PHYSICAL REVIEW D VOLUME 45, NUMBER 1 1 JANUARY 1992

Lattice QCD evaluation of baryon magnetic-moment sum rules

Derek B. Leinweber*
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia Canada V6T 2A3
(Received 17 May 1991)

Magnetic-moment combinations and sum rules are evaluated using recent results for the magnetic moments of octet baryons determined in a numerical simulation of quenched QCD. The model-independent results of the lattice calculations remove some of the confusion and contradiction surrounding past magnetic-moment sum-rule analyses. The lattice results reveal the underlying quark dynamics investigated by magnetic-moment sum rules and indicate the origin of magnetic-moment quenching for the nonstrange quarks in Σ. In contrast with previous sum-rule analyses, the lattice results indicate the magnetic moments of nonstrange quarks in Ξ are more likely enhanced than quenched relative to that in the nucleon. In most cases, the spin-dependent dynamics and center-of-mass effects giving rise to baryon dependence of the quark moments are seen to be sufficient to violate the sum rules in agreement with experimental measurements. In turn, the sum rules are used to further examine the results of the lattice simulation. The Sachs sum rule suggests that quark loop contributions, not included in present lattice calculations, may play a key role in removing the discrepancies between lattice and experimental ratios of magnetic moments. This is supported by other sum rules sensitive to quark loop contributions. A measure of the isospin symmetry breaking in the effective quark moments due to quark loop contributions is in agreement with model expectations.

PACS number(s): 13.40.Fn, 12.38Gc, 14.20.−c

I. INTRODUCTION

Magnetic-moment sum rules of octet baryons have been a useful tool in the investigation of various quark-model assumptions. While the simple quark model can qualitatively reproduce the experimental magnetic moments, it is possible to write a sum rule [1] based on SU(6) spin-flavor symmetry, broken only by the quark masses, that fails by a factor of 5. Sum-rule analyses lead to the general conclusion that a good understanding of baryon magnetic moments requires a model with large baryon-dependent nonstatic quark contributions [2].

More detailed conclusions were also drawn from sum-rule analyses. Many sum rules imply some quenching or reduction of the magnetic-moment contributions of nonstrange quarks in hyperons relative to the nucleon [1, 3]. However, it is possible to refute this conclusion with other sum rules [2]. It was also suggested that the strange quark does not contribute to the magnetic moments of Σ hyperons, while the strange quarks in Ξ were thought to give contributions considerably larger than that in Λ [3].

Of course, many mechanisms have been proposed to account for these deviations from broken SU(6) symmetry. Early calculations sought configuration mixing to quench the non-strange-quark magnetic-moment contributions in hyperons with little success [3]. One-gluon-exchange corrections in the cloudy bag model proved to be vital in reproducing the correct order of the Ξ^- and Λ magnetic moments [4]. It now appears that the experimental moments may be reproduced fairly well in an additive quark model with pion and gluon interactions between the quarks and isospin symmetry breaking giving rise to anomalous u- and d-quark magnetic moments [5, 6].

Recently the electromagnetic properties of octet baryons have been investigated in a numerical simulation of quenched QCD [7]. While the magnitudes of the magnetic moments are underestimated in comparison with the experimental moments, ratios of the magnetic moments and the lattice proton moment reproduce the experimental ratios reasonably well. The analysis has revealed a richly detailed structure in the quark magnetic-moment contributions to baryons. Evidence of relativistic motion, nonperturbative gluon dynamics, and center-of-mass effects is seen in the lattice results.

In this paper, magnetic-moment sum rules are reviewed in light of the recent lattice QCD analysis of octet-baryon magnetic moments. Some conclusions of the lattice analysis contrast those drawn from previous sum-rule analyses while other conclusions are in agreement. In Sec. II the results of the lattice calculations of magnetic moments are briefly summarized and presented in the context of an additive quark model. Here, the origin of magnetic-moment quenching for the nonstrange quarks in Σ becomes clear. These results are then used in Sec. III to reveal the underlying dynamics investigated by the magnetic-moment sum rules and describe how the sum rules are violated. The model-independent results of the lattice simulation remove some of the confusion, contradiction, and mystery surrounding past sum-rule analyses. Perhaps the most interesting sum rules are those violated by the lattice results in contradiction to the experimental measurements. These sum rules are used to further examine the results of the lattice simulation and obtain some indication of the nature of the missing dynamics. Finally the results and implications of this analysis are summarized in Sec. IV.

II. LATTICE SIMULATION SUMMARY

Investigations of the magnetic properties of octet baryons in a lattice simulation of quenched QCD [7] have revealed a richly detailed structure in the quark sector contributions to baryon magnetic moments. In this section, the evidence of relativistic motion, nonperturbative gluon dynamics and center-of-mass effects is briefly reviewed. These results are summarized in the context of an additive quark model.

The lattice calculations [7] and experimental measurements [8] of the octet baryon magnetic moments are summarized in Table I. The lattice results are obtained in a numerical simulation of quenched QCD on a $24 \times 12 \times 12 \times 24$ lattice at $\beta = 5.9$ using Wilson fermions. Twenty-eight quenched gauge configurations are used in the analysis. Statistical uncertainties in the lattice results are determined using a single elimination jackknife [9]. These uncertainties are indicated in parentheses describing the uncertainty in the last digit(s) of the results. While the order and signs of the magnetic moments are correctly reproduced, the lattice results underestimate the magnitudes of the baryon magnetic moments.

It is difficult to isolate the origin of this discrepancy. At $\beta = 5.9$ some deviations from asymptotic scaling are possible. There may be corrections to the linear extrapolation in $1/\kappa$ to the chiral limit. Finite-volume effects may also give rise to the underestimation of the magnetic moments as the baryon is restricted by its periodic images. A calculation of the proton rms electric charge radius indicates the proton largely fills the lattice in the smaller y and z spatial dimensions. Nonquenched corrections may also provide additional contributions. In quantifying this uncertainty, one would like to have some knowledge of the dependency of the magnetic moments on β, the lattice volume and κ particularly near the physical limit. Such information remains to be obtained in future lattice calculations. To reduce the effects of these uncertainties and allow a more detailed comparison of the experimental and lattice results, ratios of the lattice results and the lattice proton result are scaled to reproduce the experimental proton magnetic moment. These scaled ratios are also indicated in Table I.

To gain a deeper understanding of the quark dynamics, it is useful to consider the individual quark sector contributions to the magnetic moments. In the simple quark model, the magnetic moment of the proton is given by

TABLE I. Magnetic moments of octet baryons (μ_N).

	Lattice QCD results		
Baryon	Absolute	Scaled ratios	Experiment
p	2.26(27)	2.793	2.793
n	−1.29(21)	−1.59(21)	−1.913
Λ	−0.40(7)	−0.50(7)	−0.613(4)
Σ^+	1.91(23)	2.37(18)	2.419(22)
Σ^0	0.54(9)	0.65(6)	
Σ^-	−0.87(9)	−1.07(11)	−1.156(14)
Ξ^0	−0.95(8)	−1.17(10)	−1.253(14)
Ξ^-	−0.41(6)	−0.51(7)	−0.675(22)

$$p = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d, \tag{2.1}$$

where p indicates the magnetic moment of the proton. In the SU(2)-flavor limit where $\mu_u = -2\mu_d$, the ratio of the quark sector contributions in the simple quark model is $\frac{4}{3}\mu_u / -\frac{1}{3}\mu_d = 8$. In contrast, the lattice results indicate this ratio is 10.3(7) when determined using the heaviest quark masses considered. This ratio increases as the quarks become lighter. A ratio of 8 may be recovered for very heavy quark masses. These results give strong evidence of relativistic motion and nonperturbative gluon dynamics which are not accounted for in conventional quark models. The enhancement of the u-quark sector relative to the d-quark sector in the proton may be viewed as a spin-dependent effect as the lattice results indicate the doubly represented quarks are most often paired with their spins aligned.

Away from the SU(3)-flavor limit, one can search for quark mass effects such as a shifting of the center of mass towards the heavier strange quark(s) in hyperons. Of course, the strange-quark effective moment determined in the lattice simulation is smaller than the light quarks as expected. However there are more subtle effects seen in the effective magnetic moments of the nonstrange quarks.

For example, consider the u-quark contributions to the magnetic moments of $n(ddu)$ and $\Xi^0(ssu)$. In the SU(3)-flavor limit, the effective moments of the u quark in these two baryons are found to be the same as expected. However, with the light quarks extrapolated to the chiral limit, the u-quark contribution to the neutron magnetic moment is $-0.25(17)\ \mu_N$, while the u-quark contribution in Ξ^0 is $-0.36(5)\ \mu_N$. A calculation of the difference of these u-quark contributions indicates the magnitude of the u-sector contribution in Ξ^0 is most likely enhanced by an amount of $0.11^{+0.18}_{-0.11}\ \mu_N$. This effect, due to unequal s- and d-quark masses, contrasts the sum-rule conclusion that the magnetic moments of nonstrange quarks in hyperons are quenched.

A similar enhancement of the u-quark sector contribution to the magnetic form factor of Σ^+ is seen relative to that for p. However, the form factor is a dimensionless quantity and yields magnetic moments in units of natural magnetons, μ_B, where the mass of Σ appears in the definition of the magneton [10]. In converting from natural magnetons to nuclear magnetons the magnetic-moment contributions of the u sector in Σ^+ become quenched or reduced by a factor of M_N/M_Σ. Hence the origin of the quenching of nonstrange-quark magnetic moments in Σ is not from an exotic configuration mixing but rather from the simple realization that the baryon mass sets the scale for quark contributions to the magnetic moment. A calculation of the difference of the u-quark contributions in Σ and p indicates the u-sector contribution is larger in p by $0.14(7)\ \mu_N$.

In Ξ^0 the u-quark contribution to the magnetic form factor is much larger than that in the neutron. After unit conversion to nuclear magnetons the magnetic-moment contribution of the u sector in Ξ^0 remains larger than that in n. Similar results hold for d-quark contributions as the lattice results are obtained using isospin symmetry.

To summarize these results in an additive quark-model context the following scheme is proposed:

$$\begin{aligned}
p &= A\ \mu_u^N - B\ \mu_d^N, \\
n &= A\ \mu_d^N - B\ \mu_u^N, \\
\Sigma^+ &= A\ \mu_u^\Sigma - B\ \mu_s^\Sigma, \\
\Sigma^- &= A\ \mu_d^\Sigma - B\ \mu_s^\Sigma, \\
\Xi^0 &= A\ \mu_s^\Xi - B\ \mu_u^\Xi, \\
\Xi^- &= A\ \mu_s^\Xi - B\ \mu_d^\Xi, \\
\Sigma^0 &= A\ \frac{1}{2}\left(\mu_u^\Sigma + \mu_d^\Sigma\right) - B\ \mu_s^\Sigma, \\
\Lambda &= \frac{3}{4} A\ \mu_s^\Lambda + \delta,
\end{aligned} \tag{2.2}$$

with the isospin symmetry condition

$$\mu_u = -2\mu_d. \tag{2.3}$$

The baryon label is used to indicate the magnetic moment of the baryon. The factors A and B allow for some deviation from the simple quark model or SU(6)-spin-flavor symmetry values of 4/3 and 1/3, respectively. These factors describe the spin-dependent effects seen in the lattice results. The superscripts of the effective quark moments allow for some breaking of baryon independence in the quark magnetic-moment contributions. This effect may be associated with shifts in the system center of mass. The factor $(3A/4)$ in the definition for μ_Λ accounts for the similarity of the spin symmetry of the s quark in Λ and the doubly represented quarks in the outside members of the baryon octet. The term δ allows for u- and d-quark contributions to the Λ moment which may arise from symmetry breaking in the SU(6) wave functions. Lattice results indicate δ is small at $+0.04(4)$ μ_N, which contrasts a model estimate [11] of $-0.040(9)$ μ_N.

An estimate of the ratio B/A may be obtained using ratios of the u- and d-quark sector contributions to p or n and isospin symmetry. The lattice results indicate

$$\frac{B}{A} = 0.13^{+0.05}_{-0.11} < \frac{1}{4}, \tag{2.4}$$

where 1/4 is the SU(6) prediction. The effective magnetic moments of the u quark are ordered

$$\mu_u^\Sigma < \mu_u^N < \mu_u^\Xi, \tag{2.5}$$

and similarly for the magnitudes of d-quark effective moments. For the strange quarks

$$\mu_s^\Sigma \simeq \mu_s^\Lambda \simeq \mu_s^\Xi, \tag{2.6}$$

within statistical uncertainties. Finally, the property

$$A\mu_u^\Sigma > B\mu_u^\Xi, \tag{2.7}$$

indicates spin-dependent effects are more significant than the effects of baryon dependence of the quark moments.

III. MAGNETIC-MOMENT SUM RULES

A. Lattice evaluation

With the lattice results summarized in the form of an additive quark model, we may now use these results to reveal the quark dynamics investigated by the magnetic-moment sum rules and describe how the sum rules are violated. The manner in which each sum rule is violated is expressed by calculating the ratio of the left- and right-hand sides (LHS/RHS) of the sum rule. Experimental uncertainties are added in quadrature.

Let us begin with the sum rule that is violated by more than a factor of 5 using the present magnetic-moment measurements. It has the form [1]

$$\frac{3\,(p - \Sigma^+)}{\Xi^- - \Xi^0} = \frac{p + 3\Lambda}{p}. \tag{3.1}$$

The lattice violation of this sum rule corresponds to a ratio of 4.1 ± 1.5 and is in agreement with the experimental violation of 5.7(4). To reveal the underlying quark dynamics investigated by this sum rule we use the lattice summary of (2.2) and (2.3) to find

$$1 + 2\frac{\mu_s^\Sigma}{\mu_u^\Xi} + 2\frac{A}{B}\frac{\mu_u^N - \mu_u^\Sigma}{\mu_u^\Xi} > 1 + \frac{9}{4 + (2B/A)}\frac{\mu_s^\Lambda}{\mu_u^N} + \frac{6\delta}{(2A + B)\mu_u^N}. \tag{3.2}$$

It is clear that this sum rule is valid only with the SU(6)-symmetry conditions $A/B = 4$, $\delta = 0$ and baryon independence of the quark magnetic moments. The main source of violation in this sum rule is the third term of the LHS. Here contributions due to the reduction of μ_u^Σ relative to μ_u^N are enhanced by the large ratio A/B. Further violation of this sum rule comes from the second terms on both sides of the inequality which give negative contributions. The magnitude of the second term of the RHS is enhanced by both $\mu_u^N < \mu_u^\Xi$ and $B/A < 1/4$ relative to the LHS; however, this effect is offset somewhat by the term proportional to δ. It is not surprising that the extreme violation of this sum rule relies on both spin-dependent effects and baryon dependence of the quark magnetic moments.

There are many sum rules that rely on the full SU(6) symmetry of the wave functions. The sum rule

$$p + n = 3\Lambda + \frac{1}{2}\left(\Sigma^+ + \Sigma^-\right) - \left(\Xi^0 + \Xi^-\right) \tag{3.3}$$

was considered interesting since it was fairly accurately satisfied. The lattice ratio (LHS/RHS) is 1.45(27) in fair agreement with the experimental ratio of 1.22(5). Equations (2.2) and (2.3) indicate

$$(A - B)\,\mu_u^N > \frac{1}{2}A\mu_u^\Sigma + B\mu_u^\Xi + \frac{9}{2}A\mu_s^\Lambda - 4A\mu_s^\Xi - 2B\mu_s^\Sigma + 3\delta, \tag{3.4}$$

revealing the dominant source of violation is the spin-dependent effect giving rise to $B/A < 1/4$. Further violation is due to $\mu_u^N > \mu_u^\Sigma$. These effects also account for the inequality of the sum rule [12]

$$p - \frac{1}{2}n = 3\Lambda + \Sigma^+ - 2\Xi^0, \tag{3.5}$$

in which

$$\left(\frac{5}{4}A + B\right)\mu_u^N > A\mu_u^\Sigma + 2B\mu_u^\Xi + \frac{9}{4}A\mu_s^\Lambda - 2A\mu_s^\Xi - B\mu_s^\Sigma + 3\delta. \tag{3.6}$$

The lattice calculations indicate a ratio of 1.12(13) in agreement with the experimental ratio of 1.215(15).

With the breakdown of full SU(6)-spin-flavor symmetry firmly established, sum rules based on a more liberal parametrization of the baryon magnetic moments were investigated. Franklin [2] proposed a generalization of the SU(6) results to allow for nonstatic effects to the extent that the nonstatic components are the same for each baryon:

$$\begin{aligned}
p &= \frac{4}{3}\mu_u - \frac{1}{3}\mu_d', \\
n &= \frac{4}{3}\mu_d - \frac{1}{3}\mu_u', \\
\Sigma^+ &= \frac{4}{3}\mu_u - \frac{1}{3}\mu_s', \\
\Sigma^- &= \frac{4}{3}\mu_d - \frac{1}{3}\mu_s', \\
\Xi^0 &= \frac{4}{3}\mu_s - \frac{1}{3}\mu_u', \\
\Xi^- &= \frac{4}{3}\mu_s - \frac{1}{3}\mu_d', \\
\Sigma^0 &= \frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_s', \\
\Lambda &= \mu_s''.
\end{aligned} \tag{3.7}$$

This generalization is equivalent to (2.2) when baryon dependence of the quark moments is removed and the u-quark moment in the proton is defined by setting $A = 4/3$.

One of the more interesting sum rules consistent with this generalization is the Coleman-Glashow sum rule for magnetic moments:

$$p - n = \Sigma^+ - \Sigma^- + \Xi^- - \Xi^0. \tag{3.8}$$

This sum rule is a direct test of baryon independence of the quark moments. The experimental measurements violate this sum rule at a ratio of 1.133(10). Lattice results give a similar violation of 1.07(8). Equations (2.2) and (2.3) indicate this sum rule tests

$$(A + B)\,\mu_u^N = A\mu_u^\Sigma + B\mu_u^\Xi, \tag{3.9}$$

which is satisfied for arbitrary A and B, given the baryon independence of the quark moments. This sum rule led to the belief that both μ_u^Σ and μ_u^Ξ are quenched relative to μ_u^N. In contrast, the lattice results suggest two effects give rise to the inequality. The small value of the ratio B/A indicates that the dominant effect is $\mu_u^\Sigma < \mu_u^N$ and allows $\mu_u^\Xi > \mu_u^N$ without losing agreement with the experimental results.

Sum rules for the d-quark effective moment defined in Eqs. (3.7) also test baryon independence. Using isospin symmetry,

$$d = -\frac{1}{4}(2p + n) = \frac{1}{4}(\Sigma^- - \Sigma^+), \tag{3.10}$$

where d represents μ_d defined in (3.7). The experimental moments indicate $d = -0.918\ \mu_N < -0.894(7)\ \mu_N$. Lattice results suggest a larger violation of this sum rule where $d = -1.00(5)\ \mu_N < -0.86(6)\ \mu_N$. The dynamics probed by this sum rule are the quenching of the magnitudes of the nonstrange quarks in Σ:

$$d = \frac{3}{4}A\mu_d^N < \frac{3}{4}A\mu_d^\Sigma. \tag{3.11}$$

Similarly the d'-quark effective moment may be isolated:

$$d' = p + 2n = \Xi^0 - \Xi^-. \tag{3.12}$$

Experimental moments indicate $d' \equiv \mu_{d'} = -1.033\ \mu_N < -0.578(26)\ \mu_N$. Uncertainties in the lattice results are too large to comment on the violation of this sum rule. The d'-effective quark moment is found to be $d' = -0.40(42)\ \mu_N \simeq -0.66(12)\ \mu_N$. However, Eqs. (2.2), (2.3) and (2.5) suggest

$$d' = 3B\mu_d^N > 3B\mu_d^\Xi, \tag{3.13}$$

in contradiction to the experimental results. It should be noted that the experimental agreement of the lattice results is better for the RHS's of the previous two sum rules, (3.10) and (3.12), which involve a difference of hyperon moments. This suggests a problem with the addition of baryon moments and we will return to this sum rule at the end of this section.

While there are no sum rules for the strange quark in the generalization of SU(6) symmetry indicated in (3.7), it is possible, with isospin symmetry, to isolate the strange-quark moment in three different ways:

$$s = \frac{1}{4}(\Xi^0 + 2\Xi^-), \tag{3.14a}$$

$$s' = -\Sigma^+ - 2\Sigma^-, \tag{3.14b}$$

$$s'' = \Lambda - \delta. \tag{3.14c}$$

The results in units of μ_N for experimental and lattice baryon moments are

	s		s''		s'
Expt.	$-0.651(12)$	$<$	$-0.57(4)$	$<$	$-0.107(36)$
Lattice	$-0.55(4)$	$\simeq$	$-0.54(4)$	$<$	$-0.24(17)$

(3.15)

The lattice summary of (2.2) and (2.3) indicate the quark contributions for these results are

$$\frac{3}{4}A\mu_s^\Xi \simeq \frac{3}{4}A\mu_s^\Lambda < 3B\mu_s^\Sigma. \tag{3.16}$$

Lipkin's conclusion [3] that the strange quark does not appear to contribute to the magnetic moments of Σ is reflected here as a spin-dependent effect causing $B/A < 1/4$. On the other hand, his suggestion that the magnitude of s-quark contributions in Ξ are considerably larger than that in Λ is not reflected in either the experimental or lattice results. Equations (2.2) indicate the large magnitude of the Ξ^- magnetic moment is a consequence of the spin-dependent effect $B/A < 1/4$ which suppresses the reducing effect of the d-quark contribution to the magnitude of the Ξ^- magnetic moment. This spin-dependent effect may be associated with lattice gluon dynamics. It is interesting that the inclusion of gluon exchange in bag models is vital to reproducing the correct order of Ξ and Λ [4, 5].

Among the most mysterious of baryon moment combinations were those for the quark moment differences $s-d$ and $s'-d'$. In the simple quark model these differences are equal and have the value of approximately 0.36 μ_N. In contrast, the experimental moments indicate

$$\begin{aligned} s'-d' &= 3\left(p-\Sigma^+\right), \\ &= 1.12(7)\ \mu_N. \end{aligned} \tag{3.17a}$$

$$\begin{aligned} s-d &= \frac{3}{4}\left(\Xi^0-n\right), \\ &= 0.495(11)\ \mu_N. \end{aligned} \tag{3.17b}$$

The lattice moments give similar results of 1.3(5) μ_N and 0.32(17) μ_N, respectively. The extremely large result of (3.17a) was considerably difficult to reconcile with any simple model. We now know that (3.17a) is simply a poor way to measure the difference in s- and d-quark contributions. Using (2.2) and (2.3) we see

$$s'-d' = 3B\left(\mu_s^\Sigma-\mu_d^N\right)+3A\left(\mu_u^N-\mu_u^\Sigma\right), \tag{3.18}$$

which is dominated by contributions from the second term which were expected to vanish in the subtraction of Σ from p. In contrast,

$$s-d = \frac{3}{4}A\left(\mu_s^\Xi-\mu_d^N\right)+\frac{3}{4}B\left(\mu_u^N-\mu_u^\Xi\right). \tag{3.19}$$

This equation is dominated by the term we are trying to estimate. It is not surprising that we find a more reasonable result.

Perhaps the most interesting sum rule is the Sachs sum rule [13]

$$3\left(p+n\right) = \Sigma^+-\Sigma^-+\Xi^0-\Xi^-. \tag{3.20}$$

This sum rule is satisfied by the more general extension of SU(6) symmetry indicated in (3.7) and is therefore another test of baryon independence of the quark moments. The experimental moments indicate a ratio of 0.881(11) whereas the lattice moments yield an opposite violation of this sum rule with a ratio of 1.29(20). Equations (2.2) and (2.3) suggest the Sachs sum rule tests

$$\left(A-B\right)\mu_u^N = A\mu_u^\Sigma-B\mu_u^\Xi. \tag{3.21}$$

The experimental result cannot be reproduced without destroying the agreement with other sum rules or extensively contradicting the results of the lattice simulation summarized in (2.4) and (2.5).

A possible solution to this contradiction can be found by recognizing that the Sachs sum rule may be sensitive to dynamics not included in the lattice calculation. On the LHS of (3.20) the nucleon moments are added and enhanced by a factor of 3. In contrast, the RHS involves the *difference* of hyperon moments. Missing dynamics common to Σ or Ξ hyperons cancel on the RHS of (3.20) whereas dynamics common to the nucleons appear with a large factor of 6.

B. Quark loop contributions

In the lattice calculations of Ref. [7] the contributions of disconnected quark loops are not included. A skeleton diagram of such contributions is indicated in Fig. 1. These loop contributions are equal among octet baryons of a given strangeness when the u and d quarks are taken with the same mass. In this case an additional term μ_l^B may be added to the LHS of (3.20) accounting for quark loop effects not included in the lattice results. The superscript B allows for baryon dependence of the loop contributions. The experimental violation of (3.20) indicates

$$3\left(p+n\right)+6\mu_l^N < \Sigma^+-\Sigma^-+\Xi^0-\Xi^-, \tag{3.22}$$

for the lattice results. An estimate of μ_l^N may be obtained by equating the experimental and lattice violations of (3.20) and (3.22), respectively. This results in $\mu_l^N = -0.19(9)\ \mu_N$.

It is interesting to return to the previously discussed sum rules and assess the effects of such loop corrections. To evaluate the sum rules it is reasonable to assume a broader assumption of equal loop contributions for all octet baryons. Equating the lattice results with quark loop contributions to the experimental violations of the sum rules allows similar estimates of μ_l.

Fortunately, most of the sum rules are not as sensitive to quark loop effects as the Sachs sum rule. For example, Eqs. (3.3), (3.8), (3.17a), (3.17b) and the hyperon sides of (3.10) and (3.12) are unaffected by loop contributions. The lattice evaluations of these sum rules are in agreement with the experimental results.

Every remaining sum rule indicates that negative quark loop contributions provide better overlap of the lattice and experimental uncertainty regions. These results are summarized in Table II. Of particular interest are (3.10), (3.12), (3.14a), and (3.20) which require

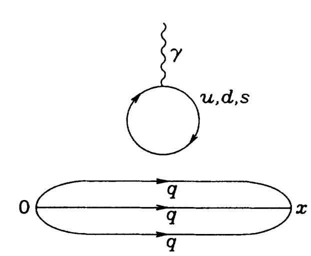

FIG. 1. A skeleton diagram of a disconnected quark loop contributing μ_l to the magnetic-moment of a baryon.

TABLE II. Estimates of quark loop contributions to baryon magnetic moments.

Equation number	μ_l (μ_N)
(3.1)	−0.13(46)
(3.5)	−0.19(28)
(3.10)	−0.11(7)
(3.12)	−0.21(14)
(3.14a)	−0.13(6)
(3.14b)	−0.04(6)
(3.14c)	−0.03(6)
(3.20)	−0.19(9)

loop contributions to restore agreement with experiment within 1.5 standard deviations. The absence of such loop contributions appears to be responsible for the contradiction between the lattice and experimental violations of (3.12) for the d'-quark moment.

Without better control of the systematic uncertainties discussed in Sec. II, it is difficult to draw any strong conclusions on the precise size of the disconnected quark loop contributions. However, it is not possible to reproduce the experimental violations of all magnetic-moment sum rules with the quark-model picture of Sec. II alone.

To obtain an optimum value for the loop contributions we consider a two-parameter fit to the individual baryon moments. A scale factor is provided to take into account lattice artifacts such as finite-volume effects. The second parameter accounts for quark loop contributions. Ratios of baryon moments with p or Σ^+ have smaller statistical uncertainties and are used in the fit. The optimum value for the quark loop contributions is

$$\mu_l = -0.10(6)\ \mu_N, \tag{3.23}$$

where the uncertainty is determined by the standard error ellipse.

The effects of quark loop contributions in the nucleon may also be equivalently described as a breaking of isospin symmetry in the effective quark magnetic moments where each quark moment is shifted by the same amount. A comparison of lattice quark loop contributions and isospin breaking in the simple quark model is possible through

$$\frac{1}{3}(\mu_u + 2\mu_d) = \mu_l = -0.031\ \mu_N, \tag{3.24}$$

where μ_u and μ_d are taken from the simple quark model parametrization of Ref. [14]. Since the coefficients $A - B = 1$ in the simple quark model, each quark moment is shifted by μ_l.

The chiral bag model [5] also indicates an enhancement of d-quark contributions relative to u quarks. Their results indicate $\mu_u/\mu_d = -1.77$, which is typical of other model estimates [15]. This ratio may be estimated with the lattice results provided we are willing to define an effective moment for the u quark. This may be done by equating the lattice u-quark sector contribution in the proton to $4\mu_u^N/3$, as in (3.7). Absorbing the quark loop contribution of (3.23) into the effective quark moments suggests a ratio of

$$\frac{\mu_u}{\mu_d} = -1.76(15), \tag{3.25}$$

for the lattice results, in good agreement with model expectations.

IV. SUMMARY

Magnetic-moment sum rules have been evaluated using the recent lattice QCD results for the magnetic moments of octet baryons. The model-independent results of the lattice calculations have been instrumental in removing some of the confusion and contradiction surrounding past sum-rule analyses. A spin-dependent effect, reflecting relativistic motion and nonperturbative gluon dynamics, in combination with a center-of-mass effect, which breaks baryon independence of the quark moments, is sufficient to account for most of the experimental violations of magnetic-moment sum rules.

The lattice results indicate that magnetic-moment quenching of the nonstrange quarks in Σ is a consequence of the larger mass of Σ which sets the scale of quark contributions to the magnetic moment. In contrast to previous sum-rule analyses, the lattice results indicate the magnetic moments of nonstrange quarks in Ξ are more likely enhanced than quenched relative to that in the nucleon. This may be associated with a large shift in the system center of mass towards the strange quarks.

The sum rules have also been used to further examine the results of the lattice simulation. The Sachs sum rule indicates the results of the lattice investigation, which are consistent with a quark-model picture, cannot reproduce the experimental violations of all magnetic-moment

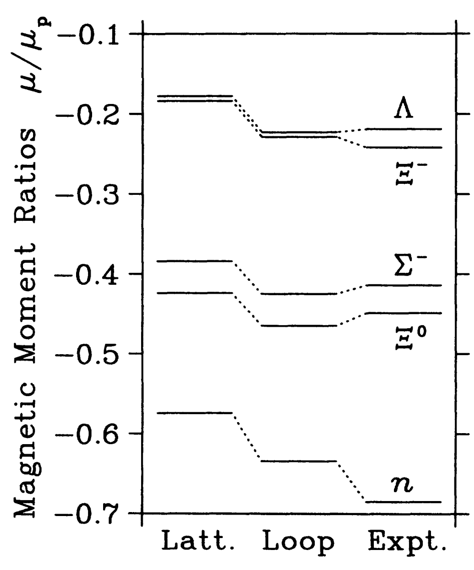

FIG. 2. Negative magnetic-moment ratios of octet baryons and the proton. The lattice ratios (Latt.) without quark loop effects, lattice ratios with loop effects (Loop) and experimental (Expt.) ratios are illustrated. Significant improvements are seen throughout the baryons, particularly for the neutron.

sum rules. Additional contributions outside of the quark-model description of Sec. II are required. The sum rules suggest that such contributions may be approximately equal for all baryons, consistent with the anticipated effects of quark loop contributions. Sum rules not sensitive to loop contributions are in agreement with the experimental results at the 1σ level. In contrast, four of the sum rules affected by disconnected quark loops require loop contributions to restore agreement with experimental measurements within $1.5\,\sigma$.

Quark loop contributions give rise to isospin-symmetry breaking in the effective u- and d-quark moments when the loop contributions are absorbed into the definition of the effective quark moment. Our estimate of the ratio of effective u- and d-quark moments is in agreement with model expectations.

The effects of quark loop contributions are most prominent for ratios of baryons with negative magnetic moments. These ratios are summarized in Fig. 2 where the loop contribution of (3.23) has been used. The addition of quark loop contributions puts the lattice and experimental ratios in agreement well within one standard deviation. However, without better control of the systematic uncertainties discussed in Sec. II, it is difficult to draw any strong conclusions on the precise size of the disconnected quark loop contributions.

A direct calculation of the magnetic moment contribution of the disconnected quark loop illustrated in Fig. 1 is possible even within a simulation of quenched QCD. Unfortunately, the reduction of statistical uncertainties in the loop contributions to a tolerable level requires considerably more computing time than is involved in calculating the three-point functions. In this investigation we have seen that disconnected quark loop contributions may provide the key to removing the remaining discrepancies between lattice and experimental ratios of magnetic moments. A calculation of quark loop contributions in quenched QCD, even on a modest size lattice, would provide considerable insight to the role of sea quarks in accounting for the magnetic moments of baryons.

ACKNOWLEDGMENTS

I thank Terry Draper, Byron Jennings, and Richard Woloshyn for helpful discussions and comments on the preparation of this manuscript. This research was supported by the Natural Sciences and Engineering Research Council of Canada.

* Present address: Nuclear Theory Group, Department of Physics, University of Maryland, College Park, MD 20742-4111.

[1] H. J. Lipkin, Phys. Rev. D **24**, 1437 (1981). Historically this sum rule failed by an order of magnitude using the experimental data available at that time.
[2] J. Franklin, Phys. Rev. D **29**, 2648 (1984).
[3] H. J. Lipkin, Nucl. Phys. **B241**, 477 (1984).
[4] K. Ushio, Phys. Lett. **158B**, 71 (1985).
[5] M. I. Krivoruchenko, Yad. Fiz. **45**, 169 (1987) [Sov. J. Nucl. Phys. **45**, 109 (1987)]. Further model calculation details may be found in the references therein.
[6] M. Krivoruchenko, B. Martemyanov, and M. Schepkin, Phys. Rev. D **41**, 997 (1990).
[7] D. B. Leinweber, R. M. Woloshyn, and T. Draper, Phys. Rev. D **43**, 1659 (1991).
[8] J. Lach, Fermi National Accelerator Laboratory Report No. FERMILAB-Conf-90/238, 1990 (unpublished); P. M. Ho *et al.*, Phys. Rev. Lett. **65**, 1713 (1990).
[9] B. Efron, SIAM Rev. **21**, 460 (1979); S. Gottlieb, P. B. MacKenzie, H. B. Thacker, and D. Weingarten, Nucl. Phys. **B263**, 704 (1986).
[10] This point is discussed at some length by A. Bohm and R. B. Teese, Phys. Rev. D **26**, 1103 (1982), and references therein.
[11] J. Franklin, D. B. Lichtenburg, W. Namgung, and D. Carydas, Phys. Rev. D **24**, 2910 (1981).
[12] J. Franklin, Phys. Rev. D **20**, 1742 (1979).
[13] R. G. Sachs, Phys. Rev. D **23**, 1148 (1981); L. Brekke and R. G. Sachs, *ibid.* **28**, 1178 (1983).
[14] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).
[15] See, for example, R. Mignani and D. Prosperi, Nuovo Cimento **75A**, 221 (1983); D. A. Geffen and W. Wilson, Phys. Rev. Lett. **44**, 370 (1980).