PHYSICAL REVIEW D VOLUME 45, NUMBER 7 1 APRIL 1992

Leading radiative corrections in two-scalar-doublet models

C. D. Froggatt and R. G. Moorhouse
Department of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, United Kingdom

I. G. Knowles
High Energy Physics Division, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439
(Received 28 October 1991)

The leading radiative corrections arising from scalar contributions to the vector-boson self-energies, oblique corrections, are analyzed for extensions of the standard model involving two scalar doublets. We employ the most general such model which is compatible with natural flavor conservation, but allowing the possibility of soft discrete-symmetry-breaking terms. Several interesting, special cases are discussed.

PACS number(s): 12.15.Cc

I. INTRODUCTION

Application of the Appelquist-Carazzone theorem [1] to a gauge theory suggests that the effects of heavy particles decouple from low-energy phenomena below threshold. However this is not the case for a broken theory such as the standard model (SM), where hard couplings grow in the large-mass limit. In this situation, the underlying symmetry ensures that while there is sufficient freedom to choose counterterms capable of removing ultraviolet divergences, it is not possible to further subtract the effects of the heavy particles. Thus we expect that the effects of heavy particles, associated with potentially new physics at a large scale Λ, will be indirectly observable via radiative corrections in SM-type theories.

In this paper we shall examine those effects due to a particularly simple extension of the SM containing two scalar doublets [2–4]. To simplify such an analysis we shall make the assumption that the leading corrections are "oblique." That is, they affect physical observables via their presence in the vacuum polarizations of the vector bosons [5]. To justify this assumption note in particular that, as in the minimal model, all the Yukawa couplings of the scalars to fermions contain a suppression factor m_f/M_W. However, in addition enhancement terms dependent on the ratio of vacuum expectation values (VEV's) also occur [2]: v/v_1 and v/v_2 for the neutral scalars and $(v_1/v_2)^{\pm 1}$ for the charged scalar. Thus to avoid sizable scalar-mediated corrections involving couplings to external fermions (tree, vertex, or box) we must impose limits on the ratio v_1/v_2. We use the bounds ($\frac{1}{7}$ or $\frac{1}{12}$) $\le v_b/v_t \le 120$, derived by requiring that the Yukawa couplings of the (≥ 89 or ≥ 50) GeV/c^2 mass top quark and 5 GeV/c^2 mass bottom quark take perturbative values: $\le\sqrt{4\pi}$. Confining ourselves to leptons and other light fermions, these bounds are quite adequate.[1]

Further additional constraints on this ratio are also available from the study of scalar corrections to a range of processes [6,7].

We take the new physics scale $\Lambda^2 \gg M_Z^2 \approx M_W^2 \ge q^2$, the scale of the relatively low-energy phenomenology. Now on dimensional grounds we expect the leading term in a one-loop self-energy to depend quadratically on Λ as $g^2\Lambda^2$ or $g^2\Lambda^2\ln(\Lambda^2/v^2)$, where g is the $SU(2)_L$ gauge coupling constant, while successive terms in the q^2-MacLaurin series expansion will be suppressed by a factor v^2/Λ^2, with possible logarithmic $\ln(\Lambda^2/v^2)$ factors always understood. In our analysis of effects due in particular to large-mass scalar particles in the two-doublet model we shall consider only the quadratic and constant (logarithmic) terms.

As an example of this structure the radiative corrections due to the (t,b) quark doublet give rise to a factor $g^2m_t^2$ in the self-energies at $q^2=0$. These for example then give a contribution to the Veltman rho parameter ρ proportional to [8] $g^2m_t^2$; it is essentially this term's quadratic dependence which allows the present determinations of the top-quark mass [9,10] within the minimal SM. At first sight a heavy Higgs particle in the SM is similar with $g^2M_H^2$ contributions to the W and Z self-energies at $q^2=0$; however in physical observables, such as ρ, this leading quadratic dependence always cancels leaving only the subleading logarithmic behavior [11] $g^2\ln(M_H^2/M_W^2)$. This cancellation of the leading power dependence on M_H^2 is explicitly known to persist to two loops [12] and is known as the Veltman screening "theorem" [13].

The origin of the cancellation of leading terms in $g^2M_H^2$ can be traced to the existence of a custodial SU(2) symmetry [14,15] and occurs to all loops [16]. In the SM the potential for the scalar field is invariant under both the usual $SU(2)_L$ and an $SU(2)_R$, which after symmetry breaking, reduces to the custodial vector $SU(2)_{L+R}$ symmetry. It should be noted though that both mass splittings in the Yukawa couplings of fermion doublets and the gauging of the scalar hypercharge coupling violate the symmetry at the tree level, the latter leading to

[1]For this purpose the b quark ought not to be considered light, which is in addition to the fact that its corrections inevitably involve its heavy isospin partner, the t quark.

$M_Z \neq M_W$. Under the residual custodial symmetry, the $SU(2)_L$ gauge fields W_1, W_2, W_3 form a triplet which, combined with the existence of charge conservation, implies $\rho=1$ at the tree level [14]. Now it is the vestiges of this "isospin" symmetry, persisting beyond the tree level, which ensure that the leading terms in $g^2 M_H^2$ go into the scalar field's wave-function renormalization constant and thus remain unobservable [15,16].

In the case of a two-scalar-doublet model such a residual custodial symmetry would also lead to the suppression of corrections to $\rho=1$ [15]; but it is not present for a general scalar potential allowing the possibility of large quadratic corrections to physical observables [3,17]. However, it still proves helpful to organize our study of the leading corrections in terms of such an isospin.

The remainder of the paper is organized as follows. First, the relevant features of our general two-doublet model are outlined; then expressions for the additional contributions to the vector-boson self-energies are presented. The corrections to physical observables are studied using the three dimensionless parameters $\{\epsilon_i\}$ introduced by Barbieri [18,19] *et al.* The SM Higgs boson and (t,b) corrections are given for comparison. We then discuss these results paying particular attention to the range of allowed values and certain special cases. Finally we comment on the relationship to experimentally favored values [9,20].

II. GENERAL TWO-SCALAR-DOUBLET MODELS

Here we briefly recall the main features of two-scalar-doublet models; see Ref. [2] for further details. The scalar potential is given by [4]

$$\begin{aligned} V(\Phi_1,\Phi_2) = & -\mu_1^2 \Phi_1^\dagger\Phi_1 - \mu_{12}^2\Phi_1^\dagger\Phi_2 - \mu_{12}^{2*}\Phi_2^\dagger\Phi_1 - \mu_2^2\Phi_2^\dagger\Phi_2 \\ & + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ & + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2] . \end{aligned} \tag{1}$$

Without loss of generality the $\{\lambda_i\}$ are taken real [4,17]. Observe that the hard mass-dimension-four terms are required to respect a custodial discrete symmetry: $\Phi_1 \mapsto -\Phi_1, \Phi_2$ invariant, which when imposed on the Yukawa couplings ensures against tree-level flavor-changing processes mediated by neutral scalars [21]. However, we do admit the possibility of a soft symmetry-breaking term: $\mu_{12}^2 \neq 0$. The parameters are chosen such that the potential's minimum corresponds to symmetry breaking with aligned VEV's. In general,

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v_1 \end{bmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v_2 e^{i\theta} \end{bmatrix},$$

where unless μ_{12}^2 is real, θ will be nonzero and CP is violated in the scalar sector. The corresponding scalar spectrum consists of the three would-be Goldstone bosons $\chi^\pm, \chi^0$; two charged scalars $H^\pm$, with mass M_+ and three neutrals h_1, h_2, h_3, with masses M_i which in general have mixed Lorentz and hence mixed CP couplings to fermions. In terms of these physical fields the two doublets may be decomposed as

$$\begin{aligned} \Phi_1 = & \frac{1}{\sqrt{2}} \left[\begin{bmatrix} 0 \\ v_1 \end{bmatrix} + c_\beta \begin{bmatrix} \chi^+\sqrt{2} \\ i\chi^0 \end{bmatrix} - s_\beta \begin{bmatrix} H^+\sqrt{2} \\ 0 \end{bmatrix} \right. \\ & \left. + \begin{bmatrix} 0 \\ (R_{i1} - i s_\beta R_{i3}) h_i \end{bmatrix} \right] , \\ \Phi_2 = & \frac{e^{i\theta}}{\sqrt{2}} \left[\begin{bmatrix} 0 \\ v_2 \end{bmatrix} + s_\beta \begin{bmatrix} \chi^+\sqrt{2} \\ i\chi^0 \end{bmatrix} + c_\beta \begin{bmatrix} H^+\sqrt{2} \\ 0 \end{bmatrix} \right. \\ & \left. + \begin{bmatrix} 0 \\ (R_{i2} + i c_\beta R_{i3}) h_i \end{bmatrix} \right] . \end{aligned} \tag{2}$$

Here β is defined by $\tan\beta = v_2/v_1$, with c_β and s_β abbreviations for $\cos\beta$ and $\sin\beta$, and R_{ij} is a 3×3 orthogonal matrix arising from the diagonalization of the neutral mass matrix [2,22]. To aid the interpretation of the fields present it is instructive to introduce a new basis due to Georgi [4], obtained from that above by the unitary rotation

$$\begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \begin{bmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\theta} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} . \tag{3}$$

These new doublets may then be written as

$$\Psi_1 = \frac{1}{\sqrt{2}} \left[\begin{bmatrix} 0 \\ v + \mathbf{H}^0 \end{bmatrix} + \begin{bmatrix} \chi^+\sqrt{2} \\ i\chi^0 \end{bmatrix} \right]$$

and (4)

$$\Psi_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} H^+\sqrt{2} \\ \mathbf{H}^1 + i\mathbf{H}^2 \end{bmatrix} ,$$

where $\mathbf{H}^0$, $\mathbf{H}^1$, and $\mathbf{H}^2$ are in general combinations of the physical neutral scalars $\{h_i\}$. It is now clear that Ψ_1 plays the role of the minimal SM doublet with H^0 the analogue of the Higgs field, while Ψ_2 represents an independent extra doublet. In the case $\theta=0$, R_{ij} and the H^a have a particularly simple form [2,3,17]:

$$R = \begin{bmatrix} c_\alpha & s_\alpha & 0 \\ -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{aligned} \mathbf{H}^0 &= c_{\alpha-\beta} h_1 - s_{\alpha-\beta} h_2 , \\ \mathbf{H}^1 &= s_{\alpha-\beta} h_1 + c_{\alpha-\beta} h_2 , \\ \mathbf{H}^2 &= h_3 , \end{aligned} \tag{5}$$

where α is a mixing angle determined by the scalar potential. Note that in this special case h_1 and h_2 are both CP even possessing scalar Yukawa couplings while h_3 is CP odd with pseudoscalar couplings. In general R controls the form of the Yukawa couplings to fermions while the $\mathbf{H}^a$ relate to the couplings to the vector gauge bosons [2].

This model is quite general, subject only to the requirement of tree-level, natural flavor conservation. As such it subsumes several other models, for example: "axion" models have $\lambda_5=0$ [23] and $\mu_{12}^2=0$ [24] while the minimal supersymmetric standard model [25] (MSSM) has $\lambda_1=(g^2+g'^2)/8=\lambda_2$, $\lambda_3=(g^2-g'^2)/4$, $\lambda_4=-g^2/2$, $\lambda_5=0$ where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively, and $\mu_1^2 = s_\beta^2 \mu^2 + c_\beta^2 M_Z^2/2$, $\mu_2^2 = c_\beta^2 \mu^2 + s_\beta^2 M_Z^2/2$, and $\mu_{12}^2 = -c_\beta s_\beta(\mu^2 + M_Z^2/2)$ for some μ^2. The presence of the μ_{12}^2 term in the potential introduces new freedom, when assigning the scalar masses and the three mixings required to specify the matrix R, which in turn characterizes the scalars couplings to fermions and vector bosons.

Radiative corrections to μ_1^2 and μ_2^2 diverge quadratically with the new physics scale Λ and, on dimensional grounds, the four soft mass parameters $\{\mu_i^2\}$ might naturally be expected to be $O(\Lambda^2)$. However, we know that the values of μ_1^2 and μ_2^2 must be fine-tuned to ensure that v_1 and v_2 take values at the weak scale v. It is further possible (i) to fine-tune the complex parameter μ_{12}^2 to the weak scale; this latter fine-tuning is technically natural as it is protected by the discrete symmetry which is only softly broken. Thus in option (i) all the $\{\mu_i^2\}$ are $O(v^2)$, so that larger scalar masses are obtained by increasing the hard parameters $\{\lambda_i\}$. Restricting the $\{\lambda_i\}$ to take finite, perturbative values, we believe, leaves sufficient freedom to choose an arbitrary set of mixing angles and scalar mass spectrum below about $10M_W \approx 800$ GeV/c^2. A second option (ii) is to allow the complex parameter μ_{12}^2 to take its value at the scale Λ; this leads, with the exception of $h_1 \approx \mathbf{H}^0$, to all scalar masses being $O(\Lambda)$. That is, inducing large scalar masses, for finite hard $\{\lambda_i\}$, by increasing the soft $\{\mu_i^2\}$ leads, up to corrections of $O(v^2/\Lambda^2)$, to an effective, one-Higgs-doublet, SM-like theory for scales below Λ. This "tree-level" decoupling of the heavy scalars will also be seen to occur in the one-loop radiative corrections.

In terms of phenomenology the real significance of μ_{12}^2 probably lies as a second source for *CP* violation within an SM-like theory [22] and its associated role in providing a potential mechanism for electroweak baryogenesis [26]. As μ_{12}^2 is a soft symmetry-breaking parameter, radiative corrections do not induce the presence of a counterterm for any neutral flavor-changing Yukawa coupling: there are no flavor-changing neutral-scalar-boson exchanges [27] in the model. *CP* violation in $K_L^0 \to 2\pi$ decay arises from the usual complex phase in the CKM mixing matrix, via charged, vector, gauge boson, and charged-scalar exchanges [28]; neutral-scalar-exchange contributions occur at higher order and are expected to be negligible. On the other hand, *CP*-violating neutral-scalar exchange should provide dominant contributions to the electric dipole moments of quarks and leptons [22,29]. The two-scalar-doublet electroweak baryogenesis scenario [26] requires the phase θ to be of order unity and consequently an electron electric dipole moment close to the present experimental upper bound.

III. SELF-ENERGIES WITH TWO SCALAR DOUBLETS

The various one-loop vector-boson propagators are decomposed into their transverse and longitudinal parts as follows:

$$\frac{-i}{k^2-M_a^2}\left[\left[\delta_{ab}+\frac{A_{ab}(k^2)}{k^2-M_b^2}\right]\left[\eta^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right]+\left[\delta_{ab}+\frac{B_{ab}(k^2)}{k^2-M_b^2}\right]\frac{k^\mu k^\nu}{k^2}\right], \tag{6}$$

where A_{ab} and B_{ab} are $-i$ times the transverse and longitudinal parts of the amputated self-energy for a goes to b. The full expressions for the transverse self-energy contributions $\tilde{A}_{ab}$ arising from the scalar particles in a two-doublet extension of the SM are now given. In the notation used s and c are the sine and cosine of the Weinberg angle[2] and the H_i^a are introduced via $\mathbf{H}^a = H_i^a h_i$; observe that the H_i^a form an orthogonal matrix. The integrals are regularized dimensionally. The contributions $\tilde{A}_{\gamma\gamma}$ and $\tilde{A}_{\gamma Z}$ are

$$\begin{aligned}
\tilde{A}_{\gamma\gamma}(k^2) &= \frac{\alpha_{\rm EM}}{4\pi}\frac{1}{3}\left[-k^2\Delta_W + k^2\ln\frac{M_+^2}{M_W^2} + (4M_+^2 - k^2)F(k^2;M_+^2,M_+^2) - \frac{2k^2}{3}\right],\\
\tilde{A}_{\gamma Z}(k^2) &= \frac{c^2-s^2}{2cs}\tilde{A}_{\gamma\gamma}(k^2)\,.
\end{aligned} \tag{7}$$

Note the neutral scalars do not contribute to these two terms. The contribution $\tilde{A}_{ZZ}$ is

$$\begin{aligned}
\tilde{A}_{ZZ}(k^2) = \frac{\alpha_{\rm EM}}{4\pi}\frac{1}{12c^2s^2}\Bigg\{ & -[(c^2-s^2)^2k^2 + 9M_Z^2 + 2k^2]\Delta_W \\
& + (c^2-s^2)^2\left[-\frac{2k^2}{3} + k^2\ln\frac{M_+^2}{M_W^2} + (4M_+^2 - k^2)F(k^2;M_+^2,M_+^2)\right] \\
& - 12M_Z^2\sum_i (H_i^0)^2\left[1 - \frac{M_i^2}{M_i^2 - M_Z^2}\ln\frac{M_i^2}{M_W^2} - \frac{M_Z^2}{M_Z^2 - M_i^2}\ln\frac{M_Z^2}{M_W^2} + F(k^2;M_i^2,M_Z^2)\right]
\end{aligned}$$

[2] The detailed definition, for example whether s_W, s_0, or $\bar{s}$, is not significant at the one-loop level.

$$
\begin{aligned}
&+\sum_i (H_i^0)^2 \Bigg[5(M_i^2+M_Z^2)-\frac{5k^2}{3}+\frac{M_i^2}{M_i^2-M_Z^2}[k^2-(3M_i^2+M_Z^2)]\ln\frac{M_i^2}{M_W^2} \\
&\qquad +\frac{M_Z^2}{M_Z^2-M_i^2}[k^2-(3M_Z^2+M_i^2)]\ln\frac{M_Z^2}{M_W^2} \\
&\qquad +\left[2(M_i^2+M_Z^2)-k^2-\frac{(M_i^2-M_Z^2)^2}{k^2}\right]F(k^2;M_i^2,M_Z^2)\Bigg] \\
&+\frac{1}{2}\sum_{i,j\neq i}(H_i^1H_j^2-H_i^2H_j^1)^2\Bigg[5(M_i^2+M_j^2)-\frac{5k^2}{3}+\frac{M_i^2}{M_i^2-M_j^2}(k^2-3M_i^2-M_j^2)\ln\frac{M_i^2}{M_W^2} \\
&\qquad +\frac{M_j^2}{M_j^2-M_i^2}(k^2-3M_j^2-M_i^2)\ln\frac{M_j^2}{M_W^2} \\
&\qquad +\left[2(M_i^2+M_j^2)-k^2-\frac{(M_i^2-M_j^2)^2}{k^2}\right]F(k^2;M_i^2,M_j^2)\Bigg] \\
&-3\sum_i M_i^2\left[1-\ln\frac{M_i^2}{M_W^2}\right]\Bigg\} \ .
\end{aligned} \tag{8}
$$

The factor $\frac{1}{2}$ in the term $\sum_{i,j\neq i}$ is included to compensate for double counting. The contribution $\tilde{A}_{WW}$ is

$$
\begin{aligned}
\tilde{A}_{WW}(k^2)=\frac{\alpha_{\rm EM}}{4\pi}\frac{1}{12s^2}&\Bigg\{-3(3M_W^2+k^2)\Delta_W-12M_W^2\sum_i (H_i^0)^2\left[1-\frac{M_i^2}{M_i^2-M_W^2}\ln\frac{M_i^2}{M_W^2}+F(k^2;M_i^2,M_W^2)\right] \\
&+\sum_i (H_i^0)^2\Bigg[5(M_i^2+M_W^2)-\frac{5k^2}{3}+\frac{M_i^2}{M_i^2-M_W^2}(k^2-3M_i^2-M_W^2)\ln\frac{M_i^2}{M_W^2} \\
&\qquad +\left[2(M_i^2+M_W^2)-k^2-\frac{(M_i^2-M_W^2)^2}{k^2}\right]F(k^2;M_i^2,M_W^2)\Bigg] \\
&+\sum_i[(H_i^1)^2+(H_i^2)^2]\Bigg[5(M_i^2+M_+^2)-\frac{5k^2}{3}+\frac{M_i^2}{M_i^2-M_+^2}(k^2-3M_i^2-M_+^2)\ln\frac{M_i^2}{M_W^2} \\
&\qquad +\frac{M_+^2}{M_+^2-M_i^2}(k^2-3M_+^2-M_i^2)\ln\frac{M_+^2}{M_W^2} \\
&\qquad +\left[2(M_i^2+M_+^2)-k^2-\frac{(M_i^2-M_+^2)^2}{k^2}\right]F(k^2;M_i^2,M_+^2)\Bigg] \\
&-3\sum_i M_i^2\left[1-\ln\frac{M_i^2}{M_W^2}\right]-6M_+^2\left[1-\ln\frac{M_+^2}{M_W^2}\right]\Bigg\} \ .
\end{aligned} \tag{9}
$$

Here,

$$\Delta_W=\frac{2}{4-d}-\gamma_E-\ln\frac{M_W^2}{4\pi\mu^2} \ ,$$

where d is the space-time dimensionality, γ_E is the Euler-Mascheroni constant, and μ^2 is the 't Hooft mass introduced to keep track of dimensions. The function $F(k^2;M_1^2,M_2^2)$ arises from the evaluation of the two-point scalar loop diagram and is given by

$$F(k^2;M_1^2,M_2^2)=-\int_0^1 dx\,\ln\left[\frac{x^2k^2-x(k^2+M_1^2-M_2^2)+M_1^2-i\epsilon}{M_1M_2}\right]-1+\frac{M_1^2+M_2^2}{M_1^2-M_2^2}\ln\frac{M_1}{M_2} \ . \tag{10}$$

It is symmetric in M_1 and M_2, and normalized so that $F(0;M_1^2,M_2^2)=0$. It will be helpful for later use to know the small-k^2 behavior

$$F(k^2;M_1^2,M_2^2)=\frac{k^2}{(M_1^2-M_2^2)^2}\left[\frac{M_1^2+M_2^2}{2}-\frac{M_1^2M_2^2}{M_1^2-M_2^2}\ln\frac{M_1^2}{M_2^2}\right]$$
$$+\frac{k^4}{(M_1^2-M_2^2)^2}\left[\frac{1}{6}+\frac{2M_1^2M_2^2}{(M_1^2-M_2^2)^2}-\frac{M_1^2M_2^2(M_1^2+M_2^2)}{(M_1^2-M_2^2)^3}\ln\frac{M_1^2}{M_2^2}\right]+\cdots\,,$$
$$F(k^2;M^2,M^2)=\frac{k^2}{6M^2}+\frac{k^4}{60M^2}+\cdots\,, \tag{11}$$

which allows us to expand out the self-energies. Observe that in the special *CP*-invariant case, Eq. (5), the more general expressions given for the self-energies reduce to the known results [17,30].

IV. THE LEADING OBSERVABLE CORRECTIONS

As indicated previously we believe the leading effects of the heavy scalars are contained within the first two terms of the MacLaurin series for the $\tilde{A}_{ab}$; thus we write the self-energies as

$$A_{ab}(k^2)=A_{ab}(0)+k^2\Pi_{ab}(k^2)\,. \tag{12}$$

Any quadratic dependence on heavy particle masses should appear in $A_{ab}(0)$ and not in $\Pi_{ab}(k^2)$. Note that $A_{\gamma\gamma}(0)=0$ is required by current conservation while $A_{\gamma Z}(0)$ only receives one nonzero contribution $-(\alpha_{EM}2M_W^2/4\pi cs)\Delta_W$ from the W loop; new physics contributions to $A_{\gamma\gamma}(0)$ and $A_{\gamma z}(0)$ vanish. Further below the particle thresholds their contributions to all the A_{ab} are real. After defining the necessary counterterms the remaining free $A_{ab}(0)$ and $\Pi_{ab}(0)$ may be combined, adopting a similar notation to Barbieri *et al.* [18,19], into the following combinations based upon the custodial SU(2) "symmetry" of the SM:

$$\epsilon_1=\frac{A_{+-}(0)-A_{33}(0)}{M_W^2}\,,$$
$$\epsilon_2=-[\Pi_{+-}(0)-\Pi_{33}(0)]\,, \tag{13}$$
$$\epsilon_3=-\frac{c}{s}\Pi_{30}(0)\,.$$

Here $W_3=cZ+sA$ and $W_0=-sZ+cA$ refer to the fields of the unbroken Lagrangian. From these definitions it is clear that ϵ_1 and ϵ_2 are proportional to the residual $SU(2)_{L+R}$-symmetry breaking and, unlike ϵ_3, will vanish as it is restored. From the above remark we expect at most quadratic dependence in ϵ_1 and logarithmic dependence in $\epsilon_{2,3}$ on heavy particle masses.

We now present the expressions for the $\{\epsilon_i\}$ arising from a heavy (t-, b-) quark doublet, the minimal SM Higgs boson, and the new expression for the two-doublet case.

A. Heavy quark doublet

The exact formulas are [8]

$$\epsilon_1=\frac{\alpha_{EM}}{4\pi}\frac{N_c}{2s^2M_W^2}f(m_t^2,m_b^2)\,,$$
$$\epsilon_2=-\frac{\alpha_{EM}}{4\pi}\frac{N_c}{3s^2}g(m_t^2,m_b^2)\,, \tag{14}$$
$$\epsilon_3=\frac{\alpha_{EM}}{4\pi}\frac{N_c}{18s^2}\left[3-\ln\frac{m_t^2}{m_b^2}\right]\,.$$

Here $N_c=3$ is the number of quark colors and we have introduced two positive functions $f(m_1^2,m_2^2)$ and $g(m_1^2,m_2^2)$ which naturally arise in our formulas:

$$f(m_1^2,m_2^2)=\frac{m_1^2m_2^2}{2}-\frac{m_1^2m_2^2}{m_1^2-m_2^2}\ln\frac{m_1^2}{m_2^2}\,,$$
$$g(m_1^2,m_2^2)=-\frac{5}{6}+\frac{2m_1^2m_2^2}{(m_1^2-m_2^2)^2}+\frac{(m_1^2+m_2^2)(m_1^4-4m_1^2m_2^2+m_2^4)}{2(m_1^2-m_2^2)^3}\ln\frac{m_1^2}{m_2^2}\,. \tag{15}$$

Note that for $m_1^2=m_2^2$ both functions vanish being proportional to $(m_1^2-m_2^2)^2$. So that when the custodial SU(2) symmetry is respected by the Yukawa couplings $m_t=m_b$, then $\epsilon_1=0=\epsilon_2$; however, $\epsilon_3=-(\alpha_{EM}/4\pi)(N_c/6s^2)$. Further $f(m_1^2,0)=m_1^2/2$ while $g(m_1^2,m_2^2\to 0)\sim\ln(m_1/m_2)-\frac{5}{6}$ diverges logarithmically to $+\infty$, so that for $m_t^2\gg m_b^2$ the $\{\epsilon_i\}$ behave as

$$\{\epsilon_1;\epsilon_2;\epsilon_3\}=\frac{\alpha_{EM}}{4\pi}\frac{N_c}{3s^2}\left\{\frac{3m_t^2}{4M_W^2};\left[\frac{5}{6}-\frac{1}{2}\ln\frac{m_t^2}{m_b^2}\right];\frac{1}{6}\left[3-\ln\frac{m_t^2}{m_b^2}\right]\right\}\,.$$

The strong quadratic dependence of ϵ_1 on the mass [8] ensures that quantities involving it lead to the best predictions for m_t; currently [9] $m_t\approx 130\pm 35$ GeV/c^2. While in ϵ_2 and ϵ_3 there is only logarithmic dependence it should be noted that (at least within the minimal SM [31]) the large bound on the ratio [32] $m_t^2/m_b^2>325$ implies that they are both numerically significant, though relatively constant.

B. Single Higgs particle

For the single Higgs particle, with a mass M_H, of the minimal SM we have the exact expressions [11]

$$\epsilon_1^{(\mathrm{SM})}(M_H^2) \equiv \frac{\alpha_{\mathrm{EM}}}{4\pi}\frac{3}{4c^2}\left[\Delta_W + \frac{1}{3} + \frac{1}{M_Z^2 - M_W^2}\left[f(M_H^2,M_Z^2) - f(M_H^2,M_W^2) - M_Z^2 \ln\frac{M_Z^2}{M_W^2}\right]\right],$$

$$\epsilon_2^{(\mathrm{SM})}(M_H^2) \equiv \frac{\alpha_{\mathrm{EM}}}{4\pi}\frac{1}{12s^2}\left[g(M_H^2,M_W^2) - g(M_H^2,M_Z^2) - \frac{1}{2}\ln\frac{M_Z^2}{M_W^2} - \frac{12M_W^2}{(M_H^2-M_W^2)^2}f(M_H^2,M_W^2)\right], \tag{16}$$

$$\epsilon_3^{(\mathrm{SM})}(M_H^2) \equiv -\frac{\alpha_{\mathrm{EM}}}{4\pi}\frac{1}{12s^2}\left[\Delta_W - g(M_H^2,M_Z^2) - \frac{1}{2}\ln\frac{M_H^2}{M_W^2} - \frac{1}{2}\ln\frac{M_Z^2}{M_W^2}\right].$$

These expressions can be greatly simplified if terms of order M_Z^2/M_H^2 are neglected:

$$\{\epsilon_1;\epsilon_2;\epsilon_3\} = \frac{\alpha_{\mathrm{EM}}}{4\pi}\frac{1}{12s^2}\left\{\frac{9s^2}{c^2};0;-1\right\}\left[\Delta_W - \ln\frac{M_H^2}{M_W^2} + \frac{5}{6}\right]. \tag{17}$$

Here ϵ_1 only depends logarithmically on M_H^2, while the leading behavior in ϵ_2 is suppressed as M_Z^2/M_H^2. As noted this is not because quadratic dependence on M_H^2 does not occur in the self-energies; for the $W^\pm$ and Z^0 both the physical-scalar–would-be-Goldstone-scalar and physical scalar "tadpole" loops depend quadratically on M_H^2. However in ϵ_1 the latter contributions cancel exactly and due to the similarity $M_W^2 \approx M_Z^2$ the former partially cancel, leaving only a logarithmic dependence. This is an example of the screening theorem, and has the immediate consequence that inferring the Higgs-boson mass from experiment becomes rather difficult. At low values of M_H^2 quadratic behavior does occur and in particular ϵ_2 is non-negligible.

Note also that ϵ_1 and ϵ_3 are not ultraviolet finite, but must be taken in conjunction with the other boson loops, with which they form a gauge-invariant set, to give a finite result. This cancellation can be checked using the results for the remaining bosons:

$$\frac{A_{+-}(k^2) - A_{33}(k^2)}{M_W^2} = -\frac{\alpha_{\mathrm{EM}}}{4\pi}\frac{3}{4c^2}\Delta_W + \text{finite}, \quad -\frac{c}{s}\Pi_{30}(k^2) = \frac{\alpha_{\mathrm{EM}}}{4\pi}\frac{1}{12s^2}\Delta_W + \text{finite}.$$

C. Two scalar doublets

In the case of two scalar doublets we find

$$\epsilon_1 = \sum_i (H_i^0)^2\epsilon_1^{(\mathrm{SM})}(M_i^2) + \frac{\alpha_{\mathrm{EM}}}{4\pi}\frac{1}{4s^2M_W^2}\left[\sum_i [(H_i^1)^2 + (H_i^2)^2] f(M_i^2,M_+^2) - \frac{1}{2}\sum_{i,j}(H_i^1H_j^2 - H_i^2H_j^1)^2 f(M_i^2,M_j^2)\right], \tag{18}$$

$$\epsilon_2 = \sum_i (H_i^0)^2\epsilon_2^{(\mathrm{SM})}(M_i^2) - \frac{\alpha_{\mathrm{EM}}}{4\pi}\frac{1}{12s^2}\left[\sum_i [(H_i^1)^2 + (H_i^2)^2] g(M_i^2,M_+^2) - \frac{1}{2}\sum_{i,j}(H_i^1H_j^2 - H_i^2H_j^1)^2 g(M_i^2,M_j^2)\right], \tag{19}$$

$$\epsilon_3 = \sum_i (H_i^0)^2\epsilon_3^{(\mathrm{SM})}(M_i^2) - \frac{\alpha_{\mathrm{EM}}}{4\pi}\frac{1}{24s^2}\left[\sum_i [(H_i^1)^2 + (H_i^2)^2]\ln\frac{M_+^2}{M_i^2} - \sum_{i,j}(H_i^1H_j^2 - H_i^2H_j^1)^2 g(M_i^2,M_j^2)\right]. \tag{20}$$

In considering these corrections it is useful to adopt the separation of fields as indicated in the Georgi basis Eq. (4). Ψ_1 containing the combination of physical scalars $\mathbf{H}^0$ is, by design, analogous to the SM's single doublet containing the Higgs particle; in particular it has the same couplings to the gauge bosons. Thus the first contributions, coming from Ψ_1, to the $\{\epsilon_i\}$ are equivalent to those of the SM. More interesting are the contributions from the additional doublet Ψ_2 which contains $H^\pm$ and the two combinations of fields $\mathbf{H}^1 + i\mathbf{H}^2$. In what follows we shall study $\{\epsilon_i^{\mathrm{extra}}\}$ defined as the difference between ϵ_i(2 doublets) and ϵ_i(1 doublet). In the large-mass limit this introduces a factor proportional to $\sum_i (H_i^0)^2 \ln(M_i^2/M_H^2)$ into ϵ_1, where it is subleading, and ϵ_3.

V. DISCUSSION OF RESULTS

To gain an insight into the $\epsilon^{\mathrm{extra}}$ we begin by considering the size of the corrections in the large-mass limit of the *CP*-invariant case $\theta = 0$, given by Eq. (5). In this situation,

$$\epsilon_{1,2}^{\mathrm{extra}} = \frac{\alpha_{\mathrm{EM}}}{4\pi}\frac{C}{12s^2}\{s_{\alpha-\beta}^2[h(M_1^2,M_+^2) + h(M_3^2,M_+^2) - h(M_1^2,M_3^2)]$$
$$+ c_{\alpha-\beta}^2[h(M_2^2,M_+^2) + h(M_3^2,M_+^2) - h(M_2^2,M_3^2)]\}, \tag{21}$$

where $C=3/M_W^2, h=f$ for ϵ_1 and $C=-1, h=g$ for ϵ_2 and

$$\epsilon_3^{\rm extra}=\frac{\alpha_{\rm EM}}{4\pi}\frac{1}{12s^2}\left[s_{\alpha-\beta}^2\left[\ln\frac{M_2^2}{M_H^2}+g(M_1^2,M_3^2)-\frac{1}{2}\ln\frac{M_+^2}{M_1^2}-\frac{1}{2}\ln\frac{M_+^2}{M_3^2}\right]\right.$$
$$\left.+c_{\alpha-\beta}^2\left[\ln\frac{M_1^2}{M_H^2}+g(M_2^2,M_3^2)-\frac{1}{2}\ln\frac{M_+^2}{M_2^2}-\frac{1}{2}\ln\frac{M_+^2}{M_3^2}\right]\right] . \quad (22)$$

We shall now examine the range of these additional corrections given by Eqs. (21) and (22), to find their maximum and minimum values and how they can be made to vanish.

We start by studying $\epsilon_1^{\rm extra}$. It is significant to note that $\epsilon_1^{\rm extra}$ can contain quadratic dependence on the scalar masses [3,35], unlike the situation in the minimal SM; thus we can anticipate larger corrections. Previously it has been noted [17] that $\epsilon_1^{\rm extra}$ is positive for $M_{1,2,3}\geq M_+$ or $M_+\geq M_{1,2,3}$ with the maximal positive corrections being given by the following two basic configurations according to whether M_+ is the smallest or largest mass:

$$s_{\alpha-\beta}=0, M_1 \text{ free: } 0\approx M_+ \ll M_2=M_3\equiv M_i \text{ or } 0\approx M_2=M_3\ll M_+ ,$$
$$c_{\alpha-\beta}=0, M_2 \text{ free: } 0\approx M_+ \ll M_1=M_3 \text{ or } 0\approx M_1=M_3\ll M_+ ,$$
$$M_1=M_2, (\alpha-\beta) \text{ free: } 0\approx M_+ \ll M_1=M_3 \text{ or } 0\approx M_1=M_3\ll M_+ .$$

In this case $\epsilon_1^{\rm extra}$ behaves as

$$\epsilon_1^{\rm max}=\frac{\alpha_{\rm EM}}{4\pi}\frac{1}{4s^2}\left[\frac{M_i^2}{M_W^2} \text{ or } \frac{M_+^2}{M_W^2}\right] . \quad (23)$$

Physically these configurations correspond to the doublet Ψ_2 having the charged-scalar mass either much less than or greater than the mass of the *CP*-even and -odd neutrals, which are mass degenerate.

Commonly new particles contribute corrections to ϵ_1 which are positive [33], though not necessarily [34]. For example, in the regions $M_{1,2}<M_+<M_3$ and $M_3<M_+<M_{1,2}$ the extra scalar contribution may be negative [35], a possibility which can be used to weaken the upper bounds on m_t which gives a positive contribution to ϵ_1, see Eq. (14). The mass configurations which minimize $\epsilon_1^{\rm extra}$ are basically all of the same form [35]:

$$s_{\alpha-\beta}=0, M_1 \text{ free: } M_3=0, M_+=\kappa(M_2\equiv M_i) \text{ or } M_2=0, M_+=\kappa(M_3\equiv M_i) ,$$
$$c_{\alpha-\beta}=0, M_2 \text{ free: } M_3=0, M_+=\kappa M_1 \text{ or } M_1=0, M_+=\kappa M_3 ,$$
$$M_1=M_2, (\alpha-\beta) \text{ free: } M_3=0, M_+=\kappa M_1 \text{ or } M_1=0, M_+=\kappa M_3 ,$$

the constant κ being chosen to minimize the form of $\epsilon_1^{\rm extra}$,

$$\epsilon_1^{\rm extra}=\frac{\alpha_{\rm EM}}{16s^2\pi}\left[\kappa^2-\frac{\kappa^2}{1-\kappa^2}\ln\frac{1}{\kappa^2}\right]\frac{M_i^2}{M_W^2}, \quad \kappa^{\rm min}=0.562, \quad \epsilon_1^{\rm min}=-\frac{\alpha_{\rm EM}}{16s^2\pi}\times 0.216\frac{M_i^2}{M_W^2} . \quad (24)$$

Physically this situation obtains when the massive, neutral component of Ψ_2 has a definite *CP* and the mass of the charged component is a fraction κ of the neutral's mass.

Finally we note the intermediate possibility that $\epsilon_1^{\rm extra}$ can be made to vanish in a number of ways. This happens if $M_1=M_+$ and $c_{\alpha-\beta}=0$ (M_2 arbitrary), or $M_2=M_+$ and $s_{\alpha-\beta}=0$ (M_1 arbitrary) or $M_1=M_+=M_2$ and $\alpha-\beta$ arbitrary; in each of these cases the *CP*-even scalar in Ψ_2 is mass degenerate with the charged field. Also $\epsilon_1^{\rm extra}$ vanishes if $M_3=M_+$ for all $M_{1,2}$ and $\alpha-\beta$. An explanation for this last option can be found in the existence of a residual SU(2) symmetry for the two-doublet model [15]. This occurs when $\lambda_4=\lambda_5$ and μ_{12}^2 is real, which implies that M_+ equals the pseudoscalar's mass M_3. In the light of this observation, it is interesting to note that the most positive values for ϵ_1 occur when there is the largest mass splitting between the neutral and charged components of Ψ_2.

Next we turn our attention to $\epsilon_2^{\rm extra}$, which given its definition, behaves in a similar way to $\epsilon_1^{\rm extra}$. In particular the minimum value of $\epsilon_2^{\rm extra}$ occurs for the same configurations as those which maximize $\epsilon_1^{\rm extra}$, with the limiting value given by

$$\epsilon_2^{\rm min}=-\frac{\alpha_{\rm EM}}{4\pi}\frac{1}{12s^2}\left[\left|\ln\frac{M_+^2}{M_i^2}\right|-\frac{5}{3}\right] , \quad (25)$$

while the maximizing configurations are of the form

$s_{\alpha-\beta}=0$, M_1 free:

$$M_3\gg M_+\gg M_2 \text{ or } M_2\gg M_+\gg M_3 ,$$

$c_{\alpha-\beta}=0$, M_2 free:

$$M_3 \gg M_+ \gg M_1 \quad \text{or } M_1 \gg M_+ \gg M_3 \ ,$$

$M_1 = M_2$, $(\alpha-\beta)$ free:

$$M_3 \gg M_+ \gg M_1 \quad \text{or } M_1 \gg M_+ \gg M_3 \ ,$$

and yield

$$\epsilon_2^{\max} = \frac{\alpha_{\rm EM}}{4\pi}\frac{1}{12s^2}\frac{5}{6} \ . \tag{26}$$

Finally we note that the same spectrum of masses which causes $\epsilon_1^{\rm extra}$ to vanish also gives $\epsilon_2^{\rm extra}=0$.

Lastly we consider $\epsilon_3^{\rm extra}$. The maximum values of $\epsilon_3^{\rm extra}$ occur when the pattern of particles and their masses take the form

$$s_{\alpha-\beta}=0\text{: } M_i \equiv M_1 \gg M_H \text{ and } 0 \approx M_+ \ll M_2 = M_3 \equiv M_j \ ,$$

$$c_{\alpha-\beta}=0\text{: } M_2 \gg M_H \text{ and } 0 \approx M_+ \ll M_1 = M_3 \ ,$$

$M_1 = M_2$, $(\alpha-\beta)$ free:

$$M_1 \gg M_H \text{ and } 0 \approx M_+ \ll M_1 = M_3 \ .$$

In this situation $\epsilon_3^{\rm extra}$ behaves as

$$\epsilon_3^{\max} = \frac{\alpha_{\rm EM}}{4\pi}\frac{1}{12s^2}\left\{\ln\frac{M_i^2}{M_H^2} - \ln\frac{M_+^2}{M_j^2}\right\} \ . \tag{27}$$

This can be understood as follows: the first inequality says that the mass of the neutral scalar in Ψ_1, which plays the same role as the SM Higgs boson, should be as large as possible. This is the exact analogue of the SM, whereas the second inequality refers to the fields which make up Ψ_2 and so maximize the extra contributions. These contributions are always positive for $M_+ \le \{M_i\}$ with the extremal configuration being that in which the charged component's mass is much less than that of the mass degenerate *CP*-even and *CP*-odd neutral components. To obtain the minimum value of $\epsilon_3^{\rm extra}$, simply reverse the inequalities in the maximizing configuration and use the expression in Eq. (27) to give $\epsilon_3^{\min}$. In order to make the extra contribution vanish we require $s_{\alpha-\beta}=0$ and $M_+ = M_2 = M_3$ or $c_{\alpha-\beta}=0$ and $M_+ = M_1 = M_3$ or $M_+ = M_1 = M_2 = M_3$ for arbitrary $(\alpha-\beta)$. Note that setting $M_+ = M_3$ is not sufficient to make $\epsilon_3^{\rm extra}$ vanish, since it is not proportional to the custodial vector isospin splitting.

In the above analysis we have identified those configurations of masses and mixing angle α which maximize and minimize the $\{\epsilon_i^{\rm extra}\}$ subject to the constraint that $\theta=0$. However we also find that when we remove this requirement, so that in general [2] there are three mixing angles $\alpha_{1,2,3}$, it is the same configurations which extremize the $\{\epsilon_i^{\rm extra}\}$. That is, the presence of a complex μ_{12}^2 in the scalar potential does not give rise to larger radiative corrections, for a given allowed range for the scalar masses.

VI. NUMERICAL EVALUATION

Numerical estimates of the potential sizes of the radiative corrections $\{\epsilon_i\}$ are clearly sensitive to the limits placed on the maximum and minimum allowed values for the particle masses. As such experimental considerations impose important restrictions. In the case of the minimal SM Higgs boson [36], or a particle with equivalent couplings to the Z^0, direct searches at the CERN e^+e^- collider LEP give the lower bound 48 GeV/c^2. In the two-doublet model lighter scalar masses can be accommodated by reducing this coupling. Similarly the lower-mass bound for the charged scalars is also [36] 48 GeV/c^2. Further constraints are implied by the failure to observe the scalar-pseudoscalar decay mode: $Z^0 \to h_{1,2} + h_3$, which we shall take to be kinematically forbidden. In addition we do not allow scalar masses below $M_W/10 \approx 8$ GeV/c^2 where constraints are dependent on details of the Yukawa couplings [7]. In the case of the top quark a direct search at LEP [37] provides the lower bound 45.8 GeV/c^2 while from hadron colliders the $W^{\pm}$ decay width implies [38] the bound is 51 GeV/c^2. Note that the limit 89 GeV/c^2 from the Collider Detector at Fermilab (CDF) Collaboration [32] is only valid within the standard model; in particular it need not hold in a two-doublet model [31]. Likewise the upper limits to the top-quark mass of ≈ 210 GeV/c^2 inferred from radiative corrections [9,10] are also model dependent and can be significantly weakened in a two-doublet model [35].

We now indicate the range of contributions to $\{\epsilon_i\}$ calculated using the exact expressions Eqs. (14), (16), and (18)–(20); in the case of the scalar contributions these are extra with respect to the contributions of a 200 GeV/c^2 SM Higgs boson and for the top quark with respect to a 150 GeV/c^2 fixed mass top quark. The top-quark mass range is taken to be 50 to 350 GeV/c^2, with $m_b = 5$ GeV/c^2, and the SM Higgs boson taken to lie between 50 and $10M_W \approx 800$ GeV/c^2. To begin with we look at option (i) of Sec. II for the two-doublet model in which all the $\{\mu_i^2\}$ are $O(v^2)$ so that large scalar masses are generated by increasing the hard couplings $\{\lambda_i\}$. After satisfying the experimental limits discussed above, this option permits sufficient freedom to choose an arbitrary mass spectrum below about $10M_W$. (The above mass bounds should not be regarded as strict, but rather as illustrative.) Table I shows our results: using $\alpha_{\rm EM}/4\pi s^2 = 0.2737\times 10^{-2}$, $M_Z = 91.17$ GeV/c^2, $M_W = 80.22$ GeV/c^2, and $c = M_W/M_Z$.

Next we look at option (ii) of Sec. II, in which the hard couplings $\{\lambda_i\}$ are held fixed and larger scalar masses are induced by increasing the soft $\{\mu_i^2\}$ or equivalently M_+. This encompasses several particularly interesting special cases: the MSSM [25], fixed-point models [39], and the related heavy-quark condensate models [40,41]. In this situation [2] the spectrum for the neutrals contains two scalars nearly degenerate in mass with the charged scalar $M_{2,3} = M_+ + O(v^2/M_+)$ and a low-mass particle h_1:

$$M_{h_1}^2 = 2v^2[\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + (\lambda_3 + \lambda_4 + \lambda_5 \cos 2\theta) c_\beta^2 s_\beta^2] \ , \tag{28}$$

which basically separates out: $H_i^0 \approx \delta_{1,i}$ and $H_1^{1,2} \approx 0$. The particle h_1, which in general has mixed *CP* couplings to fermions, is found in Ψ_1 and behaves like the SM Higgs boson. Using this spectrum of scalars it is easy to see that all the $\{\epsilon_i^{\rm extra}\}$ vanish in their respective leading orders. That is, the corrections arising from the extended

TABLE I. The maximum and minimum values of the $\epsilon_i^{\text{extra}}$ arising from the scalar and heavy-quark sectors. A common, overall factor of 10^{-2} has been extracted.

Source:	Two scalar Doublets		One scalar Doublet		Heavy-quark Doublet	
Range:	max	min	max	min	max	min
ϵ_1	6.675	-1.613	0.065	-0.134	3.187	-0.635
ϵ_2	0.032	-0.150	0.014	-0.057	0.296	-0.231
ϵ_3	0.184	-0.165	0.058	-0.034	0.100	-0.077

scalar sector (Ψ_2) are expected to be numerically small for large masses: decoupling occurs. The only scalar particle contribution to the $\{\epsilon_i\}$ comes from the relatively low, fixed mass h_1. This behavior can be seen in the MSSM [18], for example, where $M_{h_1} \approx M_Z \cos 2\theta$. The reader should be cautioned, however, first, that in a larger theory such as the MSSM, further new particle contributions may have to be included; second, that the expression for the low-lying scalar mass is based on the tree-level formulas which may be significantly modified by loop effects: again in the MSSM [42] a sizable top-quark mass can lead to the estimate for M_{h_1} being larger than M_Z; while in the fixed-point approach [39,43] new renormalization-group equations for an effective theory containing only the light scalar's self-coupling (with boundary condition at M_+ determined by the $\{\lambda_i\}$) and any large Yukawa couplings should be used to evolve down from the scale M_+ to the appropriate, low scale v.

Having chosen to work within the approximation that only oblique corrections are important, then the $\{\epsilon_i^{\text{extra}}\}$ of the two-doublet model may be directly added to the SM predictions for a Higgs boson of mass 200 GeV/c^2 and a top quark of mass 150 GeV/c^2. These SM predictions are given below, together with the experimental values for the $\{\epsilon_i\}$:

Theory ($m_t=150$, $M_H=200$)	Experiment
$\epsilon_1=+0.45\pm 0.10\times 10^{-2}$	$\epsilon_1=-0.07\pm 0.50\times 10^{-2}$
$\epsilon_2=-0.39\pm 0.03\times 10^{-2}$	$\epsilon_2=-1.00\pm 0.97\times 10^{-2}$
$\epsilon_3=+0.42\pm 0.08\times 10^{-2}$	$\epsilon_3=-0.05\pm 0.79\times 10^{-2}$.

(29)

The experimental values are based on the results quoted in Ref. [9]; see also Ref. [20]. These experimental results were derived solely from the leptonic decays of the Z^0. Including the low-energy data on neutrino-nucleon deep-inelastic scattering or atomic parity violation in cesium slightly alters the central values of the $\{\epsilon_i\}$, in particular favoring a (more) negative ϵ_3, but this is well within the quoted errors. In fact observe that the maximum and minimum contributions to ϵ_2 and ϵ_3 are less than a third of the experimental uncertainties in these quantities. The theoretical values are also taken from Ref. [9], where the $\{\epsilon_i\}$ are defined in such a way that certain nonoblique corrections arising from the minimal SM are incorporated.

VII. CONCLUSIONS

Compact expressions for the leading radiative corrections $\{\epsilon_i\}$ in the two-scalar-doublet extension of the SM have been given, in terms of a general notation designed to accommodate the *CP*-noninvariant situation. In the special situation when *CP* conservation in the scalar sector is imposed the formulas reproduce the previously known results [3,17,30]. As anticipated the scalar contributions to ϵ_1 can be relatively large, in general showing quadratic behavior, while ϵ_2 and ϵ_3 are smaller with at most logarithmic dependence on large scalar masses. The inclusion of *CP* violation in the scalar sector does not lead to increased radiative corrections; as shown the extremal values of the $\{\epsilon_i\}$, within a given mass range, occur for *CP*-invariant configurations.

As noted the only concrete lower bound to the top mass is around $M_Z/2$. Also the upper mass bound [9,10], based on the size of the radiative corrections it induces, is not in general valid; specifically not in two-doublet models. This bound is obtained by considering any observable whose radiative corrections are dominated by ϵ_1 and so depend quadratically on m_t. However [35], the effects of a heavy top in ϵ_1 can be compensated by arranging for the scalar sector to give a large, negative contribution to ϵ_1, behaving as M_i^2. Furthermore since the scalar contributions to ϵ_2 and ϵ_3 give numerically small, subdominant corrections such a choice cannot give rise to "unwanted" consequences in any other observable. This is particularly true at the present moment [9,20], when the experimental errors on ϵ_2 and ϵ_3 are so dominant. Suppose for example that within the SM a fit to the data yields the preferred values $m_t^{(\text{SM})}$ and M_H. Now consider a two-doublet theory with the following specially selected parameters: $s_{\alpha-\beta}\approx 0 \approx M_3$, $M_1=M_H$, M_2 large, and $M_+=0.562M_2$, then using Eq. (24) the new, preferred value of $m_t^{(2\text{D})}$ would be given by

$$m_t^{(2\text{D})}=\sqrt{(m_t^{(\text{SM})})^2+0.072\times M_2^2}\ .$$

As an illustration, taking $M_2=10M_W$ would entail replacing $m_t^{(\text{SM})}=150$ GeV/c^2 by $m_t^{(2\text{D})}=262$ GeV/c^2.

Interestingly when the contribution to ϵ_1 from the two-doublet scalar sector is positive, experiment does impose relatively strong constraints on the mass spectrum, as compared to the situation in the minimal SM. This is due to the possibility of quadratic mass dependence leading to significant radiative corrections. It is clear from the experimental value given in Eq. (29) that any sizable, additional, positive contribution to ϵ_1 is strongly disfavored, which in turn may be used to place limits on the allowed parameter space. For example suppose we are given that the top quark weighs at least 150 GeV/c^2. Then a set of scalar particle masses similar to those in the maximizing configuration, for which ϵ_1 is given by Eq. (23), would require the largest scalar particle mass no greater than about $2M_W$.

Such large corrections coming from the scalar sector only occur for large masses generated by increasing the hard coupling parameters $\{\lambda_i\}$. If on the other hand the large masses were achieved by increasing the soft parameters $\{\mu_i^2\}$ then the heavy scalars would essentially decouple and give vanishing contribution to the radiative corrections $\{\epsilon_i\}$. In effect the theory would behave like the SM with a comparatively light, fixed-mass Higgs boson.

ACKNOWLEDGMENTS

The authors are pleased to acknowledge the contributions made by David Sutherland to our understanding of the work presented here. The research of C.D.F. and R.G.M. was partially funded by the U.K. Science and Engineering Research Council. The work of I.G.K. was supported by the U.S. Department of Energy, Division of High Energy Physics, Contract No. W-31-109-ENG-38.

[1] T. W. Appelquist and J. Carazzone, Phys. Rev. D **11**, 2856 (1975).
[2] C. D. Froggatt, I. G. Knowles, and R. G. Moorhouse, Glasgow Report No. GUTPA/91/10-2 (unpublished).
[3] D. Toussaint, Phys. Rev. D **18**, 1646 (1978).
[4] H. Georgi, Hadr. J. Phys. **1**, 155 (1978).
[5] D. C. Kennedy and B. W. Lynn, Nucl. Phys. **B322**, 1 (1989).
[6] H. E. Haber, G. L. Kane, and T. Sterling, Nucl. Phys. **B161**, 493 (1979).
[7] J. F. Gunion, H. E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Reading, MA, 1990).
[8] M. J. G. Veltman, Nucl. Phys. **B123**, 89 (1977); M. Chanowitz, M. Furman, and I. Hinchliffe, Phys. Lett. **78B**, 285 (1978).
[9] G. Altarelli, R. Barbieri, and S. Jadach, CERN Report No. CERN-TH 6124/91, 1991 (unpublished).
[10] J. Ellis and G. L. Fogli, Phys. Lett. B **249**, 543 (1990); V. Barger, J. L. Hewett, and T. G. Rizzo, Phys. Rev. Lett. **65**, 1313 (1991).
[11] M. J. G. Veltman, Phys. Lett. **91B**, 95 (1980); F. Antoneli, G. Corbo, and M. Consoli, *ibid.* **91B**, 90 (1980).
[12] J. J. van der Bij and M. J. G. Veltman, Nucl. Phys. **B231**, 205 (1984); J. J. van der Bij, *ibid.* **B248**, 141 (1984).
[13] M. J. G. Veltman, Acta Phys. Pol. B **8**, 475 (1977); Phys. Lett. **70B**, 253 (1977).
[14] P. Sikivie, L. Susskind, M. Voloshin, and V. Zakharov, Nucl. Phys. **B173**, 189 (1980).
[15] R. S. Lytel, Phys. Rev. D **22**, 505 (1980).
[16] M. B. Einhorn and J. Wudka, Phys. Rev. D **39**, 2758 (1989).
[17] S. Bertolini, Nucl. Phys. **B272**, 77 (1986).
[18] R. Barbieri, M. Frigeni, F. Guiliani, and H. E. Haber, Nucl. Phys. **B341**, 309 (1990).
[19] G. Altarelli and R. Barbieri, Phys. Lett. B **253**, 161 (1991).
[20] P. Langacker and D. C. Kennedy, Phys. Rev. D **44**, 1591 (1991).
[21] S. L. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977); E. A. Paschos, *ibid.* **15**, 1966 (1977).
[22] S. Weinberg, Phys. Rev. Lett. **63**, 2333 (1989); Phys. Rev. D **42**, 86 (1990).
[23] R. D. Peccei and H. R. Quinn, Phys. Rev. D **16**, 1791 (1977); S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978); J. E. Kim, *ibid.* **43**, 103 (1979).
[24] D. Chang and R. N. Mohapatra, Phys. Rev. D **32**, 85 (1985).
[25] H. P. Nilles, Phys. Rep. **110**, 1 (1984); J. F. Gunion and H. E. Haber, Nucl. Phys. **B272**, 1 (1986).
[26] L. McLerran, M. Shaposhnikov, N. Turok, and M. Voloshin, Phys. Lett. B **256**, 451 (1991); A. M. Kazarian, S. V. Kuzmin, and M. E. Shaposhnikov, CERN Report No. CERN-TH 6132/91 (unpublished).
[27] J. Liu and L. Wolfenstein, Nucl. Phys. **B289**, 1 (1987).
[28] V. Barger, J. L. Hewett, and R. J. N. Phillips, Phys. Rev. D **41**, 3421 (1990); A. J. Buras, P. Krawczyk, M. E. Lautenbacher, and C. Salazar, Nucl. Phys. **B337**, 284 (1990); G. Buchalla, A. J. Buras, M. K. Harlander, M. E. Lautenbacher, and C. Salazar, *ibid.* **B355**, 305 (1991).
[29] S. M. Barr and A. Zee, Phys. Rev. Lett. **65**, 21 (1990); D. Chang, W.-Y. Keung, and T. C. Yuan, Phys. Rev. D **43**, 14 (1991); J. F. Gunion and R. Vega, Phys. Lett. B **251**, 157 (1991); R. G. Leigh, S. Paban, and R.-M. Xu, Nucl. Phys. **B352**, 45 (1991).
[30] W. Hollik, Z. Phys. C **32**, 291 (1986); **37**, 569 (1988).
[31] S. L. Glashow and E. E. Jenkins, Phys. Lett. B **196**, 233 (1987); V. Barger and R. J. N. Phillips, Phys. Rev. D **41**, 88 (1990).
[32] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **68**, 447 (1992).
[33] M. B. Einhorn, D. R. T. Jones, and M. J. G. Veltman, Nucl. Phys. **B191**, 146 (1981).
[34] S. Bertolini and A. Sirlin, Phys. Lett. B **257**, 179 (1991).
[35] A. Denner, R. J. Guth, and J. H. Kühn, Phys. Lett. B **240**, 438 (1990).
[36] ALEPH Collaboration, D. Decamp *et al.*, CERN Report No. PPE/91-149 (unpublished).
[37] ALEPH Collaboration, D. Decamp *et al.*, Phys. Lett. B **236**, 511 (1990).
[38] UA1 Collaboration, C. Albajar *et al.*, Phys. Lett. B **253**, 503 (1991).
[39] C. T. Hill, C. N. Leung, and S. Rao, Nucl. Phys. **B262**, 517 (1985); C. D. Froggatt, I. G. Knowles, and R. G. Moorhouse, Phys. Lett. B **249**, 273 (1990).
[40] M. Luty, Phys. Rev. D **41**, 2893 (1990); M. Suzuki, in *Strong Coupling Gauge Theories and Beyond,* Proceedings of the Second International Workshop, Nagoya, Japan, 1990, edited by T. Muta and K. Yamawaki (World Scientific, Singapore, 1990).

[41] T. E. Clark, S. T. Love, and W. A. Bardeen, Phys. Lett. B **237**, 235 (1990); M. Carena, T. E. Clark, C. E. M. Wagner, W. A. Bardeen, and K. Sasaki, Fermilab Report No. FERMILAB-PUB-91/96-T (unpublished).
[42] J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. B **257**, 83 (1991); H. E. Haber and R. Hampfling, Phys. Rev. Lett. **66**, 1815 (1991); A. Yamada, Phys. Lett. B **263**, 233 (1991).
[43] R. Barbieri, M. Frigeri, and F. Caravaglios, Phys. Lett. B **258**, 167 (1991).