PHYSICAL REVIEW D VOLUME 45, NUMBER 7 1 APRIL 1992

Heavy-meson form factors from QCD sum rules

Matthias Neubert
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 16 December 1991)

A consistent framework is developed for studying hadronic form factors of heavy mesons using QCD sum rules in the heavy-quark effective theory, including the next-to-leading-order renormalization-group improvement. Sum rules are derived for the asymptotic value of the meson decay constant f_P and for the universal Isgur-Wise form factor. It is shown that renormalization-group effects considerably enhance the prediction for f_P and bring its asymptotic value in accordance with recent lattice results. Including finite-mass corrections, the dependence of the physical decay constant on the meson mass is investigated. We obtain $f_D \simeq 170\pm30$ MeV and $f_B \simeq 190\pm50$ MeV. The origin of the breakdown of the heavy-quark expansion for f_D is analyzed. In the case of heavy-meson transition form factors, both the QCD and $1/m_Q$ corrections are moderate and under control. A sum rule for the renormalized Isgur-Wise function is derived and evaluated. The theoretical result is compared to experimental data.

PACS number(s): 11.50.Li, 12.38.Cy, 12.38.Lg, 13.25.+m

I. INTRODUCTION

There is recently intense interest in the hadronic form factors of particles containing a heavy quark. The reason is that, in the limit of infinite quark mass, QCD reveals a spin-flavor symmetry that is not explicit in its Lagrangian [1–6]. It implies that the spin and mass of a heavy quark decouple from the hadronic dynamics. This symmetry becomes manifest to lowest order of an effective field theory describing the strong interactions of heavy quarks [7–9]. In this effective theory, Green's functions are expanded in powers of $1/m_Q$, with m_Q being the renormalization-group invariant "physical" pole mass of the heavy quark.

The simplest type of a form factor is that describing the current-induced generation of a heavy meson out of the vacuum. Consider, for instance, the coupling of a pseudoscalar meson $P=(Q\bar{q})_{0^-}$ with momentum p to the axial-vector current

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 Q|P(p)\rangle = i f_P p_\mu \ . \tag{1.1}$$

The decay constant f_P is a measure of the strength of the quark-antiquark attraction inside the bound state. It is, therefore, a hadronic quantity of primary theoretical interest. Governing the strength of leptonic and nonleptonic weak decays of heavy mesons as well as phenomena such as B-$\bar{B}$ mixing, which provide information on the mass of the top quark and on the Cabibbo-Kobayashi-Maskawa (CKM) matrix, decay constants are also of considerable phenomenological importance. Based on the nonrelativistic constituent-quark model, it was known for a long time that, up to logarithmic corrections, f_P obeys the asymptotic scaling law

$$f_P \propto \frac{1}{\sqrt{m_P}} \ , \tag{1.2}$$

as $m_P \to \infty$. In the framework of the heavy-quark expansion, such a behavior can be shown to be a general consequence of QCD. Furthermore, the leading and next-to-leading logarithmic corrections to (1.2) have been calculated and summed to all orders in perturbation theory [4,10–12]. The asymptotic dependence of f_P on the mass of the heavy meson is thus well understood. A study of the mass dependence of physical decay constants provides, therefore, an estimate of the corrections to the infinite-quark-mass limit. In the absence of experimental information, considerable attention has been devoted to the theoretical calculation of f_P and its dependence on m_P. Besides QCD-inspired potential models [13,14], the sum-rule approach of Shifman, Vainshtein, and Zakharov [15] and lattice gauge theory are the tools that have been most extensively used for this purpose. While there is general agreement (within the intrinsic uncertainties of each method) on the value of f_D, recent lattice results indicate an unexpectedly large value $f_B > f_D$ in vast contradiction to the scaling law (1.2) [16–19]. This has been interpreted as a signal for a breakdown of the $1/m_Q$ expansion for the case of charmed particles. QCD sum-rule calculations, on the other hand, yield smaller values for f_B [20–28]. One of the purposes of this paper is to combine the sum-rule technique with the heavy-quark expansion and to understand and resolve the discrepancy between sum-rule and lattice calculations.

Probably, the most fruitful application of the heavy-quark spin-flavor symmetry is encountered in weak decays of heavy hadrons. Isgur and Wise have worked out the symmetry relations imposed on the various hadronic form factors describing current matrix elements between two heavy mesons or baryons [6]. For instance, they have shown that as $m_b, m_c \to \infty$, all the many form factors describing transitions between any two of the mesons $\bar{B}$, $\bar{B}^*$, D, and D^* become related to a single universal function $\xi(v\cdot v')$. This so-called Isgur-Wise form factor [6,29] is a universal function of QCD, summarizing all long-distance effects active in the weak transition. It only depends on the velocities of the heavy particles and is normalized at zero recoil, where $v\cdot v'=1$. The reduction of form factors implies a significant simplification of the theoretical description of semileptonic weak decay processes, as has been extensively discussed in the recent literature [30–36].

In this paper we develop a consistent framework for studying hadronic form factors of heavy mesons by combining QCD sum-rule and renormalization-group techniques with the effective theory for heavy quarks. The method is applied to calculate the asymptotic value of $f_P\sqrt{m_P}$ and the Isgur-Wise form factor. In Sec. II A we rewrite the standard Laplace sum rule for f_P in a form which is suitable for a $1/m_Q$ expansion. We then present, in Sec. II B, a rederivation of the asymptotic form of this sum rule (valid for infinitely heavy mesons) by using the effective-field-theory formalism of Georgi [7]. One advantage of this second approach is that the sum rule in the effective theory only depends on low-energy parameters, which are independent of the heavy-quark mass. These parameters are *a priori* not known in the standard approach. The most important advantage of the effective theory is, however, that a renormalization-group improvement can be performed by summing the large logarithms $(\alpha_s \ln m_Q)^n$ and $\alpha_s(\alpha_s \ln m_Q)^n$ to all orders in perturbation theory. In particular, the scale ambiguity associated with the leading QCD correction is resolved. These renormalization-group effects, which have not been taken into account in previous calculations of the asymptotic value of $f_P\sqrt{m_P}$ [21,37], turn out to be very significant. They bring the sum-rule result in accordance with lattice computations. In Sec. II C the expressions derived in the effective theory are combined with the standard Laplace sum rule for f_P to obtain an improved calculation of the physical decay constant as a function of the heavy-meson mass. Deviations from the $m_Q \to \infty$ limit and from the scaling law (1.2) are investigated in detail.

The second part of the paper is devoted to the calculation of the universal Isgur-Wise form factor. After a short introduction into the effective-field-theory formalism, we derive the Laplace sum rule for $\xi(v\cdot v')$ in Sec. III B and discuss its renormalization-group improvement in Sec. III C. This completes two recent calculations of the universal form factor, which have ignored renormalization effects [36,38]. The Isgur-Wise function is given by the ratio of a sum rule for a three-point correlator and a sum rule for a two-point correlator. A Ward identity ensures its correct normalization at zero recoil. It is shown that, unlike the situation encountered for f_P, the sum-rule calculation of $\xi(v\cdot v')$ is not affected by unusually large QCD corrections; nor does one expect large $1/m_Q$ corrections to the infinite-quark-mass limit. Our theoretical prediction for the Isgur-Wise form factor compares well with a recent extraction of this function from experimental data on $\bar{B}\to D^* l \bar{\nu}_l$ decays [34]. Section IV contains the conclusions.

II. DECAY CONSTANTS OF HEAVY MESONS

A. Laplace sum rule for f_P

The QCD sum rules proposed by Shifman, Vainshtein, and Zakharov (SVZ) [15] have proved to be a powerful phenomenological tool in the study of low-energy parameters of hadrons, such as their masses or couplings to currents. The idea is that hadronic properties may be studied in a self-consistent way by equating an integral over a physical spectral function to an approximation of the operator product expansion of the time-ordered product of two (or more) local currents. QCD sum rules for the pseudoscalar decay constant f_P have been first considered in Refs. [20,21] and [23]. One studies the two-current correlator

$$\Pi_5(q^2) = i\int d^4x\, e^{iq\cdot x}\langle 0|\mathcal{T}\{A_5(x), A_5(0)^\dagger\}|0\rangle\;, \tag{2.1}$$

with $A_5 = im_Q\bar{q}\gamma_5 Q = \partial^\mu(\bar{q}\gamma_\mu\gamma_5 Q)$ being the divergence of the axial-vector current in the limit where the light quark is massless ($m_q=0$). Since this current is partially conserved, $\Pi_5(q^2)$ is a renormalization-group-invariant quantity.

According to the philosophy of SVZ, the correlator is evaluated in two ways. In the Euclidean region $q^2 \ll 0$, it can be calculated perturbatively because of asymptotic freedom of QCD. Short-distance effects are taken care of by Wilson coefficients, while long-distance confinement effects are included as power corrections and are parametrized in terms of vacuum expectation values of local operators, the so-called condensates [15]. Hence

$$\Pi_5(q^2) = \Pi_5^{\rm pert}(q^2) + \Pi_5^{\rm cond}(q^2) \quad (q^2 \ll 0)\;. \tag{2.2}$$

On the other hand, the correlator can be expressed as a dispersion integral over a physical spectral function, which gets contributions from the ground-state meson P as well as from higher resonances. The residue of the pole is given by the decay constant f_P. Assuming quark-hadron duality, the resonance contributions are usually approximated by the perturbative continuum above a threshold s_c. Using (1.1), one thus writes

$$\Pi_5(q^2) = \frac{f_P^2 m_P^4}{m_P^2 - q^2 - i\epsilon} + \frac{1}{\pi}\int_{s_c}^\infty ds\, \frac{\mathrm{Im}\Pi_5^{\rm pert}(s)}{s - q^2 - i\epsilon} + \text{subtractions}\;. \tag{2.3}$$

Equating the two expressions for $\Pi_5(q^2)$ yields the sum rule, from which f_P can be determined. It is necessary, however, to improve the convergence by suppressing the continuum contribution to the spectral function. This is achieved by applying the Borel operator

$$\hat{B}_{M^2} = \lim_{\substack{n\to\infty \\ -q^2\to\infty \\ M^2=-q^2/n \text{ fixed}}} \frac{(q^2)^n}{\Gamma(n)}\left[-\frac{d}{dq^2}\right]^n, \tag{2.4}$$

to both sides of the sum rule. In particular, this also eliminates possible subtractions required for the convergence of the dispersion integral in (2.3). The result is the so-called Laplace sum rule

$$\frac{f_P^2 m_P^4}{M^2} e^{-m_P^2/M^2} = \frac{1}{\pi M^2}\int_{m_Q^2}^{s_c} ds\, \mathrm{Im}\Pi_5^{\rm pert}(s) e^{-s/M^2} + \hat{B}_{M^2}\Pi_5^{\rm cond}\;, \tag{2.5}$$

where the perturbatively calculated spectral function has a cut starting at $s=m_Q^2$. Explicit expressions for the functions appearing on the right-hand side of (2.5) can be found in Ref. [23]. In order to determine f_P in a self-consistent way, one tries to optimize the value of the continuum threshold s_c in such a way that the computed value of the decay constant is stable with respect to variation of the Borel parameter M^2 in a region where the theoretical calculation of $\Pi_5^{\rm pert}$ and $\Pi_5^{\rm cond}$ is reliable. For too small values of M^2, the power corrections blow up, while the continuum contribution becomes dominant at large M^2. One thus aims for stability in an intermediate region, where both the power and continuum contributions stay reasonably small.

In its above form, the Laplace sum rule is not suited for an expansion in powers of $1/m_Q$, since the dependence of the parameters M^2 and s_c on the heavy-quark mass is *a priori* not determined. It is convenient to introduce a set of new parameters by

$$\begin{aligned} m_Q T &\equiv M^2 , \\ m_Q \omega_c &\equiv s_c - m_Q^2 , \\ m_Q \bar\Lambda &\equiv m_P^2 - m_Q^2 . \end{aligned} \tag{2.6}$$

In the following section, we will show that the new variables T, ω_c, and $\bar\Lambda$ become constant low-energy parameters in the $m_Q\to\infty$ limit. Substituting them into (2.5), the Laplace sum rule takes the form

$$\begin{aligned} f_P^2 m_P \left[\frac{m_P}{m_Q}\right]^3 e^{-\bar\Lambda/T} = {} & \frac{3T^3}{8\pi^2}\int_0^{\omega_c/T} dz\, \frac{z^2 e^{-z}}{1+zT/m_Q}\left\{1+\frac{2\alpha_s}{\pi}\left[\ln\frac{m_Q}{T}+\frac{13}{6}+\frac{2\pi^2}{9}-\ln z+\frac{2}{3}K\left[z\frac{T}{m_Q}\right]\right]\right\} \\ & -\langle\bar q q\rangle(m_Q)\left[1+\frac{2\alpha_s}{3\pi}\left[1-3\frac{T}{m_Q}\int_0^\infty dz\,\frac{e^{-z}}{1+zT/m_Q}\right]\right]+\frac{\langle\alpha_s GG\rangle}{12\pi m_Q} \\ & +\frac{m_0^2\langle\bar q q\rangle(m_Q)}{4T^2}\left[1-\frac{2T}{m_Q}\right]+\frac{4\pi}{81}\frac{\alpha_s\langle\bar q q\rangle^2}{T^3}\left[1+\frac{3T}{m_Q}-\frac{12T^2}{m_Q^2}\right]+\cdots , \end{aligned} \tag{2.7}$$

where m_0^2 is defined as ratio of the so-called mixed condensate and the quark condensate, $g_s\langle\bar q\sigma_{\mu\nu}G^{\mu\nu}q\rangle \equiv m_0^2\langle\bar q q\rangle$. The ellipses stand for power corrections from high-dimensional condensates ($d>6$). These are expected to be very small. Already, the four-quark condensate is suppressed by a factor of 10^{-3} compared to the quark condensate.

In (2.7) we have included the radiative corrections to the dispersion integral and to the contribution of the quark condensate. The first one has been calculated in Ref. [39]. In our notation the function $K(x)$ is given by

$$\begin{aligned} K(x) &= 2\,{\rm Li}_2(-x)+\ln x\,\ln(1+x)-\frac{x\ln x}{1+x} \\ &\quad +\frac{1+x}{x}\ln(1+x)-1 \\ &= -\frac{3}{2}x+\left[\frac{1}{3}+\frac{\ln x}{2}\right]x^2+O(x^3) , \end{aligned} \tag{2.8}$$

with ${\rm Li}_2(z)=-\int_0^z (dt/t)\ln(1-t)$ being the dilogarithm. The correction to the quark-condensate contribution is new. In (2.7), m_Q is the physical heavy-quark mass defined as the pole of the renormalized propagator. This renormalization-group-invariant quantity is related to the running mass of the modified minimal subtraction ($\overline{\rm MS}$) scheme by [40,41]

$$\begin{aligned} \bar m_Q(\mu) &= m_Q\left[\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right]^{d_m}\left[1-\frac{4\alpha_s(m_Q)}{3\pi}\right] , \\ d_m &= \frac{12}{33-2n_f} . \end{aligned} \tag{2.9}$$

The running of the quark condensate is given by

$$\frac{\langle\bar q q\rangle(\mu)}{\langle\bar q q\rangle(m_Q)} = \left[\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right]^{-d_m} \tag{2.10}$$

and is such that the product $m_Q\langle\bar q q\rangle$ is renormalization-group invariant. In the sum rule (2.7), the condensate is to be evaluated at the scale of the heavy quark.

Equation (2.7) is completely equivalent to the more familiar form (2.5) of the sum rule, which has been first investigated in Ref. [23]. For its evaluation we use the standard values of the vacuum condensates [15,42],

$$\begin{aligned} \langle\bar q q\rangle(1\ {\rm GeV}) &= -(230\ {\rm MeV})^3 , \\ \langle\alpha_s GG\rangle &= 0.038\ {\rm GeV}^4 , \\ m_0^2 &= 0.8\ {\rm GeV}^2 , \\ \alpha_s\langle\bar q q\rangle^2 &= 6\times10^{-5}\ {\rm GeV}^6 , \end{aligned} \tag{2.11}$$

as well as the flavor-independent value $\bar\Lambda=1$ GeV, corresponding to the pole masses $m_c\simeq1.44$ GeV and $m_b\simeq4.80$ GeV. The effect of a variation of $\bar\Lambda$ is discussed below. The only free parameter in (2.7) is the threshold energy ω_c. Evaluating the radiative corrections at the scale of the heavy quark, one finds good stability in the wide region $0.5<T<2.0$ GeV for $\omega_c \simeq 2.4$ GeV for f_D and $\omega_c \simeq 2.0$ GeV for f_B, corresponding to $s_c^D \simeq 5.5$ GeV^2 and $s_c^B \simeq 33$ GeV^2, respectively. Unlike the continuum thresholds, the optimal ω_c values are rather insensitive to the mass of the heavy quark. In the stability region, the values of the decay constants are $f_D \simeq 170 \pm 30$ MeV and $f_B \simeq 140 \pm 30$ MeV. They agree with those obtained in Ref. [23]. Variation of the QCD parameters within the standard limits does not significantly change these results.

Under the premises that T, ω_c, and $\tilde{\Lambda}$ are indeed low-energy parameters, one can immediately perform the limit $m_Q \to \infty$ in (2.7). This nonrelativistic form of the sum rule was investigated as early as 1982 in a paper by Shuryak [21]. Recently, it has been rederived and discussed in Ref. [37]. At this point it is worth noting that the right-hand side of (2.7) shows a logarithmic dependence on the heavy-quark mass both in the radiative correction to the dispersion integral and in the running of the quark condensate. Eventually, one would like to separate this dependence from the sum rule and sum the large logarithms $(\alpha_s \ln m_Q)^n$ to all orders in perturbation theory. It is also obvious that the radiative correction to the dispersion integral is dangerously large, even if the strong coupling is evaluated at the scale of the heavy quark. However, it is not obvious that $\mu = m_Q$ really is the appropriate scale to use. For this reason a next-to-leading-order calculation, which resolves the scale ambiguity problem, is most desirable. In the following section, we will derive the correct asymptotic form of the sum rule and perform the complete next-to-leading-order renormalization-group improvement. To this end it is necessary to employ an effective theory for heavy quarks.

B. Sum rule in the effective theory

A convenient framework for systematically analyzing both the QCD and $1/m_Q$ corrections to the infinite-quark-mass limit is provided by the so-called heavy-quark effective theory developed by Georgi [7]. The basic observation is that as $m_Q \to \infty$, the velocity v of a heavy quark becomes a conserved quantity with respect to soft processes. It is then possible to remove the mass-dependent piece of the momentum operator by the velocity-dependent field redefinition

$$h_Q(v,x) = e^{i m_Q \not{v} \cdot x} \psi_Q(x) , \tag{2.12}$$

such that

$$i \not{\partial} h_Q(v,x) = \not{k} h_Q(v,x), \quad k_\mu = P_\mu - m_Q v_\mu , \tag{2.13}$$

where P is the total momentum of the heavy quark, m_Q is its physical pole mass, and k denotes the residual "off-shell" momentum, which is typically of order $\Lambda_{\rm QCD}$. In terms of the new fields $h_Q(v,x)$, the effective Lagrangian consists of an infinite series of operators with increasing canonical dimension, multiplied by increasing powers of $1/m_Q$. To lowest order in the $1/m_Q$ expansion, this Lagrangian explicitly exhibits the spin-flavor symmetry for heavy quarks [7–9].

The axial-vector current $A_\mu = \bar{q}\gamma_\mu\gamma_5 Q$ can be expanded in terms of operators of the effective theory as follows:

$$A_\mu \simeq D_1(m_Q/\mu)\bar{q}\gamma_\mu\gamma_5 h_Q(v) - D_2(m_Q/\mu)\bar{q} v_\mu \gamma_5 h_Q(v) + \sum_{n=1}^{\infty} \left[\frac{1}{m_Q} \right]^n \sum_i D_i^{(n)}(m_Q/\mu) \mathcal{A}_\mu^{(i,n)} , \tag{2.14}$$

where the index i labels operators of the same canonical dimension, and we have explicitly written down the two lowest-dimensional operators. The symbol "$\simeq$" means that (2.14) is an equality for matrix elements only. In the full theory, matrix elements of the axial-vector current depend on the mass of the heavy quark, but are independent of the renormalization scale since the anomalous dimension of A_μ vanishes. Matrix elements of operators in the effective theory, on the other hand, are independent of the heavy-quark mass. All reference to m_Q is either in form of powers of $1/m_Q$ that multiply the higher-dimensional operators $\mathcal{A}_\mu^{(i,n)}$, or in the short-distance coefficients $D_i^{(n)}$. These functions appear since the properties of the effective theory under renormalization differ from QCD. In particular, the effective current operators have nonzero anomalous dimensions, such that matrix elements depend on the renormalization scale μ. From the fact that the scale dependence of matrix elements must exactly cancel against that of the short-distance coefficients, one can derive the renormalization-group equation for the functions $D_i^{(n)}$. In order to solve this equation to first order in the physical coupling $\alpha_s(m_Q)$, one needs the two-loop anomalous dimensions of the current operators in the effective theory. For the lowest-dimensional operators in (2.14), the anomalous dimension is the same. It has recently been calculated in Refs. [11] and [12]. For our purpose we only need the sum of the coefficients D_1 and D_2, the complete next-to-leading-order expression for which is [43]

$$C_F(m_Q/\mu) \equiv D_1(m_Q/\mu) + D_2(m_Q/\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(m_Q)} \right]^{d_m/2} \left[1 + \frac{\alpha_s(m_Q)}{\pi} Z_{n_f} - \frac{\alpha_s(\mu)}{\pi}(Z_{n_f} + \delta) \right] , \tag{2.15}$$

where δ is a scheme-dependent constant. In the $\overline{\rm MS}$ subtraction scheme, $\delta_{\overline{\rm MS}} = \frac{2}{3}$. The constant Z_{n_f} is scheme independent and reads

$$Z_{n_f} = 3\frac{153 - 19 n_f}{(33 - 2n_f)^2} - \frac{381 - 30 n_f + 28\pi^2}{36(33 - 2n_f)} - \frac{2}{3} , \tag{2.16}$$

where n_f is the number of light-quark flavors. (The value for Z_{n_f} quoted in Ref. [11] differs from our result by $-\frac{2}{3}$. The correct next-to-leading-order correction is only half as large as stated in that paper.)

Matrix elements of operators in the effective theory respect the heavy-quark symmetries and can be most concisely computed employing a compact trace formalism [29,30]. For the relevant matrix elements of the operators appearing in (2.14), one writes

$$\langle 0|\bar{q}\Gamma h_Q(v)|P(v)\rangle = \frac{F(\mu)}{2}\mathrm{Tr}[\Gamma\mathcal{P}(v)] , \tag{2.17}$$

with Γ being an arbitrary Dirac matrix. $F(\mu)$ is a scale-dependent low-energy parameter independent of m_Q and

$$\mathcal{P}(v) = -i\sqrt{m_P}\,\frac{(1+\not{v})}{2}\gamma_5 \tag{2.18}$$

denotes the spin wave function of the pseudoscalar meson P.

Substituting the expansion (2.14) into (1.1) and carrying out the traces with (2.17), we obtain the following relation between the physical decay constant f_P and the low-energy parameter $F(\mu)$:

$$f_P m_P^{1/2} = C_F(m_Q/\mu)F(\mu) + O\left[\frac{1}{m_Q}\right] . \tag{2.19}$$

Since f_P is a physical quantity, the right-hand side of this equation must be independent of the subtraction scale and of the renormalization scheme adopted for the calculation of C_F. It is thus convenient to define the μ-independent short-distance coefficient $\hat{C}_F(m_Q)$ and the renormalized constant F_{ren} by

$$\hat{C}_F(m_Q) = [\alpha_s(m_Q)]^{-d_m/2}\left[1+\frac{\alpha_s(m_Q)}{\pi}Z_{n_f}\right] ,$$
$$F_{\mathrm{ren}} = [\alpha_s(\mu)]^{d_m/2}\left[1-\frac{\alpha_s(\mu)}{\pi}(Z_{n_f}+\delta)\right]F(\mu) , \tag{2.20}$$

such that

$$C_F(m_Q/\mu)F(\mu) = \hat{C}_F(m_Q)F_{\mathrm{ren}} . \tag{2.21}$$

These new quantities are renormalization-group invariant. F_{ren} is a universal low-energy parameter of QCD. It can only be estimated using nonperturbative techniques. In (2.20) it is to be understood that the number of active flavors, i.e., the value of n_f in Z_{n_f} and d_m, changes as one scales down from m_Q to μ. Explicitly, one has

$$\hat{C}_F(m_c) = [\alpha_s(m_c)]^{-2/9}\left[1+\frac{\alpha_s(m_c)}{\pi}Z_3\right] \simeq 1.22 ,$$
$$\hat{C}_F(m_b) = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}[\alpha_s(m_c)]^{-2/9} \times\left[1+\frac{\alpha_s(m_b)-\alpha_s(m_c)}{\pi}Z_4+\frac{\alpha_s(m_c)}{\pi}Z_3\right] \simeq 1.39 , \tag{2.22}$$

for the D and B mesons, respectively. For the numerical estimate, we have used $m_c = 1.45$ GeV and $m_b = 4.67$ GeV [41]. The running coupling is accurate to second order. In the $\overline{\mathrm{MS}}$ scheme,

$$\alpha_s(m_Q) = \frac{12\pi}{(33-2n_f)\ln\nu}\left[1-6\frac{153-19n_f}{(33-2n_f)^2}\frac{\ln\ln\nu}{\ln\nu}\right] ,\qquad \nu = \left[\frac{m_Q}{\Lambda_{n_f}^{\overline{\mathrm{MS}}}}\right]^2 . \tag{2.23}$$

We use $\Lambda_4^{\overline{\mathrm{MS}}} = 0.2$ GeV and adjust the n_f dependence of this parameter such that the running coupling is a continuous function of m_Q.

In order to compute the renormalized parameter F_{ren} using QCD sum rules in the effective theory, we investigate the correlator

$$\Gamma_5(\omega) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathcal{T}\{\mathcal{A}_5^{(v)}(x),\mathcal{A}_5^{(v)}(0)^\dagger\}|0\rangle ,\qquad \omega \equiv 2k\cdot v , \tag{2.24}$$

with $\mathcal{A}_5^{(v)} = \bar{q}\gamma_5 h_Q(v)$ being the effective pseudoscalar current. In the rest frame of the heavy quark, ω is twice the external "off-shell energy." The perturbative contribution to Γ_5 is obtained by evaluating the diagrams shown in Fig. 1(a). The Feynman rules of the effective theory are given in the Appendix. Using dimensional regularization and the $\overline{\mathrm{MS}}$ subtraction scheme, we find

$$\frac{1}{\pi}\mathrm{Im}\Gamma_5^{\mathrm{pert}}(\omega) = \frac{3\omega^2}{8\pi^2}\Theta(\omega)\left[1+\frac{2\alpha_s}{\pi}\left[\ln\frac{\mu}{\omega}+\frac{13}{6}+\frac{2\pi^2}{9}+\delta\right]\right] , \tag{2.25}$$

where δ is the same scheme-dependent constant that appears in (2.15), i.e., $\delta_{\overline{\mathrm{MS}}} = \frac{2}{3}$. Next, we compute the nonperturbative power corrections to Γ_5. The contributions involving the quark and gluon condensates are depicted

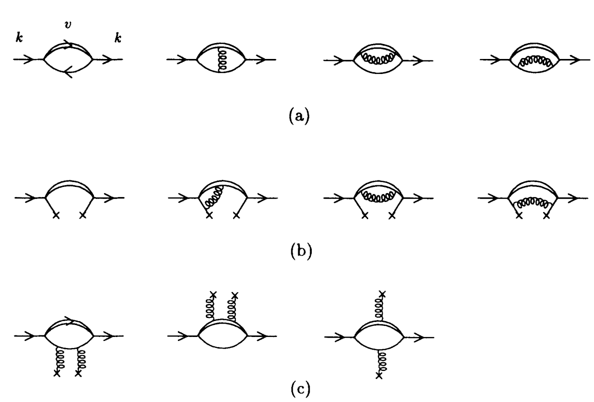

FIG. 1. Feynman diagrams for the two-current correlator $\Gamma_5(\omega)$: (a) perturbative contributions, (b) power corrections involving the quark condensate, and (c) corrections involving the gluon condensate. The heavy-quark propagators are represented by double lines.

in Figs. 1(b) and 1(c), respectively. The calculation is most easily performed in the coordinate gauge $x_\mu A^\mu(x)=0$. Including all condensates with dimension $d\le 6$, the result is

$$\begin{aligned}
&\Gamma_5^{\langle\bar qq\rangle}(\omega)=\frac{\langle\bar qq\rangle(\mu)}{\omega}\left[1+\frac{2\alpha_s}{\pi}\left[\frac{1}{3}+\delta\right]\right],\\
&\Gamma_5^{\langle GG\rangle}(\omega)=0,\\
&\Gamma_5^{\langle\bar q\sigma Gq\rangle}(\omega)=-\frac{g_s}{2}\frac{\langle\bar q\sigma_{\mu\nu}G^{\mu\nu}q\rangle}{\omega^3}=-\frac{m_0^2\langle\bar qq\rangle}{2\omega^3},\\
&\Gamma_5^{\langle\bar qq\rangle^2}(\omega)=\frac{8\pi}{27}\frac{\alpha_s\langle\bar qq\rangle^2}{\omega^4},
\end{aligned}\qquad(2.26)$$

where the factorization approximation has been used to reduce the four-quark condensates to $\langle\bar qq\rangle^2$ [15]. Note, in particular, the vanishing of $\Gamma_5^{\langle GG\rangle}$ resulting from the explicit calculation of the diagrams shown in Fig. 1(c). This is in accordance with the fact that the contribution of the gluon condensate in (2.7) is associated with a factor of $1/m_Q$ instead of $1/T$.

Concerning the pole contribution of the pseudoscalar meson P to the phenomenological side of the sum rule, we note that according to (2.12) the total external momentum in (2.24) is $P=m_Qv+k$, such that the propagator of the heavy meson becomes $(P^2-m_P^2)^{-1}\to[m_Q(\omega-\bar\Lambda)]^{-1}$, with $\bar\Lambda$ as defined in (2.6). The hadronic matrix elements in the effective theory are readily evaluated using (2.17). Approximating higher-resonance contributions by the perturbative continuum above a "threshold energy" ω_c, which is the analogue of the continuum threshold s_c, the phenomenological expression for the correlation function becomes

$$\Gamma_5^{\rm phen}(\omega)=\frac{F^2(\mu)}{\bar\Lambda-\omega-i\epsilon}+\frac{1}{\pi}\int_{\omega_c}^\infty d\omega'\frac{{\rm Im}\Gamma_5^{\rm pert}(\omega')}{\omega'-\omega-i\epsilon}+\text{subtractions}\,.\qquad(2.27)$$

The Laplace sum rule for $F^2(\mu)$ is obtained by applying the Borel operator with respect to ω,

$$\hat B_T=\lim_{\substack{n\to\infty\\ -\omega\to\infty\\ T=-\omega/n\ \text{fixed}}}\frac{\omega^n}{\Gamma(n)}\left[-\frac{d}{d\omega}\right]^n,\qquad(2.28)$$

and equating the different expressions for the correlator. The result is

$$\begin{aligned}F^2(\mu)e^{-\bar\Lambda/T}=&\frac{3T^3}{8\pi^2}\int_0^{\omega_c/T}dz\,z^2e^{-z}\left[1+\frac{2\alpha_s}{\pi}\left[\ln\frac{\mu}{T}+\frac{13}{6}+\frac{2\pi^2}{9}+\delta-\ln z\right]\right]\\
&-\langle\bar qq\rangle(\mu)\left[1+\frac{2\alpha_s}{\pi}\left[\frac{1}{3}+\delta\right]-\frac{m_0^2}{4T^2}-\frac{4\pi}{81}\frac{\alpha_s\langle\bar qq\rangle}{T^3}\right].\end{aligned}\qquad(2.29)$$

We observe that the sum rule in the effective theory precisely corresponds to the leading term in the $1/m_Q$ expansion of (2.7), showing that the parameters T, ω_c, and $\bar\Lambda$ introduced in (2.6) were in fact properly chosen low-energy parameters. The only difference is the replacement of the heavy-quark mass m_Q by the subtraction scale μ and the appearance of the scheme-dependent terms proportional to δ. These terms cancel if one computes the renormalized quantity $F_{\rm ren}^2$.

With the help of (2.20), one can readily perform the renormalization-group improvement of (2.29). The logarithmic dependence on μ can be summed to all orders to produce a factor $[\alpha_s(\mu)/\alpha_s(T)]^{-d_m}$. For the quark condensate, this is already known from (2.10). The next-to-leading-order corrections split into a contribution $2[\alpha_s(\mu)/\pi](Z_{n_f}+\delta)$ and a μ-independent correction. For this latter the running coupling is to be evaluated at a characteristic low-energy scale of the effective theory. We choose $\bar\Lambda$ for this scale, since it provides a measure of the average off-shell energy ω of the heavy quark in the meson. An alternative choice would be the Borel parameter T. The differences are formally of order α_s^2 and hence beyond the accuracy of the present calculation. Because of the size of the order-α_s correction, the numerical results are not insensitive to this choice, however. We shall comment on this below.

Putting everything together, we obtain the following renormalization-group-improved sum rule for the renormalized parameter $F_{\rm ren}^2$ defined in (2.20):

$$\begin{aligned}F_{\rm ren}^2e^{-\bar\Lambda/T}=[\alpha_s(T)]^{4/9}\left[1-\frac{2\alpha_s(\bar\Lambda)}{\pi}Z_3\right]\Bigg\{&\frac{3T^3}{8\pi^2}\int_0^{\omega_c/T}dz\,z^2e^{-z}\left[1+\frac{2\alpha_s(\bar\Lambda)}{\pi}\left[\frac{13}{6}+\frac{2\pi^2}{9}-\ln z\right]\right]\\
&-\langle\bar qq\rangle(T)\left[1+\frac{2\alpha_s(\bar\Lambda)}{3\pi}-\frac{m_0^2}{4T^2}-\frac{4\pi}{81}\frac{\alpha_s\langle\bar qq\rangle}{T^3}\right]\Bigg\},\end{aligned}\qquad(2.30)$$

where we have used the fact that the number of light-quark flavors in the effective low-energy theory is $n_f=3$. In Fig. 2 the function $F_{\rm ren}(T)$ is shown for $\bar\Lambda=1.25$ GeV and various values of the threshold energy ω_c. We find good stability over a wide range of values of the Borel parameter T. For other choices of $\bar\Lambda$, the resulting curves look very similar. The values of ω_c and T, providing optimal stability, approximately scale with $\bar\Lambda$. We obtain

$$F_{\rm ren}\simeq\begin{cases}0.34\pm0.03\ {\rm GeV}^{3/2}, & \bar\Lambda=1.0\ {\rm GeV},\\ 0.41\pm0.04\ {\rm GeV}^{3/2}, & \bar\Lambda=1.25\ {\rm GeV},\\ 0.47\pm0.05\ {\rm GeV}^{3/2}, & \bar\Lambda=1.5\ {\rm GeV},\end{cases} \qquad (2.31)$$

for the renormalized low-energy parameter. The quoted errors reflect the variation of the results with respect to changes in ω_c and in the values of the vacuum condensates. The intrinsic uncertainty of the sum-rule calculation might be considerably larger, however. This can be inferred from the following observation. If the scale $\bar\Lambda$ in the next-to-leading logarithmic corrections is replaced by T, the running of $\alpha_s(T)$ leads to a suppression of $F_{\rm ren}(T)$ at large values of the Borel parameter. This effect, which is formally of order α_s^2, is so significant that it changes the continuum threshold providing best stability from $\omega_c\simeq2.3$ to 3.0 GeV (for $\bar\Lambda=1.25$ GeV). As a consequence, the result for $F_{\rm ren}$ increases by almost 20%. This is in conflict with the SVZ philosophy that the stability of the sum rule should result from a balance between its perturbative and nonperturbative parts. In order to avoid this effect, we evaluate the next-to-leading-order corrections at the fixed scale $\bar\Lambda$.

Equation (2.31) exhibits a rather strong dependence of $F_{\rm ren}$ on the mass difference $\bar\Lambda$, which is a low-energy parameter that can again in principle be determined using nonperturbative techniques such as lattice gauge theory or QCD sum rules. While no lattice results are available today, an estimate of the pole mass of the b quark can be obtained from a QCD-sum-rule analysis of the bottomonium spectrum. The extracted values range from $m_b=4.80\pm0.03$ GeV [44] to $m_b=4.67\pm0.10$ GeV [41] and $m_b=4.55\pm0.05$ GeV [26], corresponding to $\bar\Lambda_{(b)}=1.01\pm0.06$, 1.30 ± 0.20, and 1.57 ± 0.10 GeV, respectively. This is the range of values covered in (2.31), and we shall assume that $\bar\Lambda_{(b)}$ is not very different from the asymptotic value that $\bar\Lambda$ acquires in the infinite-quark-mass limit.

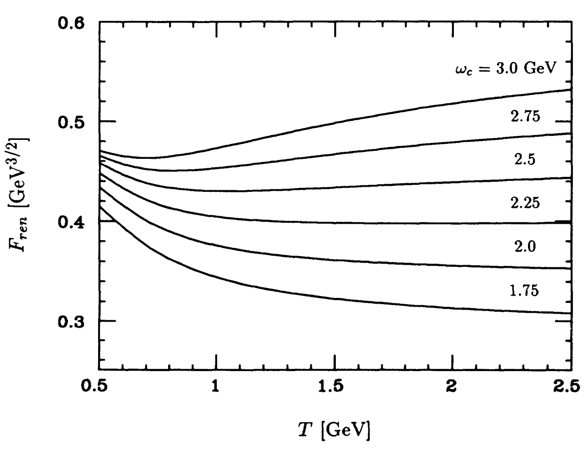

FIG. 2. Numerical evaluation of the sum rule (2.30) for $\bar\Lambda=1.25$ GeV and various values of the threshold parameter ω_c.

From (2.31) and (2.22), one can compute the so-called static limit of the decay constant of the B meson, which is given by the first term on the right-hand side of (2.19). We find

$$f_B^{\rm stat}=\frac{\hat C_F(m_b)}{\sqrt{m_B}}F_{\rm ren}\simeq200-300\ {\rm MeV}\ . \qquad (2.32)$$

These sum-rule results can be compared to recent lattice calculations of the decay constants of heavy mesons, which use the static approximation for the heavy-quark propagator [1]. In these computations one determines the parameter $F(\mu=a^{-1})$ in (2.19) in units of the inverse lattice spacing $a^{-3/2}$. The numerical result can be converted into physical units by normalizing to the decay constant of a light pseudoscalar meson (f_π or f_K). Furthermore, a renormalization factor is required to relate the lattice result to the renormalized parameter $F_{\rm ren}$ defined in (2.20). In next-to-leading order, the relation is

$$F_{\rm ren}=[\alpha_s(a^{-1})]^{d_m/2}\left[1-\frac{\alpha_s(a^{-1})}{\pi}(Z_{n_f}+\delta_{\overline{\rm MS}}+\delta_{\rm latt})\right]\times F_{\rm latt}(a^{-1})\ , \qquad (2.33)$$

where $\delta_{\overline{\rm MS}}=\frac{2}{3}$ accounts for the matching between QCD and the effective theory for heavy quarks, while $\delta_{\rm latt}$ provides the matching between the effective theory in the continuum and on the lattice. For Wilson fermions (with $r=1$), the value of this constant is $\delta_{\rm latt}\simeq4.38$ [45,46]. For a lattice spacing corresponding to $a^{-1}\simeq2$ GeV, the relation (2.33) thus reads $F_{\rm ren}\simeq0.48F_{\rm latt}(a^{-1})$. We note that in next-to-leading order of renormalization-group-improved perturbation theory, the correction factor in (2.33) is 20% smaller than that used in three recent lattice computations of $f_B^{\rm stat}$ [16–18]. Rescaling the values obtained there, we find (the result of Ref. [16] increases to 0.43 ± 0.07 if the lattice spacing is determined without reference to f_π and f_K)

$$F_{\rm ren}\simeq\begin{cases}0.36\pm0.06\ {\rm GeV}^{3/2} & (\text{Ref. [16]}),\\ 0.50\pm0.08\ {\rm GeV}^{3/2} & (\text{Ref. [17]}),\\ 0.44\pm0.07\ {\rm GeV}^{3/2} & (\text{Ref. [18]}),\end{cases} \qquad (2.34)$$

corresponding to $f_B^{\rm stat}=218\pm36$, 302 ± 48, and 266 ± 42 MeV, respectively. Comparison with (2.31) and (2.32) shows that the values obtained from our improved sum rule are completely consistent with the lattice results.

Before proceeding, we would like to make a comment on the size of the QCD correction in (2.30). After the renormalization-group improvement, the scale ambiguity associated with the next-to-leading logarithmic corrections has been resolved. The radiative correction to the dispersion integral is proportional to the physical coupling $\alpha_s(\bar\Lambda)$ in the low-energy theory and is accompanied by the large coefficient

$$\frac{13}{3}+\frac{4\pi^2}{9}-2Z_3=\frac{887}{162}+\frac{122}{243}\pi^2\simeq 10.43\ , \qquad (2.35)$$

which (in the Feynman gauge) is mainly due to the gluon exchange between the heavy and the light quark. With $\alpha_s(\tilde{\Lambda})/\pi\simeq 0.1$, the radiative correction amounts to a 100% enhancement of $F_{\rm ren}^2$, corresponding to a 50% enhancement of f_P. We would like to stress that this is a purely perturbative result not specific for QCD sum rules. It is rather a general property of the two-current correlator Γ_5. The physical origin of this effect is likely to be the Coulomb interaction between the quarks. We believe that the size of the correction might indicate an interesting nonperturbative enhancement of decay constants, which could ultimately limit any perturbative approach to calculate f_P. This should be kept in mind when considering the significance of the sum-rule results (2.31) and (2.32).

C. Improved sum rule for f_P

After this *caveat* we proceed by deriving an improved sum rule for f_P, which allows for a quantitative estimate of finite-mass corrections to the infinite-quark-mass limit. To this end we combine the renormalization-group-improved sum rule (2.30) derived in the effective theory with the standard sum rule (2.7) by adding back those term that vanish in the $m_Q\to\infty$ limit. Let us define the quantity $G(m_Q)$ by

$$f_P\sqrt{m_P}\left[\frac{m_P}{m_Q}\right]^{3/2}\equiv\hat{C}_F(m_Q)G(m_Q)\ ,$$
$$\lim_{m_Q\to\infty}G(m_Q)=F_{\rm ren}\ . \qquad (2.36)$$

This function is free of large logarithms and has a well-defined behavior in the infinite-quark-mass limit. From (2.30) and (2.7), we obtain the sum rule

$$G^2(m_Q)e^{-\tilde{\Lambda}/T}=[\alpha_s(T)]^{4/9}\left[1-\frac{2\alpha_s(\tilde{\Lambda})}{\pi}Z_3\right]\times\left[\frac{3T^3}{8\pi^2}\int_0^{\omega_c/T}dz\frac{z^2e^{-z}}{1+zT/m_Q}\left\{1+\frac{2\alpha_s(\tilde{\Lambda})}{\pi}\left[\frac{13}{6}+\frac{2\pi^2}{9}-\ln z+\frac{2}{3}K\left[z\frac{T}{m_Q}\right]\right]\right\}\right.$$
$$-\langle q\bar{q}\rangle(T)\left[1+\frac{2\alpha_s(\tilde{\Lambda})}{3\pi}\left[1-3\frac{T}{m_Q}\int_0^\infty dz\frac{e^{-z}}{1+zT/m_Q}\right]-\frac{m_0^2}{4T^2}\left[1-\frac{2T}{m_Q}\right]\right]$$
$$\left.+\frac{\langle\alpha_s GG\rangle}{12\pi m_Q}+\cdots\right]\ , \qquad (2.37)$$

where we have neglected the tiny contribution of the four-quark condensate. To first order in α_s, Eqs. (2.36) and (2.37) are equivalent to (2.7). In the improved sum rule, however, the large logarithms $(\alpha_s\ln m_Q)^n$ and $\alpha_s(\alpha_s\ln m_Q)^n$ are correctly summed to all orders in perturbation theory and factorized into the short-distance coefficient $\hat{C}_F(m_Q)$. The leading corrections not taken into account are of order α_s^2 or $(1/m_Q)\,(\alpha_s\ln m_Q)^2$, neither one of which becomes large as $m_Q\to\infty$.

In Table I we investigate the m_Q dependence of $G(m_Q)$ and related quantities for $\tilde{\Lambda}=1$ and 1.25 GeV. Shown are the values providing best stability only. The optimal value of ω_c is found to slightly increase as the heavy-quark mass becomes smaller. This effect stabilizes the mass dependence of $G(m_Q)$. As a consequence, we find a moderate mass dependence of this function even in the region of the charm quark. Typically, $G(m_b)$ and $G(m_c)$ are 5–10% and 15–20% smaller than the asymptotic value $G(\infty)=F_{\rm ren}$, respectively. The $1/m_Q$ corrections being of the naively expected order of magnitude ($\sim\Lambda_{\rm QCD}/m_Q$), we conclude that the heavy-quark expansion works well for the quantity $G(m_Q)$.

The situation changes if one considers the quantity $(m_Q/m_P)^{3/2}G(m_Q)$, which up to the short-distance correction determines the size of $f_P\sqrt{m_P}$. The additional mass ratio amounts to a further suppression of f_B by $\sim 15\%$ and f_D by $\sim 40\%$. Therefore the decay constants themselves are subject to very large finite-mass corrections. It is important to remember, however, that the explicit appearance of quark masses is a specific feature of the pseudoscalar decay constants. Such an effect does not occur, e.g., in the case of heavy-quark transition form factors. The fact that the decay constants are subject to large scaling violations should therefore not be considered as an indication of a general failure of the heavy-quark expansion for the case of the charm quark.

Using the pole masses given in Ref. [41], $m_b=4.67\pm0.10$ GeV and $m_c=1.45\pm0.05$ GeV, our final results are $f_D\simeq 170\pm30$ MeV and $f_B\simeq 190\pm50$ MeV; i.e., the decay constant of the B meson is indeed found to be larger than (or at least comparable to) that of the D meson. The quoted errors mainly reflect the uncertainty in the value of $\tilde{\Lambda}$ and in the scale used in the next-to-leading logarithmic corrections. Two effects are responsible for the large value of f_B (as compared with the standard sum-rule estimate at the end of Sec. II A), the most

TABLE I. Mass dependence of the sum-rule results for (a) $\tilde{\Lambda}=1$ GeV and (b) $\tilde{\Lambda}=1.25$ GeV.

(a)					
m_Q (GeV)	$m_c=1.44$	3.0	$m_b=4.80$	20.0	∞
m_P (GeV)	$m_D=1.87$	3.46	$m_B=5.28$	20.5	∞
ω_c (GeV)	2.4	2.2	2.1	2.0	1.9
s_c (GeV^2)	5.5	15.6	33.1	440	∞
$G(m_Q)$ (GeV$^{3/2}$)	0.286	0.310	0.317	0.335	0.340
$\left[\frac{m_Q}{m_P}\right]^{3/2} G(m_Q)$ (GeV$^{3/2}$)	0.193	0.251	0.275	0.323	0.340
f_P (MeV)	172	180	166	109	0
(b)					
m_Q (GeV)	$m_c=1.35$	3.0	$m_b=4.69$	20.0	∞
m_P (GeV)	$m_D=1.87$	3.57	$m_B=5.28$	20.6	∞
ω_c (GeV)	3.0	2.6	2.5	2.4	2.3
s_c (GeV^2)	5.9	16.8	33.7	448	∞
$G(m_Q)$ (GeV$^{3/2}$)	0.330	0.361	0.373	0.404	0.411
$\left[\frac{m_Q}{m_P}\right]^{3/2} G(m_Q)$ (GeV$^{3/2}$)	0.203	0.278	0.312	0.386	0.411
f_P (MeV)	179	196	189	130	0

important one being that after renormalization-group improvement the radiative corrections have to be evaluated at a low-energy scale instead of at the scale of the heavy-quark mass. A second increase is due to the different quark masses used and the strong sensitivity of f_P to $\tilde{\Lambda}\simeq 2\,(m_P-m_Q)$. The decay constant of the D meson, on the other hand, agrees with our earlier estimate.

III. SUM-RULE CALCULATION OF THE ISGUR-WISE FORM FACTOR

A. Current matrix elements in the effective theory

We now turn to the study of the form factors describing current-induced transitions between two heavy mesons P_1 and P_2. Specifically, consider transitions between any two of the ground-state pseudoscalar and vector mesons D, D^*, $\bar{B}$, and $\bar{B}^*$. In general, the corresponding matrix elements involve a large set of *a priori* unrelated form factors. Exploiting the consequences of the spin-flavor symmetry for the heavy quarks, however, Isgur and Wise have shown that in the limit $m_b, m_c \to \infty$ all these form factors become proportional to a single universal function $\xi(v\cdot v')$ [6]. This so-called Isgur-Wise form factor is independent of the masses of the heavy quarks. It only depends on the velocity transfer and is normalized at zero recoil.

The relations among form factors that the heavy-quark symmetries generate can again be most concisely worked out by using the trace formalism already discussed in Sec. II B. In the effective theory, the matrix element describing the transition of a heavy meson P_1 with velocity v to a heavy meson P_2 with velocity v' is given by [29]

$$\langle P_2(v')|\bar{h}_{Q_2}(v')\Gamma h_{Q_1}(v)|P_1(v)\rangle = -\xi(v\cdot v',\mu)\mathrm{Tr}[\bar{\mathcal{P}}_2(v')\Gamma\mathcal{P}_1(v)]\ , \quad (3.1)$$

where Γ is an arbitrary Dirac matrix, and the scale-dependent function $\xi(v\cdot v',\mu)$ is the analogue of the low-energy parameter $F(\mu)$ in (2.17). By evaluating the current matrix element for identical mesons, one can readily show that this function satisfies the zero-recoil normalization $\xi(1,\mu)=1$. The spin wave function $\mathcal{P}(v)$ of a pseudoscalar meson has been given in (2.18). For a vector meson with polarization ϵ_μ, one has instead

$$\mathcal{P}(v)=\sqrt{m_P}\,\frac{(1+\not{v})}{2}\not{\epsilon}\ . \quad (3.2)$$

The current $\bar{Q}_2\Gamma Q_1$ of the full theory is related to the effective heavy-quark current in (3.1) by an expansion similar to (2.14). In this case, however, the associated short-distance coefficients C_i are not only functions of the two heavy-quark masses and the renormalization scale, but also of the velocity transfer [29]. These coefficients have recently been calculated in next-to-leading order of renormalization-group-improved perturbation theory [43]. As in (2.20), they can be factorized into scale-independent functions $\hat{C}_i(m_{Q_1},m_{Q_2},v\cdot v')$ and a μ-dependent factor, which is independent of the heavy-quark masses and precisely cancels the scale dependence of matrix elements in the effective theory. With the help of this factor, we define the renormalized Isgur-Wise function in analogy to the definition of F_{ren} in (2.20). Putting everything together, one obtains for the full QCD matrix element

$$\langle P_2(v')|\bar{Q}_2\Gamma Q_1|P_1(v)\rangle = -\xi_{\mathrm{ren}}(y)\sum_i \hat{C}_i(m_{Q_1},m_{Q_2},y)\mathrm{Tr}[\bar{\mathcal{P}}_2(v')\Gamma_i\mathcal{P}_1(v)] + O\left[\frac{1}{m_{Q_i}}\right]\ , \quad (3.3)$$

where for abbreviation $y=v\cdot v'$, and Γ_i are Dirac matrices with the same canonical dimension as Γ. For the vector current, e.g., it is convenient to choose $\Gamma_1=\Gamma=\gamma_\mu$, $\Gamma_2=-v_\mu$, and $\Gamma_3=-v'_\mu$. The renormalized Isgur-Wise form factor is defined as [43]

$$\xi_{\rm ren}(y)=[\alpha_s(\mu)]^{-a_L(y)}\left[1-\frac{\alpha_s(\mu)}{\pi}\delta_Z(y)\right]\xi(y,\mu)\;. \qquad (3.4)$$

The velocity-dependent anomalous dimension $a_L(y)$ is given by [29,47]

$$a_L(y)=\frac{8}{33-2n_f}[yr(y)-1]\;,\qquad r(y)=\frac{1}{(y^2-1)^{1/2}}\ln[y+(y^2-1)^{1/2}]\;, \qquad (3.5)$$

with $n_f=3$ in the low-energy theory. The next-to-leading-order correction $\delta_Z(y)$ in (3.4) is a complicated, scheme-dependent function [47,43]. In the $\overline{\rm MS}$ renormalization scheme, one has

$$\delta_Z^{\overline{\rm MS}}(y)=\frac{8(109-5n_f-9\pi^2)}{729}(y-1)+O[(y-1)^2]\;, \qquad (3.6)$$

which is a sufficient approximation for $y<1.5$. Note that both $a_L(y)$ and $\delta_Z(y)$ vanish at $y=1$. As a consequence, the renormalization prescription (3.4) preserves the normalization of the Isgur-Wise form factor at zero recoil, i.e., $\xi_{\rm ren}(1)=1$. The renormalized form factor is an observable, universal function of QCD, which contains all long-distance dynamics relevant to the hadronic transition.

The $1/m_Q$ corrections in (3.3) can be systematically classified using the effective-field-theory approach [48,49,35]. In particular, Luke has shown that there are no leading $1/m_Q$ corrections to (3.3) at zero recoil [48]. This is very different from the case of meson decay constants, where no such restriction holds. As a consequence, the $1/m_Q$ corrections to $b\to c$ transitions are expected to be small for basically all values of $v\cdot v'$ that are kinematically accessible. This is indeed confirmed by model estimates [35], and the heavy-quark symmetries have proved to be a useful tool in the theoretical description of weak decays of heavy mesons [31,34,36].

B. Laplace sum rule for the Isgur-Wise function

In order to calculate the universal function $\xi_{\rm ren}(v\cdot v')$ using QCD sum rules, we study the following correlator of currents in the effective theory:

$$i^2\int d^4x\,d^4y\,e^{i(k'\cdot x-k\cdot y)}\langle 0|\mathcal{T}\{\mathcal{A}_5^{(v')}(x),\mathcal{J}_\Gamma^{(v,v')}(0),\mathcal{A}_5^{(v)}(y)^\dagger\}|0\rangle\equiv\Xi(\omega,\omega',v\cdot v')\,{\rm Tr}\left[\frac{\not v'+1}{2}\Gamma\frac{\not v+1}{2}\right],\quad \omega^{(\prime)}\equiv 2k^{(\prime)}\cdot v^{(\prime)}\;, \qquad (3.7)$$

where $\mathcal{J}_\Gamma^{(v,v')}=\bar h_Q(v')\Gamma h_Q(v)$ and, as previously, $\mathcal{A}_5^{(v)}=\bar q\gamma_5 h_Q(v)$. In defining the three-point function Ξ, we have factored out the trace which determines its Lorentz structure in the effective theory. Ignoring QCD corrections, the perturbative contribution to Ξ is obtained be evaluating the triangle diagram shown in Fig. 3(a). The result can be written as a double dispersion integral

$$\Xi^{\rm pert}(\omega,\omega',v\cdot v')=\int d\varpi\,d\varpi'\frac{\rho(\varpi,\varpi',v\cdot v')}{(\varpi-\omega-i\epsilon)(\varpi'-\omega'-i\epsilon)}+\text{subtractions}\;. \qquad (3.8)$$

Setting again $y=v\cdot v'$, the spectral density is given by [36,38]

$$\rho(\varpi,\varpi',y)=\frac{3}{16\pi^2}\frac{\varpi+\varpi'}{(y+1)(y^2-1)^{1/2}}\Theta(\varpi)\Theta(\varpi')\Theta(2y\varpi\varpi'-\varpi^2-\varpi'^2)\;. \qquad (3.9)$$

For the nonperturbative power corrections to the correlator, we find

$$\begin{aligned}\Xi^{\langle\bar qq\rangle}(\omega,\omega',y)&=-\frac{\langle\bar qq\rangle}{\omega\omega'}\;,\\ \Xi^{\langle GG\rangle}(\omega,\omega',y)&=\frac{\langle\alpha_s GG\rangle}{12\pi}(y-1)I_G(\omega,\omega',y)\;,\\ \Xi^{\langle\bar q\sigma Gq\rangle}(\omega,\omega',y)&=\frac{m_0^2\langle\bar qq\rangle}{2\omega\omega'}\left[\frac{1}{\omega^2}+\frac{1}{\omega'^2}+\frac{4y-1}{3\omega\omega'}\right]\;.\end{aligned} \qquad (3.10)$$

The diagram involving the quark condensate is shown in Fig. 3(b). In contrast to the sum rule for the decay constant f_P, we also find a nonvanishing contribution from the gluon condensate. It arises from the diagram depicted in Fig. 3(c) (in coordinate gauge) and is proportional to the parameter integral

$$I_G(\omega,\omega',y)=\int_0^\infty du\frac{u}{1+u^2+2yu}\left[\frac{3u(\omega+\omega')}{(\omega+u\omega')^4}-\frac{2(1+u)}{(\omega+u\omega')^3}\right]\;. \qquad (3.11)$$

The phenomenological side of the sum rule is as usual obtained by saturating the correlator with the lowest intermediate pseudoscalar-meson states and approximating the contributions of higher resonances by the perturbative continuum. Using (2.17) and (3.1), we find

$$\Xi^{\text{phen}}(\omega,\omega',y)=\frac{\xi(y,\mu)F^2(\mu)}{(\bar\Lambda-\omega-i\epsilon)(\bar\Lambda-\omega'-i\epsilon)}+\int_{\omega_c}^{\infty}d\tilde\omega\int_{\omega_c}^{\infty}d\tilde\omega'\frac{\rho(\tilde\omega,\tilde\omega',y)}{(\tilde\omega-\omega-i\epsilon)(\tilde\omega'-\omega'-i\epsilon)}+\text{subtractions}\ . \tag{3.12}$$

To obtain the Laplace sum rule for the Isgur-Wise function, one equates the above expressions after applying the Borel operator with respect to both ω and ω'. In particular, this greatly simplifies the contribution proportional to the gluon condensate. Since the correlator is symmetric in ω and ω', we set the associated Borel parameters τ and τ' equal:

$$\tau=\tau'\equiv 2T\ . \tag{3.13}$$

The factor 2 is chosen for later convenience. The resulting sum rule reads

$$\xi(y,\mu)F^2(\mu)e^{-\bar\Lambda/T}=\frac{3}{8\pi^2}\frac{T^3}{(y+1)(y^2-1)^{1/2}}\int_0^{\omega_c/T}dz\int_0^{\omega_c/T}dz'\frac{z+z'}{2}e^{-(1/2)(z+z')}\Theta(2yzz'-z^2-z'^2)$$
$$-\langle\bar qq\rangle\left[1-\frac{(2y+1)}{3}\frac{m_0^2}{4T^2}\right]+\frac{\langle\alpha_s GG\rangle}{48\pi T}\left[\frac{y-1}{y+1}\right]\ . \tag{3.14}$$

Let us evaluate this equation in the zero-recoil limit $y=v\cdot v'\to 1$. Because of the normalization of the Isgur-Wise function, it then becomes a sum rule for $F^2(\mu)$, which must agree with that derived in Sec. II B. As $y\to 1$, the Θ function in the dispersion integral reduces to $(y^2-1)^{1/2}(z+z')\delta(z-z')$, and it is readily seen that, apart from QCD corrections, one indeed recovers (2.29). This is a consequence of a Ward identity that relates the three-point function Ξ to the derivative of the two-point correlator Γ_5 [50]. It was for this reason that we introduced the factor 2 in (3.13). The empirical observation that the scaled Borel parameter in a three-point sum rule should be chosen approximately twice as large as that in the corresponding two-point sum rule has been first made in Ref. [51]. In the infinite-quark-mass limit and for $v\cdot v'=1$, this relation becomes exact.

It is clear from the above discussion that if one divides the sum rule (3.14) by (2.29), one obtains an expression for the Isgur-Wise function which explicitly obeys the normalization condition $\xi(1,\mu)=1$, independent of the value of T. The resulting equation is furthermore independent of $\bar\Lambda$ and $F^2(\mu)$. This is welcome since the variation of the decay constants with $\bar\Lambda$ was rather strong (see Table I), and $F^2(\mu)$ was associated with the factor $(m_Q/m_P)^3$, which induced large $1/m_Q$ corrections. In addition, we shall see in the following section that also the uncomfortably large QCD corrections to $F^2(\mu)$ do not affect the final expression for the Isgur-Wise function.

Before proceeding, however, it is necessary to modify the sum rule (3.14) in two respects. The first one concerns the simulation of higher-resonance contributions. The integration domain for the dispersion integral is the "kitelike" area shown in Fig. 4. The separation between the pole and continuum contributions appears to be rather crude and to a large extent arbitrary. Performing the integral over the region specified by (3.14), one can show that the derivative of the Isgur-Wise form factor with respect to $v\cdot v'$ diverges at zero recoil, a result which is certainly unphysical. One should, therefore, change the integration domain. To this end it is convenient to introduce new variables $x=(z+z')/2$ and $q=z-z'$ and integrate over a symmetric triangle (see Fig. 4), such that the q integration becomes trivial. Instead of the double

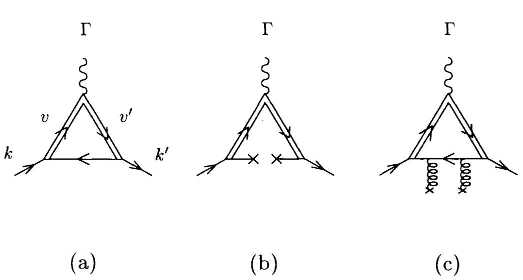

FIG. 3. Lowest-order diagrams for the three-current correlator $\Xi(\omega,\omega',v\cdot v')$: (a) perturbative contribution, (b) correction involving the quark condensate, and (c) correction involving the gluon condensate. In (c) only the diagram yielding a nonvanishing contribution (in the coordinate gauge) is shown.

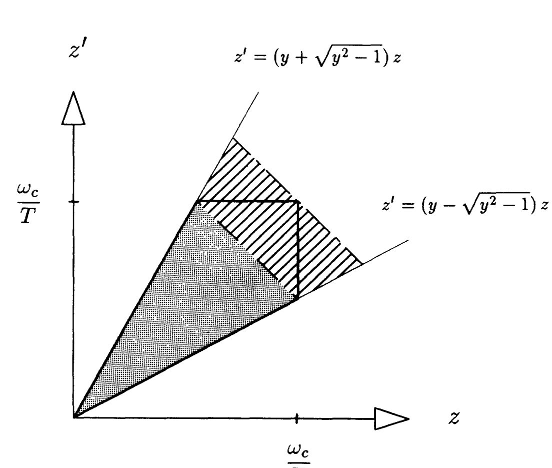

FIG. 4. Integration domain for the double integral in (3.14). The original "kitelike" region is replaced by a symmetric triangle. The small (shaded) triangle corresponds to the lower bound for $\sigma(y)$ in (3.16), the large (shaded + ruled) one to $\sigma(y)=1$.

integral in (3.14), we then obtain

$$\int_0^{\omega_c/T} dz \int_0^{\omega_c/T} dz' \cdots \to 4\left[\frac{y-1}{y+1}\right]^{1/2} I\left[\sigma(y)\frac{\omega_c}{T}\right], \tag{3.15}$$

$$I(z)=\int_0^z dx\, x^2 e^{-x}=2-(2+2z+z^2)e^{-z}.$$

According to Fig. 4, any choice for $\sigma(y)$ in the range

$$\tfrac{1}{2}[y+1-(y^2-1)^{1/2}]<\sigma(y)<1 \tag{3.16}$$

is reasonable. Note, however, that $\sigma(1)=1$ in any case. A finite slope of the Isgur-Wise function is obtained, e.g., for $\sigma(y)=(y+1)/2y$. The arbitrariness of $\sigma(y)$ yields to a significant uncertainty of the sum-rule prediction for the universal form factor, as will be discussed in Sec. III C below. Facing the lack of information on the structure of higher-resonance contributions to the spectral function Ξ, this uncertainty is unavoidable.

A second modification is required to improve the large-recoil behavior of the sum rule (3.14). For large values of $y=v\cdot v'$ (corresponding to large negative values of q^2), the Isgur-Wise function should tend to zero, whereas the nonperturbative contributions in (3.14) stay constant or even increase with y. Hence higher-order power corrections must compensate the contributions of the lowest-dimensional ones. In order to model this cancellation and to simulate the effect of higher-order corrections, one approximates the nonlocal quark condensate by a Gaussian distribution in Euclidean spacetime [52–54]:

$$\langle \bar{q}(x)q(0)\rangle \simeq \langle \bar{q}q\rangle e^{-x^2/\lambda^2}, \quad \lambda^2=\frac{16}{m_0^2}. \tag{3.17}$$

The damping length λ is chosen to be consistent with the short-distance expansion

$$\langle \bar{q}(x)q(0)\rangle = \langle \bar{q}q\rangle - \frac{x^2}{16} g_s \langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle + \cdots . \tag{3.18}$$

The standard value $m_0^2 \simeq 0.8$ GeV^2 corresponds to $\lambda \simeq 1$ fm. For the simple Gaussian ansatz (3.17), the contribution of the diagram shown in Fig. 3(b) can be calculated in closed form. Denoting the "double-borelized" correlator by $\hat{\hat{\Xi}}$, the result is

$$4T^2 \hat{\hat{\Xi}}^{\langle \bar{q}q\rangle} = -\langle \bar{q}q\rangle \exp\left[-\frac{m_0^2}{4T^2}\left[\frac{y+1}{2}\right]\right]. \tag{3.19}$$

In contrast to the old result, the contribution of the nonlocal condensate becomes exponentially small at large recoil. Comparison with (3.14) shows that part of the contribution of the mixed condensate is taken into account by (3.19). In the following we shall assume that all nonperturbative contributions have the same exponential damping factor at large values of y. It must be noted that for physical values of $y=v\cdot v'$ relevant for $\bar{B}\to D^{(*)}$ decays, the numerical effect of the exponentiation is less than 3%.

Putting everything together, we obtain the following expression for the universal form factor [36,38]:

$$\xi(y,\mu)=\frac{\dfrac{3T^3}{8\pi^2}\left[\dfrac{2}{y+1}\right]^2 I\left[\sigma(y)\dfrac{\omega_c}{T}\right]+C(T,y)}{\dfrac{3T^3}{8\pi^2} I\left[\dfrac{\omega_c}{T}\right]+C(T,1)}. \tag{3.20}$$

where the condensate contributions are given by (the scale in the quark condensate is chosen for later convenience)

$$C(T,y)=\left[-\langle \bar{q}q\rangle(T)\left[1-\frac{(y-1)}{6}\frac{m_0^2}{4T^2}\right]+\left[\frac{y-1}{y+1}\right]\frac{\langle \alpha_s GG\rangle}{48\pi T}\right]\exp\left[-\frac{m_0^2}{4T^2}\frac{(y+1)}{2}\right]. \tag{3.21}$$

This result explicitly obeys the normalization condition $\xi(1,\mu)=1$. Since, up to now, QCD corrections have been ignored, our theoretical expression does not yet reproduce the proper scale dependence of $\xi(y,\mu)$. The calculation of the renormalized form factor defined in (3.4) is the subject of the next section.

C. Calculation of the renormalized Isgur-Wise form factor

In order to calculate the renormalized Isgur-Wise function, one has to include the QCD corrections to both the numerator and denominator of (3.20). The corrections to the denominator have been calculated in Sec. II B and are given in (2.29). The corrections to the three-point correlator Ξ involve loop integrals in the effective theory with two heavy quarks, which depend on ω, ω', and $v\cdot v'$. As an example, we have calculated the radiative corrections to the contribution involving the quark condensate [see Fig. 3(b)]. After applying the double Borel transformation, we find, using the formulas given in the Appendix,

$$\hat{\hat{\Xi}}^{\langle \bar{q}q\rangle} = -\frac{\langle \bar{q}q\rangle(\mu)}{4T^2}\left[1+\frac{2\alpha_s}{3\pi}\left[1+3\delta+2[yr(y)-1]\ln\frac{T}{\mu}+c(y)\right]\right]. \tag{3.22}$$

where $\delta_{\overline{\rm MS}}=\frac{2}{3}$, and $c(y)$ is a scheme-dependent function, which vanishes at zero recoil. In the $\overline{\rm MS}$ scheme,

$$c_{\overline{\rm MS}}(y)=[yr(y)-1][\ln 8(y+1)-2\gamma_E]-(y-1)r(y)+2 - \frac{y}{(y^2-1)^{1/2}}[L_2(1-y_-^2)-L_2(1-y_+^2)+L_2(1-y_+)-L_2(1-y_-)] = \tfrac{4}{3}(\ln 4-\gamma_E-\tfrac{7}{3})(y-1)+O[(y-1)^2]\,, \tag{3.23}$$

where $y_\pm=y\pm(y^2-1)^{1/2}, \gamma_E\simeq 0.5772$, and $r(y)$ has been defined in (3.5). The important observations are the following:

(i) At zero recoil the radiative correction is precisely that encountered in (2.29) in the calculation of the correlator Γ_5. The equivalence of the sum rules at zero recoil is preserved by renormalization.

(ii) The logarithmic dependence on μ in (3.22) can be summed to yield

$$\langle\bar{q}q\rangle(\mu)\left[1+\frac{4\alpha_s}{3\pi}[yr(y)-1]\ln\frac{T}{\mu}\right] \to \langle\bar{q}q\rangle(T)\left[\frac{\alpha_s(\mu)}{\alpha_s(T)}\right]^{-4/9+a_L(y)} . \tag{3.24}$$

The factor $[\alpha_s(\mu)/\alpha_s(T)]^{-4/9}$ also appears in the renormalization-group-improved version of the denominator in (3.20) and drops out of the ratio. The remaining μ-dependent anomalous scaling factor is precisely canceled if one computes the renormalized Isgur-Wise form factor from (3.4).

(iii) The functions $c(y)$ from (3.22) and $\delta_Z(y)$ from (3.4) combine to give a scheme-independent result

$$\tfrac{2}{3}c(y)-\delta_Z(y)\simeq -1.42(y-1)+O[(y-1)^2]\,. \tag{3.25}$$

As a consequence of the restriction that it must vanish at $y=1$, this velocity-dependent next-to-leading-order correction stays small for all relevant values of $y=v\cdot v'$. We shall neglect it from now on.

The radiative corrections to the triangle diagram of Fig. 3(a) involve two-loop diagrams and are hard to calculate. However, they must exhibit the same structures as encountered above. The logarithmic dependence on μ/T must be of the same form as in (3.24), and the correction must reduce to that given in (2.29) at zero recoil. There is thus no need to calculate the two-loop diagrams as long as one neglects those next-to-leading-order corrections which vanish at $y=1$. In view of the uncertainties associated with the simulation of higher-resonance contributions to the perturbative part of the three-point sum rule, this is a safe approximation. In total, we obtain, from (3.4) and (3.14), for the renormalized Isgur-Wise function

$$\xi_{\rm ren}(y)=[\alpha_s(T)]^{-a_L(y)}\frac{\frac{3T^3}{8\pi^2}\left[\frac{2}{y+1}\right]^2\tilde{I}\left[\sigma(y)\frac{\omega_c}{T}\right]+\eta_{\rm QCD}^{-1}C(T,y)}{\frac{3T^3}{8\pi^2}\tilde{I}\left[\frac{\omega_c}{T}\right]+\eta_{\rm QCD}^{-1}C(T,1)}\,, \tag{3.26}$$

where

$$\eta_{\rm QCD}=1+\frac{\alpha_s(\tilde{\Lambda})}{\pi}\left[\frac{11}{3}+\frac{4\pi^2}{9}\right]\simeq 1.8 \tag{3.27}$$

is essentially the difference of the QCD corrections to the dispersion integral and to the quark condensate, and we have introduced the new function

$$\tilde{I}(z)=\int_0^z dx\, x^2 e^{-x}\left[1-\frac{2\alpha_s(\tilde{\Lambda})}{\pi}\ln x\right]. \tag{3.28}$$

Our final result (3.26) for the universal form factor has an appealing structure. The normalization condition $\xi_{\rm ren}(1)=1$ is explicitly fulfilled. In contrast to the sum rule for the decay constant f_P discussed in Sec. II B, the large QCD corrections to the perturbative contribution cancel in the ratio, since they are y independent. The only remnant is the QCD factor $\eta_{\rm QCD}^{-1}\simeq 0.6$, which leads to a significant suppression of the nonperturbative contributions. This is still a large correction, but only affects the small power corrections to the leading perturbative contribution, which therefore determines the shape of the universal form factor. The corresponding y dependence is approximately of the form $[2/(y+1)]^2$, corresponding to a double pole at $q^2=(m_{Q_1}+m_{Q_2})^2$. Corrections to a pure pole behavior arise, however, from the function $\sigma(y)$.

As mentioned previously, the final result for the heavy-quark form factor would be unaffected by mass factors of the type m_Q/m_P even if $1/m_Q$ corrections were included. In the framework of QCD sum rules, we therefore do not expect these corrections to be unusually large. This is very different form the situation encountered for f_P.

For the numerical evaluation of (3.26), we vary the continuum threshold ω_c between 1.9 and 2.5 GeV and the Borel parameter T between 1.0 and 2.5 GeV, corresponding to the range of values providing stability of the two-point sum rule investigated in Sec. II B. The radiative corrections in (3.27) and (3.28) are evaluated at the scale $\tilde{\Lambda}=1.25$ GeV. For the function $\sigma(y)$ in the dispersion integral, we consider the two extreme choices

$\sigma_{\min}(y)=\frac{1}{2}[y+1-(y^2-1)^{1/2}]$ and $\sigma_{\max}=1$ [cf. (3.16)], as well as $\sigma_0(y)=(y+1)/2y$, which for all $y>1$ lies in between these limits. The latter ansatz gives results very similar to those obtained evaluating the original form of the dispersion integral in (3.14), but, as discussed earlier, yields a finite slope of the universal form factor at zero recoil. The results are summarized in Fig. 5. The three bands correspond to the three choices for $\sigma(y)$. The width of the bands arises from the variation of the sum-rule parameters ω_c and T in the limits specified above. Obviously, the dependence on these parameters is very weak. Also, a variation of $(\eta_{\rm QCD}-1)$ by $\pm 50\%$ changes the results by less than 4%, indicating that the QCD corrections are under control. The main uncertainty is due to the arbitrariness of the choice of $\sigma(y)$. We consider the band obtained using $\sigma_0(y)$ as a reasonable estimate, whereas the results obtained using $\sigma_{\min}(y)$ and $\sigma_{\max}$ should be regarded as very conservative lower and upper limits, respectively.

Also shown in Fig. 5 are "data" for the universal form factor $\xi_{\rm ren}(v\cdot v')$ that have been extracted from an experimental measurement of the differential branching ratio for the semileptonic decay $\bar{B}^0\to D^{*+}l\bar{\nu}_l$ by the CLEO and ARGUS collaborations [55], accounting for the leading QCD and $1/m_Q$ corrections to the infinite-quark-mass limit [34]. The normalization of the data corresponds to $|V_{cb}|\tau_{B^0}^{1/2}=0.044\times(1.18\ {\rm ps})^{1/2}$. It is seen that the sum-rule result obtained using $\sigma_0(y)$ nicely compares to the data. Defining the slope parameter ρ by $\xi'_{\rm ren}(1)=-\rho^2$, we obtain the prediction $\rho_{\rm SR}=1.13\pm0.11$. This is in accordance with previous estimates of this parameter such as $\rho=1.20\pm0.17$ in Ref. [31] and $\rho=1.13\pm0.23$ in Ref. [34]. A simple parametrization of the sum-rule result in terms of a pole-type function is

$$\xi_{\rm ren}^{\rm SR}(y)=\left[\frac{2}{y+1}\right]^{\beta(y)}, \quad \beta(y)\simeq 2+\frac{0.6}{y}\ . \tag{3.29}$$

It exhibits dipole behavior at large recoil.

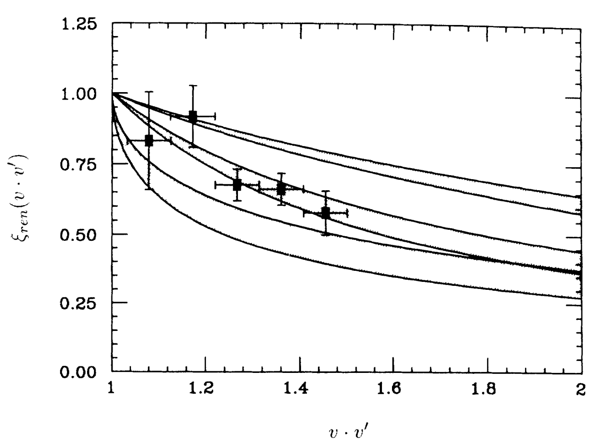

FIG. 5. Sum-rule result for the renormalized Isgur-Wise form factor. The three bands correspond, from top to bottom, to $\sigma(y)=1$, $\sigma_0(y)$, and $\sigma_{\min}(y)$, respectively. The data points are taken from Ref. [34].

IV. CONCLUSIONS

We have presented a consistent framework for the calculation of hadronic form factors of heavy mesons using QCD sum rules in the effective theory for heavy quarks. Since the effective currents have nonvanishing anomalous dimensions, a renormalization of the form factors is required. We have included the complete next-to-leading-order renormalization-group improvement, thereby summing the logarithms $(\alpha_s \ln m_Q)^n$ and $\alpha_s(\alpha_s \ln m_Q)^n$ to all orders in perturbation theory. This procedure resolves the scale-ambiguity problem associated with the order-α_s corrections. We have applied this technique to derive sum rules for the asymptotic value of the scaled meson decay constant $f_P\sqrt{m_P}$ and of the universal Isgur-Wise form factor, which describes current matrix elements between two heavy mesons in the infinite-quark-mass limit.

In the case of f_P, the renormalization-group improvement turns out to be most important and resolves the discrepancy between previous sum-rule and lattice calculations. The large radiative correction to the correlator of two axial currents has to be evaluated at a low-energy scale rather than at the scale of the heavy quark. While this effect is unimportant for the decay constant of the D meson, it considerably enhances previous estimates of the value of f_B. In the static approximation (i.e., neglecting $1/m_b$ corrections), we find $f_B^{\rm stat}\simeq 200-300$ MeV, in agreement with recent lattice results.

Combining the renormalization-group-improved sum rule derived in the effective theory with the standard Laplace sum rule for f_P, we have investigated the effect of finite heavy-quark masses. The appearance of an explicit factor $(m_Q/m_P)^3$ in front of the sum rule is found to be the origin of unnaturally large finite-mass corrections to the decay constant of the D meson. This effect is, however, specific for decay constants of pseudoscalar particles and should not be considered as an indication for a general breakdown of the heavy-quark expansion for charm. Taking into account the full dependence on m_Q, we find the physical decay constants $f_D\simeq 170\pm30$ MeV and $f_B\simeq 190\pm50$ MeV. For large values of m_Q, the decay constants depend rather strongly on the mass difference $\bar{\Lambda}\simeq 2(m_P-m_Q)$. The quoted value for f_B refers to $m_b=4.67\pm0.10$ GeV.

The sum rule for the renormalized Isgur-Wise form factor $\xi_{\rm ren}(v\cdot v')$ is obtained from the study of a three-current correlation function in the effective theory. At zero recoil a Ward identity relates this function to the two-current correlator from which one derives the sum rule for f_P. As a consequence, the Isgur-Wise function can be expressed as the ratio of two sum rules in such a way that its normalization at zero recoil is explicitly obeyed. This ratio is independent of the parameter $\bar{\Lambda}$ and, after renormalization-group improvement, is not affected by uncomfortably large QCD corrections. The main uncertainty arises from the way in which higher-resonance contributions to the perturbative part of the sum rule are approximated. We have given a prescription that ensures a finite slope of the universal form factor at zero recoil and discussed conservative lower and upper limits for $\xi_{\rm ren}(v\cdot v')$. Our final result compares nicely to data on the form factor extracted from the differential decay rate for $\bar{B}\to D^* l\bar{\nu}_l$ decays.

ACKNOWLEDGMENTS

It is a pleasure to thank J. D. Bjorken, A. Falk, M. Peskin, Y. Nir, B. Stech, and N. Tina for helpful discussion. Financial support from the BASF Aktiengesellschaft and the German National Scholarship Foundation is gratefully acknowledged. This work was also supported by the Department of Energy, contract DE-AC03-76SF00515.

APPENDIX

The Feynman rules of the heavy-quark effective theory can be readily obtained from the effective Lagrangian derived by Georgi [7]. The momentum of the heavy quark is split into an on-shell and an off-shell piece, $P=m_Q v+k$, where k acts as an infrared cutoff and is of order $\Lambda_{\rm QCD}$. In momentum space the propagator of the heavy quark is then given by $(i/v\cdot k)(\not{v}+1)/2$. The heavy-quark–gluon coupling is $ig_s v_\mu t_a$. Master integrals for the calculation of one- and two-loop integrals involving only a single heavy quark are given in Ref. [12]. The master one-loop integral for diagrams involving two heavy quarks with velocities v,v' and off-shell energies $\omega=2v\cdot k$, $\omega'=2v'\cdot k'$ is (in D space-time dimensions)

$$I_{\alpha\beta\gamma}(\omega,\omega',v\cdot v')=\int d^Dt\left[-\frac{1}{t^2}\right]^\alpha\left[\frac{1}{\omega+2v\cdot t}\right]^\beta\left[\frac{1}{\omega'+2v'\cdot t}\right]^\gamma$$
$$=i\pi^{D/2}I(\alpha,\beta,\gamma)\int_0^\infty du\frac{u^{\gamma-1}}{[\Omega(u)]^{\beta+\gamma}}\left[-\frac{\Omega(u)}{V(u)}\right]^{D-2\alpha},$$

with $(v\cdot v'\equiv\cosh\theta)$

$$\Omega(u)=\omega+u\omega',$$
$$V(u)=(1+2uv\cdot v'+u^2)^{1/2}=(u+e^\theta)^{1/2}(u+e^{-\theta})^{1/2}$$

and coefficients

$$I(\alpha,\beta,\gamma)=\frac{\Gamma(2\alpha+\beta+\gamma-D)\Gamma(D/2-\alpha)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}.$$

It is convenient to apply the double Borel transformation with respect to ω and ω' before performing the integral over the Feynman parameter u. Useful formulas are

$$\hat{B}_{\tau'}\hat{B}_\tau[-\Omega(u)]^{-\alpha}=\frac{1}{\Gamma(\alpha)\tau^{\alpha-1}\tau'}\delta(u-\tau/\tau'),$$
$$\hat{B}_{\tau'}\hat{B}_\tau\frac{1}{\omega\omega'}[-\Omega(u)]^{-\alpha}=\frac{1}{\Gamma(\alpha+1)\tau\tau'}\left[\frac{\Theta(\tau/\tau'-u)}{\tau^\alpha}+\frac{\Theta(u-\tau/\tau')}{(u\tau')^\alpha}\right].$$

Additional powers of ω' or $\omega=\Omega(u)-u\omega'$ in the numerator can be generated by derivatives with respect to the Feynman parameter u.

[1] E. Eichten and F. Feinberg, Phys. Rev. D **23**, 2724 (1981); E. Eichten, in *Field Theory on the Lattice,* Proceedings of the International Symposium, Seillac, France, 1987, edited by A. Billoire *et al.* [Nucl. Phys. B (Proc. Suppl.) **4**, 170 (1988)]; E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990).

[2] G. P. Lepage and B. Thacker, in *Field Theory on the Lattice* [1], p. 119.

[3] B. Grinstein, Nucl. Phys. **B339**, 253 (1990).

[4] M. B. Voloshin and M. A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)].

[5] M. B. Voloshin and M. A. Shifman, Yad. Fiz. **47**, 801 (1988) [Sov. J. Nucl. Phys. **47**, 511 (1988)].

[6] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990); Nucl. Phys. **B348**, 276 (1991).

[7] H. Georgi, Phys. Lett. B **240**, 447 (1990).

[8] J. G. Körner and G. Thompson, Phys. Lett. B **264**, 185 (1991).

[9] T. Mannel, W. Roberts, and Z. Ryzak, Harvard Report No. HUTP-91/A017, 1991 (unpublished).

[10] H. D. Politzer and M. B. Wise, Phys. Lett. B **206**, 681 (1988); **208**, 504 (1988).

[11] X. Ji and M. J. Musolf, Phys. Lett. B **257**, 409 (1991).

[12] D. J. Broadhurst and A. G. Grozin, Phys. Lett. B **267**, 105 (1991).

[13] For a review, see S. Capstick and S. Godfrey, Phys. Rev. D **41**, 2856 (1990).

[14] P. Colangelo, G. Nardulli, and M. Pietroni, Phys. Rev. D **43**, 3002 (1991).

[15] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).

[16] C. R. Allton *et al.*, Nucl. Phys. **B349**, 598 (1991).

[17] C. Alexandrou *et al.*, Phys. Lett. B **256**, 60 (1991).
[18] C. Bernard, J. Labrenz, and A. Soni, in *Lattice '90*, Proceedings of the International Symposium, Tallahassee, Florida, 1990, edited by U. M. Heller, A. D. Kennedy, and S. Sanielevici [Nucl. Phys. B (Proc. Suppl.) **20** (1990)].
[19] For a review, see L. Maiani, Helv. Phys. Acta **64**, 853 (1991).
[20] V. A. Novikov *et al.*, Phys. Rev. Lett. **38**, 626 (1977).
[21] E. V. Shuryak, Nucl. Phys. **B198**, 83 (1982).
[22] V. L. Chernyak, A. R. Zhitnitsky, and I. R. Zhitnitsky, Yad. Fiz. **38**, 1303 (1983) [Sov. J. Nucl. Phys. **38**, 774 (1983)].
[23] T. M. Aliev and V. L. Eletskii, Yad. Fiz. **38**, 1572 (1983) [Sov. J. Nucl. Phys. **38**, 936 (1983)].
[24] V. S. Mathur and T. Yamawaki, Phys. Rev. D **29**, 2057 (1984).
[25] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[26] L. J. Reinders, Phys. Rev. D **38**, 947 (1988).
[27] C. A. Dominguez and N. Paver, Phys. Lett. B **197**, 423 (1987); **199**, 596(E) (1987); **246**, 493 (1990).
[28] S. Narison, Phys. Lett. B **218**, 238 (1989).
[29] A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. **B343**, 1 (1990).
[30] J. D. Bjorken, invited talk given at Les Recontre de la Valle d'Aoste La Thuile, Aosta Valley, Italy, 1990, SLAC Report No. SLAC-PUB-5278, 1990 (unpublished); in *Proceedings of the Eighteenth SLAC Summer Institute on Particle Physics,* Stanford, California, 1990, edited by J. Hawthorne (SLAC, Stanford, in press).
[31] J. L. Rosner, Phys. Rev. D **42**, 3732 (1990); in *Research Directions for the Decade,* Proceedings of the Snowmass Summer Study on High Energy Physics, Snowmass, Colorado, 1990, edited by E. L. Berger and I. Butler (World Scientific, Singapore, 1991).
[32] N. Isgur and M. B. Wise, Phys. Rev. D **42**, 2388 (1990).
[33] T. Mannel, W. Roberts, and Z. Ryzak, Phys. Lett. B **248**, 392 (1990); **255**, 593 (1991); **259**, 359 (1991); Nucl. Phys. **B355**, 38 (1991).
[34] M. Neubert, Phys. Lett. B **264**, 455 (1991).
[35] M. Neubert and V. Rieckert, Heidelberg Report No. HD-THEP-91-6, 1991 (unpublished).
[36] M. Neubert, V. Rieckert, B. Stech, and Q. P. Xu, in *Heavy Flavours,* edited by A. J. Buras and M. Lindner, Advanced Series on Directions in High Energy Physics (World Scientific, Singapore, in press).
[37] C. A. Dominguez and N. Paver, Phys. Lett. B **269**, 169 (1991).
[38] A. V. Radyushkin, Phys. Lett. B **271**, 218 (1991).
[39] D. J. Broadhurst, Phys. Lett. **101B**, 423 (1981); D. J. Broadhurst and S. C. Generalis, Open University Report No. OUT-4102-8/R, 1982 (unpublished).
[40] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).
[41] S. Narison, Phys. Lett. B **197**, 405 (1987), and references therein; **216**, 191 (1989).
[42] S. Narison, *QCD Spectral Sum Rules* (World Scientific, Singapore, 1989).
[43] M. Neubert, Heidelberg Report No. HD-THEP-91-30, 1991 (unpublished).
[44] M. B. Voloshin and Y. M. Zaitsev, Usp. Fiz. Nauk **152**, 361 (1987) [Sov. Phys. Usp. **30**, 553 (1987)].
[45] Ph. Boucaud, C. L. Lin, and O. Pène, Phys. Rev. D **40**, 1529 (1989); **41**, 3541(E) (1990).
[46] E. Eichten and B. Hill, Phys. Lett. B **240**, 193 (1990).
[47] G. P. Korchemsky and A. V. Radyushkin, Nucl. Phys. **B283**, 342 (1987); G. P. Korchemsky, Mod. Phys. Lett. A **4**, 1257 (1989).
[48] M. E. Luke, Phys. Lett. B **252**, 447 (1990).
[49] A. F. Falk, B. Grinstein, and M. E. Luke, Nucl. Phys. **B357**, 185 (1991).
[50] V. N. Baier and A. G. Grozin, Z. Phys. C **47**, 669 (1990).
[51] B. L. Ioffe and A. V. Smilga, Phys. Lett. **114B**, 353 (1982); Nucl. Phys. **B216**, 373 (1983).
[52] S. V. Mikhailov and A. V. Radyushkin, Yad. Fiz. **49**, 794 (1989) [Sov. J. Nucl. Phys. **49**, 494 (1989)].
[53] E. V. Shuryak, Nucl. Phys. **B328**, 85 (1989).
[54] P. Ball, V. M. Braun, and H. G. Dosch, Phys. Rev. D **44**, 3567 (1991).
[55] D. Bortoletto and S. Stone, Phys. Rev. Lett. **65**, 2951 (1990).

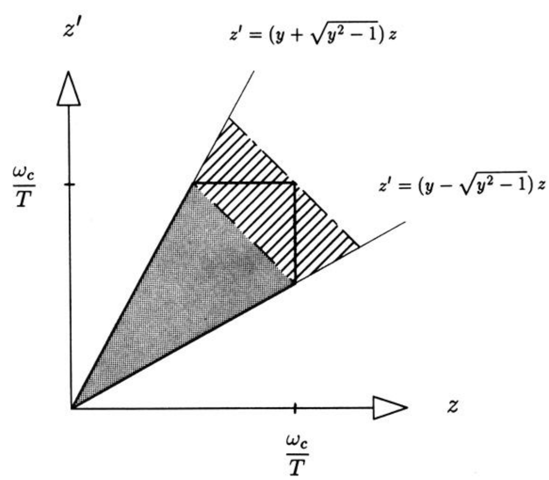

FIG. 4. Integration domain for the double integral in (3.14). The original "kitelike" region is replaced by a symmetric triangle. The small (shaded) triangle corresponds to the lower bound for $\sigma(y)$ in (3.16), the large (shaded + ruled) one to $\sigma(y)=1$.

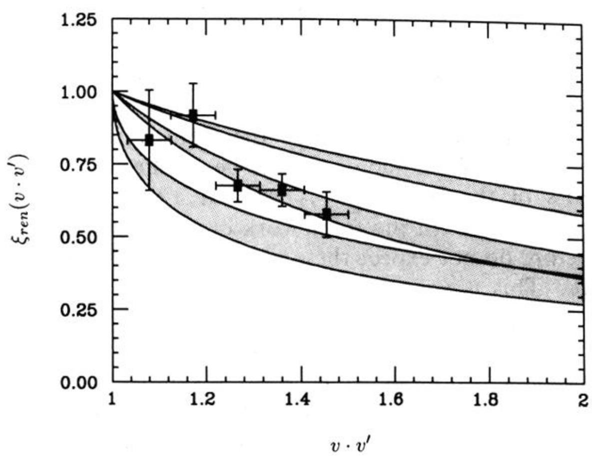

FIG. 5. Sum-rule result for the renormalized Isgur-Wise form factor. The three bands correspond, from top to bottom, to $\sigma(y)=1$, $\sigma_0(y)$, and $\sigma_{\min}(y)$, respectively. The data points are taken from Ref. [34].