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Symmetric Cabibbo-Kobayashi-Maskawa matrix and quark mass matrices

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In this article we aim to find the constraints on the quark mass matrices for the symmetric Cabibbo-Kobayashi-Maskawa (CKM) matrix V. We work in the bases, where (i) M_u is diagonal, (ii) M_d is diagonal, and (iii) $M_d = f(M_u^*)$, i.e., $U = D^* P$, where U and D are matrices that diagonalize the up- and down-quark mass matrices, respectively, and P is the phase matrix. We find that none of the moduli of the off-diagonal elements of these interesting forms of the quark mass matrices M_u and M_d, which lead to the symmetric CKM matrix, are consistent with zero for these *Ansätze*, which means that such forms for mass matrices are difficult to obtain from any symmetry. We then give the symmetry constraint for V written in terms of the mass eigenvalues in a basis-independent form.

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I. INTRODUCTION

The importance of studying mass matrices lies in the fact that the structure of quark and lepton mass matrices determines the flavor dynamics of the standard electroweak theory. However, the elements of these matrices cannot be predicted within the standard model as quark and lepton masses are free parameters within the model. Furthermore, there exists an infinite number of mass matrices, related to each other by unitarity transformations, which yield the same physics. Apart from the masses, the other existing free parameters in the standard model are the three mixing angles and a CP-violating phase, which are incorporated into the quark sector of the standard model via the Cabibbo-Kobayashi-Maskawa (CKM) [1] matrix V. All the presently available data [2] are consistent with having symmetric moduli for the CKM matrix: i.e.,

$$|V_{ij}| = |V_{ji}|. \tag{1}$$

It should be noted that, for three generations, the assumption that V has symmetric moduli implies a single constraint on the matrix V because the unitarity requirement alone yields

$$A = |V_{12}|^2 - |V_{21}|^2 = |V_{31}|^2 - |V_{13}|^2 = |V_{23}|^2 - |V_{32}|^2 \tag{2}$$

for three generations. The fact that experimentally the asymmetry parameter A is, in general, small, i.e., $A < 10^{-4}$ and, in particular, $|V_{12}|$ and $|V_{21}|$ are quoted to be the same modulo the errors and both of them lie between 0.217 and 0.223 prompted us to believe that V has a symmetric modulus. It has been shown [2] that for three generations, symmetric moduli of the CKM matrix lead, through unitarity, to the vanishing of the imaginary part of a rephasing-invariant sextet consisting of the off-diagonal matrix elements of V: namely,

$$\mathrm{Im}(V_{12}V_{23}V_{31}V_{21}{}^*V_{13}{}^*V_{32}{}^*) = 0. \tag{3}$$

This in turn implies that if V has a symmetric modulus, then it is always possible to choose the phases of the quark fields so that V is also symmetric, since under rephasing of the up- and down-quark fields the nonphysical individual phases γ_j and β_i of V_{ij} transform as

$$V_{ij} \to (V_{ij})' = V_{ij}\exp(\gamma_j - \beta_i). \tag{4}$$

Recently there has been some work done assuming the CKM matrix to be symmetric [2, 6, 9]. In Sec. II, we give the notation used and the preliminaries required in the subsequent sections. In Sec. III, we write the symmetry requirement of V in terms of the flavor projection operators of Jarlskog, without using any parametrization of V, and obtain a general constraint relating all the parameters of the mass matrices in a basis where the matrix M_u is diagonal. We give the necessary condition in terms of the matrices U and D for the matrix V to be symmetric in general and then provide the ranges for the elements of the mass matrices that lead to the symmetric CKM matrix for an interesting choice of U in Sec. IV. In Sec. V we give the symmetry constraint written in a basis-independent form. Section VI consists of our conclusions.

II. NOTATIONS AND PRELIMINARIES

Since the fermion mass term in the standard model arises from the Yukawa couplings due to the assumption of a nonzero vacuum expectation value by the Higgs field, it is not diagonal in the fermions and not even Hermitian. The mass term written in terms of the weak basis (denoted by prime) reads

$$L_m = \overline{u'_L} M_u u'_R + \overline{d'_L} M_d d'_R + \text{H.c.}, \tag{5}$$

where the matrices M_u and M_d denote the quark mass matrices for charge $\frac{2}{3}$ (up-type) and $-\frac{1}{3}$ (down-type) quarks respectively. In order to find the physical fields, the quark mass matrices M_u and M_d must be diagonalized. As is well known from the theory of matrices, any square matrix (Hermitian or not) can be diagonalized by a biunitary transformation. Since the mutually exclusive left- and right-handed fields in the standard model can be rotated differently, we can find four matrices such that, for three generations,

$$U_L M_u U_R{}^\dagger = \widehat{M_u} = \mathrm{diag}(m_u, m_c, m_t),$$
$$D_L M_d D_R{}^\dagger = \widehat{M_d} = \mathrm{diag}(m_d, m_s, m_b). \tag{6}$$

Multiplying the first line of the above equation with its Hermitian conjugate, we can see that the matrix U_L diagonalizes the Hermitian matrix S $(= M_u M_u{}^\dagger)$. Similarly the matrix U_R diagonalizes the Hermitian matrix $M_u{}^\dagger M_u$. These conclusions can be simply extended to the down-quark sector. Since the weak eigenstates d'_α $(\alpha = d, s, b)$ are related to the mass eigenstates d_i $(i = 1, 2, 3)$ through the relation

$$d'_\alpha = \sum_{i=1}^{3} V_{\alpha i} d_i, \tag{7}$$

where $V_{\alpha i}$ is the CKM mixing matrix and the charge currents in the standard model only involve left-handed fields, we have

$$V = U_L D_L^\dagger. \tag{8}$$

In the basis where the up-quark fields are mass eigenstates, M_u is diagonal, i.e.,

$$\widehat{M_u} = \mathrm{diag}(m_u, m_c, m_t). \tag{9}$$

In general the matrix M_d is not Hermitian, but we assume M_d to be Hermitian and write the most general Hermitian M_d as

$$M_d = h\widehat{M_u} + A, \tag{10}$$

where

$$A = \begin{pmatrix} 0 & R_1 e^{i\rho_1} & R_2 e^{i\rho_2} \\ R_1 e^{-i\rho_1} & f & R_3 e^{i\rho_3} \\ R_2 e^{-i\rho_2} & R_3 e^{-i\rho_3} & d \end{pmatrix}. \tag{11}$$

Thus the mass matrices are a ten-parameter family determined by $m_u, m_c, m_t, h, f, d, R_{1,2,3}$ and the invariant phase $(\rho_1 + \rho_3 - \rho_2)$ [3]. Taking the trace of both the sides of the equation, we obtain the constant h in terms of parameters of mass matrices as

$$h = \frac{(m_d + m_s + m_b) - f - d}{(m_u + m_c + m_t)}. \tag{12}$$

Since the identity of the quarks is defined in the basis where the mass matrix is diagonal, the flavor projection operators [4], denoted by P_α and P'_j $(\alpha, j = 1, 2, ..., n)$, are introduced to keep track of the identity of quarks in any arbitrary basis, where the mass matrices are arbitrary, by projecting out the appropriate flavor. They are given by

$$P_\alpha(S) = v_\alpha(S)/v,$$
$$P'_j(S') = v'_j(S')/v', \tag{13}$$

where the Hermitian matrices S $(= M_u M_u^\dagger)$ and S' $(= M_d M_d^\dagger)$ have non-negative eigenvalues

$$(x_1, x_2, ..., x_n) = (m_u{}^2, m_c{}^2, ...),$$
$$(x'_1, x'_2, ..., x'_n) = (m_d{}^2, m_s{}^2, ...), \tag{14}$$

respectively, and v is a Vandermonde-type determinant given by

$$v = v(x_1, x_2, ..., x_n) = \prod_{\beta,\alpha} (x_\beta - x_\alpha), \quad \beta > \alpha. \tag{15}$$

The quantity v' is the primed version of v, whereas the quantity v_α is obtained from the v by replacing x_α with the matrix S and all other x_β, $\beta \neq \alpha$ by $x_\beta I$, where I is the unit matrix. Thus v_α is an $n \times n$ matrix. For example, for $n = 3$ we have

$$v = v(x_1, x_2, x_3) = (x_3 - x_1)(x_3 - x_2)(x_2 - x_1) \tag{16}$$

and

$$v_1(S) = (x_3 - x_2)(x_3 - S)(x_2 - S). \tag{17}$$

These projection operators are Hermitian and have unit traces. They can be used to express the measurable combinations of the CKM matrix elements in terms of invariant functions of the mass matrices.

In QCD, the quark masses are running parameters; i.e., they depend on the renormalization point at which they are computed. The physical mass of a particle is its value calculated at the same scale. Although the determination of the light-quark masses involves larger errors, still they are best estimated by the use of chiral QCD perturbation theory as well as meson and baryon spectroscopy [5]:

$$m_u = 5.1 \pm 1.5 \ \text{ MeV},$$
$$m_d = 8.9 \pm 1.5 \ \text{ MeV}, \tag{18}$$
$$m_s = 175 \pm 55 \ \text{ MeV}.$$

Similarly the physical masses of the charm and bottom quarks are obtained from e^+e^- data by using QCD sum rules for the vacuum-polarization amplitude. The running masses at 1 GeV and $\Lambda_{\rm QCD} = 100$ MeV [5] are

$$m_c(1 \text{ GeV}) = 1.35 \pm 0.5 \ \text{ GeV},$$
$$m_b(1 \text{ GeV}) = 5.3 \pm 0.1 \ \text{ GeV}. \tag{19}$$

The limit that we have used for m_t is

$$180 \text{ GeV} < m_t(1 \text{ GeV}) < 280 \text{ GeV}, \tag{20}$$

which is consistent with the phenomenological constraints imposed on the symmetric CKM matrix [6]. The ranges for the individual matrix elements of the CKM matrix that have been used in our calculations are

$$V = \begin{pmatrix} 0.9759\text{–}0.9747 & 0.2240\text{–}0.2180 & 0.0070\text{–}0.0010 \\ 0.2240\text{–}0.2180 & 0.9752\text{–}0.9734 & 0.0580\text{–}0.0300 \\ 0.0190\text{–}0.0030 & 0.0580\text{–}0.0290 & 0.9996\text{–}0.9983 \end{pmatrix}, \tag{21}$$

which are taken from Particle Data Group tables [7].

III. THE SYMMETRY CONSTRAINT IN THE BASIS WHERE M_u IS DIAGONAL

To incorporate the constraint due to the symmetry of CKM, we use Jarlskog's flavor projection operators to express the mod square elements of V in terms of the matrices S and S' as

$$|V_{ij}|^2 = \text{tr}[P_i(S)P'_j(S')], \tag{22}$$

where the first and the second indices denote the up- and down-quark sectors, respectively, and the flavor projection operator in S is given as

$$P_i(S) = \frac{(S-x_1)(S-x_2)\cdots(S-x_n)}{[(x_i-x_1)(x_i-x_2)\cdots(x_i-x_n)]'}, \tag{23}$$

where $[\cdots]'$ means that the factor $(S-x_i)$ in the numerator and the factor (x_i-x_i) in the denominator must be left out. The expression for $P_j(S')$ is obtained by replacing i, S, and x_n with j, S', and x'_n, respectively. Then, the symmetry condition

$$|V_{ij}|^2 = |V_{ji}|^2 \tag{24}$$

is translated into a relation involving the matrices S and S' as

$$\text{tr}[P_i(S)P'_j(S')] = \text{tr}[P_j(S)P'_i(S')]. \tag{25}$$

Since the matrices M_u and M_d of our choice are Hermitian, we have done all the calculations in terms of invariant functions of the matrices M_u and M_d instead of S and S'. Considering, in particular,

$$|V_{12}|^2 = |V_{21}|^2 \tag{26}$$

we obtain the constraint condition due to symmetry of the CKM matrix involving the parameters of the mass matrices as

$$[R_1^2 + R_3^2 + (hm_c + f - m_s)(hm_c + f - m_b)] + \frac{m_b - m_d}{m_b - m_s}[R_1^2 + R_2^2 + (hm_u - m_d)(hm_u - m_b)] = 0. \tag{27}$$

In general, it was not possible to find out the form of M_d based on the general consraint mentioned above. But, an interesting point was noticed. When we calculated the CP violation measuring plaqeutte J in terms of S and S' using [8]

$$\pm J = \text{Im}\frac{\text{tr}[v_1(S)v'_2(S')v_3(S)v'_1(S')]}{vv'} \tag{28}$$

it was found that if any of the R_1, R_2, R_3 are chosen to be zero along with M_u being diagonal, then J is zero implying such a choice is not allowed for three generations. Thus,we note that in the basis in which M_u is diagonal, no off-diagonal elements of M_d can be made zero consistent with the CP violation in the quark sector for three generations.

The numerical calculation was done to find out whether or not any of the off-diagonal elements of the mass matrix M_d are consistent with zero. To find out numerically the allowed ranges for the elements of the mass matrix M_d we note that M_d can be written as

$$M_d = D^\dagger \widehat{M}_d D = V\widehat{M}_d V^\dagger \tag{29}$$

because a diagonal form for M_u implies $U = \mathbf{I}$ and $D = V^\dagger$. For a symmetric V it reduces to

$$M_d = V\widehat{M}_d V^*. \tag{30}$$

Since any unitary matrix that diagonalizes a Hermitian matrix can be written as the product of an orthogonal matrix and a phase matrix, we write V, in this basis, as

$$V = O_v P_v, \tag{31}$$

where the phase matrix P_v carries all the information regarding CP violation in the quark sector for three generations. Then, the ranges for the elements of the M_d were calculated using the eigenvalues of M_d and the moduli of the elements of V. The allowed ranges for the elements of M_d in GeV are found to be

$$M_d = \begin{pmatrix} 0.0117\text{–}0.0052 & 0.0549\text{–}0.0207 & 0.0409\text{–}0.0059 \\ 0.0549\text{–}0.0207 & 0.2374\text{–}0.1186 & 0.3261\text{–}0.1591 \\ 0.0409\text{–}0.0059 & 0.3261\text{–}0.1591 & 5.3962\text{–}5.1824 \end{pmatrix} \tag{32}$$

in the basis where M_u is diagonal. Similarly the allowed ranges for the elements of M_u are found to be

$$M_u = \begin{pmatrix} 0.1137\text{–}0.0657 & 0.4209\text{–}0.2818 & 1.9774\text{–}0.1881 \\ 0.4209\text{–}0.2818 & 2.2736\text{–}1.3885 & 16.312\text{–}5.4287 \\ 1.9774\text{–}0.1881 & 16.312\text{–}5.4287 & 279.78\text{–}179.38 \end{pmatrix} \tag{33}$$

in the basis where M_d is diagonal. It is apparent from these numbers that when M_u is diagonal none of the modulus of the off-diagonal terms for the matrix M_d is consistent with zero. This means that it is difficult to get such a form for M_d from any symmetry. Thus any generalization of the Stech form of mass matrices to lead to the symmetric CKM matrix should be such that the mod elements of the matrix M_d lie in the ranges given above.

The above conclusion can be extended to the case when the down-quark eigenstates are the mass eigenstates; i.e., the matrix M_d is diagonal. In such a basis, it will be difficult to obtain the form of M_u as given above from any symmetry because none of the modulus of the off-diagonal elements are consistent with zero.

IV. AN *ANSATZ* FOR QUARK MASS MATRICES LEADING TO SYMMETRIC CKM MATRIX

Since the CKM matrix $V = UD^\dagger$, where U and D are unitary matrices that diagonalize the mass matrices M_u and M_d, respectively, then the symmetry condition for V, i.e.,

$$V = V^T \tag{34}$$

will be satisfied by the necessary and sufficient condition involving the matrices U and D:

$$D = U^* P, \tag{35}$$

where the matrix P ($= U^T D$) is a symmetric, unitary matrix. Then the symmetric CKM matrix V is

$$V = UD^\dagger = UP^* U^T. \tag{36}$$

Writing the unitary matrix U as the product of a phase matrix P_u and an orthogonal matrix O_u, i.e.,

$$U = O_u P_u, \quad U^\dagger = P_u^* O_u^T, \tag{37}$$

the symmetric V can be rewritten as

$$V = U(U^* P)^\dagger = O_u P_u P^* P_u O_u^T. \tag{38}$$

Consider the matrix P to be a phase matrix. Such a choice is a special but interesting case because, as a consequence we can write either

$$M_d = f(M_u^*) \text{ or } M_u = g(M_d^*). \tag{39}$$

Then one of the possible choices for the mass matrix M_d as a function of M_u^* is

$$M_d = p(M_u^*)^2 + qM_u^* + rI, \tag{40}$$

where the parameters p, q, r are introduced to retain the mass hierarchy for the down-quark sector. Upon diagonalization of both sides of the above equation, we obtain three equations involving six quark masses and three unknown parameters p, q, r which can be determined uniquely. These three parameters are given in terms of the quark masses as

$$\begin{aligned} p &= \frac{m_s}{m_c m_t}, \\ q &= \frac{m_s}{m_c}, \\ r &= m_d - m_u \frac{m_s}{m_c}. \end{aligned} \tag{41}$$

For such a choice of P, we write the CKM matrix as

$$V = O_u \tilde{P} O_u^T, \tag{42}$$

where $\tilde{P}$ is a phase matrix. To get the ranges of the mod elements of the mass matrices for the case when P is a phase matrix we proceed with the numerical calculation using a convenient parametrization. For three generations, the three orthonormalized complex eigenvectors of V are determined up to a phase. We can thus choose one nonvanishing component of each vector to be real. The two remaining arbitrary phases can be chosen in such a way that one eigenvector is real. We use the following parametrization [9] of the three eigenvectors with the above properties:

$$\begin{aligned} w_1 &= \begin{pmatrix} c_1 \\ s_1 c_2 \\ s_1 s_2 \end{pmatrix}, \quad w_2 = \begin{pmatrix} -s_1 c_3 \\ c_1 c_2 c_3 - s_2 s_3 e^{i\alpha} \\ c_1 s_2 c_3 + c_2 s_3 e^{i\alpha} \end{pmatrix}, \\ w_3 &= \begin{pmatrix} s_1 s_3 \\ -c_1 c_2 s_3 - s_2 c_3 e^{i\alpha} \\ -c_1 s_2 s_3 + c_2 c_3 e^{i\alpha} \end{pmatrix}, \end{aligned} \tag{43}$$

where $c_i = \cos\beta_i$ and $s_i = \sin\beta_i$. The reparametrized CKM matrix V may then be written in terms of its eigenvalues λ_i and eigenvectors w_i as

$$V = \sum_{i=1}^{3} \lambda_i w_i \otimes w_i^\dagger = W \Lambda W^\dagger, \tag{44}$$

where

$$\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3) \tag{45}$$

and W is the matrix of the eigenvectors:

$$W = (w_1, w_2, w_3). \tag{46}$$

Comparing this general form with the form of symmetric V, we see that if Λ is recognized as $\tilde{P}$ then the general form is reducible to a symmetric form only if W is real. Thus we conclude that the reality of W is a necessary and sufficient condition for having a symmetric CKM matrix. This result was reported [2] based on a different and simple proof. Then it is evident that the choice $\alpha = 0$ will make V symmetric within the above parametrization. As is well known, the eigenvalues of the CKM matrix depend on the choice of the phases of the quark fields. If we choose three out of five arbitrary phases of the quark fields in such a way that the CKM matrix satisfies the condition

$$\mathrm{tr}V = xe^{i\phi/3}, \tag{47}$$

then it follows that the eigenvalues of the CKM matrix for such a choice of the quark fields phases are

$$\lambda_1 = \tfrac{1}{2}e^{-(i\phi/3)}(x - 1 - i\sqrt{3 + 2x - x^2}), \quad -1 \le x \le 3,$$
$$\lambda_2 = \tfrac{1}{2}e^{-(i\phi/3)}(x - 1 + i\sqrt{3 + 2x - x^2}), \quad -1 \le x \le 3,$$
$$\lambda_3 = e^{i\phi/3}. \tag{48}$$

In this parametrization [10] all the mod elements of V were written in terms of x and the angles. Consider the case when $\beta_3 = 0$ and α drops out. Then the mod elements of CKM matrix relevant to our discussion are

$$|V_{11}| = \sqrt{1 - \tfrac{1}{4}\sin^2(2\beta_1)(3 + 2x - x^2)},$$
$$|V_{12}| = \tfrac{1}{2}\sin(2\beta_1)\cos(\beta_2)\sqrt{(3 + 2x - x^2)}, \tag{49}$$
$$|V_{13}| = \tfrac{1}{2}\sin(2\beta_1)\sin(\beta_2)\sqrt{(3 + 2x - x^2)}.$$

The experimental constraints, i.e., the values of the magnitudes of the elements of V, $\rho = |V_{13}/V_{23}|$, and J imply [6] that x must lie between -0.882 and 0.02. We then solve for β_1 and β_2 by inverting the above equation and using the magnitudes of the first row of V and find the allowed ranges to be

$$\beta_1 = 0.1265 \text{ to } 0.3605,$$
$$\beta_2 = 0.0040 \text{ to } 0.0320. \tag{50}$$

Consider the case of $\alpha = 0$. Then the elements of the matrix W are functions of three mixing angles β_1, β_2, and β_3 out of which two are independent and we recognize $O_u = W$. Then, the unitary matrix U is given as

$$U = O_u P_u = W P_u,$$
$$P_u = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}). \tag{51}$$

Using the matrix U and assuming the matrix M_u to be Hermitian, we can write the mass matrix M_u as

$$M_u = U^\dagger \widehat{M}_u U = P_u^* W^T \widehat{M}_u W P_u \tag{52}$$

and the mass matrix M_d as

$$M_d = D^\dagger \widehat{M}_d D = P^* P_u W^T \widehat{M}_d W P_u^* P. \tag{53}$$

In our numerical calculation, we use the above-mentioned ranges of the angles β_1 and β_2 to calculate the ranges for the mod elements of the mass matrices using the above equations in this two-angle parametrization of the CKM matrix. The allowed ranges in GeV for the mod elements of M_u and M_d in GeV are, respectively,

$$M_u = \begin{pmatrix} 0.1799\text{–}0.0242 & 0.4609\text{–}0.1617 & 0.0147\text{–}0.0006 \\ 0.4609\text{–}0.1617 & 1.6636\text{–}1.1413 & 8.9172\text{–}0.7141 \\ 0.0147\text{–}0.0006 & 8.9172\text{–}0.7141 & 279.93\text{–}179.87 \end{pmatrix} \tag{54}$$

and

$$M_d = \begin{pmatrix} 0.0386\text{–}0.0081 & 0.0738\text{–}0.0135 & 0.0023\text{–}0.00005 \\ 0.0738\text{–}0.0135 & 0.2318\text{–}0.1059 & 0.1692\text{–}0.0198 \\ 0.0023\text{–}0.00005 & 0.1692\text{–}0.0198 & 5.3995\text{–}5.1947 \end{pmatrix}. \tag{55}$$

In this basis neither the mod element of any off-diagonal term of the matrix M_u nor that of the matrix M_d is consistent with zero, which implies that it is difficult to get such forms of the mass matrices that lead to a symmetric CKM matrix in a natural way from any symmetry. Although the mod of the $|M_{d13}|$ element is small compared to that of other elements of M_d, it still is considerably different from zero. It is interesting to note that the mass matrices, apart from the phase factor, are symmetric.

V. BASIS-INDEPENDENT SYMMETRY CONSTRAINT

In the preceding sections, we have given the ranges of the elements of the mass matrices M_u and M_d allowed by the symmetric CKM in two different bases. In this section we give the symmetry constraint written in a basis-independent form. As we have seen in the previous section, the condition $|V_{12}| = |V_{21}|$ implies

$$\text{tr}[P_1(M_u)P_2(M_d)] = \text{tr}[P_2(M_u)P_1(M_d)], \tag{56}$$

which can be rewritten as

$$\text{tr}[c_1 V^\dagger \widehat{P}_1(M_u) V \widehat{P}_2(M_d) - c_2 V^\dagger \widehat{P}_2(M_u) V \widehat{P}_1(M_d)] = 0, \tag{57}$$

where the constants c_1 and c_2 are functions of the mass eigenvalues and

$$\widehat{P}_{1,2}(M_u) = U P_{1,2}(M_u) U^\dagger,$$
$$\widehat{P}_{1,2}(M_d) = D P_{1,2}(M_d) D^\dagger. \tag{58}$$

Consider going from an unprimed basis to a primed basis by the transformations

$$U' = AU, \quad D' = BD, \tag{59}$$

where A and B are unitary matrices. Then the CKM matrix in the primed basis is

$$V' = AVB^\dagger. \tag{60}$$

Requiring $V' = V$ relates A and B through the matrix V as follows:

$$A = VBV^{\dagger}. \tag{61}$$

Then use of symmetry of the CKM matrix in the primed basis yields

$$A = VBV^{*}. \tag{62}$$

The mass matrices transform under this basis transformation as

$$M_u' = AM_uA^{\dagger}, \quad M_d' = BM_dB^{\dagger}. \tag{63}$$

But the difficulty in using these expressions to find out how the mod elements of the mass matrices transform under this basis transformation is that it is not possible to separate out the phase from the the mass matrices in the primed basis for any general unitary matrix A and B after the transformation.

VI. CONCLUSIONS

We have tried to find the constraints imposed on the form of the mass matrices due to the symmetric CKM matrix. First, we wrote the symmetry constraint as an equation involving the parameters of the mass matrices using flavor projection operators in a basis where M_u is diagonal. We gave the numerical ranges for the mod elements of M_d in this basis. We have repeated this procedure in the basis where M_d is diagonal. Then, we wrote the necessary condition for having a symmetric V in terms of the matrices U and D. We chose a particularly interesting basis where $U = D^{*}P$, P being a phase matrix, and gave the ranges for the mod elements of M_u, M_d in that basis using a convenient parametrization for V. We noticed that none of the off-diagonal elements of M_u and M_d are consistent with zero for a symmetric V, which means such forms for mass matrices cannot be obtained from any symmetry. But, in principle there exists an infinite number of other bases related to each other by similarity transformations. So it is apparent that the numbers we provided for the allowed ranges of the mod elements of mass matrices are not basis independent. Finally we wrote down the symmetry constraint in a basis-independent form.

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