Higgs effect in SU(15) grand unified theory

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(Received 11 October 1991)

We analyze proton decay and the choice of Higgs fields in grand unified theories where the baryon number is a gauge symmetry [e.g., SU(15)]. Although Higgs effects forbid low-energy unification in the SU(15) model (within the extended survival hypothesis) as claimed by Frampton and Lee, other breaking patterns exist which allow unification at 10^9 GeV, as well as chiral color symmetry, quark-lepton ununified symmetry, and baryon-number symmetry breaking at the TeV scale, without any observable proton decay.

PACS number(s): 12.10.Dm, 11.30.Ly, 12.15Cc, 14.80.Gt

The most important prediction of grand unified theories (GUT's) is proton decay. In models such as SU(5) or SO(10), the coupling constants for the $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$ groups evolve to $M_u \sim 10^{14}$ GeV or higher before they get unified. Proton decay in these models is predicted to occur with a rate of the order of the present experimental limit for such M_u. However, the nonobservation of proton decay has made these theories less attractive.

Recently interest in GUT's has been revived [1–4] again following an observation that at least one symmetry-breaking chain of a GUT based on the group SU(15) can be unified at a very low energy $M_u \sim 10^7$ GeV [1]. Because the baryon number B is a gauge symmetry in this model, proton decay can be suppressed, and one possible Higgs structure has been proposed to this end [2]. This was true for SU(16) GUT [5] also, where it was also known that the proton lifetime can be large [6]. Low-energy unification makes these models free from problems of grand unified monopoles [3] and the gauge hierarchy problem is also much less severe.

All the present activity on SU(15) GUT relies on two important claims: namely, (i) there exists at least one symmetry-breaking pattern of SU(15) grand unification, where the gauge coupling constants evolve very fast and can be unified at an energy scale $M_u \sim 10^7$ GeV and (ii) there exists at least one choice of Higgs fields which can (a) allow the above symmetry-breaking chain, (b) forbid any gauge-boson-mediated proton decay, (c) suppress Higgs-boson-mediated proton decay, and (d) make this low-energy unification consistent with the nonobservation of proton decay.

Here we analyze these two claims. We discuss in a general way proton decay and the choice of Higgs fields required for any symmetry breaking in these GUT's along with their effect on the evolution of the gauge coupling constants. We find this cannot be neglected: for SU(15), unification below $M_u \sim 10^{14}$ GeV is impossible for the breaking pattern proposed by Frampton and Kephart (FK) [2], if one assumes the extended survival hypothesis [7] to calculate the effect of the Higgs boson in the evolution of the coupling constants. However, other interesting patterns exist which yield unification at $\sim 10^9$ GeV and violate baryon-number symmetry $U(1)_B$ at about the electroweak breaking scale, although there is no proton decay. The low-energy (~ 250 GeV) symmetry includes phenomenologically interesting chiral color symmetry [8] and quark-lepton non-unified electroweak symmetry [9].

We denote the group $SU(n)^q_L \times SU(m)^l_R \times U(1)_X$ as n^q_L–m^l_R–1_X where the subscript implies either the charge of the U(1) group or that right- (left-) handed particles are nonsinglets under $SU(n)$ ($SU(m)$) and the superscript q (l) means that only quarks (leptons) tranform under this group. The gauge coupling constants of the groups $SU(n)^q_L$ and $U(1)_X$ will be writen as $\alpha_{nqL} = g^2_{nqL}/4\pi$ and $\alpha_{1X} = g^2_{1X}/4\pi$ respectively. For the breaking $G_i \longrightarrow G_{i-1}$, the G_{i-1} singlet component of the Higgs field ϕ_i acquires a vacuum expectation value (VEV) at a scale M_i. 1^n denotes a totally antisymmetric nth-rank tensor; hence $1^m 1^n$ denotes a Young tableaux of m and n in the first and second columns respectively.

We consider the pattern $G_1[15] \xrightarrow{\langle\phi_1\rangle} G_2[12^q\text{–}3^l] \xrightarrow{\langle\phi_2\rangle} G_3[6_L\text{–}6_R\text{–}1_B\text{–}3_l] \xrightarrow{\langle\phi_3\rangle} G_4[3_{cL}\text{–}2^q_L\text{–}6_R\text{–}1_B\text{–}3_l] \xrightarrow{\langle\phi_4\rangle} G_5[3_{cL}\text{–}2^q_L\text{–}3_R\text{–}1_R\text{–}1_B\text{–}2^l_L\text{–}1^l_Y] \xrightarrow{\langle\phi_5\rangle} G_6[3_c\text{–}2_L\text{–}1_B\text{–}1_{Y'}] \xrightarrow{\langle\phi_6\rangle} G_7[3_c\text{–}2_L\text{–}1_Y] \xrightarrow{\langle\phi_7\rangle} G_8[3_c\text{–}1_Q]$, with $\langle\phi_i\rangle = M_i$. We shall denote this pattern by {1234567}; the pattern of Ref. [1] is {1267}, for which $M_3 = M_4 = M_5 = M_6$.

We turn next to the Higgs fields required to ensure this pattern, taking minimal representations whenever possible. Our Higgs structure is very similar to that of FK [2]. We choose ϕ_1 to be a 1^3, i.e., a **455**-plet. The

G_2 singlet component of ϕ_1 can then acquire a VEV to break the group $G_1 \longrightarrow G_2$. The VEV of the G_3 singlet component of $1^{14}1$ (**224**-plet) can break G_2, leaving $U(1)_B$ unbroken. Breaking $SU(6)_L$ to its special maximal subalgebra $SU(3)_{cL} \otimes SU(2)_L^q$ requires a somewhat large Higgs representation. Although self-conjugate representations can break any group to its maximal subalgebra, in this case the adjoint representation does not work and the next higher dimensional self-conjugate representation is required. These are contained in the self-conjugate representations of the higher groups, and the particular $6_L \longrightarrow 3_{cL}$-$2_L^q$ symmetry breaking can be accomplished with a **10800**-dimensional ($1^{13}1^2$) Higgs field of SU(15) which is contained in $\mathbf{105} \otimes \overline{\mathbf{105}}$. This is the lowest-dimensional Higgs field to break $G_3 \longrightarrow G_4$; FK considered a **14175**-plet $(1^{14}1^{14}11) \subset \mathbf{120} \otimes \overline{\mathbf{120}}$, i.e., the next-highest one. Appropriate components of the adjoint (**224**-plet) can break $G_4 \longrightarrow G_5$. For the next stage a 1^3 ($\phi_5 \equiv$**455**-plet) can be used; this breaks global lepton number in addition to the local groups.

The surviving group is now G_6 [3_c-2_L-$1_{Y'}$- 1_B]. Note that $U(1)_{Y'}$ is orthogonal to $U(1)_B$, while the hypercharge Y in the standard model does not commute with B. In fact Y is a linear combination of B and Y'. FK break G_6 with a 1^5 (**3003**-plet) by giving a VEV to the $Y = 0$ component labeled (10,11,12,13,14). To find out whether there exists any lower dimensional Higgs representation one can check that it is not possible to write any B-violating operator only with the fermions invariant under G_6. However with a 1^3 (**455**-dimensional) or a 1^4 (**1365**-dimensional) Higgs field there exists a G_6-invariant B-violating dimension-7 operator. But under G_7 one can write down B-violating dimension-6 operators only with fermions. Hence one can have $\phi_6 = \mathbf{455}$ or **1365**. Both have B and Y' nonzero; the $Y = 0$ component can acquire a VEV. Either of $\phi_7 = \mathbf{105}$ or a **120** can be used to break the standard electroweak symmetry; FK had considered both for this purpose, but this is not necessary.

Considering next proton decay, since quark-lepton unification is broken at a scale M_1, the leptoquark gauge bosons (X_μ) acquire a mass $\approx M_1$, while the diquark bosons (Y_μ) acquire mass at a scale where the quark-antiquark unification is broken ($\approx M_2$). Since $U(1)_B$ is a local gauge symmetry X_μ and Y_μ do not mix at this level. These transform under G_3 as $X_\mu \equiv [(6,1,\frac{1}{3},\overline{3})+(1,\overline{6},-\frac{1}{3},\overline{3})+(1,6,\frac{1}{3},3)+(\overline{6},1,-\frac{1}{3},3)]$ and $Y_\mu \equiv [(6,6,\frac{2}{3},1)+(\overline{6},\overline{6},-\frac{2}{3},1)]$, with $m_X^2 \sim \langle\phi_1\rangle$ and $m_Y^2 \sim \langle\phi_2\rangle$. The mixing between X_μ and Y_μ takes place when the Higgs fields ϕ_a and ϕ_b acquire VEV's in the term $X_\mu\phi_a Y^\mu\phi_b \subset D_\mu\phi_a D^\mu\phi_b$. Since X_μ and Y_μ carry different B, the mixing can occur only at M_6, suppressing the amplitude for gauge-boson-mediated proton decay $= O(M_5M_6/M_1^2M_2^2)$. Thus if $M_1 \approx M_2 \approx M_u$ and $M_5 \approx M_6 \approx 10^2$ GeV, then $M_u \geq 10^9$ GeV from the present limit on the proton lifetime.

Now both X_μ and Y_μ are contained in the SU(15) gauge boson G_μ, which transforms as a self-adjoint **224**-plet of SU(15). As a result, the SU(15) multiplets $\phi_a(\supset \Phi_a)$ and $\phi_b(\supset \Phi_b)$ can allow the coupling $X^\mu\Phi_a Y_\mu\Phi_b$ iff $\phi_a = \phi_b^\dagger(\equiv \phi)$. If only one component of ϕ acquires a VEV, i.e., the Higgs multiplet that breaks $U(1)_B$ takes part in no other symmetry breaking, then $\langle\Phi_a\rangle$ and $\langle\Phi_b\rangle = \langle\Phi_a^\dagger\rangle$ will carry equal and opposite B, forbidding mixing between X_μ and Y_μ. Gauge-boson-mediated proton decay is then absent, at least to this order. Couplings of $\phi_a^\dagger$ with other Higgs fields will determine the higher-order terms. Since ϕ_a is the Higgs field which breaks $U(1)_B$, in our case $\phi_a = \phi_6 = \mathbf{1365}$. The couplings of $\mathbf{1365}^\dagger$ of the form $\langle\mathbf{1365}\rangle\langle\mathbf{1365}^\dagger\rangle$ with other Higgs fields cannot have any B-violating effect. If we also consider $\phi_7 = \mathbf{120}$ then the only $U(1)_B$-breaking term is of the form $\langle\mathbf{1365}\rangle\langle\mathbf{1365}\rangle\langle\mathbf{1365}\rangle\langle\mathbf{455}\rangle$, for which $B = 3$. Thus this also cannot contribute to proton decay. Since there is no linear coupling of **1365** with other Higgs fields, in this scenario there is absolutely no gauge-boson-mediated proton decay with $\phi_6 = \mathbf{1365}$ and $\phi_7 = \mathbf{120}$. If different components of the same Higgs field [which break $U(1)_B$] acquire VEV's, then there can be gauge-boson-mediated proton decay: for example, if $\phi_6 = \mathbf{455}$, then since $\phi_5 = \mathbf{455}$, mixing between X_μ and Y_μ will occur. The amplitude will be proportional to $\sim \langle\phi_5\rangle\langle\phi_6\rangle/M_1^2M_2^2$, which is not suppressed by Yukawa couplings.

There is no straightforward way to understand the Higgs-boson-mediated proton decay; such processes will depend on the choice of all the Higgs fields in the theory. For ϕ_6, the types of operators which can lead to proton decay are of the form $\psi\psi\psi\psi\langle\phi_6\rangle$. But the Higgs fields necessary to couple the fermions with $\phi_6 = \mathbf{1365}$ are **105** dimensional, and ϕ_6 does not have any linear couplings with combinations of other Higgs fields; hence this operator cannot give rise to proton decay. Considering higher-dimensional operators, with one ϕ_6 there does not exist any other higher dimensional operator, and as a result there is also no Higgs-boson-mediated proton decay for this choice. Hence to avoid proton decay we choose $\phi_6 = \mathbf{1365}$ and $\phi_7 = \mathbf{120}$.

We next compute the effect of the Higgs fields considered in the evolution of the coupling constants [10]. We use the one-loop renormalization-group equations which have the form

$$\mu\frac{d\alpha_i(\mu)}{d\mu} = 2\beta_i\alpha_i^2(\mu), \tag{1}$$

where $\alpha_i = g_i^2/4\pi$, the β functions are defined as, $\beta_i = -b_i/(4\pi)$, and $b_i = T_g[i] - \frac{4}{3}T_f[i] - \frac{1}{6}T_s[i]$, corresponding to the contributions from gauge bosons, fermions, and Higgs scalars, respectively. The fermionic contributions to the various subgroups are the same and are given by $T_f = n_f$, where n_f is the number of generations; these cancel out in the equation of $\sin^2\theta_w$ and $(1-\frac{8}{3}\alpha/\alpha_s)$. The gauge contributions are

$$\begin{gathered}
T_g[12] = 176,\\
2T_g[3_{cL}] = 2T_g[3_R] = T_g[3^l] = 4T_g[3_c] = 44,\\
T_g[6_L] = T_g[6_R] = 88,\\
3T_g[2_L^q] = T_g[2_L^l] = 4T_g[2_L] = \tfrac{88}{3},
\end{gathered} \tag{2}$$

TABLE I. Contributions to $T_s[n]$ at various scales.

$M_1 \to M_2$	$M_2 \to M_3$	$M_3 \to M_4$	$M_4 \to M_5$	$M_5 \to M_6$	$M_6 \to M_7$
$[12] = 3052$	$[6_L] = 264$	$[3_{cL}] = [2^q_L] = 48$	$[3_{cL}] = [2^q_L] = [3_R] = 18$	$[3_c] = 0$	$[3_c] = 0$
$[3_l] = 608$	$[6_R] = 114$	$[6_R] = 114$	$[1_R] = 18.33$	$[2_L] = 0.5$	$[2_L] = 0.5$
	$[1_B] = 93$	$[1_B] = 93$	$[1_B] = 1.5$	$[1_B] = 1.5$	$[1_Y] = .3$
	$[3_l] = 136$	$[3_l] = 136$	$[1_l] = 13.33$	$[1_{Y'}] = .5$	
			$[2^l_L] = 36$		

with $T_g = 0$ for all U(1) groups. For our choice of Higgs fields the T_s are given in Table I. To include the Higgs contributions we assumed the extended survival hypothesis [7] and the Appelquist-Carazzone decoupling theorem [11] (standard assumptions made in calculating Higgs effects in evolution of coupling constants).

Denoting $\alpha_G^{-1}(M_J)$ by $\mathcal{A}_G(J)$, we employ the appropriate boundary conditions: (i) $\mathcal{A}_{12}(1) = \mathcal{A}_{3l}(1) = \mathcal{A}_{15}(1)$, (ii) $\mathcal{A}_{6L}(2) = \mathcal{A}_{6R}(2) = \mathcal{A}_{1B}(2) = \mathcal{A}_{12}(2)$, (iii) $\mathcal{A}_{3cL}(3) = \mathcal{A}_{2qL}(3) = \mathcal{A}_{6L}(3)$, (iv) $\mathcal{A}_{1R}(4) = \mathcal{A}_{3R}(4) = \mathcal{A}_{6R}(4)$, and $\mathcal{A}_{2l}(4) = \mathcal{A}_{1l}(4) = \mathcal{A}_{3l}(4)$, (v) $\mathcal{A}_{3c}(5) = \frac{1}{2}\mathcal{A}_{3cL}(5) + \frac{1}{2}\mathcal{A}_{3R}(5)$; $\mathcal{A}_{2L}(5) = \frac{3}{4}\mathcal{A}_{2qL}(5) + \frac{1}{4}\mathcal{A}_{2l}(5)$ and $\mathcal{A}_{1Y'}(5) = \frac{1}{2}\mathcal{A}_{1R}(5) + \frac{1}{2}\mathcal{A}_{1l}(5)$, (vi) $\mathcal{A}_{1Y}(6) = \frac{9}{10}\mathcal{A}_{1Y'}(6) + \frac{1}{10}\mathcal{A}_{1B}(6)$. With this information we can relate the SU(15) coupling constants (at energy $M_u \approx M_1$) to the low-energy ($M_7 \approx M_w \approx 10^2$ GeV) $SU(3)_c \times SU(2)_L \times U(1)_Y$ coupling constants:

$$\alpha_{3c}^{-1}(M_W) = \alpha_{15}^{-1}(M_1) + 2\beta_{12}\ln\left(\frac{M_1}{M_2}\right) + (\beta_{6L} + \beta_{6_R})\ln\left(\frac{M_2}{M_3}\right) + (\beta_{3cL} + \beta_{6R})\ln\left(\frac{M_3}{M_4}\right) + (\beta_{3cL} + \beta_{3R})\ln\left(\frac{M_4}{M_5}\right) + 2\beta_{3c}\ln\left(\frac{M_5}{M_6}\right) + 2\beta_{3c}\ln\left(\frac{M_6}{M_W}\right), \quad (3)$$

$$\alpha_{2L}^{-1}(M_W) = \alpha_{15}^{-1}(M_1) + (\tfrac{3}{2}\beta_{12} + \tfrac{1}{2}\beta_{3l})\ln\left(\frac{M_1}{M_2}\right) + (\tfrac{3}{2}\beta_{6L} + \tfrac{1}{2}\beta_{3l})\ln\left(\frac{M_2}{M_3}\right) + (\tfrac{3}{2}\beta_{2qL} + \tfrac{1}{2}\beta_{3l})\ln\left(\frac{M_3}{M_4}\right) + (\tfrac{3}{2}\beta_{2qL} + \tfrac{1}{2}\beta_{2lL})\ln\left(\frac{M_4}{M_5}\right) + 2\beta_{2L}\ln\left(\frac{M_5}{M_6}\right) + 2\beta_{2L}\ln\left(\frac{M_6}{M_W}\right), \quad (4)$$

$$\alpha_{1Y}^{-1}(M_W) = \alpha_{15}^{-1}(M_1) + (\tfrac{11}{10}\beta_{12} + \tfrac{9}{10}\beta_{3l})\ln\left(\frac{M_1}{M_2}\right) + (\tfrac{9}{10}\beta_{6R} + \tfrac{1}{5}\beta_{1B} + \tfrac{9}{10}\beta_{3l})\ln\left(\frac{M_2}{M_3}\right) + (\tfrac{9}{10}\beta_{6R} + \tfrac{1}{5}\beta_{1B} + \tfrac{9}{10}\beta_{3l})\ln\left(\frac{M_3}{M_4}\right) + (\tfrac{9}{10}\beta_{1R} + \tfrac{1}{5}\beta_{1B} + \tfrac{9}{10}\beta_{1l})\ln\left(\frac{M_4}{M_5}\right) + (\tfrac{9}{5}\beta_{1Y'} + \tfrac{1}{5}\beta_{1B})\ln\left(\frac{M_5}{M_6}\right) + 2\beta_{1Y}\ln\left(\frac{M_6}{M_W}\right). \quad (5)$$

The relevant linear combinations are those which yield $\sin^2(\theta_W) = \frac{3}{8} - \frac{5}{8}\alpha(\alpha_{1Y}^{-1} - \alpha_{2L}^{-1})$ and $(1 - \frac{8}{3}\alpha/\alpha_s) = \alpha(\alpha_{2L}^{-1} + \frac{5}{3}\alpha_{1Y}^{-1} - \frac{8}{3}\alpha_{3c}^{-1})$: namely,

$$2.4(16\pi^2) = (52.8 - 162.9h)\ln(M_{12}) + (35.2 - 36.7h)\ln(M_{23}) + (17.2h - 82.1)\ln(M_{34}) + (29.3 - 2.7h)\ln(M_{45}) + (14.7 + 0.1h)\ln(M_{56}) + (14.7 - 0.1h)\ln(M_{6W}), \quad (6)$$

$$8.3(16\pi^2) = (264 - 814.7h)\ln(M_{12}) + (117.3 - 23h)\ln(M_{23}) + (58.7 - 19h)\ln(M_{34}) + (88 - 0.5h)\ln(M_{45}) + (44 + 0.5h)\ln(M_{56}) + (44 + 0.3h)\ln(M_{6W}), \quad (7)$$

where $h = 0$ denotes the pure gauge case and $h = 1$ includes Higgs effects. Here $M_{ij} \equiv M_i/M_j$ and the current experimental values [12] of $\sin^2\theta_W (= 0.232)$ and $\alpha_s (= 0.11)$ have been used.

For the pattern {1267} the unification scale $M_1 \approx M_u \approx 10^7$ GeV in the pure gauge case, which is the FL result [1]. Large gauge contributions to the evolution equations enhance the coefficients of the first two terms; as a result, unification is reached faster than in the usual GUT's such as SU(5) (for which $M_1 = M_2 = M_A = M_u \sim 10^{14}$ GeV). However, when Higgs effects are included ($h = 1$) we find no solution to (6) and (7) for the {1267} scenario other than $M_1 = M_u \geq 10^{14}$ GeV, forbidding the low-energy unification of FL.

For $h = 1$ we find three other interesting three-stage patterns: (a) {2467} with $M_1 = M_2 = M_u$, $M_3 = M_4 =$

TABLE II. Mass scales (in GeV) for patterns (A)-(C).

	{2467}		{3467}		{2567}	
M_y	M_u	M_x	M_u	M_x	M_u	M_x
250	7.91×10^8	2.96×10^8	8.87×10^8	3.50×10^2	1.97×10^{14}	1.77×10^3
500	1.11×10^9	4.06×10^8	1.25×10^9	7.05×10^2	1.98×10^{14}	3.53×10^3
1000	1.56×10^9	5.56×10^8	1.76×10^9	1.42×10^3	1.98×10^{14}	7.05×10^3
1500	1.91×10^9	6.68×10^8	2.15×10^9	2.15×10^3	1.99×10^{14}	1.06×10^4

M_x, $M_5 = M_6 = M_y$; (b) {3467} with $M_1 = M_2 = M_3 = M_u$, $M_4 = M_x$, $M_5 = M_6 = M_y$; and (c) {2567} with $M_1 = M_2 = M_u$, $M_3 = M_4 = M_5 = M_x$, $M_6 = M_y$ each having a 1-parameter family of solutions for M_y. [Although (c) does not have full unification at low energy, it does have interesting TeV physics.] Sample values are given in Table II.

The most interesting pattern is {3467}, which has both low-energy unification at $\sim 10^9$ GeV and interesting TeV physics. We can decouple the electroweak breaking scale with the other symmetry breakings and have TeV scale chiral color symmetry and the quark-lepton ununified electroweak symmetry breaking, which will raise the unification scale a little. The existence of chiral color symmetry at the TeV scale or lower will imply the presence of axigluons, whose phenomenological consequences have been studied [13]. The presence of the ununified electroweak symmetry at low energy will imply the existence of extra charged and neutral gauge bosons, whose mixing with the Z boson will affect various asymmetry parameters in the e^+e^- deep-inelastic scattering [14].

To summarize, we have shown that Higgs fields play a significant role in the evolution of gauge coupling constants in GUT's where baryon number is a symmetry. The consistency of the symmetry-breaking scenario presented here with present-day proton-decay data along with its interesting TeV scale physics make it a model worthy of further investigation.

Note added. After completion of this work we received a paper from Dr. Palash Pal [15], who has also studied the question of proton decay in SU(15). However, he did not consider the Higgs **1365**-plet, and instead avoided proton decay by imposing a discrete symmetry.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada. We would like to thank Dr. Palash Pal for helpful correspondence.

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