Baryon magnetic moments and the spin of the proton

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I derive a set of generalized Sehgal equations (GSE) which link octet-baryon magnetic moments to quark axial-vector currents in the proton. The equations are shown to be valid in a class of shell models, containing both nonrelativistic and relativistic quark models. The GSE fit magnetic-moment data better than naive quark model formulas, but not perfectly. In best fits to magnetic moments quarks carry a small fraction of the angular momentum of the proton. The precise value of this fraction is poorly determined by these fits: $\Delta u+\Delta d+\Delta s=0.27\pm0.23$; this fraction is small compared to unity, but consistent with the recent European Muon Collaboration experiments.

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It has been known for a long time that the proton is a composite system containing quarks. This raises immediately the question of how the spin of the proton is shared among these constituents. In the last few years, this question has led to a "spin crisis" which was triggered by the important polarized muon (deep-inelastic) scattering experiments at CERN by the European Muon Collaboration (EMC) [1]. This experiment, with information from baryon semileptonic decays, led to the surprising conclusion that only a small fraction of the spin of the proton is carried by the spin of the light quarks (and antiquarks) it contains.

This conclusion contradicts much prejudice enshrined in the naive quark model (NQM), where it was assumed, on the contrary, that the entire spin of the proton was carried by three (valence) quarks. Models for the baryon magnetic moments have been constructed which (using this assumption) fit the experimental data quite well [2]. One can therefore ask the question whether or not there is a contradiction between baryon magnetic moments and the EMC results, which might perhaps occur because there is some error in these new and difficult measurements. This is the subject of our paper, and we find that not only is there no contradiction, but the best fits to the baryon magnetic moments and semileptonic decay data are in remarkable agreement with analyses based on the EMC measurements.

To start with we discuss how to link baryon magnetic moments to quark axial-vector currents in the proton. The magnetic moment of a system is more model dependent than axial-vector currents, and therefore one is forced to consider models or classes of models if one wishes to link these quantities. I derive a set of equations, generalized Sehgal equations (GSE), which are valid in a class of (relativistic and nonrelativistic) quark models; this class is defined in detail in the next section. The class of models used is a simple generalization of the naive quark model and a version of the bag model.

In the second part of this paper this set of equations, together with experimental data on baryon magnetic moments and semileptonic decays, is used in an error minimization procedure to obtain a "best" fit to the data, which determines a favored set of parameters. We discuss how well the various parameters are fitted and also compare the best fit obtained in this fashion with fits constrained to follow the naïve quark model. We compare the parameters obtained from magnetic moments with those obtained using the EMC measurements. We also discuss alternative ways of analyzing the information from the magnetic moment and semileptonic decays. We present a summary of our conclusions and a discussion at the end of the paper.

I. LINKING MAGNETIC MOMENTS TO QUARK AXIAL-VECTOR CURRENTS

In general the magnetic-moment operator and the axial-vector-current operator are unrelated. However, we describe a class of shell models in which the matrix elements of these operators are closely connected. This class of models will then be used throughout the paper.

We assume that the proton contains quarks, antiquarks, and other electrically neutral particles. Only quarks and antiquarks contribute to the magnetic moment, and we assume only three flavors of quark. The magnetic-moment operator of the proton $M_z(p)$ is a sum of one-body operators, representing the contribution of every quark or antiquark. We *assume that all quarks and antiquarks of a given flavor, say u, are in a single mode of angular momentum one-half of some potential, or spherical cavity.* The quarks and antiquarks may be at rest, or moving relativistically or nonrelativistically. Then the contribution of these quarks and antiquarks to the expectation value of the magnetic moment of the proton is

$$M_z(p)=M_z(p;u)+M_z(p;d)+M_z(p;s)\ , \qquad (1)$$

where

$$M_z(p;u)=[n(u_\uparrow)-n(u\downarrow)-n(\bar{u}_\uparrow)+n(\bar{u}_\downarrow)]\tilde{\mu}_u\ , \qquad (2)$$

where $\tilde{\mu}_u$ is the expectation value of the one-body magnetic-moment operator of u quarks in the mode chosen, and $n(u_\uparrow)$ is the occupation number of u quarks with $J_z=+\frac{1}{2}$, $n(\bar{u}_\uparrow)$ the number of u antiquarks with

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$J_z=+\frac{1}{2}$, etc. The simplest case of Eq. (2) is for static quarks, when $\tilde{\mu}_u$ is the actual magnetic moment of the u quark at rest. But we could instead choose a spherical cavity, and ultrarelativistic quarks, their $\tilde{\mu}_u$ being the one-body moment in the $J=\frac{1}{2}$ mode chosen.

The contribution of these u quarks and antiquarks to the axial-vector current $A_z(p;u)$ of the proton has a similar expression:

$$\Delta u \equiv A_z(p,u)=[n(u_\uparrow)-n(u_\downarrow)+n(\bar{u}_\uparrow)-n(\bar{u}_\downarrow)]a_u \ , \qquad (3)$$

where the antiquarks contribute with different signs than in the magnetic moment (2) because of the difference under charge conjugation. In Eq. (3) a_u is the one-body matrix element of the axial-vector current for a u quark in the model. We can now use Eqs. (2) and (3) to rewrite the magnetic moment $M_z(p,u)$ in terms of Δu, if we do a little algebra. We denote the ratio of antiquark to quark polarizations by λ_μ,

$$n(\bar{u}_\uparrow)-n(\bar{u}_\downarrow)=\lambda_u[n(u_\uparrow)-n(u_\downarrow)] \ , \qquad (4)$$

so that we have

$$\Delta u = A_z(p;u)=[n(u_\uparrow)-n(u_\downarrow)](1+\lambda_u)a_u \ . \qquad (5)$$

We then substitute (4) and (5) into Eq. (2), to obtain

$$M_z(p;u)=\Delta u\frac{1-\lambda_u}{1+\lambda_u}\frac{\tilde{\mu}_u}{a_u}=\mu_u\Delta u \ , \qquad (6)$$

where we introduced the notation

$$\mu_u=\frac{1-\lambda_u}{1+\lambda_u}\frac{\tilde{\mu}_u}{a_u} \qquad (7)$$

for the "effective" magnetic moment μ_u per unit axial-vector current. Note that Eqs. (6) and (7) are not valid in the special case $\lambda_u=-1$, when the axial-vector current Δu vanishes. If Δu vanishes, the equations are invalid, since the magnetic moment $M_z(p;u)$ need not vanish.

If we now add the contribution of the other two quark flavors, we obtain the total magnetic moment of the proton

$$\mu(p)\equiv M_z(p)=\mu_u\Delta u+\mu_d\Delta d+\mu_s\Delta s \ . \qquad (8a)$$

The generalization to the neutron is simple, if SU(2) symmetry holds,

$$\mu(N)\equiv M_z(N)=\mu_d\Delta u+\mu_u\Delta d+\mu_s\Delta s \ , \qquad (8b)$$

as is the extension to the other members of the baryon octet, again assuming that SU(3) symmetry is valid for the wave functions of the baryon-octet states [such as $n(u_\uparrow)_{\text{proton}}=n(s_\uparrow)_\Xi$, etc.] in our class of models:

$$\mu(\Sigma^+)\equiv M_z(\Sigma^+)=\mu_u\Delta u+\mu_s\Delta d+\mu_d\Delta s \ , \qquad (8c)$$

$$\mu(\Sigma^-)\equiv M_z(\Sigma^-)=\mu_d\Delta u+\mu_s\Delta d+\mu_u\Delta s \ , \qquad (8d)$$

$$\mu(\Xi^0)\equiv M_z(\Xi^0)=\mu_s\Delta u+\mu_u\Delta d+\mu_d\Delta s \ , \qquad (8e)$$

$$\mu(\Xi^-)\equiv M_z(\Xi^-)=\mu_s\Delta u+\mu_d\Delta d+\mu_u\Delta s \ , \qquad (8f)$$

$$\mu(\Lambda)\equiv M_z(\Lambda)=\tfrac{1}{6}(\Delta u+4\Delta d+\Delta s)(\mu_u+\mu_d) + \tfrac{1}{6}(4\Delta u-2\Delta d+4\Delta s)\mu_s \ , \qquad (8g)$$

$$\mu(\Sigma\Lambda)\equiv M_z(\Sigma\Lambda)=-(1/2\sqrt{3})(\Delta u-2\Delta d+\Delta s)(\mu_u-\mu_d). \qquad (8h)$$

Equations (8) will be referred to as the generalized Sehgal equations (GSE) since their simplest form, for the proton and neutron, in the limit $\Delta s=0$, were given by Sehgal [3]. The NQM equations are a special case: for $\Delta u=\frac{4}{3}$, $\Delta d=-\frac{1}{3}$. The formulas (8c)–(8f) for particles at the periphery of the octet are obtained by substitutions from (8a), but the formula (8g) for Λ and (8h) for the $(\Lambda\Sigma)$ transition moment require Clebsch-Gordan coefficients or tensor methods for SU(3). We shall not present these derivations here. As explained above, the GSE hold also for bag-model wave functions with ultrarelativistic quarks and antiquarks, provided we keep the same radius R for all quarks of a given flavor, in all baryons. If the radius R is allowed to change from baryon to baryon, however, the resultant breaking of SU(3) violates Eqs. (8). This example shows that Eqs. (8), while approximate, apply to both nonrelativistic and relativistic quarks (and antiquarks); it also shows that the class of models we consider is a simple generalization of the NQM and bag model that allows antiquarks in the proton.

Versions of these equations were published by Bartelski and Rodenberg [4], Decker, Nowakowski, and Stahov [5], by Gerasimov (in a reference not accessible to this author) [5], by the authors of Ref. [6], by Choudhuri, O'Donnell, and Sarkar [7], and more implicitly by Close [8]. Carlson and Milana [4] discussed magnetic moments without these formulas, but in a similar spirit. The derivation presented here is more general than in these references.

II. FITTING MAGNETIC MOMENTS WITH THE GENERALIZED SEHGAL EQUATIONS

In this section, armed with the approximate equations (8), we shall interrogate the data, and obtain information about the various parameters in these equations. We first show that there is no exact solution to Eqs. (8) which fits all experimental data (within experimental errors). This conclusion follows from the failure of the experimental data to obey sum rules which are consequences of these equations.

The generalized Sehgal equations are formally similar to the equations for the baryon-octet masses in chiral perturbation theory [9]. In the case of the baryon masses, these are SU(3) mass-sum rules, and the sum rules for magnetic moments here are very analogous.

The first sum rule is analogous to the Gell-Mann–Okubo mass formula [10]:

$$\mu(\Sigma^+)+6\mu(\Lambda)+\mu(\Sigma^-)=2\mu(p)+2\mu(n) +2\mu(\Xi^0)+2\mu(\Xi^-) \ . \qquad (9)$$

The left-hand side of this equation is $(-2.34\pm0.06)\mu_N$ while the right-hand side equals $(-2.10\pm0.04)\mu_N$.

The second sum rule is analogous to the Coleman-Glashow sum rule [11] for the masses, and is

$$\mu(n)-\mu(p)+\mu(\Sigma^+)-\mu(\Xi^0)+\mu(\Xi^-)-\mu(\Sigma^-)=0 , \quad (10)$$

where the left-hand side of this equation is actually $(-0.49\pm0.06)\mu_N$. The failure of the data to obey the two relations (9) and (10) shows that there is no set of parameters in (8) which can fit the data perfectly. This conclusion, of course, holds also in the special case $\Delta u=\frac{4}{3}$, $\Delta d=-\frac{1}{3}$, $\Delta s=0$, which reduces formulas (8) to the usual quark model formulas.

Equations (8) appear to contain six parameters, three μ's and three Δq_i, which could be extracted from the eight experimental data points. Because of the symmetric form of the equations, this counting is wrong. The experimental quantities on the left, depend on five parameters, one being [7] the product $(\mu_u+\mu_d+\mu_s)(\Delta u+\Delta d+\Delta s)$ while the other four can be chosen to be $(\Delta u-\Delta d)$, $(\Delta u+\Delta d-2\Delta s)$, $(\mu_u-\mu_d)$, and $(\mu_u+\mu_d-2\mu_s)$. However, even this counting is somewhat deficient as it ignores the bilinear character of the equations: every term is the product of a μ_i with a Δq_i. To avoid the possibility of rescaling, we have to *fix* one of the four quantities enumerated above. We do this here by choosing [12]

$$a^{(3)}=\Delta u-\Delta d=1.26 , \quad (11)$$

the experimental value of the neutron axial-vector charge. (This is an arbitrary choice as far as the GSE's are concerned: we can just as well choose $a^{(8)}=0.6$, or some other fixed value for μ, Δq.) Then we are left with four parameters: $(\mu_u+\mu_d+\mu_s)(\Delta u+\Delta d+\Delta s)$, $(\Delta u+\Delta d-2\Delta s)$, $(\mu_u-\mu_d)$, and $(\mu_u+\mu_d-2\mu_s)$. If we make the assumption

$$\mu_u=-2\mu_d \quad (12)$$

the *number* of parameters remains unchanged, but we can now solve for

$$a^{(1)}=\Delta u+\Delta d+\Delta s \quad (13)$$

and we have three more parameters: μ_d, μ_s and

$$a^{(8)}=\Delta u+\Delta d-2\Delta s . \quad (14)$$

Therefore there is no loss of generality in making the assumption (12) for the GSE. There are *four* independent parameters which we choose to be $a^{(1)}$, $a^{(8)}$, μ_d, μ_s while $a^{(3)}$ is fixed by Eq. (11). This is to be contrasted with the NQM equations which have *three* parameters μ_d, μ_u, μ_s. In the NQM equations the restriction (12) leads to a reduction in the number of parameters from three to two. Note that the quantity $a^{(1)}$ is called in the literature $\Delta\Sigma$, a notation I avoid because of possible confusion in the baryon context.

We now proceed to find a "best" set of parameters to minimize the error in fitting Eq. (8) to the data. Before proceeding we recognize that our formulas cannot fit the experimental data with an accuracy approaching the experimental error, and therefore we assign a theoretical error which should express how close we hope to come to the experimental measurements. We estimate the theoretical error from the sum rule (10): the best fit cannot hope to be better than $0.49/6\simeq0.08\mu_N\simeq0.1\mu_N$ to any individual magnetic moment if the error is equally shared among the six baryons. We add this theoretical error to the experimental error in quadrature, and the data we fit is shown in Table I.

We search in the four-dimensional space μ_d, μ_s, $a^{(1)}$, $a^{(3)}$ to find the minimum of the quantity χ^2

$$\chi^2=\sum_{i=1}^{8}\frac{[\text{Eq. (8)}-\text{expt.}]_i^2}{(\text{error})_i^2} . \quad (15)$$

With the values from Table I we find a best fit (fit 1) given in Table II, which lists the values of the parameters obtained, and the fitted magnetic moments. The errors on individual magnetic moments are at most $0.11\mu_N$, in line with our expectations. The χ^2 for this fit is 4.36 for four degrees of freedom (eight data points minus four parameters). The confidence level for this fit is about 35%. The most interesting feature is that the minimum is reached for a rather small value of $a^{(1)}\simeq0.27$. We can ask how well the magnetic moments determine the value of $a^{(1)}$ by repeating the minimization with different values of $a^{(1)}$, and looking at the change in χ^2. If we take $a^{(1)}=0.04$ or $a^{(1)}\simeq0.50$ the χ^2 increases to 5.42 (from 4.42). So we see that $a^{(1)}$ is very poorly determined from magnetic moments, and we write

$$a^{(1)}=0.27\pm0.23 . \quad (16)$$

This result is compatible with analyses using the EMC effect [1]. For example, Ross [1] quotes (0.12 $\pm0.09\pm0.14$) for this quantity. The value of $a^{(8)}$ we find from the fits is much better determined and it is

$$a^{(8)}\simeq0.86\pm0.05 . \quad (17)$$

This value of $a^{(8)}$ differs significantly from the values inferred from hyperon beta decay, where typically $a^{(8)}\simeq0.60\pm0.05$. The only point to note about the parameters μ_d and μ_s, is that their ratio remains rather close to 0.6, the value traditionally found in quark model fits, which equals the constituent mass ratio.

We compare this best fit, discussed above (fit 1) with the best naïve quark model fit (fit 2), where we insist that $a^{(1)}=1$, $a^{(8)}=1$, $g_A=\frac{5}{3}$, the traditional values. We then

TABLE I. Data on magnetic moments, in nuclear magnetons.

Particle	Magnetic moment	Exptl. error	Total error[a]
p	2.79	±0.00	±0.10
n	−1.91	±0.00	±0.10
Σ^+	2.48	±0.05	±0.11
Σ^-	−1.16	±0.03	±0.10
Ξ^0	−1.25	±0.03	±0.10
Ξ^-	−0.68	±0.03	±0.10
Λ	−0.61	±0.01	±0.10
$\Lambda\Sigma$	−1.61	±0.08	±0.13

[a] A theoretical error of $0.1\mu_N$ has been added in quadrature to all experimental errors.

TABLE II. Best fits to magnetic moments (in μ_B). $N_{\rm DF}$ denotes number of degrees of freedom.

Particle	Magnetic moment	Fit 1 GSE	Fit 2 NQM
p	2.79 ± 0.10	2.69	2.68
n	-1.91 ± 0.10	-1.85	-1.92
Σ^+	2.48 ± 0.11	2.59	2.55
Σ^-	-1.16 ± 0.10	-1.22	-1.13
Ξ^0	-1.25 ± 0.10	-1.33	-1.40
Ξ^-	-0.68 ± 0.10	-0.61	-0.48
Λ	-0.61 ± 0.10	-0.59	-0.59
$\Lambda\Sigma$	-1.6 ± 0.13	-1.53	-1.60
$\chi^2/N_{\rm DF}$		4.42/4	7.53/5
$a^{(1)}=\Delta u+\Delta d+\Delta s$:	0.12 ± 0.17	0.28	1.00 (input)
$a^{(8)}=\Delta u+\Delta d-2\Delta s$:	0.60 ± 0.05	0.86	1.00 (input)
$g_A=\Delta u-\Delta d$:	1.26 ± 0.01	1.26 (input)	1.67 (input)
μ_u		$+2.42$	1.76
μ_d:		-1.21	-1.00
μ_s:		-0.71	-0.61

search for a minimum of χ^2 by varying μ_u, μ_d, μ_s. The value of χ^2 at the minimum is significantly larger, $\chi^2\simeq 7.53$, corresponding to a confidence level of only 17% for this fit. The worst fitted magnetic moment (Ξ^-) is about $0.2\mu_N$ in error from the experimental value. The confidence level of this fit is about two times smaller than that of the best fit. In addition fit 1 has the virtue of having the correct axial-vector coupling for the neutron, $g_A=1.26$, whereas for fit 2 we choose $g_A\simeq 1.67$. If we would include the contribution of g_A to the χ^2 evaluated in (15), the NQM value would jump by 16 units, making the confidence level of the NQM fit 0.001, while the confidence level of the GSE fit would remain at 35%. With the condition (12) and only two parameters μ_d, μ_s the NQM equations give a fit with $\chi^2\simeq 10.9$ and a confidence level of less than 10%.

The quantity $a^{(1)}$ can be computed also directly from Eqs. (8) if we form the linear combinations:

$$\mu(p)+\mu(\Xi^0)+\mu(\Sigma^-)=(\mu_u+\mu_d+\mu_s)(\Delta u+\Delta d+\Delta s)\ , \tag{18}$$

$$\mu(n)+\mu(\Sigma^+)+\mu(\Xi^-)=(\mu_u+\mu_d+\mu_s)(\Delta u+\Delta d+\Delta s)\ . \tag{19}$$

These equations lead immediately to the Coleman-Glashow sum rule (10). Within SU(3) symmetry both sums in (18) and (19) would have to vanish because the sum of magnetic moments ($\mu_u+\mu_d+\mu_s$) is proportional to the sum of the electric charges ($Q_u+Q_d+Q_s$) which vanishes. In actual fact, both sums when evaluated from the data do not vanish, and are not equal to one another since (10) is violated. We can evaluate $a^{(1)}$, if we assume Eqs. (8), and estimate ($\mu_s-\mu_d$) from the difference of magnetic moments of Σ^+ and the proton, and the axial-vector matrix element for the semileptonic decay ($\Sigma^-\rightarrow N$). This gives the formula

$$a^{(1)}=\frac{\mu(p)+\mu(\Sigma^+)+\mu(\Xi^0)+\mu(\Xi^-)+\mu(\Sigma^-)+\mu(n)}{2[+\mu(p)-\mu(\Sigma^+)]/g_A(\Sigma^-\rightarrow n)}\ , \tag{20}$$

which gives the numerical estimate

$$a^{(1)}=\frac{-0.34\pm0.24}{2\times(0.31/-0.34)}\simeq 0.19\pm0.24 \tag{21}$$

where we have used in the numerator of expression (20) the *theoretical* error ±0.10, for the magnetic moment of each baryon. The estimate (21) is similar to the estimate (16), which was obtained from fitting all magnetic moments.

Before ending this section, we can solve Eqs. (11), (16), and (17) to obtain the individual Δu, Δd, Δs:

$$\begin{aligned}\Delta u &\simeq 0.86\pm0.12\ ,\\ \Delta d &\simeq -0.40\pm0.12\ ,\\ \Delta s &\simeq -0.20\pm0.05\ .\end{aligned} \tag{22}$$

These values are quite similar to figures found in the literature, based on the EMC effect [1].

III. DISCUSSION AND SUMMARY

In the first two sections of this paper we presented a proof of the GSE, and used them to extract information from magnetic moments. Of course, just the fact that we can prove some equations in the context of a particular class of quark models does not imply that the models and the equations are reasonable approximations to real protons. We stress first of all that the models we consider include the bag model (in a simple version), the NQM, and their simplest extensions which contain antiquarks, so that they are in no way exotic or contrived. Second, the result obtained for $a^{(1)}$ is fairly consistent with information based on the EMC. Thus there is a reasonable chance that these models, and the GSE are on the right track.

We next ask about the relationship of the SU(3) singlet $a^{(1)}$ extracted from magnetic moments with the gluon anomaly expected in QCD [13]. The result (16) or (21) is specific to the models in which the GSE are valid. But these models are independent of QCD, as they did not even assume the existence of gluons. And so at first sight there is no relation to the gluon anomaly. However, one can make more speculative comments. If *these* models are good approximations for the physical proton, some other, neutral particles *must* make up the missing angular momentum, since the quarks and antiquarks have been fully taken into account. Therefore a sizable fraction of the angular momentum of the proton even at rest, must be in neutral particles.

We also mention that we have carried out numerical fits without the introduction of "theoretical error." The conclusions drawn do not change significantly, but the fits are then better for the baryons with the best measured magnetic moments, p, n, Λ, at the expense of the other baryons. There is no particular merit in fitting data in this way: one would in fact *guess* that the GSE should fit better the Ξ's which have more "heavy" quarks, rather than the p, n which have more light quarks.

Our numerical fits may be extended in several ways. For example, we can include in the data set the quantity measured in the EMC experiments $(4\Delta u+\Delta d+\Delta s)/9$ to be fitted simultaneously with the magnetic moments. This gives a slightly tighter determination of the quantity $a^{(1)}=0.24\pm0.21$. We do not present this fit, since the magnetic moments are not changed significantly from the fit presented in Table II. A more interesting exercise is to include $g_A\equiv a^{(3)}$ and $a^{(8)}$ both into χ^2 and as parameters to be fitted, again assuming some theoretical errors. We have done this as well, and one obtains a "compromise" set of values for $a^{(3)}=1.08$, $a^{(8)}=0.76$ which "split" the error between the two observables: again the magnetic moments in this fit are very similar to the fit of Table II. The fit to magnetic moments is only sensitive [14] to the ratio $a^{(8)}/a^{(3)}$, which is about 0.675 in Table II (fit 1), whereas it is 0.695 for the values of $a^{(3)}$, $a^{(8)}$ given above. This may also help explain why the NQM (fit 2) is reasonable, since $a^{(8)}/a^{(3)}=0.60$, not too far from the GSE fit.

A summary of our main results is as follows

(1) The magnetic moments of baryons can be linked to quark axial-vector currents within a well-defined framework.

(2) The magnetic moments of the octet baryons are compatible with the EMC results.

(3) The best fits to magnetic moments favor a small fraction of the proton spin carried by quark spins, but the precise value is badly determined.

(4) These best fits to magnetic moments are significantly better than the naïve quark model fits, even though the fits incorporate information about axial-vector currents.

Our main conclusion, Eqs. (16) and (21), are both unexpected and unwelcome. They indicate that the source of the weakness of the NQM as an approximation to baryons stems from the weakness of the NQM as an approximation to constituent quarks. It would be interesting to construct models which correspond to fit 1 in Table II.

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[1] J. Ashman *et al.*, Phys. Lett. B **206**, 364 (1988); Nucl. Phys. **B328**, 1 (1989). For reviews, see G. G. Ross, in *Proceedings of the XIVth International Symposium on Lepton and Photon Interactions,* Stanford, California, edited by M. Riordan (World Scientific, Singapore, 1990); R. L. Jaffe and A. Manohar, Nucl. Phys. **B337**, 509 (1990).

[2] M. A. B. Beg, B. W. Lee, and A. Pais, Phys. Rev. Lett. **13**, 514 (1964); G. Morpurgo, Physics (N.Y.) **2**, 95 (1965); W. Thirring, Acta Phys. Austriaca, Suppl. **II**, 205 (1965). For a recent review, see Lee Brekke and Jonathan L. Rosner, Comments Nucl. Part. Phys. **18**, 83 (1988).

[3] L. M. Sehgal, Phys. Rev. D **10**, 1663 (1974).

[4] J. Bartelski and Rudolf Rodenberg, Phys. Rev. D **41**, 2800 (1990); see also Carl E. Carlson and Joseph Milana, *ibid.* **40**, 3122 (1989).

[5] R. Decker, M. Nowakowski, and J. Stahov, Nucl. Phys. **A512**, 626 (1990); S. Gerasimov, International Meeting on Elementary Particle Physics, Alushta, U.S.S.R., 1987 as quoted by M. D. Scadron, Arizona report, 1991 (unpublished).

[6] G. Karl and M. D. Scadron, in Proceedings of the Twelfth Annual Montreal-Rochester-Syracuse-Toronto Meeting, Montreal, Canada, 1990, edited by B. Margolis and P. Valin (unpublished), p. 105.

[7] D. Choudhuri, P. J. O'Donnell, and V. Sarkar, Toronto Report No. UTPT-91-02 (unpublished).

[8] F. E. Close, Nucl. Phys. **A497**, c109 (1989).

[9] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[10] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Prog. Theor. Phys. **27**, 949 (1962); J. Franklin, Phys. Rev. D **20**, 1742 (1979).

[11] S. Coleman and S. L. Glashow, Phys. Rev. Lett. **6**, 423 (1961); J. Franklin, Phys. Rev. **182**, 1607 (1969).

[12] J. D. Bjorken, Phys. Rev. **148**, 1479 (1966).

[13] G. Altarelli and G. G. Ross, Phys. Lett. B **212**, 391 (1988); R. D. Carlitz, J. C. Collins, and A. H. Mueller, *ibid.* **214**, 229 (1988).

[14] C. Avenarius, Phys. Lett. B (to be published).