.50	-4.96	-4.66

malized at scale μ. $\langle \bar{\psi}\psi \rangle_\mu$ along with the three quark masses in M and the constituent-quark mass m makes a total of five parameters They are determined along with $L_4 - L_8$ by fitting to five physical quantities; the resulting values of all these quantities are found in the erratum to [4].

Since $L_1 = \frac{1}{2} L_2$, there are four independent numbers to be calculated in the chiral-symmetry limit: L_2, L_3, L_9, L_{10}. They are independent of the five parameters; in particular, they are independent of m since the L_i's are dimensionless. To calculate them we need vertices in addition to those listed above; e.g., for L_1, L_2, and L_3, we also need $\Lambda_{\pi\pi\pi}$ and $\Lambda_{\pi\pi\pi\pi}$, and for L_9 we also need $\Lambda_{V\pi\pi}$. L_{10} may be extracted from the p^2 term of the VV-AA two-point function. (The formula for f_π^2 may be extracted from the p^0 term.)

We are associating these L_i's to physical L_i's renormalized at $\mu = 2m$. Values of L_i's quoted in the literature are renormalized at some other scale, in particular $\mu = m_\eta$, and to make the comparison we must run our L_i's appropriately [5]. We determine m for each choice of A from the Pagels-Stokar formula (41) for f_π. For that formula we must take a value of f_π corresponding to the chiral-symmetry limit; we take $f_\pi = 84$ MeV.

Our results for L_2, L_3, L_9, L_{10} renormalized at $\mu = m_\eta$ are presented in Table I and compared to the experimental values [5,10]. L_3 does not receive a correction from running; and the uncorrected values L_2^0, L_9^0, L_{10}^0 are also given. L_{10} is the same as in [4], but L_1, L_2, L_3, and L_9 are different.

We see that the $A = 2$ or 3 results compare quite favorably with the data. And it is of interest that a model based on low-lying QCD resonances also gives values for L_2 and L_3 ($L_2 = 2.2$, $L_3 = -5.5$ in [11]), which are somewhat larger in absolute value than the present data. We comment more on the meaning of these results in the conclusions.

IV. RELATION TO QCD USING THE AUXILIARY-FIELD METHOD

In this section we discuss the model more in the spirit of other attempts in the literature to "derive" a nonlocal quark model from QCD [2,3]. We find such arguments to be rather crude, but the simplified picture of QCD dynamics they present may at times be illuminating. We will concentrate on the chiral-symmetry-breaking dynamics responsible for the generation of $\Sigma(p)$ and its possible effect, if any, on the above results.

We will make use of the nonlocal auxiliary-field method [12–15]. Let us ignore the cubic and quartic gluon self-couplings in QCD (we will reintroduce some of their effects in a moment), thus allowing us to integrate the gluons out of the theory. We would obtain the following action, with the external-field (V, A, S, P) terms implicitly present:

$$\int d^4x\, \bar{q}(x) \partial\!\!\!/ q(x) + \frac{1}{2} \int d^4x\, d^4y\, \bar{q}(x) \lambda^a \gamma^\mu q(x) \times g^2 D_{\mu\nu}^{ab}(x-y) \bar{q}(y) \lambda^b \gamma^\nu q(y) \ . \tag{44}$$

But instead of using the bare gluon propagator which would appear here, let us use instead [2]

$$g^2 D_{\mu\nu}^{ab}(x) = \delta^{ab} \int \frac{d^4k}{(2\pi)^4} \left[\frac{k^2 \delta_{\mu\nu} - k_\mu k_\nu}{k^4} F_T(k^2) + \frac{k_\mu k_\nu}{k^4} F_L(k^2) \right] e^{ik\cdot x} \ . \tag{45}$$

In this way the effects of a running gauge coupling, including those contributions to the β function from gluon self-couplings previously omitted, can be introduced through the functions $F_T(k^2)$ and $F_L(k^2)$.

We Fierz transform the four-fermion interaction

$$\frac{1}{2} \int d^4x\, d^4y\, q(y)_{j,f} \bar{q}(x)_{i,c} K(x-y) q(x)_{i,d} \bar{q}(y)_{j,e} \lambda_{cd}^a \lambda_{ef}^a \ , \tag{46}$$

here (i,j) and (c,d,e,f) are flavor and color indices and K is a nontrivial matrix in Lorentz spinor space. A Gaussian term of the following form involving a bilocal auxiliary field $\chi(x,y)$ may be added to the action to cancel the four-fermion term:

$$-\frac{1}{2} \mathrm{Tr} \left[\left[\frac{\chi}{C_2} - K q\bar{q} \right] \lambda^a K^{-1} \left[\frac{\chi}{C_2} - K q\bar{q} \right] \lambda^a \right] \ . \tag{47}$$

C_2 is the color Casimir invariant. The resulting action takes the form

$$-\frac{1}{2C_2^2}\,\mathrm{Tr}(\chi\lambda^a K^{-1}\chi\lambda^a)+\bar{q}\left[\partial\!\!\!/+\frac{\lambda^a\chi\lambda^a}{C_2}\right]q$$
$$+V,A,S,P \text{ terms} . \qquad (48)$$

Now the quarks may be integrated out, turning the second term and external-field terms into a Tr ln term. But an intractable integration over $\chi(x,y)$ remains.

It is popular at this stage to consider the tree approximation and to study the tree-level action $A^{\text{tree}}[\chi;V,A,S,P]$. It is found that the stationary condition for $A^{\text{tree}}[\chi;0,0,0,0]$ with respect to χ reproduces the SD equation in ladder approximation, which is "improved" since the bare gluon propagator is replaced with (45). Let the solution of the SD equation be given by $\Sigma(p)$ and $Z(p)=1$ (in Landau gauge); then the vacuum value of χ is

$$\langle\chi(x,y)_{ij,cd,\alpha\beta}\rangle_{V,A,S,P=0}=\delta_{ij}\delta_{cd}\delta_{\alpha\beta}\Sigma(x-y) . \qquad (49)$$

Diagonalizing the classical quadratic fluctuations around this stationary point corresponds to solving the ladder Bethe-Salpeter equations [13,15].

Quantum corrections beyond the tree approximation will effectively reintroduce QCD corrections. But this is not a simple perturbative expansion in the QCD coupling; the new corrections are those not already included in the sum of ladder graphs inherent in the solution of the ladder SD equation. Thus the importance of the quantum corrections is related to the reliability of the ladder approximation.

The above picture is based on $A^{\text{tree}}[\chi;0,0,0,0]$, and we must try to extend it in two ways. First, we require nonvanishing external gauge fields since we seek the generating functional $\Gamma(V,A,S,P)$. And second we want to retain the PGB degrees of freedom in χ as dynamical fields in the low-energy theory.

Since we are interested in the low-energy theory we may consider the external fields to be small in magnitude and slowly varying compared to the mass scale of the heavy degrees of freedom in χ. We are then led to the following construction. We will set the heavy degrees of freedom in χ in the presence of the external fields to their vacuum values, as determined in the ladder approximation with *vanishing* external fields. The trick is that this must be done in a way consistent with the local chiral symmetry, thus ensuring that this symmetry remains in the low-energy theory.

We accomplish this by imposing a constraint on $\chi(x,y)$, which is chirally invariant. This will constrain $\chi(x,y)$ to depend only on π, V_μ, A_μ in such a way that it still transforms properly under local chiral transformations. Basically, we are generalizing the constraint $U^\dagger U=1$ in the local nonlinear σ model. The constrained $\chi(x,y)$ is proportional to δ_{cd} in color space; with this in mind, the color indices are implicit below. We consider the following constraints, where tr denotes a trace only over the Lorentz indices:

$$\mathrm{tr}[\gamma_\mu\chi(x,y)]=0 , \qquad (50)$$

$$\mathrm{tr}[\gamma_\mu\gamma_5\chi(x,y)]=0 , \qquad (51)$$

$$\mathrm{tr}[\sigma_{\mu\nu}\chi(x,y)]=0 , \qquad (52)$$

$$\mathrm{tr}[\chi(x,y)_{ik}]\,\mathrm{tr}[\chi(y,x)_{kj}]+\mathrm{tr}[i\gamma_5\chi(x,y)_{ik}]\,\mathrm{tr}[i\gamma_5(y,x)_{kj}]$$
$$=16\delta_{ij}\Sigma(x-y)^2 . \qquad (53)$$

Equation (53) can also be written as

$$\mathrm{tr}[\chi(x,y)_{ik}|_{\gamma_5\to-\gamma_5}\chi(y,x)_{kj}]=4\delta_{ij}\Sigma(x-y)^2 . \qquad (54)$$

We now note that a solution to these constraints is

$$\chi(x,y)_{ij}^{\text{constrained}}=\Sigma(x-y)[\xi(x)X(x,y)\xi(y)]_{ij} , \qquad (55)$$

where $X(x,y)$ is the path-ordered exponential defined in (15). To check, insert this into the LHS of (54), and dropping $\Sigma(x-y)^2$, we have

$$\mathrm{tr}[\xi(x)^\dagger X(x,y)\xi^\dagger(y)\xi(y)X(y,x)\xi(x)]_{ij}$$
$$=\mathrm{tr}[\xi(x)^\dagger X(y,x)^\dagger X(y,x)\xi(x)]_{ij}=4\delta_{ij} . \qquad (56)$$

We have used the fact that Γ_μ appearing in the definition of $X(x,y)$ is even in γ_5 and Hermitian. And $X(x,y)$ considered as a matrix in flavor/Lorentz space is unitary.

We now insert $\chi^{\text{constrained}}$ into $A^{\text{tree}}[\chi;VA,S,P]$. We first put the K^{-1} term in a more explicit form. From the QED result in [14] and by using the constraints (50)–(52), we find

$$-\frac{1}{2C_2^2}\,\mathrm{Tr}(\chi\lambda^a K^{-1}\chi\lambda^a)$$
$$=-\frac{N_c}{2}\int d^4x\, d^4y\{\sigma_{ij}(x,y)[g^2D(x-y)]^{-1}\sigma_{ji}(y,x)$$
$$+\Pi_{ij}(x,y)[g^2D(x-y)]^{-1}$$
$$\times\Pi_{ji}(y,x)\} , \qquad (57)$$

where

$$\sigma=\frac{\mathrm{tr}(\chi)}{4},\quad \Pi=\frac{\mathrm{tr}(i\gamma_5\chi)}{4} , \qquad (58)$$

and

$$g^2D(x)=\frac{C_2}{4}\int\frac{d^4k}{(2\pi)^4}\,\frac{3F_T(k^2)+F_L(k^2)}{k^2}e^{ik\cdot x} . \qquad (59)$$

The constraint (53) then implies that

$$-\frac{1}{2C_2^2}\,\mathrm{Tr}(\chi\lambda^a K^{-1}\chi\lambda^a)$$
$$=-\frac{N_cN_f}{2}\int d^4x\, d^4y\;\Sigma(x-y)^2[g^2D(x-y)]^{-1} . \qquad (60)$$

This is a contribution to an "effective potential" times the usual infinite volume factor.

The main point is that it is independent of PGB's, and it thus makes no contribution to the chiral Lagrangian. The chiral Lagrangian is therefore completely due to the Tr ln term in $A^{\text{tree}}[\chi;V,A,A,P]$. Upon substituting $\chi^{\text{constrained}}$ into this term, we recover our model A_{GNC} described above with $Z(q)=1$. We note the emergence of $g_A=1$ from this new perspective.

This truncation of QCD leading to our model is interesting for two reasons. One is that the quantity $\Sigma(x-y)$ is now determined dynamically; it is no longer an arbitrary quantity as when we introduced A_{GNC}. The essential dynamics is introduced by the K^{-1} term. But with the heavy degrees of freedom in χ constrained in the manner above, we find that this term does not contribute to the chiral Lagrangian. This suggests that QCD dynamics may influence the low-energy chiral Lagrangian mainly via the form of $\Sigma(x-y)$, as determined by the Schwinger-Dyson equation.

The other point of interest is that our truncation of the theory is related to (but not identical to) the tree approximation, since the heavy degrees of freedom in χ are set to the vacuum values determined in tree approximation. But the tree approximation is equivalent to the improved ladder approximation. Our approach may therefore be more reliable in those cases where the ladder approximation is successful. This may be the case for example in gauge theories having small β functions [16].

V. CONCLUSIONS

The motivation for this study should be made clear. *A priori,* our model has nothing to do with QCD. In fact, it is quite clear that a compelling derivation directly from QCD, of any such model, does not presently exist. What we have done is to construct a theory of fermions with momentum-dependent mass and to determine a minimal set of nonlocal and nonlinear couplings to Goldstone bosons consistent with local $SU(3)\times SU(3)$ chiral symmetry. As such the model shares a few key features with the theory of QCD in the presence of external chiral gauge fields.

We have compared the low-energy effective theory of our minimal model to that of QCD in the chiral-symmetry limit. We are intrigued that with a simple choice of the dynamical mass function $\Sigma(p)$, rather remarkable agreement exists for a number of dimensionless constants. The model is also amenable to the study of explicit chiral-symmetry-breaking effects. For example, explicit quark masses are naturally introduced since the momentum-dependent constituent mass is clearly distinguished from the essentially momentum-independent current quark mass. The values of m_u, m_d, and m_s may then be extracted if the model is to reproduce the physical decay constants and masses of kaons and pions. Using the same $\Sigma(p)$ as before, the quark masses obtained agree quite well with accepted values [4].

All this demonstrates the following fact, which has yet to be fully explained. Chiral dynamics, at least to $O(p^4)$ in the energy expansion, is well described by a loop of free, or weakly interacting, constituent quarks with a momentum-dependent mass.

We are induced to consider the other source of explicit chiral-symmetry breaking: that due to the weak interactions. Again, the standard local four-quark terms may be directly introduced into the model. The result in the effective theory is a large set of new terms at $O(p^4)$ in the energy expansion. Work is underway to extract their coefficients, both to compare with those already known and to estimate those not yet determined.

Another application of the model is in the extrapolation of results from QCD to other gauge theories, most notably in the technicolor context. The formula of Pagels and Stokar [9] expressing f_π in terms of $\Sigma(p)$ has proven useful in this connection, in particular in studies of theories which produce a different behavior for $\Sigma(p)$, such as walking technicolor. Our model expresses other low-energy parameters, as well as f_π, in terms of $\Sigma(p)$. As an application in the context of technicolor-induced electroweak corrections, the analogue of L_{10} plays the role of the S parameter [17,18]. Our model was used in [17] to show that S from a walking technicolor theory is expected to be smaller than the S from a QCD-like technicolor theory. This tendency has also been noted in a quite different analysis [19].

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[1] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); D. Espriu, E. de Rafael, and J. Taron, *ibid.* **B345**, 22 (1990), and references therein.
[2] C. D. Roberts, R. T. Cahill, and J. Praschifka, Ann. Phys. (N.Y.) **138**, 20 (1988), and references therein.
[3] D. W. McKay, H. Munczek, and B. L. Young, Phys. Rev. D **37**, 195 (1988); H. Munczek and D. W. McKay, *ibid.* **39**, 888 (1989); R. D. Ball, Mod. Phys. A **5**, 4391 (1990).
[4] B. Holdom, J. Terning, and K. Verbeek, Phys. Lett. B **245**, 612 (1990); **273**, 549(E) (1991).
[5] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
[6] J. Terning, Phys. Rev. D **44**, 887 (1991); Ph.D. thesis, University of Toronto, 1990.
[7] J. S. Ball and T.-W. Chiu, Phys. Rev. D **22**, 2542 (1980).
[8] S. Weinberg, Phys. Rev. Lett. **65**, 2542 (1980).
[9] H. Pagels and S. Stokar, Phys. Rev. D **20**, 2947 (1979).
[10] C. Riggenbach, J. Gasser, J. F. Donoghue, and B. R. Holstein, Phys. Rev. D **43**, 127 (1991).
[11] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, Phys. Lett. B **223**, 425 (1989).
[12] H. Kleinert, Phys. Lett. **62B**, 429 (1976).
[13] T. Kugo, Phys. Lett. **76B**, 625 (1978).
[14] T. Morozumi and H. So, Prog. Theor. Phys. **77**, 1434 (1987).
[15] K.-I. Aoki, M. Bando, T. Kugo, M. G. Mitchard, and H. Nakatani, Prog. Theor. Phys. **84**, 683 (1990).
[16] T. Appelquist, K. Lane, and U. Mahanta, Phys. Rev. Lett. **61**, 1553 (1988); B. Holdom, *ibid.* **62**, 997 (1989); U. Mahanta, *ibid.* **62**, 2349 (1989).
[17] B. Holdom and J. Terning, Phys. Lett. B **247**, 88 (1990).
[18] M. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990).
[19] R. Sundrum and S. Hsu, Lawrence Berkeley Laboratory Report No. LBL-31066, 1991 (unpublished).