Relevance of a dilute instanton ensemble to light hadrons

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We investigate the relevance of a dilute instanton ensemble to the masses and structure of light hadrons by using the lattice-cooling method to suppress all short-wavelength modes in the quenched QCD vacuum while leaving the well-separated instantons more or less intact. Our hadron model-independent results indicate that the masses and sizes of the pion, ρ, and nucleon are dominated by dilute-instanton configurations and insensitive to perturbative gluon exchange and confinement.

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I. INTRODUCTION

Despite a common belief that quantum chromodynamics (QCD) is the underlying theory of strong interaction, there has not been a satisfactory derivation of hadronic properties starting from QCD. The connections between QCD and various successful phenomenological models are not even clear, although the basic ingredients in these models are believed to originate from QCD. The lattice QCD method provides a systematic, nonperturbative framework to calculate matrix elements in hadronic states and thus the hope to study hadron structure in a model-independent way.

A lot of effort in lattice QCD calculations has been spent in calculating as accurately and realistically as possible various physical observables and comparing them directly with experimental results. Here we take the attitude that the lattice method is basically all right, and can be used as a laboratory to study hadron physics where we can dial parameters, such as the coupling constant, to investigate the behavior of hadrons at artificial situations never realized in real experiments. In other words, we are reporting here results of some "experiments" with simulated hadrons.

These experiments with simulated hadrons are useful in identifying the dominant modes in the QCD vacuum, which contribute to various observables. In particular, we are interested in the relevance of the dilute instantons to the light hadron masses and structure. By light hadrons, we mean the ground states of hadrons made up of u and d quarks, such as the pion, ρ, and nucleon. We used the technique of "cooling" to suppress perturbative gluon exchanges and confinement in the quenched QCD vacuum. Then we calculated the masses and the Bethe-Salpeter amplitudes of the cooled hadrons. By comparing the cooled hadrons to the normal ones we can then study the relevance of the dilute instanton ensemble.

We emphasize that the method we used and the results we obtained are independent of hadron models. No artificial assumptions, such as the bag boundary conditions or some carefully selected effective degrees of freedom, are involved. In addition, no new term was added to the QCD action in cooling. By carefully monitoring the string tension, the action and the topological charges in the configurations, we could indeed focus on the modes we want to study. Similar strategies for studying the relevance of certain modes to a specific phenomenon by suppressing or enhancing their statistical weights in the ensemble average have been used before, such as in Ref. [1].

II. COOLING

Cooling [2,3] is a sequence of operations that updates an equilibrated field configuration by minimizing *locally* the action density. The minimization can be achieved by a Monte Carlo updating with $\beta=\infty$ or a Cabibbo-Marinari pseudo-heat-bath updating [4] without thermal noise. In a loose sense, cooling is nothing but *locally* smoothing the field configurations. Locality usually implies that the short-wavelength modes are eliminated more efficiently than the long-wavelength ones, analogous to the phenomenon of critical slowing down. Although there is no clear consensus on how to understand cooling precisely in terms of field theory, it is sufficient for our purpose to regard cooling as a technical means to switch off the short-wavelength modes while leaving the long-wavelength modes almost intact in each given equilibrated field configuration. In contrast with the renormalization-group transformations where the effect of the integrated-out short-wavelength modes is taken into account by the modified operators, cooling only suppresses the weights of the short-wavelength modes in the partition function. No operator modification is involved in the cooling process.

To put this in mathematical terms, let us first schematically consider two averages: $\langle A \rangle = \sum_\lambda P(\lambda) A(\lambda)$ and $\langle A \rangle' = \sum_\lambda C(\lambda) P(\lambda) A(\lambda)$, where $P(\lambda)$ and $A(\lambda)$ are some arbitrary distribution function and the amplitude, in terms of wavelength λ, respectively, and $C(\lambda)$ is a smeared steplike function, being one when λ is large and close to zero when λ is small. If $A(\lambda)$ is strongly peaked in large (small) λ, $\langle A \rangle'$ will be close to (different from) $\langle A \rangle$; so, by comparing these two averages we can learn how $A(\lambda)$ is peaked. In the case of the path integral, the ensemble average of an operator A is defined by

$$\langle A \rangle \equiv \int [dU]\exp(-\beta S[U])A(U)$$

where U indicates the gauge fields, β relates to the inverse of the coupling constant, and $S[U]$ is the action. Usually, $S[U]$ is written as a sum of the action density (or Lagrangian) over spacetime: $S[U]=\sum_x L[U(x)]$. However, we can also write the action as a sum over the wavelength $S[U]=\sum_\lambda L[U_\lambda]$ through a Fourier transform. To smooth a field configuration we make the high-frequency modes cost more energy to create than they would have cost in the normal case. Cooling can then be understood as to average A in the sense

$$\langle A \rangle' \equiv \int [dU]\exp(-\beta S'[U])A(U_\lambda) ,$$

where $S'[U]=\sum_\lambda L[U_\lambda]/C(\lambda)$, with $C(\lambda)$ being a similar steplike function mentioned earlier. Notice that $L[U_\lambda]$ remains the same in both averages. The only difference between $S[U]$ and $S'[U]$ is in the short-wavelength part. So the comparison between $\langle A \rangle$ and $\langle A \rangle'$ can give us the information on whether the operator A is dominated by the short- or long-wavelength modes. Operationally, the factor $C(\lambda)$ is inserted after the normal Monte Carlo updating for gauge fields, and it is not the same as if we truly use $S'[U]$ as the action to do the Monte Carlo updating.

A priori, we do not know how cooling changes the configurations. We therefore monitor the cooling process by measuring a set of physical observables, which correspond to a particular set of modes in the field configuration. The changes in each observable then tell us which modes are eliminated after a certain number of cooling steps. We use the total action to monitor the perturbative gluon exchange, since the total action in the weak-coupling limit can be written as $S_{\text{total}}=bg^2V+$ nonperturbative contributions, where g is the coupling constant, V the space-time volume, and b a numerical constant. The monitor for confinement is the string tension, and the long-wavelength modes, such as the instantons, are monitored by plateaus in the total action and the topological charge.

Alternatively, we can also regard the cooling as a process where the coupling constant pertinent to each mode becomes cooling time dependent, $g_\lambda=g_\lambda(t)$, with the initial condition that $g_\lambda(0)=g_0$, the normal coupling constant. This is so because we can absorb $C(\lambda)$, which is obviously cooling time dependent, into the definition of the coupling $\beta_\lambda \equiv \beta/C(\lambda)$. For example, when the total action is still dominated by the perturbative part, the perturbative coupling constant can be defined by

$$g^2_{\text{pert}}(t) \equiv S_{\text{total}}(t)/bV . \qquad (1)$$

The consistency of this definition has been partially checked in Ref. [5]. Likewise, the coupling constant for the instantons can be defined by the action plateau

$$g^2_{\text{inst}}(t) \equiv n\,8\pi^2/S_{\text{total}}(t) , \qquad (2)$$

where n is an integer. Since g_λ characterizes the strength of mode λ, a diminishing weight in the distribution function can be viewed as decreasing the corresponding coupling. We emphasize that Eqs. (1) and (2) do not need to have a precise field-theoretical meaning; they only help us to identify the relevant physics during cooling.

We used a $8^3\times 16$ lattice at $\beta=5.7$. Ten SU(3) gauge configurations were generated using the Cabibbo-Marinari pseudo-heat-bath method [4], separated by 500 iterations each. We cooled these configurations with the method mentioned above, while monitoring the action, the topological charge ($Q_t(x) \equiv -\epsilon_{\mu\nu\rho\sigma}\text{Re}\,\text{Tr}[U_{\mu\nu}(x)U_{\rho\sigma}(x)]/32\pi^2$, where $U_{\mu\nu}$ is the plaquette variable), and the string tension (σ) as a function of cooling steps (Fig. 1). As can be seen from Fig. 1(a), the action decreases very rapidly in the initial 30 cooling steps, settling down at some plateaus corresponding to a few instantons, anti-instantons or pairs, in good agreement with the topological charges shown in Fig. 1(b). By the one-hundredth cooling step where we stopped, the perturbative gluon exchange and confinement are both strongly suppressed, as shown by S_{total} [or g^2_{pert} according to Eq. (1)] and σ, being smaller by a factor of 100 and 10, respectively. The fact that Eq. (2) becomes a good approximation and the instanton and anti-instanton numbers are consistent with being Poisson distributed [6] suggest that only dilute instantons are left in the cooled configurations.

According to Ref. [6], the ratio of the topological susceptibility (obtained with the cooling method) to the string tension has already attained its scaling limit for $\beta \geq 5.7$. We could therefore use either the string tension for the uncooled configuration or the topological susceptibility (believed to be the same before and after the cooling) to fix the lattice constant pertinent to large instantons. Using the string tension, the inverse lattice spacing was determined to be $1/a=1$ GeV (see Ref. [7]). On the other hand, fitting the topological susceptibility observed on the lattice after 100 cooling steps,

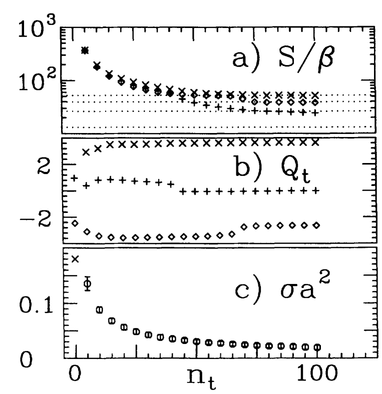

FIG. 1. Three observables vs number of cooling steps (n_t): (a) total action for three configurations; the dotted lines indicate the integer multiples of a single instanton action; (b) the corresponding total topological charge; (c) the string tension averaged over ten configurations; the uncooled value [14] ($\times$) is shown at $n_t=0$.

$\chi_t \equiv \langle Q_t^2 \rangle / V = 5.2 \pm 2.0$, to the phenomenological value of (190 MeV)^4 gives $1/a = 1.2$ GeV. The close agreement between the two ways to fix a indicates that the distortion of the long-wavelength modes due to the cooling process is small. The small difference in a could be caused by statistical fluctuations. We will use the value given by the topological susceptibility in this paper, because it is directly related to the long-range modes and also the value we measured directly using our ten configurations. We emphasize here that our conclusions are not changed with either choice of a.

III. CALCULATION AND RESULTS

From the cooled configurations, we obtained the fermion propagators using the conjugate gradient method to invert the Wilson fermion matrix, at the hopping parameters $\kappa = 0.122$, 0.124, 0.126, and 0.128, roughly corresponding to quark masses, $m_q \equiv 1/2\kappa - 1/2\kappa_c$, of about 360, 280, 200, and 130 MeV, respectively.

To build a hadron, we simply put together quarks to form a state of the appropriate quantum number and make use of Euclidean time filtering. After a few time slices, we are left with the ground-state hadron, and its mass could be extracted from the exponential decay of its propagator in Euclidean time. The results are listed in Table I, and the extrapolation of these masses to the chiral limit is shown in Fig. 2, assuming chiral-symmetry breaking [8]. The extrapolated values of the proton and the ρ masses are 900 and 560 MeV, respectively (with typical statistical errors about 10%), using our estimate of the lattice spacing, whereas the uncooled values [9] are 900 and 460 MeV; apparently, the proton and the ρ masses are dominated by the instanton sector. The slope of the pion mass squared vs bare-quark mass is related to the chiral condensate, $m_\pi^2/m_q = -2\langle 0|\bar{q}q|0\rangle / f_\pi^2$, and we extracted from Fig. 2 $\langle 0|\bar{q}q|0\rangle \approx -(200 \text{ MeV})^3$, assuming the uncooled value of f_π. We also measured the chiral condensate directly and found it to be consistent with the above value, though the use of the Wilson fermion rendered it to be not quantitatively reliable. The critical hopping parameter $\kappa_c = 0.131$ is very close to the free-field value of 0.125 as one would have expected, since the cooled configurations have little renormalization effect.

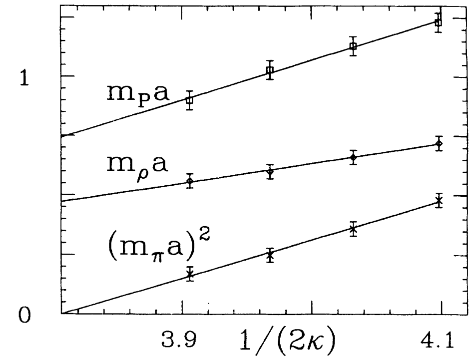

FIG. 2. Hadron masses vs hopping parameters. The error bars are statistical only.

To investigate the spatial distribution of quarks in hadrons, we calculated the Bethe-Salpeter amplitudes, defined as a function of the interquark separation y:

$$\Psi^{\text{BS}}_{\pi,\rho}(y) \equiv \int d\mathbf{x} \langle \Omega | \bar{d}(\mathbf{x}) \gamma_{5,1} U(\mathbf{x} \to \mathbf{x}+\mathbf{y}) u(\mathbf{x}+\mathbf{y}) | \pi, \rho \rangle ,$$

$$\Psi^{\text{BS}}_{P}(y) \equiv \int d\mathbf{x} \langle \Omega | \epsilon^{ijk} u^i_\alpha(\mathbf{x},t) [u^j(\mathbf{x},t) C\gamma_5 U^{kk'}(\mathbf{x} \to \mathbf{x}+\mathbf{y}) d^{k'}(\mathbf{x}+\mathbf{y},t)] | P \rangle .$$

Here, $|\Omega\rangle$ is the vacuum, $C \equiv \gamma_2\gamma_4$, and $U(\mathbf{x} \to \mathbf{y})$ is the product of the link variables from $\mathbf{x}$ to $\mathbf{y}$ inserted to make the amplitudes gauge invariant. The results, which are insensitive to κ, are shown in Fig. 3. The cusp behavior at $y = 0$ in the uncooled amplitudes [9] disappears after cooling, presumably due to the suppression of Coulomb interaction and the perimeter law. The cooled amplitudes also decay slower in y, and they probably suffer some finite-size effects, especially for the pion and the ρ. The fact that we are only measuring the diquark component of the proton amplitude probably accounts for the apparent smaller size of the proton compared to the pion and the ρ. Even though confinement is strongly suppressed after cooling, the Bethe-Salpeter amplitudes

TABLE I. Hadron masses corresponding to four values of the hopping parameters and their extrapolated values at $m_\pi = 0$.

κ	$m_\pi a$	$m_\rho a$	$m_P a$
0.122	0.69±0.02	0.72±0.03	1.23±0.04
0.124	0.60±0.02	0.66±0.03	1.13±0.04
0.126	0.50±0.03	0.60±0.03	1.03±0.04
0.128	0.41±0.04	0.56±0.03	0.90±0.05
0.131	0	0.47±0.05	0.75±0.08

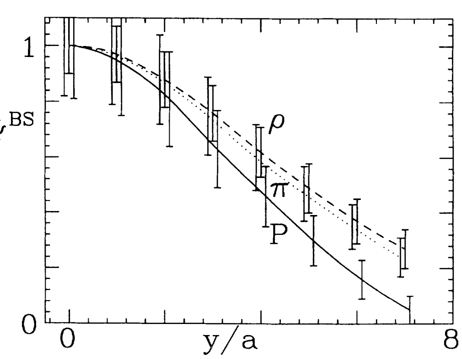

FIG. 3. The Bethe-Salpeter amplitudes for the pion (dotted line), the ρ (dashed line), and the proton (solid line) vs quark spatial separation. The error bars indicate estimated statistical errors. The curves are smooth interpolations of the points.

still show that the quarks are well localized inside the hadrons. The characteristic sizes as given by the half-widths of $|\Psi|^2$ are about 0.6 to 0.8 fm for all three particles, which are not far from the experimental values of the charged radii.

As a "control experiment," we inverted the free fermion matrix at the same κ's as above and the κ's that correspond to the same quark masses in lattice units. This will check that our results are not artifacts of the relatively heavy-quark masses used in the calculations but are due to the instanton configurations. We found that $m_\pi = m_\rho = 2m_P/3 = 2m_q$ as expected, and $\Psi(y)$ are essentially flat for all cases. These results are qualitatively different from those of the interacting configurations.

There have been many works devoted to investigating the properties of the weakly interacting dilute instanton ensembles, with various approximation schemes [10–12]. Among other things, many low energy hadronic phenomena, such as chiral-symmetry breaking and the topological susceptibility, were attributed to the instantons. We found that many results from instanton phenomenology [11,12] are amazingly close to ours. These include the average distance between instantons, the diluteness of instantons, the topological susceptibility, the chiral and gluon condensates in the QCD vacuum, and the pion charged radius. It is also interesting to note that the QCD sum rules [13] predict that the dominant contribution to light hadron masses is related to $\langle \bar{q}q \rangle$, not to α_s or $\langle g^2 G_{\mu\nu} G^{\mu\nu} \rangle$, which is again very close in spirit to our results. These consistencies lend an "experimental" support to the phenomenological approaches.

IV. SUMMARY AND DISCUSSION

We have shown, at least semiquantitatively and in the quenched approximation, that the dilute instantons are the relevant modes in the QCD vacuum for the light hadrons. As long as the chiral symmetry is broken by the instantons, the light hadrons fall into the pattern dictated by the chiral-symmetry breaking. The short wavelength ($< a \approx 0.2$ fm) fluctuations, including perturbative gluon exchange, confinement, as well as small instantons and maybe some of the interinstanton interactions, which are all eliminated by cooling, play little role in the masses and sizes of the light hadrons. However, they could be crucial for stabilizing the dilute instanton ensemble in the QCD vacuum. Our results are consistent with models that treat chiral symmetry as an essential ingredient, such as the QCD sum rules, Nambu–Jona-Lasinio- and Skyrme-types of models, as far as the light hadron masses and sizes are concerned. Models that require one-gluon exchange to split the hadron multiplet masses and explicit confinement to control the hadron sizes, despite their numerical successes, are not consistent with our findings. Since there are no phenomenological assumptions and no free parameters involved in our work, our results should be treated as those of "experimental" observations.

An intuitive way to understand our lattice results is the following. Since the current quark has a small mass or a long Compton wavelength and therefore cannot be localized well, the short-wavelength fluctuations, which are relatively weak compared with the instanton field strengths, are averaged out. Therefore the current quark practically does not feel the short-range fluctuations. It is important to note that the quark mass plays a special role here. Heavy quarks certainly feel completely different modes. For example in the $\bar{c}c$ and $\bar{b}b$ mesons perturbative gluon exchange becomes very important.

In addition to obvious technical improvements, such as to enlarge the box size and to decrease the lattice spacing, further checks should be done on the scaling behavior and the effects of dynamical fermions. Other instanton-related physics, such as the mass of η', which is supposed to be solely due to the instantons, can be studied in a similar fashion. It will also be interesting to repeat our calculation using the staggered fermion formalism, which is better suited for studying the chiral property than the Wilson fermion formalism. Finally, the question of how the dilute instanton ensemble gets stabilized, which is opaque in our present numerical work, may be answered by a detailed analysis of the cooled gluonic configurations.

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