Neutron electric dipole moment in chiral quark-meson models

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The neutron electric dipole moment is calculated in models of the quark structure of nucleons. The models studied incorporate chiral symmetry and the axial anomaly. The results are shown to satisfy the requirement of vanishing if either the strength of the anomaly or any current quark mass vanishes. In the cloudy-bag model both quark and pion-loop contributions to the dipole moment are found and these reinforce. In the color-dielectric model, which is consistent with partial conservation of the axial-vector current, there is only a pion-loop contribution.

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I. INTRODUCTION

Recently interest has reawakened in an old problem: the calculation of the strong CP-violating contribution to the neutron electric dipole moment (d_n) in models of nucleon structure. This quantity is proportional to the coefficient θ of the CP-violating term in the QCD Lagrangian:

$$\mathcal{L}_{\rm QCD}^{CP} = -\theta \frac{g^2}{32\pi^2} F^{\mu\nu} F^*_{\mu\nu} \ . \tag{1}$$

It is important as it provides the best experimental limits on θ, and hence places stringent constraints on theories of CP violation. For reviews see Refs. [1–3]. The topological density in Eq. (1),

$$Q(x) = \frac{g^2}{32\pi^2} F^{\mu\nu} F^*_{\mu\nu} \ , \tag{2}$$

also appears as the anomalous divergence of the $U(1)_A$ current in QCD [4]. Because of the nonperturbative structure of the QCD vacuum [5], this anomaly generates various physical effects such as the mass of the η' meson [6,7] as well as the dependence of observables on θ. It also makes it possible to transfer strong CP violation to the quark mass matrix by making an appropriate axial rotation of the quark fields [8].

Various authors [9–11] have stressed the need to incorporate both the $U(1)_A$ anomaly and chiral-symmetry breaking correctly in models of low-energy QCD. This has been discussed in some detail by Aoki and Hatsuda [9] and Cheng [12]. In particular the effects of strong CP violation should vanish if either the anomaly or any quark mass vanishes. Many earlier calculations of d_n [8,13–16] fail to satisfy this requirement.

Moreover there is complete confusion in the literature as to the sign of d_n, as has been noted elsewhere [12,17]. The sign is crucial in models where there are both direct quark and pion-loop contributions to d_n. If these tend to cancel, the limits on θ are relaxed by about an order of magnitude [16].

Morgan and Miller [16], working in the cloudy-bag model [18], calculated both contributions and did indeed find a cancellation. However, their model was not fully consistent, as an *ad hoc* volume-coupling form was used for the CP-violating quark-pion interaction. In addition the model made no direct reference to the $U(1)_A$ anomaly. As a result their work has recently been criticized by two groups [9,11], both of whom claim that the direct term is obtained by faulty handling of the $U(1)_A$ anomaly. Nevertheless an approach based on a baryon-level Lagrangian does find two contributions and an apparent cancellation between them [9].[1] As we discuss later, the cancellation found in the three-flavor model of Ref. [9] stems from its arbitrary choice of chiral-symmetry-breaking parameters. The simpler, and more realistic, two-flavor model in Ref. [9] gives reinforcement.

Here we calculate the neutron electric dipole moment in a version of the cloudy-bag model in which the axial anomaly is consistently incorporated. By making an axial rotation to a CP-invariant vacuum, we get a quark Lagrangian which is similar to that of Baluni [8] except for an extra factor. This factor has the form $1-m_\pi^2/m_{\eta'}^2$ in a two-flavor model, and it vanishes if the strength of the anomaly vanishes. Thus physical observables calculated with this Lagrangian satisfy the requirement noted in Refs. [9–11]. In practice this factor is very close to unity and so makes little difference to the results of earlier calculations which ignored it, such as the quark contribution to d_n of Ref. [16].

In the cloudy-bag model we find that there are indeed two contributions to d_n whose magnitudes are in fact very close to those given in Ref. [16]. However, we find that they have the *same* sign, and so reinforce one another, in agreement with the two-flavor model of Ref. [9]. Moreover the direct quark contribution can be expressed in terms of the anomalous magnetic moment of the neu-

[1]The sign of the direct term obtained from the three-flavor model in Ref. [9] is given incorrectly there. This was noted by Cheng [12], although he quotes the pion term with the wrong sign [17].

tron and so has precisely the form obtained from baryon-level models [9,15].

A tree-level contribution to d_n arises only if there are explicit fermion mass terms which break chiral symmetry. Such terms are need with a nonlinear realization of chiral symmetry, but they violate partial conservation of the axial-vector current (PCAC). In linear σ models explicit chiral-symmetry breaking can be introduced through terms linear in the scalar fields. Separate fermion mass terms do not appear and PCAC is respected. Hence in models with linear realizations of chiral symmetry, there is only a pion-loop contribution to d_n. As an example of such a model, we calculate d_n in the color-dielectric model [19,20]. We find a loop contribution to d_n similar to that of the cloudy-bag model.

II. CLOUDY-BAG MODEL

A full three-flavor version of the cloudy-bag model [18] including the nonet of pseudoscalar fields, SU(3)-symmetry breaking and the effect of the anomaly can be described by the Lagrangian

$$\begin{aligned}\mathcal{L}=&\bar{\psi}(i\partial\!\!\!/-M)\psi\theta_V-\frac{1}{2}\bar{\psi}U_5\psi\delta(r-R)\\&+\frac{f_\pi^2}{4}\mathrm{Tr}(\partial^\mu U\partial_\mu U)+\frac{f_\pi^2 v}{4}\mathrm{Tr}(MU+MU^\dagger)\\&+f_\pi^2 a\,\mathrm{Re}(e^{-i\theta}\det U)\ ,\end{aligned}\tag{3}$$

where $U=\exp(i\lambda^\alpha\phi^\alpha/f_\pi)$ and $U_5=\exp(i\lambda^\alpha\phi^\alpha\gamma_5/f_\pi)$ and the pion decay constant f_π is 93 MeV. Explicit chiral-symmetry breaking comes from the current quark mass matrix $M=\mathrm{diag}(m_n,m_n,m_s)$, where m_n $(=m_\pi^2/v)$ and m_s $[=(2m_K^2-m_\pi^2)/v]$ are the nonstrange and strange current quark masses, respectively; the value of v is not required here. The determinant term models the nonperturbative effects of the $\mathrm{U}(1)_A$ anomaly and has the form suggested by 't Hooft's instanton arguments [6]. The physical effects of the anomaly are governed by the parameter a. Strong CP violation is introduced by the phase $e^{-i\theta}$ in this term.

Other workers prefer to include the anomaly via a "Tr ln$U$" term [21]. The distinction is unimportant for our present purpose since what matters is the expansion to second order in the meson fields, which is the same in both cases

$$f_\pi^2 a\,\mathrm{Re}(e^{-i\theta}\det U)\approx-\tfrac{3}{2}a\phi_0^2+(\tfrac{3}{2})^{1/2}af_\pi\phi_0\theta\ .\tag{4}$$

This agrees with the form used in Ref. [12]. Consistency with $\mathcal{L}_{\mathrm{QCD}}^{CP}$ is obtained either with the signs above, or (as in Ref. [9]) with the opposite sign for the $\phi_0\theta$ term and with $U_5^\dagger$ in the quark-meson coupling.

If SU(3) symmetry is broken, ϕ_8 and ϕ_0 are not mass eigenstates since, in this basis, the mass matrix is

$$\mathbf{N}=\begin{bmatrix}\tfrac{1}{3}(4m_K^2-m_\pi^2) & \dfrac{2\sqrt{2}}{3}(m_\pi^2-m_K^2)\\ \dfrac{2\sqrt{2}}{3}(m_\pi^2-m_K^2) & \tfrac{1}{3}(2m_K^2+m_\pi^2)+3a\end{bmatrix}.\tag{5}$$

The mass eigenstates can be written

$$\eta=\phi_8\cos\phi-\phi_0\sin\phi,\quad \eta'=\phi_8\sin\phi+\phi_0\cos\phi\ ,\tag{6}$$

where their masses and the angle ϕ depend on the strength of the anomaly a. As is well known, m_η and $m_{\eta'}$ cannot both be fit to their experimental values. A best fit can be obtained by using the experimental masses to fix $a=\frac{1}{3}(m_\eta^2+m_{\eta'}^2-2m_K^2)$. With this choice, diagonalizing $\mathbf{N}$ gives

$$\begin{aligned}&a=12.5m_\pi^2,\quad \phi=-18.3^\circ\ ,\\&m_\eta=500\ \mathrm{MeV},\quad m_{\eta'}=984\ \mathrm{MeV}\ .\end{aligned}\tag{7}$$

In the following, m_η, $m_{\eta'}$, and ϕ will refer to the model values, not the experimental ones, and as such implicitly depend on a.

As the Lagrangian of Eq. (3) stands, the vacuum is not CP invariant since the vacuum expectation values of ϕ_8 and ϕ_0 do not vanish. It is most convenient to work in a chirally rotated frame in which there are no expectation values for the pseudoscalar fields. The chiral invariance of the quark-photon coupling means that the final result will be the same whatever rotation is performed, and so we are free to choose the most convenient basis. The desired axial rotation has the form

$$V=e^{-i(\epsilon_0\lambda_0+\epsilon_8\lambda_8)/2}\ ,\tag{8}$$

under which

$$\begin{aligned}&\psi\to V_5\psi,\quad U\to V^\dagger UV^\dagger\ ,\\&\bar{\psi}M\psi\to\bar{\psi}M[1-i(\epsilon_0\lambda_0+\epsilon_8\lambda_8)\gamma_5]\psi\ ,\\&\phi_0\to\phi_0+f_\pi\epsilon_0\ ,\quad \phi_0\to\phi_8+f_\pi\epsilon_8\ ,\end{aligned}\tag{9}$$

where V_5 bears the same relation to V as U_5 does to U. The angles ϵ_0 and ϵ_8, which make the vacuum expectation values of ϕ_0 and ϕ_8 vanish, are

$$\begin{aligned}\epsilon_8=-\frac{\mathbf{N}_{12}}{\mathbf{N}_{11}}\epsilon_0,\quad \epsilon_0&=\left[\frac{3}{2}\right]^{1/2}\frac{a\mathbf{N}_{11}}{\det\mathbf{N}}\theta\\&=\left[\frac{1}{6}\right]^{1/2}(1-H)\theta\ ,\end{aligned}\tag{10}$$

where

$$H=m_\pi^2(2m_K^2-m_\pi^2)/m_\pi^2 m_{\eta'}^2\ .\tag{11}$$

By making this rotation we remove all terms linear in ϕ_0 and ϕ_8 from the mesonic part of the Lagrangian. When U is expanded up to second order in the meson fields, the *only* source of CP violation in the Lagrangian is the CP-odd quark mass term. The relation between ϵ_8 and ϵ_0 is just that required to make this term a flavor singlet. In this frame the CP-violating term in the quark sector is

$$\begin{aligned}\mathcal{L}_B&=v^{-1}Ha\theta\bar{\psi}i\gamma_5\psi\\&=\bar{m}(1-H)\theta\bar{\psi}i\gamma_5\psi,\end{aligned}\tag{12}$$

where

$$\bar{m}=\left[\sum_{i=1}^{3}m_i^{-1}\right]^{-1}.\tag{13}$$

This rotation thus achieves Baluni's aim of transferring the strong CP violation to the quark sector [8]. Like his original form, this term vanishes if any quark mass vanishes, but unlike the original form it also vanishes in the absence of the anomaly. This rotated basis, with a CP-invariant vacuum, has the further advantage that current algebra can be used to evaluate matrix elements involving pseudoscalar mesons.

In fact Eq. (12) differs from Baluni's expression only by the factor of $1-H$. It is significant in that it vanishes if the strength of the anomaly a vanishes and so satisfies the requirement noted in Refs. [9–11]. Indeed many previously obtained results need only to be multiplied by this factor to be correct. In practice the corrections it produces are very small. For example in a two-flavor model, with pions and one isoscalar meson (η') only, H is given by $m_\pi^2/m_{\eta'}^2$. For the experimental values of the masses this is 0.021. It has the same form in the SU(3)-symmetric three-flavor model and in the case of very strong SU(3) breaking ($m_K^2 \gg a$). For realistic symmetry breaking H can be expressed either as in Eq. (11) or in terms of the η and η' masses and mixing angle as given by Eq. (3.15) of Ref. [12].[2] With the values given in Eq. (7), $H=0.037$ and so the factor $1-H$ is again very close to unity.

We now return to the cloudy-bag model [18] and follow the standard practice in this model, keeping only contributions from nonstrange quarks and pions since those from the strange degrees of freedom are significantly smaller. If we regard all heavier mesons as frozen out, the Lagrangian for a spherical bag reduces to

$$\mathcal{L}=\bar{\psi}[i\partial\!\!\!/-m_n+\bar{m}(1-H)\theta i\gamma_5]\psi\Theta(R-r) -\tfrac{1}{2}\bar{\psi}U_5\psi\delta(r-R)+\tfrac{1}{2}(\partial^\mu\boldsymbol{\phi}\cdot\partial_\mu\boldsymbol{\phi})-\tfrac{1}{2}m_\pi^2\boldsymbol{\phi}^2 , \tag{14}$$

where $U_5=\exp(i\boldsymbol{\phi}\cdot\boldsymbol{\tau}/f_\pi)$. At this stage we can set the quark mass m_n (but not $\bar{m}$) to zero since its effect on the quark wave functions is very small. Our results for CP-violating quantities will then be valid to first order in the current quark masses. With the exception of the factor $1-H$, this Lagrangian is just the one used by Morgan and Miller [16] to calculate the direct contribution to d_n.

There are two contributions to the neutron electric dipole moment in this model. One is the direct or tree-level contribution, and the other comes from pion loops in which one of the couplings to the nucleon is CP violating. We consider first the direct or tree-level contribution which Morgan and Miller [16] obtained using perturbation theory in $\mathcal{L}_B$.

An equivalent, but more transparent, way to obtain this is to solve the Dirac equation for the quark wave function to first order in θ directly. In the rotated frame where there are no pseudoscalar vacuum expectation values, the quark wave functions satisfy the usual MIT bag boundary condition [22] for a spherical bag:

$$-i\hat{\mathbf{r}}\cdot\gamma q=q , \tag{15}$$

but within the bag there is now a CP-violating quark-mass term $\mathcal{L}_B$.

The ground-state quark orbital in the absence of $\mathcal{L}_B$ is

$$q_0(\mathbf{r})=\left[\frac{N_q}{4\pi}\right]^{1/2}\begin{bmatrix} j_0(\omega r/R)\chi \\ i\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}j_1(\omega r/R)\chi\end{bmatrix} , \tag{16}$$

where R is the radius of the bag, $N_q^{-1}=2R^3(\omega-1)\times j_0^2(\omega)/\omega$, and χ is a Pauli spinor. This wave function satisfies the free, massless Dirac equation with eigenvalue ω/R, where $\omega=2.04$. It is easily seen that $\gamma_0\gamma_5 q_0$ is another solution of the same equation with energy $-\omega/R$. The full Dirac equation with CP violation, to order θ, reads

$$[-\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}-i\bar{m}(1-H)\theta\gamma_0\gamma_5]q=\frac{\omega}{R}q , \tag{17}$$

and it is easy to verify that the solution to this equation subject to the boundary condition Eq. (15) is

$$q=\left[1-i\frac{\bar{m}(1-H)\theta R}{2\omega}\gamma_0\gamma_5\right]q_0 . \tag{18}$$

CP violation mixes positive- and negative-energy states but does not mix in excited states, as was first noted in Ref. [16].

Using the wave function of Eq. (18), it is now straightforward to calculate the direct contribution to the dipole moment of a spin-up neutron:

$$\begin{aligned} d_n^{\text{tree}}&=\left\langle n\uparrow\left|\int d^3r\, z\sum_q q^\dagger\hat{Q}q\right|n\uparrow\right\rangle \\ &=\frac{\bar{m}(1-H)\theta R}{\omega}j_\omega R\left\langle n\uparrow\left|\sum_q\hat{Q}\sigma_3\right|n\uparrow\right\rangle_{sf} \\ &=-\frac{2}{3}e\frac{\bar{m}(1-H)\theta R}{\omega}j_\omega R , \end{aligned} \tag{19}$$

where $\hat{Q}$ is the quark charge operator, $\frac{1}{2}e(\tau_3+\frac{1}{3})$ and $j_\omega=\frac{1}{3}(\omega-\frac{3}{4})/(\omega^2-\omega)$. Except for the factor $1-H$, that is the result obtained by Morgan and Miller [16]. If we compare this with the expression for the magnetic moment in the cloudy-bag model,

$$\begin{aligned} \mu_n&=\left\langle n\uparrow\left|\int d^3r\,\mathbf{r}\sum_q q^\dagger\gamma\hat{Q}q\right|n\uparrow\right\rangle \\ &=j_\omega R\left\langle n\uparrow\left|\sum_q\hat{Q}\sigma_3\right|n\uparrow\right\rangle_{sf} \\ &=-\tfrac{2}{3}ej_\omega R , \end{aligned} \tag{20}$$

we see that the direct quark contributions to the neutron electric and magnetic dipole moments are closely related:

$$d_n^{\text{tree}}=\bar{m}(1-H)\theta R\mu_n/\omega . \tag{21}$$

It makes little difference whether $\bar{m}(1-H)$ is taken from a two- or three-flavor model, but here we use the

[2]The quantity called H in Eq. (3.4) of Ref. [12] is wrong; Eqs. (3.4) and (3.15) of that paper should be identical and can be shown to be equivalent to Eq. (11) if the model masses are used [17].

three-flavor version and take the value $\overline{m}=5.3$ MeV from Ref. [16]. For a bag radius of 1 fm, this gives $d_n^{\rm tree}=-1.77\times10^{-16}\theta\ e$ cm. Diagrams in which the photon couples to the quark current while a virtual pion is in flight lead to the same correction factor as for the magnetic moment [18]. Their effect on the result is small except for small bag radii where the quark piece is in any case much smaller than that due to pion loops. For $R=1$ fm it reduces the quark contribution to $-1.69\times10^{-16}\theta\ e$ cm.

Note that our result is chirally invariant: it does not depend on our choice of the rotation in Eq. (9). We could equally well work with the unrotated Lagrangian Eq. (3) and obtain the same result. In that case the vacuum expectation values of the pseudoscalar fields lead to *CP* violation only in the boundary condition at the bag surface. The wave function obtained by solving the free, massive Dirac equation subject to that condition is just an axial rotation of the one above, Eq. (16). This does not alter the dipole moment since the vector coupling of the photon to the quarks is chirally invariant.

It is instructive to compare this result with ones obtained using baryon-level model Lagrangians. Chiral symmetry ensures that there is no direct contribution to d_n if the only photon-baryon coupling is vector, but such a contribution can be obtained if the tensor coupling to the anomalous magnetic moment is included [15]. Aoki and Hatsuda [9] have developed such Lagrangians incorporating chiral symmetry and the anomaly. In these models there is a tree-level contribution to the nucleon electric dipole moment proportional to its anomalous magnetic moment. The simple two-flavor model of Ref. [9], for instance, gives

$$d_n^{\rm tree}=\frac{m_{\rm SB}}{2M_N}\left[1-\frac{m_\pi^2}{m_{\eta'}^2}\right]\theta\mu_n\ , \tag{22}$$

where $m_{\rm SB}$ is that part of the nucleon mass attributable to chiral-symmetry breaking, given approximately by the pion-nucleon σ term [23]. This has exactly the structure of the cloudy-bag result expressed in the form given by Eq. (21).[3]

In Refs. [9,12] an attempt was made to find a three-flavor version of Eq. (22). The Lagrangian used in Refs. [12,24] does not incorporate the effects of chiral-symmetry breaking in a realistic way. This is remedied in Ref. [9] at the expense of introducing further parameters which are not uniquely determined by the baryon masses. The arbitrary choice used gives a contribution to d_n with the opposite sign to the more trustworthy two-flavor case.[4] A different choice could have reversed the sign. We believe our present calculation provides a more reliable three-flavor result; as remarked before it does not differ significantly from the two-flavor case.

The second contribution to the dipole moment in this model comes from pion-loop diagrams in which one vertex is *CP* violating. To obtain such an interaction we calculate the πNN vertex function using the quark wave function derived above. As is usual in the cloudy-bag model, we write the pion-baryon interaction Hamiltonian as [18]

$$H_{\rm int}=\sum_{ABj}\int d^3k[v_j^{AB}(\mathbf{k})A^\dagger Ba_j(\mathbf{k})+v_j^{AB*}(\mathbf{k})B^\dagger Aa_j^\dagger(\mathbf{k})]\ , \tag{23}$$

where $a_j(\mathbf{k})$ annihilates a pion with momentum $\mathbf{k}$, and A and B are baryon annihilation operators. With the wave function of Eq. (18) the vertex function is given by

$$\begin{aligned} v_j^{AB}(\mathbf{k})&=\frac{1}{2f_\pi}\int\frac{d^3r}{\sqrt{(2\pi)^2 2\omega_k}}\times\langle A|\bar{q}i\gamma_5\tau_j q|B\rangle e^{i\mathbf{k}\cdot\mathbf{r}}\delta(r-R)\\ &=-i\frac{f_q}{f_\pi}\frac{u(kR)}{\sqrt{(2\pi)^2 2\omega_k}}\left\langle A\left|\sum_q\sigma\cdot\mathbf{k}\tau_j\right|B\right\rangle_{sf}\\ &\quad-\frac{g_x\bar{u}(k)}{\sqrt{(2\pi)^2 2\omega_k}}\left\langle A\left|\sum_q\tau_j\right|B\right\rangle_{sf}\ , \end{aligned} \tag{24}$$

where $\omega_k^2=k^2+m_\pi^2$, $f_q=\omega/6(\omega-1)$, and

$$g_x=\frac{\overline{m}(1-H)\theta}{2f_\pi(\omega-1)}\ . \tag{25}$$

Here $u(k)$ is the usual form factor $3j_1(kR)/kR$, while the form factor for the odd-parity coupling $\bar{u}(k)$ is just $j_0(kR)$. The g_x of Ref. [16] enters $v_j(\mathbf{k})$ with the same sign as ours. If we compare this with the Lagrangian

$$\mathcal{L}_{\pi NN}=-\overline{N}(i\gamma_5 g_{\pi NN}+\bar{g}_{\pi NN})\tau\cdot\phi N\ , \tag{26}$$

we can identify the normal- and abnormal-parity πNN coupling constants. The former is

$$\frac{g_{\pi NN}}{2M_N}=\frac{5f_q}{3f_\pi}=0.544f_\pi^{-1}\ , \tag{27}$$

to be compared with the most recent experimental value $g_{\pi NN}/2M_N=0.641f_\pi^{-1}$ [25]. The abnormal, even-parity coupling is

$$\bar{g}_{\pi NN}=-g_x=-0.0266\theta\ . \tag{28}$$

An alternative derivation of the expression for $\bar{g}_{\pi NN}$ can

[3] The cloudy-bag result can be extended to the proton provided one takes account of the fact that the proton is charged. The wave function of Eq. (18) would seem to lead to a dipole moment proportional to the full proton magnetic moment. If one corrects for the displacement of the charge with respect to the center of mass, the proton dipole moment is equal and opposite to that of the neutron. This agrees with the experimental observation that the nucleon anomalous magnetic moment is also almost purely isovector.

[4] In Ref. [9] the sign appears to be the same in the two- and three-flavor cases, but this is because the choice of signs in the mesonic and three-flavor baryonic Lagrangians are incompatible. In Ref. [12] this is corrected.

be obtained from the current-algebra relation

$$\bar{g}_{\pi NN}=\frac{-i}{f_\pi}\langle N|[Q_5^3,-\mathcal{L}_B]|N\rangle$$
$$=-\frac{\overline{m}(1-H)\theta}{f_\pi}\left\langle N\left|\sum_q \bar{q}\tau_3 q\right|N\right\rangle , \quad (29)$$

where the wave function q_0 is used to evaluate the last matrix element. Thus the abnormal-parity coupling constant is negative (in agreement with Ref. [12]).

The dipole moment can be found by inserting the vertex function of Eq. (25) in the standard cloudy-bag expressions for the pion electromagnetic current [26]. As we have described in detail elsewhere [17], this gives

$$d_n^{\rm loop}=\frac{20ef_q g_x}{3(2\pi)^3 f_\pi}\lim_{q\to 0}\int d^3k\frac{u(k)k_3}{\omega_k}\times\frac{\partial}{\partial q_3}\left[\frac{\bar{u}(k')}{\omega_{k'}(\omega_{k'}+\omega_k)}\right]_{k'=k+q} . \quad (30)$$

The most frequently quoted expression for the loop contribution of the dipole moment is the leading term in a chiral perturbation expansion [27]:

$$d_n^{\rm loop}\simeq\frac{e}{4\pi^2 M_N}g_{\pi NN}\bar{g}_{\pi NN}\ln\frac{M_N}{m_\pi} . \quad (31)$$

Our expression has the same leading term, as can be seen by extracting the potential infrared singularity which is cut off by the finite pion mass. At low momenta the form factors $u(k)$ and $\bar{u}(k)$ can be replaced by unity, and Eq. (30) yields

$$d_n^{\rm loop}\simeq\frac{e}{4\pi^2 M_N}g_{\pi NN}\bar{g}_{\pi NN}\int\frac{k^4dk}{\omega_k^5} . \quad (32)$$

Assuming that the form factors cut this integral off at momenta of the order of M_N, as obtain the usual result Eq. (31).

In the full calculation with form factors we get

$$d_n^{\rm loop}=-\frac{5ef_q g_x}{\pi^2 f_\pi}\int dz\, j_1(z)\times\left[\frac{\frac{1}{3}z^2 j_1(z)}{(z^2+\alpha^2)^{3/2}}+\frac{\frac{1}{2}z^3 j_0(z)}{(z^2+\alpha^2)^{5/2}}\right] , \quad (33)$$

where $\alpha=m_\pi R$. This integral is dependent on R, increasing for small bag radii. As a result the sum of the direct quark and pion-loop contributions is only rather weakly depended on R, as can be seen in Fig. 1. For $r=1.0$ fm, we have $d_n^{\rm loop}=-1.14\times 10^{-16}\theta$ e cm. (If we use the experimental value for $g_{\pi NN}$, we get $-1.31\times 10^{-16}\theta$ e cm.) The total dipole moment in this model is $d_n=-2.83\times 10^{-16}\theta$ e cm.

Morgan and Miller [16] assumed a volume coupling for the abnormal-parity piece, and took the value of $\bar{g}_{\pi NN}$ from Crewther *et al.* [27], who used Eq. (29) and estimated the matrix element from the baryon-mass spectrum. This gives a value extremely close to ours:

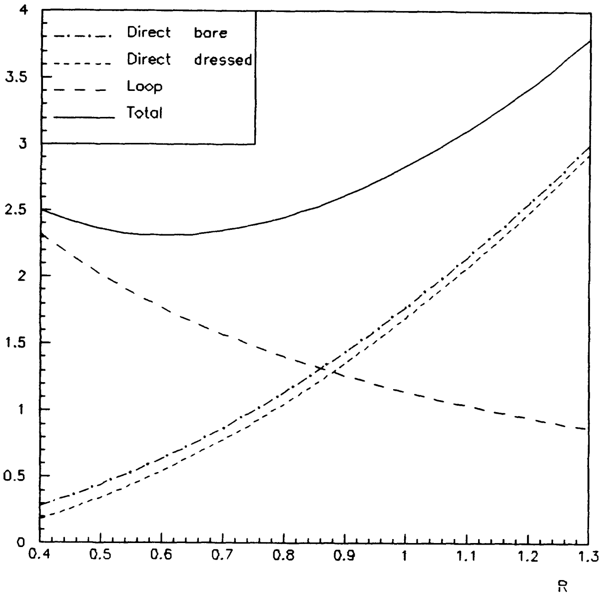

FIG. 1. The two contributions to the neutron electric dipole moment in the cloudy-bag model as a function of R. The dressed direct contribution includes corrections from diagrams in which the photon couples to the quark current while a virtual pion is in flight.

$|\bar{g}_{\pi NN}|=0.027\theta$. Although the model of Ref. [16] is not consistent with chiral symmetry, in practice it only differs from ours in the form for the abnormal-parity coupling. Thus the magnitude of $d_n^{\rm loop}$ they obtain is similar to ours.

The corresponding expression given in Ref. [16] differs from our Eq. (30) by an overall minus sign, leading to cancellation between the tree and loop contributions to d_n. However this cannot be correct. The *CP*-violating term in Eq. (18) (for positive θ) interferes constructively with the normal term for **r** in the direction of the quark spin, and destructively in the opposite direction. In a spin-up neutron the down quarks have spin up on average and so *CP* violation displaces them along the positive z direction, while the up quark is moved in the opposite direction. Hence the direct quark contribution to d_n is negative. The pion-loop term is due to virtual negative pions. These are emitted from the down quarks in the neutron and so their source is displaced in the positive z direction. Hence these also make a negative contribution to d_n.

III. COLOR-DIELECTRIC MODEL

The appearance of a direct quark contribution to d_n in the cloudy-bag model can be directly related to the presence of two scalar terms in the Dirac equation: a current quark mass term as well as the confining MIT boundary condition (which is equivalent to an infinite scalar well). These terms have different radial dependences and so if *CP* violation induces a pseudoscalar part in either, it is not possible to find an axial rotation which makes both purely scalar. If such a rotation did not exist the direct contribution to d_n would vanish. Similarly in the baryon-level models of Ref. [9], an explicit symmetry-breaking mass was necessary to generate a tree-level dipole moment.

Such explicit quark or baryon mass terms are required within the framework of a nonlinear σ model. However they violate the principle of partial conservation of the axial-vector current (PCAC), which has proved phenomenologically very successful [28]. In particular the Goldberger-Treiman relation is very well satisfied by the most recent values for g_A [29], $g_{\pi NN}$ [25], and f_π [30]. This leaves very little room for such PCAC-violating terms. To model quark-mass effects in a manner consistent with PCAC, one has to include scalar-meson degrees of freedom and work with models based on the linear σ model [31]. With a linear realization of chiral symmetry, explicit symmetry breaking can be introduced through terms in the meson sector which change the vacuum expectation values of the scalar fields. These in turn change the effective masses of the fermions without any need for explicit mass terms.

This can be seen by considering a two-flavor version in which the relevant fields are the pions and a flavor singlet σ subject to a "Mexican hat" potential:

$$V(\sigma,\boldsymbol{\phi})=\tfrac{1}{4}\lambda^2(\sigma^2+\boldsymbol{\phi}^2-\nu^2)^2 \ . \tag{34}$$

Without explicit symmetry breaking there is a "chiral circle" of degenerate minima, and the pions are massless. The addition of the term $-c\sigma$ to the potential tips the Mexican hat slightly, making the vacuum unique and giving the pions a mass. PCAC requires the vacuum expectation value of σ to be f_π and $c=f_\pi m_\pi^2$. The parameters λ and ν can then be fixed in terms of the pion and σ masses. In a model with, say, a nucleon doublet coupled to the chiral fields,

$$\mathcal{L}_{\rm int}=-g_{\pi NN}\bar{N}(\sigma+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi})N \ , \tag{35}$$

it would be possible to identify two components of the nucleon mass: one, due to spontaneous symmetry breaking, of $g\nu$ and another, due to explicit breaking (ultimately, to the quark mass terms in QCD), of $g(f_\pi-\nu)$. Thus separate fermion mass terms are not required to model the effects of chiral-symmetry breaking in the linear σ model. An axial transformation of the meson fields will rotate the spontaneous and explicit symmetry-breaking contributions in the same way. Hence it is always possible to find a frame in which the fermion mass is real. There is thus no direct term in d_n, only a pion-loop term. The abnormal-parity πN coupling arises in a different manner from the nonlinear case since there is no CP violation in the quark wave functions. Instead it is due to mixing between the pion and its scalar partner [not included above, but essential in the linear model if the $\mathrm{U}(1)_A$ anomaly is to be incorporated].

As an example of a model of this type, we consider the color-dielectric model [19]. In this quarks are confined through a coupling of the form

$$\mathcal{L}_{q\chi}=\frac{m\bar{q}q}{g_\chi \chi} \tag{36}$$

to a scalar, chirally invariant field χ, whose vacuum expectation vanishes. The soliton solutions to this model are discussed extensively elsewhere [20]. The model can be made chirally symmetric by replacing the mass in Eq. (36) by a coupling to the fields of a σ model [32]. The pion fields in these solitons are sufficiently weak that they can be treated perturbatively [33] as in the cloudy-bag model.

A linear realization of $\mathrm{SU}(3)_L\times\mathrm{SU}(3)_R$ chiral symmetry requires 18 mesons—a singlet and an octet of scalars (ξ_a) and a similar set of pseudoscalars (ϕ_a) [34]. Only the pseudoscalar octets are Goldstone bosons in the case of exact chiral symmetry; the scalars are all heavy and contribute little to the dynamics. The full three-flavor model Lagrangian is [35,36]

$$\mathcal{L}=\bar{\psi}\left[i\gamma^\mu\partial_\mu+\frac{g_\sigma}{\chi}\left[\sum_{a=0}^{8}(\xi_a+i\phi_a\gamma_5)\lambda^a\right]\right]\psi \\ +\tfrac{1}{2}\partial_\mu\chi\partial^\mu\chi-\tfrac{1}{2}M_\chi^2\chi^2+\tfrac{1}{4}\mathrm{Tr}(\partial_\mu U^\dagger\partial^\mu U)-V(M) \ , \tag{37}$$

where the matrix of meson fields is

$$U=\sum_{a=0}^{8}(\xi_a+i\phi_a)\lambda^a \ . \tag{38}$$

(Note that U is not a unitary matrix in this model.) The potential for the mesons $V(U)$ is given by

$$V=V_0(U^\dagger U)-3(\tfrac{3}{2})^{1/2}\gamma\,\mathrm{Re}[e^{-i\theta}\det(U)] \\ -\tfrac{1}{4}\mathrm{Tr}(CU+C^\dagger U^\dagger) \ . \tag{39}$$

Here

$$V_0=\tfrac{1}{4}\lambda^2[\tfrac{1}{2}\mathrm{Tr}(U^\dagger U)-\nu^2]^2 \\ +\tfrac{1}{16}\kappa^2\{3\,\mathrm{Tr}(U^\dagger UU^\dagger U)-[\mathrm{Tr}(U^\dagger U)]^2\} \tag{40}$$

is a generalized Mexican hat which is $\mathrm{U}(3)\times\mathrm{U}(3)$ symmetric; the determinant term models the effects of the anomaly as described above and determines the masses and mixing angle of η and η'. Explicit chiral-symmetry breaking is produced by terms linear in the neutral scalar fields, with coefficients

$$C=\mathrm{diag}(c_n,c_n,\sqrt{2}c_s) \ . \tag{41}$$

SU(3) symmetry is broken if the strange explicit symmetry-breaking parameter differs from the nonstrange one ($\sqrt{2}c_s\neq c_n$).

The constants in the mesonic potential (six in all) are determined by fitting the charged-pion mass, the average kaon mass, and charged pion and kaon decay constants f_π and f_K, an unimportant mixing angle between the neutral scalar mesons, and a best fit to the η and η' masses. This is described in detail in Ref. [35]. In particular, PCAC gives

$$c_n=f_\pi m_\pi^2,\quad c_s=\sqrt{1/2}(2f_K m_K^2-f_\pi m_\pi^2) \\ \text{and} \\ \sigma_v=f_\pi,\quad \xi_v=\sqrt{1/2}(2f_K-f_\pi) \tag{42}$$

where σ_v and ξ_v are the vacuum expectation values of the combinations of scalar fields which couple to nonstrange and strange quarks respectively:

$$\sigma=\sqrt{1/3}(\sqrt{2}\xi_0+\xi_8),\quad \zeta=\sqrt{1/3}(\xi_0-\sqrt{2}\xi_8)\ . \tag{43}$$

Choosing $\gamma=1.635\ \mathrm{fm}^{-1}$ gives η and η' masses of 537 and 958 MeV, which are close to the experimental ones, but a mixing angle of $-5.1°$ which is rather small.

As before, strong CP violation is introduced by the phase in the anomaly term and leads to a vacuum which is not CP invariant. The rotation required to produce a CP-invariant vacuum is similar to Eq. (9), but with rather more complicated expressions for ϵ_0 and ϵ_8:

$$\epsilon_0=\frac{1}{\sqrt{6}}(1-H)\theta,\quad \epsilon_8=\sqrt{2}\frac{2c_s\zeta_v-c_n\sigma_v}{4c_s\zeta_v+c_n\sigma_v}\epsilon_0\ , \tag{44}$$

where

$$H=\frac{c_nc_s}{\sigma_v\zeta_n m_{\eta'}^2 m_\eta^2}\ . \tag{45}$$

Under this rotation the Lagrangian is unchanged except that

$$C\rightarrow C\exp[i(\epsilon_0\lambda_0+\epsilon_8\lambda_8)],\quad \theta\rightarrow H\theta\ . \tag{46}$$

The detailed form of the U(3)$\times$U(3)-symmetric part of the potential is irrelevant, provided the parameters have been chosen to give the correct pion and kaon masses. The quantity H plays the same role as did Eq. (11) in the nonlinear case; in particular it vanishes if either of the explicit symmetry-breaking parameters vanishes. In the SU(3)-symmetry limit ($\sqrt{2}c_s=c_n$) the ϕ_8 has the same mass as the pion, and there is no mixing between it and the ϕ_0; in that case H is simply $m_\pi^2/m_{\eta'}^2$ as before.

In the SU(3)-symmetric limit, it is easy to calculate the $\eta\rightarrow\pi\pi$ amplitude with this chirally rotated Lagrangian. Since the vacuum is CP invariant we can use the current-algebra relation

$$\begin{aligned}\langle\pi^0\pi^0|\mathcal{L}|\eta\rangle&=\left[\frac{-i}{f_\pi}\right]^3\langle 0|[Q_5^3,[Q_5^3,[Q_5^8,\mathcal{L}]]]|0\rangle\\&=\frac{i\sqrt{2/3}c_0\epsilon_0}{f_\pi^2}\langle 0|[Q_5^3,[Q_5^3,[Q_5^8,\phi_0]]]|0\rangle\\&=\frac{m_\pi^2}{3\sqrt{3}f_\pi}(1-H)\theta\ ,\end{aligned} \tag{47}$$

which is the same as the prediction of the nonlinear model.

By rotating to the CP-invariant vacuum, we ensure that there is no pseudoscalar potential in the mean-field Dirac equation. We can therefore take the s-wave quark wave function

$$q(\mathbf{r})=\begin{bmatrix}G(r)\\ i\hat{\mathbf{r}}\cdot\sigma F(r)\end{bmatrix}\ , \tag{48}$$

along with the mean χ field, found numerically by solving the Euler-Lagrange equations of the color-dielectric model [20].

At first sight there is no abnormal-parity coupling between the quarks and the pions in this model. However, although the fields ϕ_i are CP eigenstates, they are not mass eigenstates. After rotation to the CP-invariant vacuum, there is still a phase $H\theta$ in the determinant term, and this gives rise to mixing between each pion and its scalar partner, ξ_i. Thus the mass eigenstate ϕ_i' can be written as

$$\phi_i'=\phi_i-\alpha\xi_i,\quad \alpha=\frac{\Delta}{m_{\xi_i}^2-m_\pi^2}\ , \tag{49}$$

where Δ is the vacuum expectation value of the second derivative of the potential V with respect to ϕ_i and ξ_i:

$$\Delta=6\sqrt{3}\gamma H\theta\zeta_v\ . \tag{50}$$

The combination $6\sqrt{3}\gamma\zeta_v$ corresponds to a of the nonlinear model since both are half the difference between the mass of the pion and the nonstrange η. Thus the numerator is analogous to $aH\theta$ and has the required property of vanishing if either explicit symmetry-breaking parameter or the anomaly vanishes.

The mass of the scalar pion is completely determined in the model, independently of the neutral scaling mixing angle: $m_{\xi_i}=1027$ MeV. In the SU(3)-symmetric limit $aH=\frac{1}{3}m_\pi^2(1-H)$. For realistic symmetry breaking $a=16.5m_\pi^2$ and $H=0.029$, so $aH=0.494m_\pi^2(1-H)$. We can see that $aH/(m_{\xi_i}^2-m_\pi^2)$ plays the role of $\overline{m}/f_\pi$ in the cloudy-bag model, and again it matters little if we use two or three flavors.

We can now rewrite the quark-pion interaction term in the color-dielectric Lagrangian in terms of the meson-mass eigenstates as

$$\frac{g_\sigma}{\chi}\overline{q}(\xi+i\gamma_5\boldsymbol{\phi})\cdot\tau q=\frac{g_\sigma}{\chi}\overline{q}[(1-i\alpha\gamma_5)\xi'+(i\gamma_5+\alpha)\boldsymbol{\phi}']\cdot\tau q\ . \tag{51}$$

As in the cloudy-bag model, we can express the pion-nucleon Hamiltonian in the form of Eqs. (24) and (25). The coupling constants are now given by

$$\begin{aligned}f_q&=\frac{8\pi}{3}f_\pi g_\sigma\int r^3dr\frac{GF}{\chi}\ ,\\ g_x&=4\pi g_\sigma\alpha\int r^2dr\frac{G^2-F^2}{\chi}\ ,\end{aligned} \tag{52}$$

and their values are $g_{\pi NN}/2M_N=0.660/f_\pi$ and $\overline{g}_{\pi NN}=-0.0208$. The corresponding form factors are

$$\begin{aligned}u(k)&=\frac{\int r^3dr(GF/\chi)(3j_1(kr)/kr)}{\int r^3dr(GF/\chi)}\ ,\\ \overline{u}(k)&=\frac{\int r^2dr\, j_0(kr)(G^2-F^2)/\chi}{\int r^2dr(G^2-F^2)/\chi}\ .\end{aligned} \tag{53}$$

These can be inserted in the expression for d_n^{loop}, Eq. (30), to get

$$d_n^{\mathrm{loop}}=-\frac{5ef_qg_x}{\pi^2f_\pi}\int k^4dk\ u(k)\left[\frac{\frac{1}{3}\mathcal{D}\overline{u}(k)}{\omega_k^3}+\frac{\frac{1}{2}\overline{u}(k)}{\omega_k^5}\right]\ , \tag{54}$$

where

$$\mathcal{D}\bar{u}(k)=\frac{\int r^4dr[j_1(kr)/kr](G^2-F^2)/\chi}{\int r^2dr(G^2-F^2)/\chi} . \quad (55)$$

Provided the χ mass is chosen to fit observed nucleon properties, the dependence of the results on the coupling constant g_σ is very weak. For a typical choice ($\beta=0.028$, where $M_\chi^2\beta^2=g_\sigma f_\pi$) we get $d_n^{\text{loop}}=-1.17\times10^{-16}\theta$ e cm.

IV. CONCLUSIONS

We have calculated the neutron electric dipole moment in two widely used models for the quark structure of baryons; the cloudy-bag and color-dielectric models. Straightforward extensions can be made to these models to describe strong *CP* violation. We have shown that the versions of the models used correctly incorporate the axial anomaly, and so the effects of *CP* violation satisfy the requirements noted in Refs. [9–11]. All observable effects of strong *CP* violation vanish if either the strength of the $U(1)_A$ anomaly or any quark mass vanishes.

In order to work in the most convenient and physical basis, we make an axial rotation to a *CP*-invariant vacuum. In models based on a nonlinear realization of chiral symmetry, such as the cloudy-bag model (in which symmetry breaking is included via explicit quark mass terms), this leads to a *CP*-violating term in the quark Lagrangian, which is similar to that of Baluni [8], except for a factor which vanishes in the absence of the anomaly. This term leads to a direct quark contribution to d_n (as found by Morgan and Miller [16] apart from the anomaly factor). We have shown that the quark contribution is proportional to the neutron anomalous magnetic moment and so has the same form as the tree-level contribution calculated in baryonic models [9,15].

Explicit quark-mass terms violate the principle of PCAC, which is phenomenologically very successful. We have, therefore, looked at a model with a linear realization of chiral symmetry, the color-dielectric model, in which explicit chiral-symmetry breaking is included through terms which change the vacuum expectation values of the scalar fields. In such models there is no direct or tree-level contribution to d_n.

The *CP*-violating term in the Lagrangian also gives rise to an abnormal-parity πN coupling in both models and hence to a pion-loop contribution to d_n. The magnitude of the loop term we get is similar to that of Morgan and Miller [16] but has the opposite sign. Hence where both are present, the quark and pion-loop terms reinforce. The reason for this reinforcement can be seen from the way in which strong *CP* violation admixes negative-energy states into the quark wave function.

Neither of the models we consider leads to a cancellation between two contributions to d_n. Hence the limits we can impose on θ are as stringent as those from most other estimates of d_n. From the most recent experimental limit on the dipole moment, $d_n<1.2\times10^{-25}$ e cm [37], our calculation of d_n in the color-dielectric model gives an upper bound of $\theta<10^{-9}$. This would be about halved for the cloudy-bag model.

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