Study of T violation in $K^+ \to \pi^+\mu^+\mu^-$ decays

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We study the possibility of observing T violation in the process $K^+ \to \pi^+\mu^+\mu^-$. To this end we define appropriate T-violation indicators. We find that the measurements of the two-spin correlations can provide a reliable test of the Cabibbo-Kobayashi-Maskawa mechanism of T violation. The final-state interactions mainly contribute to indicators involving only one spin. We also estimate the size of the contribution of a representative set of extensions of the standard model to T-violation indicators. We find that their contribution can be important only in models with loosely constrained new parameters.

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I. INTRODUCTION

Among a number of phenomena which we observe in the decays of various particles, the phenomenon of T violation or CP violation remains intriguing and fascinating. In the framework of local quantum field theories, with Lorentz invariance and the usual spin-statistics connection, T violation implies CP violation (and vice versa), because of the CPT invariance of such theories [1]. Experimentally only CP violation has been observed so far and this only in kaons [2]. Empirically the violation of CPT symmetry has not been observed [3]. Still, it will be worthwhile to remember that outside this framework of local quantum field theories, there is no reason for the two symmetries to be linked. Their *a priori* linkage through CPT invariance itself is certainly very profound. Therefore, it would be interesting to directly investigate T violation, rather than inferring it as a consequence of CP violation. In this paper, our discussion is within the context of local quantum field theories; therefore we shall not make any distinction between the CP and T violation. But the quantities we shall be calculating will be a measure of T violation.

Over the years, a number of processes have been suggested where one can look for either T or CP violation, e.g., kaon decays, B meson decays, rare Z^0 decays, hyperon decay asymmetries, and electric dipole moments of the electron and the neutron [2]. Until now CP violation has been observed only in a few decay modes of the kaon [3]. Because of the new generation of kaon experiments that are ongoing and the possibility of a kaon factory on the horizon, various kaon decays will continue to provide the most fertile territory for such observations.

The standard model (SM) has been unsurpassed in its simplicity, internal consistency, and its power to shed light on experimental and observational data [4]. With at least three families of quarks and leptons, it has a "natural" place for the T (or CP) violation, in terms of the mixing of quarks; however, a more satisfactory and better understanding of this phenomenon is still lacking. One impediment in this regard has been that this phenomenon has only been observed in a very limited number of processes. The benchmark process for CP violation has been the $K \to \pi\pi$ decay [2]. Here CP violation is quantified in terms of the parameters ε and ε'. To date only $|\varepsilon|$ has been measured and the experimental status of ε' is still confused [5–7]. To enhance our ability to understand this phenomenon, and to test whether CP violation arises solely from the phase in quark mixings, or from more than one source, it is imperative to explore other avenues where the possibility of observing T (or CP) violation exists. To achieve this better understanding, the uncertainties in the model's prediction of T-violation indicators must be under control. At the moment the techniques for the computation in the nonperturbative domain of a realistic model, such as the SM, are not yet well developed. This leads to uncertainties, which are not usually under control, in the predictions for the nonleptonic decay modes of the kaon or other hadrons [8]. Unfortunately, the above-mentioned CP-violating process is plagued with such difficulties.

We have already mentioned that the only direct evidence of CP violation comes from kaon decays. In the K-$\bar{K}$ system, CP violation can come about in two ways. First, it can occur in the wave function through K-$\bar{K}$ mixing which is parametrized by ε. Second, it can appear directly through the decay amplitude of a process. The former is often referred to as indirect CP violation, and the latter as direct CP violation. A charged-kaon decay mode is a better place to look for direct CP violation since, in general, in a neutral-kaon decay mode, indirect CP-violation effects can be significant (one exception is $K_L \to \pi^0\nu\bar{\nu}$ which, however, has the drawback of being hard to measure as well as being very rare). Until now, there has been no conclusive experimental evidence [5–7] about the existence of direct CP violation, i.e., a nonvanishing value of ε'. It would in any case be desirable to observe T violation *directly* and test whether the sources of the two violations (i.e., T and CP) are identical.

From the above discussion we establish the following criteria for a process to be considered as a good candidate for the study of the mechanism of T violation: (a) it has a large enough branching ratio so that enough events can be collected in a given experiment; (b) the appropriate T-violation indicator in the process is "large"; (c) the prediction of the model for such indicators is reasonably unambigous, i.e., the theoretical uncertainties are under control; (d) the experiment is feasible in the near future. We note that these conditions are not completely independent. A number of processes have been suggested in the past to look for signals of CP violation. These include the searches for the rare decays $K_L \to \pi^0 \nu\bar{\nu}, K_L \to \pi^0 l\bar{l}$, and the measurement of the longitudinal polarization of μ^+ in the decay $K_L \to \mu^+\mu^-$. None of these meet the above criteria completely.

Recently, we analyzed the process [9]

$$K^+ \to \pi^+ l\bar{l} \ , \tag{1}$$

and investigated the possibility of observing T violation in this decay within the context of the standard model. Here l can be either an electron or a muon. We found that the two-spin correlations in the muon decay mode are the best T-violation indicators for this process. Furthermore, from a theoretical point of view, such correlations provide a clean test of the Cabibbo-Kobayashi-Maskawa (CKM) mechanism of CP violation. In this paper, we present the details of the above-mentioned analysis and discuss the importance of the measurements. We show that final-state interactions do *not* contribute significantly to the two-spin correlations. However, they can contribute significantly to one-spin-dependent T-violation indicators. We also analyze the prospects of probing the physics beyond the standard model using such indicators. To this end, we estimate contributions to these indicators in some of the more popular extensions of the standard model. We find that typically such models do not give large T-violation effects with the exception of leptoquark models.

The process $K^+ \to \pi^+ l\bar{l}$ is a rare decay process. In the SM, such a process cannot occur at the tree level due to the absence of flavor-changing neutral currents. However, this decay can take place at the one-loop level via the electroweak penguin and box diagrams as displayed in Fig. 1. Since in the SM, the weak interaction leads to T violation, we shall focus on the electroweak contribution to this process. The strong-interaction corrections to this process have the potential of introducing theoretical uncertainties that are difficult to control. Our strategy is to use as much experimental input as possible, and not confront these issues directly. Details are given in Sec. II. For many T-violation indicators we are studying, this can be done, and hence they can be calculated fairly reliably in the standard model. The predictions then come relatively free from theoretical ambiguities, and this process can provide a good test of the mechanism of T violation in the standard model.

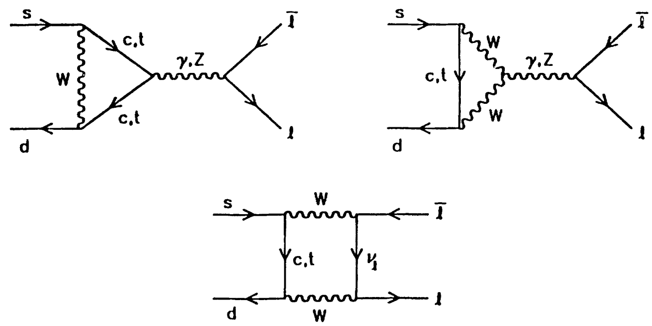

FIG. 1. Some one-loop diagrams for the short-distance contribution to the $K^+ \to \pi^+ l\bar{l}$ decay.

The quantities which we compute suffer from a potential drawback from an experimental point of view. It is hard to measure the polarization of fast-moving leptons. In the case of the electron decay mode, it may be impossible to make the required measurements. However, in the case of the muon decay mode, it may be possible to carry out such an experiment at a kaon factory, e.g., the proposed KAON. Therefore, in this paper our focus is on the muon decay mode. The polarization measurements on the μ^+ have been carried out in the past. The main experimental challenge will be the measurement of the polarization of μ^-.

In Sec. II, we carry out a general analysis of the decay $K^+ \to \pi^+ l\bar{l}$ and define various quantities. In Sec. III, we compute and analyze these quantities in the standard model. In Sec. IV, we calculate the contribution from final-state interactions. In Sec. V, we carry out the analysis for some extensions of the standard model, putting in the most recent constraints on the myriads of parameters in these models. Finally, in Sec. VI, we address some theoretical and experimental issues, and present our conclusions.

II. GENERAL ANALYSIS

We first carry out a general analysis of the process (1) based on Lorentz invariance. The amplitude for such a process can be written as [9]

$$\begin{aligned}\mathcal{M} = {} & F_S \bar{u}(p_l,s) v(\bar{p}_l,\bar{s}) + F_P \bar{u}(p_l,s)\gamma_5 v(\bar{p}_l,\bar{s}) \\ & + F_V p_k^\mu \bar{u}(p_l,s)\gamma_\mu v(\bar{p}_l,\bar{s}) + F_A p_k^\mu \bar{u}(p_l,s)\gamma_\mu\gamma_5 v(\bar{p}_l,\bar{s}) \ .\end{aligned} \tag{2}$$

Here F_S, F_P, F_V, and F_A are scalar, pseudoscalar, vector, and axial-vector form factors, respectively. These form factors are functions of Lorentz-invariant quantities. The p_k, p_π, p_l, and $\bar{p}_l$ are the four-momenta of K^+, π^+, l, and $\bar{l}$, respectively, while the s and $\bar{s}$ are the polarization vectors of the l and $\bar{l}$, respectively. Thus to compute a physical quantity for such a process, we must have prior knowledge of these form factors either experimentally or through some dynamical model.

The invariant amplitude squared is given by

$$\mathcal{P}=|\mathcal{M}|^2=|F_S|^2[\tfrac{1}{2}(L^2-4m_l^2)(1-s\cdot\bar{s})+\bar{p}_l\cdot s\, p_l\cdot\bar{s}]+|F_P|^2[\tfrac{1}{2}L^2(1+s\cdot\bar{s})-\bar{p}_l\cdot s\, p_l\cdot\bar{s}]$$
$$+|F_V|^2[2p_l\cdot p_k\,\bar{p}_l\cdot p_k(1-s\cdot\bar{s})-\tfrac{1}{2}m_K^2L^2(1-s\cdot\bar{s})$$
$$+2p_k\cdot s\,p_k\cdot\bar{p}_l\,p_l\cdot\bar{s}+2p_k\cdot\bar{s}\,p_k\cdot p_l\,\bar{p}_l\cdot s-m_K^2\bar{p}_l\cdot s\,p_l\cdot\bar{s}-L^2p_k\cdot s\,p_k\cdot\bar{s}]$$
$$+|F_A|^2[2p_l\cdot p_k\,\bar{p}_l\cdot p_k(1+s\cdot\bar{s})-\tfrac{1}{2}m_K^2(L^2-4m_l^2)(1+s\cdot\bar{s})$$
$$-2p_k\cdot s\,p_k\cdot\bar{p}_l\,p_l\cdot\bar{s}-2p_k\cdot\bar{s}\,p_k\cdot p_l\,\bar{p}_l\cdot s+m_K^2\bar{p}_l\cdot s\,p_l\cdot\bar{s}+(L^2-4m_l^2)p_k\cdot s\,p_k\cdot\bar{s}]$$
$$+2\,\mathrm{Im}(F_SF_P^*)\varepsilon^{\mu\nu\rho\sigma}\bar{p}_{l\mu}\bar{s}_\nu s_\rho p_{l\sigma}-2\,\mathrm{Re}(F_SF_P^*)m_l(\bar{s}\cdot p_l+\bar{p}_l\cdot s)$$
$$+2\,\mathrm{Re}(F_SF_V^*)m_l[p_k\cdot(\bar{p}_l-p_l)(1-s\cdot\bar{s})+\bar{s}\cdot p_k\,s\cdot\bar{p}_l-s\cdot p_k\,\bar{s}\cdot p_l]$$
$$+2\,\mathrm{Im}(F_SF_V^*)\varepsilon^{\mu\nu\rho\sigma}p_{k\mu}p_{l\nu}\bar{p}_{l\rho}(s+\bar{s})_\sigma+2\,\mathrm{Re}(F_SF_A^*)[\tfrac{1}{2}(L^2-4m_l^2)p_k\cdot(s+\bar{s})-\bar{p}_l\cdot s\,p_l\cdot p_k-\bar{p}_l\cdot p_k\,p_l\cdot\bar{s}]$$
$$+2\,\mathrm{Im}(F_SF_A^*)m_l\varepsilon^{\mu\nu\rho\sigma}p_{k\mu}\bar{s}_\nu s_\rho(\bar{p}_l-p_l)_\sigma-2\,\mathrm{Im}(F_PF_V^*)m_l\varepsilon^{\mu\nu\rho\sigma}p_{k\mu}s_\nu\bar{s}_\rho(\bar{p}_l+p_l)_\sigma$$
$$-2\,\mathrm{Re}(F_PF_V^*)[\tfrac{1}{2}L^2p_k\cdot(\bar{s}-s)-p_k\cdot\bar{p}_l\,p_l\cdot\bar{s}+p_k\cdot p_l\,\bar{p}_l\cdot s]-2\,\mathrm{Im}(F_PF_A^*)\varepsilon^{\mu\nu\rho\sigma}p_{k\mu}p_{l\nu}\bar{p}_{l\rho}(s-\bar{s})_\sigma$$
$$-2\,\mathrm{Re}(F_PF_A^*)m_l[-\tfrac{1}{2}(m_K^2-m_\pi^2+L^2)(1+s\cdot\bar{s})+\bar{s}\cdot p_l\,s\cdot p_k+s\cdot\bar{p}_l\,\bar{s}\cdot p_k]$$
$$+2\,\mathrm{Re}(F_VF_A^*)m_l[-2\bar{s}\cdot p_k\,p_l\cdot p_k+2s\cdot p_k\,\bar{p}_l\cdot p_k-m_K^2(s\cdot\bar{p}_l-\bar{s}\cdot p_l)]$$
$$+2\,\mathrm{Im}(F_VF_A^*)[\bar{p}_l\cdot p_k\varepsilon^{\mu\nu\rho\sigma}p_{k\mu}p_{l\nu}\bar{s}_\rho s_\sigma+p_l\cdot p_k\varepsilon^{\mu\nu\rho\sigma}p_{k\mu}\bar{p}_{l\nu}\bar{s}_\rho s_\sigma$$
$$+s\cdot p_k\varepsilon^{\mu\nu\rho\sigma}\bar{p}_{l\mu}p_{l\nu}p_{k\rho}\bar{s}_\sigma+\bar{s}\cdot p_k\varepsilon^{\mu\nu\rho\sigma}\bar{p}_{l\mu}p_{l\nu}p_{k\rho}s_\sigma]\ . \tag{3}$$

In the above L is the invariant mass of the dilepton system.

We split $\mathcal{P}$ in two pieces. One contains the pieces contributing to T violation and the other does not. Thus, we write

$$\mathcal{P}=\mathcal{P}_{\rm TC}+\mathcal{P}_{\rm TV}\ , \tag{4}$$

where

$$\mathcal{P}_{\rm TV}=2\,\mathrm{Im}(F_SF_P^*)(\bar{E}_l\mathbf{p}_l\cdot\bar{\mathbf{s}}\times\mathbf{s}+E_l\bar{\mathbf{p}}_l\cdot\mathbf{s}\times\bar{\mathbf{s}}+s^0\bar{\mathbf{s}}\cdot\mathbf{p}_l\times\bar{\mathbf{p}}_l+\bar{s}^0\mathbf{s}\cdot\bar{\mathbf{p}}_l\times\mathbf{p}_l)$$
$$-2\,\mathrm{Im}(F_SF_V^*)m_K(\mathbf{s}+\bar{\mathbf{s}})\cdot\bar{\mathbf{p}}_l\times\mathbf{p}_l+2\,\mathrm{Im}(F_SF_A^*)m_lm_K(\mathbf{p}_l-\bar{\mathbf{p}}_l)\cdot\mathbf{s}\times\bar{\mathbf{s}}$$
$$-2\,\mathrm{Im}(F_PF_V^*)m_lm_K(\bar{\mathbf{p}}_l+\mathbf{p}_l)\cdot\mathbf{s}\times\bar{\mathbf{s}}-2\,\mathrm{Im}(F_PF_A^*)m_K(\mathbf{s}-\bar{\mathbf{s}})\cdot\mathbf{p}_l\times\bar{\mathbf{p}}_l$$
$$+2\,\mathrm{Im}(F_VF_A^*)m_K^2[\bar{E}_l\mathbf{p}_l\cdot\bar{\mathbf{s}}\times\mathbf{s}+E_l\bar{\mathbf{p}}_l\cdot\bar{\mathbf{s}}\times\mathbf{s}+s^0\bar{\mathbf{s}}\cdot\bar{\mathbf{p}}_l\times\mathbf{p}_l+\bar{s}^0\mathbf{s}\cdot\bar{\mathbf{p}}_l\times\mathbf{p}_l]\ . \tag{5}$$

Here the spin vectors are defined in the rest frame of the kaon. Therefore they depend on the momenta of the leptons. For a lepton with momentum p, the spin four-vector s is

$$s=\left[\frac{\mathbf{p}\cdot\hat{\mathbf{n}}}{m_l},\hat{\mathbf{n}}+\frac{\mathbf{p}\cdot\hat{\mathbf{n}}}{m_l(E+m_l)}\mathbf{p}\right]\ , \tag{6}$$

where $\hat{\mathbf{n}}$ is a unit vector in the direction of the spin of the lepton in its rest frame.

Using this we can write the pieces of the amplitude squared that lead to T violation as

$$\mathcal{P}_{\rm TV}=2\,{\rm Im}(F_SF_P^*)\left[-\bar{E}_l|\mathbf{p}_l|\,\hat{\mathbf{p}}_l\cdot\hat{\mathbf{n}}\times\hat{\bar{\mathbf{n}}}+E_l|\bar{\mathbf{p}}_l|\,\hat{\bar{\mathbf{p}}}_l\cdot\hat{\mathbf{n}}\times\hat{\bar{\mathbf{n}}}-\frac{|\bar{\mathbf{p}}_l|^2|\mathbf{p}_l|}{\bar{E}_l+m_l}\hat{\bar{\mathbf{p}}}_l\cdot\hat{\bar{\mathbf{n}}}\,\hat{\mathbf{n}}\cdot\hat{\mathbf{p}}_l\times\hat{\bar{\mathbf{p}}}_l+\frac{|\mathbf{p}_l|^2|\bar{\mathbf{p}}_l|}{E_l+m_l}\hat{\mathbf{p}}_l\cdot\hat{\mathbf{n}}\,\hat{\bar{\mathbf{n}}}\cdot\hat{\mathbf{p}}_l\times\hat{\bar{\mathbf{p}}}_l\right]$$

$$-2\,{\rm Im}(F_SF_V^*)m_K|\mathbf{p}_l||\bar{\mathbf{p}}_l|(\hat{\mathbf{n}}+\hat{\bar{\mathbf{n}}})\cdot\hat{\bar{\mathbf{p}}}_l\times\hat{\mathbf{p}}_l+2\,{\rm Im}(F_SF_A^*)m_lm_K$$

$$\times\left[|\mathbf{p}_l|\,\hat{\mathbf{p}}_l\cdot\hat{\mathbf{n}}\times\hat{\bar{\mathbf{n}}}-|\bar{\mathbf{p}}_l|\,\hat{\bar{\mathbf{p}}}_l\cdot\hat{\mathbf{n}}\times\hat{\bar{\mathbf{n}}}+\frac{|\bar{\mathbf{p}}_l|^2|\mathbf{p}_l|}{m_l(\bar{E}_l+m_l)}\hat{\bar{\mathbf{p}}}_l\cdot\hat{\bar{\mathbf{n}}}\,\hat{\mathbf{n}}\cdot\hat{\bar{\mathbf{p}}}_l\times\hat{\mathbf{p}}_l-\frac{|\mathbf{p}_l|^2|\bar{\mathbf{p}}_l|}{m_l(E_l+m_l)}\hat{\mathbf{p}}_l\cdot\hat{\mathbf{n}}\,\hat{\bar{\mathbf{n}}}\cdot\hat{\bar{\mathbf{p}}}_l\times\hat{\mathbf{p}}_l\right]$$

$$-2\,{\rm Im}(F_PF_V^*)m_lm_K\left[|\mathbf{p}_l|\,\hat{\mathbf{p}}_l\cdot\hat{\mathbf{n}}\times\hat{\bar{\mathbf{n}}}+|\bar{\mathbf{p}}_l|\,\hat{\bar{\mathbf{p}}}_l\cdot\hat{\mathbf{n}}\times\hat{\bar{\mathbf{n}}}+\frac{|\bar{\mathbf{p}}_l|^2|\mathbf{p}_l|}{m_l(\bar{E}_l+m_l)}\hat{\bar{\mathbf{p}}}_l\cdot\hat{\bar{\mathbf{n}}}\,\hat{\mathbf{n}}\cdot\hat{\bar{\mathbf{p}}}_l\times\hat{\mathbf{p}}_l+\frac{|\mathbf{p}_l|^2|\bar{\mathbf{p}}_l|}{m_l(E_l+m_l)}\hat{\mathbf{p}}_l\cdot\hat{\mathbf{n}}\,\hat{\bar{\mathbf{n}}}\cdot\hat{\bar{\mathbf{p}}}_l\times\hat{\mathbf{p}}_l\right]$$

$$+2\,{\rm Im}(F_PF_A^*)m_K|\bar{\mathbf{p}}_l||\mathbf{p}_l|(\hat{\mathbf{n}}-\hat{\bar{\mathbf{n}}})\cdot\hat{\bar{\mathbf{p}}}_l\times\hat{\mathbf{p}}_l-2\,{\rm Im}(F_VF_A^*)m_K^2\left[\bar{E}_l|\mathbf{p}_l|\,\hat{\mathbf{p}}_l\cdot\hat{\mathbf{n}}\times\hat{\bar{\mathbf{n}}}+E_l|\bar{\mathbf{p}}_l|\,\hat{\bar{\mathbf{p}}}_l\cdot\hat{\mathbf{n}}\times\hat{\bar{\mathbf{n}}}\right.$$

$$+\frac{|\bar{\mathbf{p}}_l|^2|\mathbf{p}_l|}{\bar{E}_l+m_l}\hat{\bar{\mathbf{p}}}_l\cdot\hat{\bar{\mathbf{n}}}\,\hat{\mathbf{n}}\cdot\hat{\mathbf{p}}_l\times\hat{\bar{\mathbf{p}}}_l$$

$$\left.+\frac{|\mathbf{p}_l|^2|\bar{\mathbf{p}}_l|}{E_l+m_l}\hat{\mathbf{p}}_l\cdot\hat{\mathbf{n}}\,\hat{\bar{\mathbf{n}}}\cdot\hat{\mathbf{p}}_l\times\hat{\bar{\mathbf{p}}}_l\right]. \qquad (7)$$

We notice that $\mathcal{P}$ contains two types of T-violation indicators: namely, one-spin and two-spin correlations. To estimate the strength of these T-violation indicators, we define the quantities $I_{1,2\rm TV}$ to characterize the different kind of correlations. These quantities represent the relative strength of the coefficient of the appropriate correlation in the distribution $d\Gamma/d^3p_ld^3\bar{p}_l$ in the rest frame of the kaon.

The first category of indicators involves only one spin and can be defined as

$$I_{1\rm TV}^1=\left|\frac{2[{\rm Im}(F_SF_V^*)-{\rm Im}(F_PF_A^*)]|\mathbf{p}_l||\bar{\mathbf{p}}_l|\sin\theta_{l\bar{l}}}{m_K[(|F_V|^2+|F_A|^2)(E_l\bar{E}_l+\mathbf{p}_l\cdot\bar{\mathbf{p}}_l)-m_l^2(|F_V|^2-|F_A|^2)]}\right|, \qquad (8)$$

$$I_{1\rm TV}^2=\left|\frac{2[{\rm Im}(F_SF_V^*)+{\rm Im}(F_PF_A^*)]|\mathbf{p}_l||\bar{\mathbf{p}}_l|\sin\theta_{l\bar{l}}}{m_K[(|F_V|^2+|F_A|^2)(E_l\bar{E}_l+\mathbf{p}_l\cdot\bar{\mathbf{p}}_l)-m_l^2(|F_V|^2-|F_A|^2)]}\right|. \qquad (9)$$

There are four different indicators in the second category that involve two spin correlations:

$$I_{2\rm TV}^1=\left|\frac{2\{\bar{E}_l\,{\rm Im}(F_SF_P^*)-m_lm_K[{\rm Im}(F_SF_A^*)-{\rm Im}(F_PF_V^*)]+m_K^2\bar{E}_l\,{\rm Im}(F_VF_A^*)\}|\mathbf{p}_l|}{m_K^2(|F_V|^2+|F_A|^2)(E_l\bar{E}_l+\mathbf{p}_l\cdot\bar{\mathbf{p}}_l)-m_l^2(|F_V|^2-|F_A|^2)}\right|, \qquad (10)$$

$$I_{2\rm TV}^2=\left|\frac{2\{E_l\,{\rm Im}(F_SF_P^*)-m_lm_K[{\rm Im}(F_SF_A^*)+{\rm Im}(F_PF_V^*)]-m_K^2E_l\,{\rm Im}(F_VF_A^*)\}|\bar{\mathbf{p}}_l|}{m_K^2(|F_V|^2+|F_A|^2)(E_l\bar{E}_l+\mathbf{p}_l\cdot\bar{\mathbf{p}}_l)-m_l^2(|F_V|^2-|F_A|^2)}\right|, \qquad (11)$$

$$I_{2\rm TV}^3=\left|\frac{2\{{\rm Im}(F_SF_P^*)+m_K[{\rm Im}(F_SF_A^*)+{\rm Im}(F_PF_V^*)]-m_K^2\,{\rm Im}(F_VF_A^*)\}|\mathbf{p}_l|^2|\bar{\mathbf{p}}_l|\sin\theta_{l\bar{l}}/(E_l+m_l)}{m_K^2(|F_V|^2+|F_A|^2)(E_l\bar{E}_l+\mathbf{p}_l\cdot\bar{\mathbf{p}}_l)-m_l^2(|F_V|^2-|F_A|^2)}\right|, \qquad (12)$$

$$I_{2\rm TV}^4=\left|\frac{2\{{\rm Im}(F_SF_P^*)+m_K[{\rm Im}(F_SF_A^*)-{\rm Im}(F_PF_V^*)]+m_K^2\,{\rm Im}(F_VF_A^*)\}|\bar{\mathbf{p}}_l|^2|\mathbf{p}_l|\sin\theta_{l\bar{l}}/(\bar{E}_l+m_l)}{m_K^2(|F_V|^2+|F_A|^2)(E_l\bar{E}_l+\mathbf{p}_l\cdot\bar{\mathbf{p}}_l)-m_l^2(|F_V|^2-|F_A|^2)}\right|. \qquad (13)$$

Here indicators $I_{2\rm TV}^1$ and $I_{2\rm TV}^2$ correspond to the correlations $\hat{\mathbf{p}}_l\cdot\hat{\mathbf{n}}\times\hat{\bar{\mathbf{n}}}$ and $\hat{\bar{\mathbf{p}}}_l\cdot\hat{\mathbf{n}}\times\hat{\bar{\mathbf{n}}}$, respectively, while $I_{2\rm TV}^3$ and $I_{2\rm TV}^4$ correspond to the correlations $(\hat{\mathbf{p}}_l\cdot\hat{\mathbf{n}})\,(\hat{\bar{\mathbf{n}}}\cdot\hat{\mathbf{p}}_l\times\hat{\bar{\mathbf{p}}}_l)$ and $(\hat{\bar{\mathbf{p}}}_l\cdot\hat{\bar{\mathbf{n}}})\,(\hat{\mathbf{n}}\cdot\hat{\mathbf{p}}_l\times\hat{\bar{\mathbf{p}}}_l)$, respectively. In the above $\theta_{l\bar{l}}$ is the angle between $\bar{\mathbf{p}}_l$ and $\mathbf{p}_l$. We have chosen to normalize these quantities with respect to the dominant part of $\mathcal{P}_{\rm TC}$ which is proportional to the total width of the process $K^+\to\pi^+l\bar{l}$ after phase-space integration. These indicators are basically the relative probability for the correlations to exist.

At this stage, these quantities look quite complex. However, within the context of a specific model, as we shall see, they can be simplified considerably. For example, in the SM the dominating amplitude is proportional to F_V, and the most important T-violation signature comes from the interference between the F_V and F_A terms. Notice that these indicators are proportional to the relative phase between any of the two form factors, so we have to focus on models where such a relative phase can appear.

There are a number of different sources that might give rise to a relative phase between the form factors, the most important one being a phase in the CKM matrix [10]. The electromagnetic interaction among the final-state particles (FSI), and the strong interaction among the particles in the intermediate state, can also make contributions. The last two contributions are usually less interesting, and they could even hide the standard-model signal. Finally, we consider the models that go beyond the SM (BSM). In these models there are numerous possibilities for a relative phase between any of the two form factors, leading to nonvanishing T-violation indicators. We shall only briefly comment on the potential relative phase from the strong interaction, but will consider other contributions in more detail.

To facilitate our discussion, we define

$$\begin{aligned} F_S &= F_S^{\rm CKM}+F_S^{\rm FSI}+F_S^{\rm BSM} ,\\ F_P &= F_P^{\rm CKM}+F_P^{\rm FSI}+F_P^{\rm BSM} ,\\ F_V &= F_V^{\rm CKM}+F_V^{\rm FSI}+F_V^{\rm BSM} ,\\ F_A &= F_A^{\rm CKM}+F_A^{\rm FSI}+F_A^{\rm BSM} . \end{aligned} \tag{14}$$

Here various symbols are self-explanatory. In the next section we shall estimate the CKM contribution to these form factors.

III. THE CKM CONTRIBUTION IN THE STANDARD MODEL

As indicated in the Introduction, the electroweak and box diagrams, displayed in Fig. 1, are the main contributors to the form factors in the SM. These diagrams do not give rise to the form factor F_S. The scalar form factor receives a contribution from a diagram with a two-photon intermediate state. However, this contribution is of order $G_F\alpha^2$ and it is expected to be much smaller than the penguin and box contributions to the other form factors which are of order $G_F\alpha$. We will neglect altogether the two-photon intermediate state and therefore F_S will not be considered further in this section. The penguin and box diagrams have been calculated in Refs. [11–14]. Using their results, we find

$$\begin{aligned} F_P^{\rm CKM} &= m_l V_{is} V_{id}^* \frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi} f_+ \left[1-\frac{f_-}{f_+}\right] L_I(x_i) ,\\ F_V^{\rm CKM} &= V_{is} V_{id}^* \frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi} f_+ I_L(x_i) ,\\ F_A^{\rm CKM} &= V_{is} V_{id}^* \frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi} f_+ L_I(x_i) , \end{aligned} \tag{15}$$

where

$$\begin{aligned} I_L(x_i) &= I_L^\gamma(x_i)+I_L^Z(x_i)+I_L^B(x_i) ,\\ L_I(x_i) &= L_I^Z(x_i)+L_I^B(x_i) . \end{aligned} \tag{16}$$

In the above equations, $x_i=m_i^2/M_W^2$, where i can be t or c, corresponding to the exchange of an internal top quark and charm quark, respectively. A sum over i is implied in the expressions for F_P, F_V, and F_A. The V_{ij} are the CKM matrix elements, f_- and f_+ are the form factors which are the same as in K_{l3} decays [3]. The functions, I_L and L_I, are [11–14]

$$I_L^\gamma(x_i) = \frac{(25-19x_i)x_i^2}{36\pi(x_i-1)^3} - \frac{(3x_i^4-30x_i^3+54x_i^2-32x_i+8)\ln x_i}{18\pi(x_i-1)^4} , \tag{17}$$

$$I_L^Z(x_i) = \frac{4\sin^2\theta_W-1}{\sin^2\theta_W}\frac{x_i}{8\pi} \times \left[\frac{(x_i-6)(x_i-1)+(3x_i+2)\ln x_i}{(x_i-1)^2}\right] , \tag{18}$$

$$I_L^B(x_i) = \frac{1}{\sin^2\theta_W}\frac{x_i}{4\pi}\left[\frac{1-x_i+\ln x_i}{(x_i-1)^2}\right] , \tag{19}$$

and

$$L_I^Z(x_i) = \frac{I_L^Z(x_i)}{4\sin^2\theta_W-1}, \quad L_I^B(x_i) = -I_L^B(x_i) . \tag{20}$$

Some of these form factors can receive large strong-interaction corrections. This is especially true of F_V, as we discuss below. Since the long-distance part of such corrections cannot be computed in a model-independent fashion, we shall have to find a way to get its value from some other means to get a model-independent estimate. To this end we can use appropriate experimental measurements. In fact F_V can be determined by comparing the theoretical calculation with the experimental value for the width of the decay mode $K^+\to\pi^+e^+e^-$.

After a standard calculation, we find that the differential decay rate in the K^+ rest frame to be given by

$$\begin{aligned} \frac{d\Gamma}{dE_\pi} = \frac{1}{96\pi^3}\frac{|\mathbf{p}_\pi|}{m_K}\left[1-\frac{4m_l^2}{L^2}\right]^{1/2} \times \Big[& 3|F_P|^2L^2+12\,{\rm Re}(F_PF_A^*)m_l m_K(m_K-E_\pi) \\ & +12|F_A|^2m_l^2m_K^2 \\ & +2(|F_V|^2+|F_A|^2)m_K^2|\mathbf{p}_\pi|^2\left[1+2\frac{m_l^2}{L^2}\right]\Big] \end{aligned} \tag{21}$$

To get the width we have to carry out the remaining integration. This integral does not give any simple closed analytical result. However, in the case of the electron decay mode, where $m_e\simeq 0$, we get

$$\Gamma_e=(|F_V|^2+|F_A|^2)\frac{m_K^5}{384\pi^3}\times\left\{\frac{1}{8}\left[1-\frac{m_\pi^4}{m_K^4}\right]\left[\left[1-\frac{m_\pi^2}{m_K^2}\right]^2-6\frac{m_\pi^2}{m_K^2}\right]+3\frac{m_\pi^4}{m_K^4}\ln\left[\frac{m_k}{m_\pi}\right]\right\}. \quad (22)$$

As we see this width depends on F_V and F_A, which are given in Eq. (14); the major contribution is from $F_i^{\rm CKM}$ given in Eq. (15). The other contributions to these form factors are of higher order in the perturbative expansion. In this section, F_i stands for $F_i^{\rm CKM}$. The contribution of the diagram with the top quark in the intermediate state is negligible even for a large top-quark mass. This is due to the CKM matrix suppression (the contribution of the diagram with the top quark is proportional to λ^5, while with the charm quark it is proportional to λ. Here $\lambda=0.22$ is a parameter in the CKM matrix). Therefore, the width of the process $K^+\to\pi^+e^+e^-$ is dominated by the one-photon-exchange diagram with the charm quark in the intermediate state. The one-photon-exchange diagram only gives a significant contribution to F_V and not F_A. So F_A can be neglected in the above expression. The one-photon-exchange diagram is also expected to have large long-distance corrections, which remain uncertain. This contribution can only be calculated in the context of a model, which will require additional assumptions. However, since Γ_e now principally depends on only one form factor F_V, we can use the experimental value of Γ_e and Eq. (21) to obtain F_V, denoted by $F_V^{\rm expt}$. Experimentally, $B(K^+\to\pi^+e^+e^-)=(2.7\pm0.5)\times10^{-7}$ [3]. We find that [15] $|F_V^{\rm expt}|=(9\pm1)\times10^{-15}$ $({\rm MeV}/c^2)^{-2}$. We can use this value to estimate the branching ratio of the muon decay mode $K^+\to\pi^+\mu^+\mu^-$. We find $B(K^+\to\pi^+\mu^+\mu^-)=(5\pm1)\times10^{-8}$. This value agrees with the value estimated by Beder and Dass [16]. We note that here we have assumed that F_V is a constant, in fact there is a momentum dependence in this form factor. However, one should be able to measure this dependence of F_V by analyzing the dilepton distributions of the decay $K^+\to\pi^+e^+e^-$.

Unlike F_V, the long-distance corrections to F_A are expected to be small. This is the case since it contains no large logarithm [13, 17, 18] of the form $\ln(M_W^2/\mu^2)$ as the dominant term arises from t-quark exchanges, where μ is the QCD scale. Hence, there are no large strong-interaction corrections to F_A. From Eqs. (15)–(20), we notice that F_A depends on m_t,ρ, and η, where ρ and η are the yet undetermined CKM parameters in the Wolfenstein-Maiani parametrization [19]. Until now we only have bounds on these parameters. A precise measurement of F_A can help us to tighten these bounds. In fact ${\rm Im}(F_A)$ depends on η, while ${\rm Re}(F_A)$ depends on ρ. In the following we will discuss how to extract η from the T-violation indicators.

We saw in the last section that there are basically two types of muon spin correlations in the decay $K^+\to\pi^+\mu^+\mu^-$: one-spin and two-spin. To avoid a cumbersome discussion we will concentrate on one representative of each category: $I_{1\rm TV}^1$ and $I_{2\rm TV}^4$. As argued earlier, the denominator of these indicators will be dominated by F_V, so we can always neglect F_A in the denominator. We also set F_V to be equal to its experimental value $F_V^{\rm expt}$ in order to avoid the theoretical ambiguities. From Eq. (15), we see that if ${\rm Im}(f_-/f_+)\simeq0$, as expected from K_{l3}, there is almost no relative phase between $F_P^{\rm CKM}$ and $F_A^{\rm CKM}$. Therefore, we can write

$$I_{1\rm TV}^1=\left|\frac{{\rm Im}(F_SF_V^*)}{m_K|F_V^{\rm expt}|^2}\right|\left[\frac{2|\mathbf{p}_\mu||\bar{\mathbf{p}}_\mu|\sin\theta_{\mu\bar\mu}}{E_\mu\bar E_\mu+\mathbf{p}_\mu\cdot\bar{\mathbf{p}}_\mu-m_\mu^2}\right], \quad (23)$$

and,

$$I_{2\rm TV}^4=\left|\frac{-{\rm Im}(F_P)+m_K\,{\rm Im}(F_A^*)}{m_KF_V^{\rm expt}}\right|\times\left[\frac{2|\bar{\mathbf{p}}_\mu|^2|\mathbf{p}_\mu|\sin\theta_{\mu\bar\mu}}{(\bar E_\mu+m_\mu)(E_\mu\bar E_\mu+\mathbf{p}_\mu\cdot\bar{\mathbf{p}}_\mu-m_\mu^2)}\right]. \quad (24)$$

The correction to these will be of $O(|F_A|^2/|F_V|^2)$, which is several orders of magnitude smaller than the above leading term. Because of four-momentum conservation, these quantities depend on two variables which can be taken to be E_μ and $\bar E_\mu$.

To measure the indicator $I_{1\rm TV}^1$, one will have to measure the polarization of the μ^+ perpendicular to the decay plane. However, for $I_{2\rm TV}^4$, one will have to measure in addition the helicity of the muon. Obviously the former will be easier to measure. Unfortunately in the standard model it is very small because to this order $F_S^{\rm CKM}\simeq0$. Therefore the CKM contribution to the one-spin correlations is not likely to be accessible to experiments. The situation is, however, different in the case of two-spin correlation.

Using the Wolfenstein-Maiani parametrization [19] of the CKM matrix, and the values $\lambda=0.22$, $|F_V^{\rm expt}|=(9\pm1)\times10^{-15}$ $({\rm MeV}/c^2)^{-2}$, and $f_+=1$, we find, to the leading order,

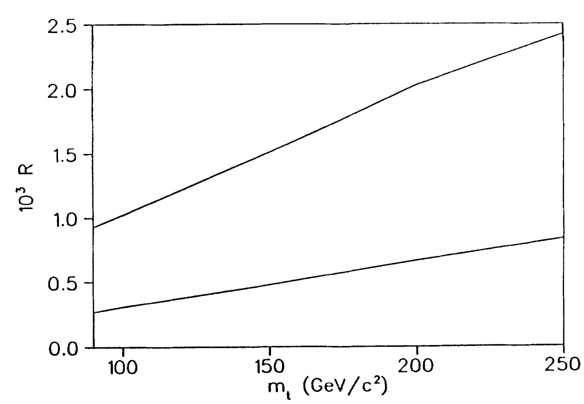

FIG. 2. The allowed region of R as a function of m_t.

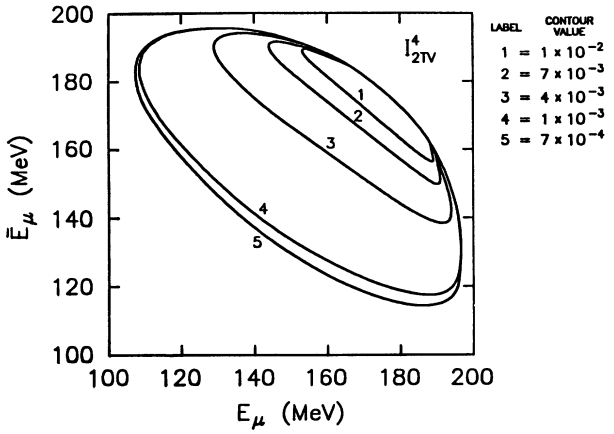

FIG. 3. Dalitz plot for the indicator I^4_{2TV}.

$$R \equiv \left| \frac{-\mathrm{Im}(F_P^{\mathrm{CKM}})+m_K\,\mathrm{Im}(F_A^{\mathrm{CKM}*})}{m_K F_V^{\mathrm{expt}}} \right| = (4.1\pm0.3)\times10^{-3}A^2\eta L_I(x_t)\ . \qquad (25)$$

Thus we see that I^4_{2TV} is directly proportional to η. We emphasize that this quantity is relatively free of hadronic uncertainties. In Fig. 2, we show the dependence of R on the top-quark mass taking into account one standard deviation on the parameters of the CKM matrix [20]. If we choose $m_t=200$ GeV/c^2, we find the maximum value for the parameter $R\simeq2.1\times10^{-3}$ leading to a value for the muon decay mode of $I^4_{2TV}\simeq0.5\times10^{-2}$, when $E_\mu=\bar{E}_\mu=m_K/3$. In Fig. 3 we give a Dalitz plot of this indicator which shows its values over the allowed phase space. From there one can see where this indicator has the largest value, $I^4_{2TV}\simeq7.5\times10^{-2}$, and where a prospective experiment should focus.

Naturally, the fixing of the CKM parameters is in itself an important issue. Once we know m_t, the above indicators can then give another data point to constrain the parameter η. We note here that R depends linearly on η, that is expected to be less than 1. On the other hand the branching ratio of $K_L\to\pi^0\nu\bar{\nu}$ depends on η^2. Therefore, in principle, R is more sensitive to the measurement of the CKM phase. We note that there are other T-violation indicators involving two-spin correlations given by Eqs. (10) and (11). These indicators are expected to be somewhat larger than those estimated above, and were discussed in Ref. [9]. However, these indicators may be harder to measure unless one can measure the spins of the muons event by event.

IV. THE FINAL-STATE-INTERACTION CONTRIBUTION

Within the standard model, there is another contribution to the T-violation indicators. It comes from the electromagnetic interaction between two charged particles in the final state. The diagrams making such contributions are displayed in Fig. 4. As we see these are higher-order diagrams in a perturbative expansion; therefore, their contribution is expected to be smaller. Furthermore we are mostly interested only in imaginary part of these contributions.

To estimate the final-state-interaction contribution, we only need to calculate the imaginary part of the $F_S^{\mathrm{FSI}}, F_P^{\mathrm{FSI}}, F_V^{\mathrm{FSI}}$, and F_A^{FSI}. We follow a procedure analogous to the one used in the case of the $K_{\mu3}$ decay in Ref. [21]. After some lengthy calculation we get

$$\begin{aligned}
\mathrm{Im}(F_S^{\mathrm{FSI}}) &= \alpha F_S^{\mathrm{CKM}}\left[\frac{1}{p_1^2D_1}[p_1\cdot p_\mu(p_1^2+m_\mu^2)-2m_\mu^2p_1^2]+\frac{1}{p_2^2D_2}[p_2\cdot\bar{p}_\mu(p_2^2+m_\mu^2)-2m_\mu^2p_2^2]+\frac{1}{p_3^2D_3}2m_\mu^2p_3^2\right]\\
&\quad+\alpha F_V^{\mathrm{CKM}}\left[-\frac{1}{p_1^2D_1}m_\mu(p_1^2-m_\mu^2)p_1\cdot(p_1-p_\mu)+\frac{1}{p_2^2D_2}m_\mu(p_2^2-m_\mu^2)p_2\cdot(p_2-\bar{p}_\mu)\right.\\
&\quad\left.+\frac{1}{p_3^2D_3}m_\mu\frac{1}{2}p_3^2p_K\cdot(p_\mu-\bar{p}_\mu)\right],\\
\mathrm{Im}(F_P^{\mathrm{FSI}}) &= \alpha F_P^{\mathrm{CKM}}\left[\frac{1}{p_1^2D_1}p_1\cdot p_\mu(p_1^2-m_\mu^2)+\frac{1}{p_2^2D_2}p_2\cdot\bar{p}_\mu(p_2^2+m_l^2)\right]\\
&\quad-\alpha F_A^{\mathrm{CKM}}\left[\frac{1}{p_1^2D_1}m_\mu(p_1^2-m_\mu^2)p_1\cdot(p_\mu-p_1)+\frac{1}{p_2^2D_2}m_\mu(p_2^2-m_\mu^2)p_2\cdot(\bar{p}_\mu-p_2)\right.\\
&\quad\left.+\frac{1}{p_3^2D_3}m_\mu p_K\cdot p_3(-\tfrac{3}{2}p_3^2+2m_\mu^2)\right],\\
\mathrm{Im}(F_V^{\mathrm{FSI}}) &= \alpha F_S^{\mathrm{CKM}}m_\mu\left[\frac{1}{p_1^2D_1}p_1\cdot(p_1-p_\mu)+\frac{1}{p_2^2D_2}p_2\cdot(p_2-\bar{p}_\mu)\right]\\
&\quad+\alpha F_V^{\mathrm{CKM}}\left[\frac{1}{p_1^2D_1}p_1\cdot p_\mu(p_1^2-m_\mu^2)+\frac{1}{p_2^2D_2}p_2\cdot\bar{p}_\mu(p_2^2-m_\mu^2)+\frac{1}{p_3^2D_3}3D_3^2\right],
\end{aligned}$$

$$\mathrm{Im}(F_A^{\mathrm{FSI}})=\alpha F_P^{\mathrm{CKM}} m_\mu \left[\frac{1}{p_1^2 D_1} p_1\cdot(p_1-p_\mu)+\frac{1}{p_2^2 D_2} p_2\cdot(p_2-\bar{p}_\mu)\right] +\alpha F_A^{\mathrm{CKM}}\left[\frac{1}{p_1^2 D_1}[p_1\cdot p_\mu(p_1^2-m_\mu^2)-2m_\mu^2 p_1^2]+\frac{1}{p_2^2 D_2}[p_2\cdot\bar{p}_\mu(p_2^2-m_\mu^2)-2m_\mu^2 p_2^2] +\frac{1}{p_3^2 D_3}(3D_3^2+2m_\mu^2 p_3^2)\right], \tag{26}$$

where

$$p_1=p_\pi+p_\mu,\quad p_2=p_\pi+\bar{p}_\mu,\quad p_3=p_\mu+\bar{p}_\mu, \tag{27}$$

and

$$D_1^2=(p_\pi\cdot p_\mu)^2-m_\mu^2 m_\pi^2,\quad D_2^2=(p_\pi\cdot\bar{p}_\mu)^2-m_\mu^2 m_\pi^2,\quad D_3^2=(\bar{p}_l\cdot p_\mu)^2-m_\mu^4. \tag{28}$$

In the above α is the fine-structure constant. Here we have taken the electromagnetic form factor of the pion, F_π, to be one. It is straightforward to include its momentum dependence [21]. It is not expected to change our estimates significantly.

In the standard model, as we saw, F_V has the largest value among the form factors. Therefore only those corrections that are proportional to it are significant. We are assuming here that the largest contribution to F_V comes from the standard model. As we shall see in the next section, the contribution from extensions of the standard model are indeed small. Therefore although we are fitting F_V to experimental data, the resulting value can be considered to be the standard model value. Thus, we see that only F_S receives substantial contribution from the final-state interactions. Therefore only the indicators that are proportional to the imaginary part of F_S are significantly affected. Consequentially it is the one-spin correlations rather than the two-spin ones that are sensitive to this mainly QED effect.

One can see from Eq. (13) that F_S contributes to $I_{2\mathrm{TV}}^4$. However, the terms proportional to F_S are expected to be smaller by a factor of α relative to other terms. Hence the inclusion of FSI's will not alter much the estimate we have given in the previous section for this indicator. It is easy to show that this is also the case for other indicators involving two spins.

In contrast with this, one-spin correlations, which are negligible in the CKM model, are dominated by the FSI's. Because of such contributions, one-spin correlations may even be experimentally observable. If we consider that all form factors with the exception of F_S are the same as in the CKM description it becomes clear that the main contribution to $I_{1\mathrm{TV}}^{1,\mathrm{CKM}}$ has the form given in Eq. (23). With the explicit form for F_S and neglecting as before F_A in the denominator, we get

$$I_{1\mathrm{TV}}^{1,\mathrm{FSI}}=\left[\alpha\frac{m_\mu}{m_K}\right]\left|\left[-\frac{1}{p_1^2 D_1}(p_1^2-m_\mu^2)p_1\cdot(p_1-p_\mu)+\frac{1}{p_2^2 D_2}(p_2^2-m_\mu^2)p_2\cdot(p_2-\bar{p}_\mu)+\frac{1}{p_3^2 D_3}\frac{1}{2}p_3^2 p_K\cdot(p_\mu-\bar{p}_\mu)\right]\right| \times\left[\frac{2|\mathbf{p}_l||\bar{\mathbf{p}}_\mu|\sin\theta_{\mu\bar{\mu}}}{(E_\mu\bar{E}_\mu+\mathbf{p}_\mu\cdot\bar{\mathbf{p}}_\mu-m_\mu^2)}\right]. \tag{29}$$

We notice that the form factor F_V has dropped out, and this indicator is proportional to $\alpha(m_\mu/m_K)\simeq 1.5\times 10^{-3}$. This sets the scale for the final-state-interaction contributions. Figure 5 is a Dalitz plot of this indicator, and we find that in some region of phase space we could have $I_{1\mathrm{TV}}^1\simeq 10^{-2}$.

V. BEYOND THE STANDARD-MODEL CONTRIBUTIONS

Despite the successes of the SM, it leaves a number of questions unanswered. In particular the origin of *CP*

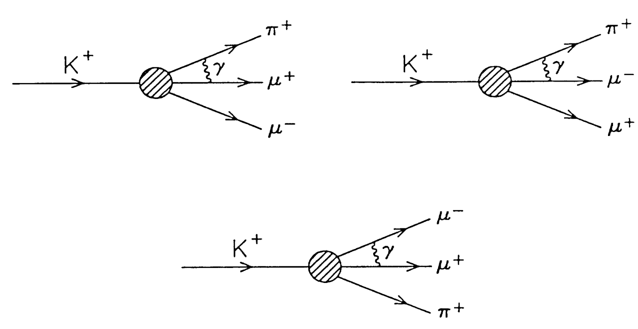

FIG. 4. The final-state-interaction diagrams.

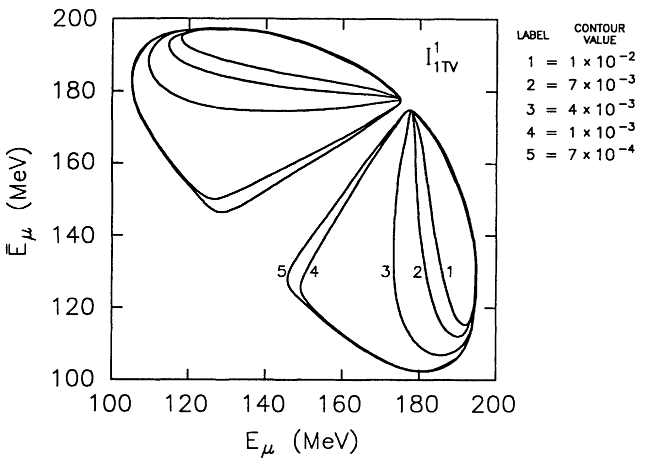

FIG. 5. Dalitz plot for the indicator $I^{1,\mathrm{FSI}}_{1TV}$.

violation is not well understood. In search of answers, many extensions of the SM, which include the multi-Higgs-doublet, leptoquark, left-right-symmetric, and supersymmetry (SUSY) models, have been put forward. In this section we will examine the contributions to T-violation indicators in these extensions of the SM to see how that might affect our predictions in Sec. III. Since many models can be constructed within each class, we shall focus on the simplest or/and more popular models relevant to our discussions. In this section, we first study the charged-Higgs-boson effect in a two-Higgs-doublet model. Then we explore leptoquark models which contribute to the $K^+\to\pi^+\mu^+\mu^-$ decay at tree level.

We also discuss the predictions for the T-violation indicators in various CP-violation models studied by two of us in Ref. [22]. Here, a class of models which give contributions to a CP-violation indicator, the longitudinal muon polarization P_L, in the $K_L\to\mu^+\mu^-$ decay through one-loop diagrams were studied. The models include the multi-Higgs-doublet models with CP-violating neutral-Higgs-boson exchanges, SUSY models, and left-right-symmetric models. It is straightforward to show that all the loop diagrams which contribute to the $K_L\to\mu^+\mu^-$ decay will contribute to the $K^+\to\pi^+\mu^+\mu^-$ decay. As shown in Ref. [22], these models could lead to sizable $P_L(K_L\to\mu^+\mu^-)$. Thus it is interesting to examine whether or not they could also induce large T-violation indicators in the charged K decays. We will give a detailed discussion on the effect of the CP violating neutral-Higgs-boson exchanges while the effects of other models will be generalized from it.

We will see that in some cases the T-violation indicators are enhanced. We will in general assume the standard CKM description of CP violation and estimate the additional T-violating effects typical of each model.

A. Charged-Higgs-boson effects

It is well known that the simplest extension of the standard model is to enlarge the Higgs sector to include an extra Higgs doublet. Many two-Higgs-doublet models have been constructed in the literature [23]. Among them, the most popular one has one doublet ϕ_1 that couples to up-type quarks and the other, ϕ_2, to down-type ones, leading to the so-called natural flavor conservation (NFC) [24]. It is also necessary to introduce this type of Higgs doublet in the supersymmetric extension of the standard model and in models using U(1) Peccei-Quinn symmetry to solve the strong CP problem. The two-Higgs-doublet model has one physical charged Higgs scalar H whose coupling to fermions is given by

$$\mathcal{L}_Y^{\pm}=\frac{g}{2\sqrt{2}M_W}\left[\frac{v_2}{v_1}H^+\bar{U}_R M_U V D_L+\frac{v_1}{v_2}H^+\bar{U}_L M_D V D_R+\frac{v_1}{v_2}H^+\bar{N}_L M_E E_R+\mathrm{H.c.}\right], \tag{30}$$

where v_i are the vacuum expectation values (VEV's) of ϕ_i, V is the CKM matrix,

$$U=\begin{bmatrix}u\\c\\t\end{bmatrix},\quad D=\begin{bmatrix}d\\s\\b\end{bmatrix},\quad N=\begin{bmatrix}\nu_e\\\nu_\mu\\\nu_\tau\end{bmatrix},\quad E=\begin{bmatrix}e\\\mu\\\tau\end{bmatrix}, \tag{31}$$

and

$$M_U=\begin{bmatrix}m_u & & 0\\ & m_c & \\ 0 & & m_t\end{bmatrix},\quad M_D=\begin{bmatrix}m_d & & 0\\ & m_s & \\ 0 & & m_b\end{bmatrix},\quad M_E=\begin{bmatrix}m_e & & 0\\ & m_\mu & \\ 0 & & m_\tau\end{bmatrix}. \tag{32}$$

We now discuss the charged-Higgs-boson effects on the form factors $F_i(i=S,P,V,A)$. The charged-Higgs-boson contributions to the decay $K^+\to\pi^+\mu^+\mu^-$ come from the penguin and box diagrams, analogous to the SM, with the charged Higgs boson replacing the W boson in the loop. These diagrams make no contribution to the form factor F_S just as in the SM. It is easy to show that

$$F_V^{\mathrm{CH}}\ll F_V^{\mathrm{expt}}. \tag{33}$$

This is essentially because the coupling of the charged-Higgs boson to quarks is proportional to the quark mass, and therefore we can take $F_V^{\mathrm{CH}}\sim 0$ in our discussions.

The contributions to $F_A^{\rm CH}$ and $F_P^{\rm CH}$ are dominated by the Z penguin graphs since the box diagram contributions have a suppression factor of m_μ^2/M_W^2 and they are given by

$$F_P^{\rm CH}=m_l V_{is}V_{id}^*\frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi}f_+\left[1-\frac{f_-}{f_+}\right]L_I^{\rm CH}(x_i,y_i)\ ,$$
$$F_A^{\rm CH}=V_{is}V_{id}^*\frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi}f_+L_I^{\rm CH}(x_i,y_i)\ , \tag{34}$$

where, $y_i=m_i^2/M_H^2$ and M_H is the charged-Higgs-boson mass, and

$$L_I^{\rm CH}(x_i,y_i)=\frac{-1}{\sin^2\theta_W}\left[\frac{v_2}{v_1}\right]^2\frac{x_i}{2}\frac{y_i}{4\pi}\left[\frac{1-y_i+\ln y_i}{(y_i-1)^2}\right]\ . \tag{35}$$

Since there is still no relative phase between $F_P^{\rm CH}$ and $F_A^{\rm CH}$ in Eq. (34) as in the CKM form factors, $F_P^{\rm CKM}$ and $F_A^{\rm CKM}$ in Eq. (14), we expect that there is no significant contribution to the one-spin T-violation indicators, $I_{1\rm TV}$, from the charged-Higgs boson. For the case of two-spin correlation, using the same notations as the SM, we have

$$I_{2\rm TV}^4=\frac{2|\bar{\mathbf{p}}_\mu|^2|\mathbf{p}_\mu|\sin\theta_{\mu\bar\mu}R_{\rm CH}}{(\bar{E}_\mu+m_\mu)(E_\mu\bar{E}_\mu+\mathbf{p}_\mu\cdot\bar{\mathbf{p}}_\mu-m_\mu^2)}\ , \tag{36}$$

with

$$R_{\rm CH}=(4.1\pm0.3)\times10^{-3}A^2\eta[L_I(x_t)+L_I^{\rm CH}(x_t,y_t)]\ . \tag{37}$$

We note that the charged-Higgs-boson term in Eq. (37) has the same sign as the standard one [25]. Thus we expect that the value of $I_{2\rm TV}^4$ may increase due to charged-Higgs-boson effects. For instance, with parameters obtained by fitting the experimental constraints such as $|V_{ub}/V_{cb}|$, the CP-violating parameter ϵ and the B_d^0-$\bar{B}_d^0$ mixing in Ref. [26], we find that

$$I_{2\rm TV}^4\lesssim0.05\quad\text{and}\quad I_{2\rm TV}^4\lesssim0.1\ , \tag{38}$$

for $(M_H,\ m_t,\ v_2/v_1)=(150$ GeV, 100 GeV,1) and (150 GeV, 200 GeV,1), respectively.

B. Leptoquark models

There are four scalar leptoquarks which could give contributions to the decay $K^+\to\pi^+\mu^+\mu^-$ through the tree diagrams. The quantum numbers of these leptoquarks under the standard group $SU(3)_C\times SU(2)_L\times U(1)_Y$ are $\phi_1=(3,1,-\frac{8}{3})$, $\phi_2=(3,3,-\frac{2}{3})$, $\phi_3=(3,2,\frac{7}{3})$, $\phi_4=(3,2,\frac{1}{3})$. To see the possible large effect on T-violation indicators we will concentrate on ϕ_1 and we will remark on the other leptoquarks effects at the end. The general coupling of ϕ_1 with fermions is given by

$$\mathcal{L}=\sum_{i,j}\lambda^{ij}\bar{d}_R^{\,i}e_R^{cj}\phi_1+\text{H.c.}\ , \tag{39}$$

where i,j are family indices. Here the coupling constants λ^{ij} can be complex and thus T violation could arise from either the Yukawa interactions in Eq. (39), or from the phase in the CKM matrix. We assume that CP violation in $K\to\pi\pi$ decays can be accounted for by the nonvanishing CKM phase, and investigate the effect on the correlations of the additional CP-violation interaction terms given in Eq. (39).

The contributing Feynman diagram is shown in Fig. 6. The effective interaction from this diagram is

$$\mathcal{L}_{\rm eff}=\frac{\lambda^{22}(\lambda^{12})^*}{4M_{\phi_1}^2}\bar{s}(1-\gamma_5)\mu^c\bar{\mu}^c(1+\gamma_5)d+\text{H.c.}\ , \tag{40}$$

where M_{ϕ_1} is the mass of ϕ_1. Using the Fierz transformations,

$$2\bar{s}(1\pm\gamma_5)a\bar{b}(1\mp\gamma_5)d=-\bar{s}\gamma_\mu(1\mp\gamma_5)d\bar{b}\gamma^\mu(1\pm\gamma_5)a\ , \tag{41}$$

we obtain the form factors

$$F_S^{\rm LQ}=0,\quad F_P^{\rm LQ}=m_\mu\ ,$$
$$F_V^{\rm LQ}=-m_\mu,\quad F_A^{\rm LQ}=\frac{\lambda^{22}(\lambda^{12})^*}{4M_{\phi_1}^2}f_+m_\mu\ . \tag{42}$$

We note that the CP-violating muon polarization in the $K_L\to\mu^+\mu^-$ decay cannot be induced by the effective interaction in Eq. (40) because of the vanishing F_S form factor. From Eqs. (40)–(42), we find

$$I_{1\rm TV}^{1,\rm LQ}\simeq\left|\frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi}L_I(x_t)\frac{m_\mu A^2\lambda^5}{2m_K(F_V^{\rm expt})^2}\left[(1-\rho)\frac{\text{Im}[\lambda^{22}(\lambda^{12})^*]}{M_{\phi_1}^2}-\eta\frac{\text{Re}[\lambda^{22}(\lambda^{12})^*]}{M_{\phi_1}^2}\right]\right|\left[\frac{2|\mathbf{p}_\mu||\bar{\mathbf{p}}_\mu|\sin\theta_{\mu\bar\mu}}{E_\mu\bar{E}_\mu+\mathbf{p}_\mu\cdot\bar{\mathbf{p}}_\mu-m_\mu^2}\right] \tag{43}$$

and

$$I_{2\rm TV}^{4,\rm LQ}\simeq\left|\frac{\text{Im}[\lambda^{22}(\lambda^{12})^*]}{M_{\phi_1}^2}\frac{1}{4F_V^{\rm expt}}\right|\left[\frac{2|\bar{\mathbf{p}}_\mu|^2|\mathbf{p}_\mu|\sin\theta_{\mu\bar\mu}}{(\bar{E}_\mu+m_\mu)(E_\mu\bar{E}_\mu+\mathbf{p}_\mu\cdot\bar{\mathbf{p}}_\mu-m_\mu^2)}\right]\ , \tag{44}$$

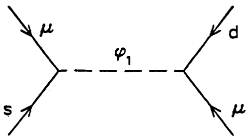

FIG. 6. A diagram due to leptoquark interactions.

where we have assumed $|F_V^{\rm LQ}|<F_V^{\rm expt}$. To estimate $I_{\rm 1TV}^{1,\rm LQ}$ and $I_{\rm 2TV}^{4,\rm LQ}$ we need to find out the bounds on the parameters:

$$\frac{\mathrm{Im}[\lambda^{22}(\lambda^{12})^*]}{M_{\phi_1}^2} \quad\text{and}\quad \frac{\mathrm{Re}[\lambda^{22}(\lambda^{12})^*]}{M_{\phi_1}^2}\ . \tag{45}$$

The coupling in Eq. (39) could induce several rare processes such as $(g-2)_\mu$, $\mu\to e\gamma$, $\mu N\to eN$, $K_L\to\mu\bar{e}$, $\mu^+\mu^-$, and the CP-violating parameter ϵ which may put constraints on the parameters in Eq. (45).

Generally speaking, Eq. (45) contains five independent parameters. One can put tight constraints on these parameters using existing experimental data, provided we make some additional assumptions. For example, we can assume $\lambda\sim|\lambda^{ij}|$, i.e., ignore the generation index. Then using the experimental bound [3] of $\Gamma(\mu Ti\to eTi)/\Gamma(\mu Ti\to \mathrm{all})|_{\rm expt}<4.6\times10^{-12}$ one finds that [27,28]

$$\frac{\lambda^2}{M_{\phi_1}^2}<10^{-11}\ \mathrm{GeV}^{-2}\ . \tag{46}$$

This constraint in turn implies very small contribution to the T-violation indicators. We get

$$I_{\rm 1TV}^{1,\rm LQ}<3.4\times10^{-6}\ , \tag{47}$$

and

$$I_{\rm 2TV}^{4,\rm LQ}<2.0\times10^{-2}\ , \tag{48}$$

from Eqs. (43) and (44), respectively, for $m_t<200$ GeV. Of course, the parameters in Eq. (45) could escape some experimental constraints since *a priori* there are no relations among the couplings corresponding to either flavor-diagonal or flavor-changing interactions. For instance, the limit in Eq. (46) depends on $\lambda^{11}(\lambda^{12})^*/M_{\phi_1}^2$; to get the direct constraints on $\lambda^{22}(\lambda^{12})^*$ we must know the rates of $K^+\to\pi^+\mu^+\mu^-$, $K_L\to\mu^+\mu^-$, and $K_S\to\mu^+\mu^-$.

For the decay $K_L\to\mu^+\mu^-$, we use the limit of the short-distance contribution to the branching ratio given in Ref. [20], which is

$$B(K_L\to\mu\bar{\mu})_{\rm SD}<2\times10^{-9}\ . \tag{49}$$

This leads to

$$\frac{|\mathrm{Re}[\lambda^{22}(\lambda^{12})^*]|}{M_{\phi_1}^2}<2\times10^{-10}\ \mathrm{GeV}^{-2}\ . \tag{50}$$

Similarly we find

$$\frac{|\mathrm{Im}[\lambda^{22}(\lambda^{12})^*]|}{M_{\phi_1}^2}<6\times10^{-8}\ \mathrm{GeV}^{-2}\ , \tag{51}$$

by using $B(K_S\to\mu\bar{\mu})_{\rm expt}<3.2\times10^{-7}$ [3]. The assumption $|F_V^{\rm LQ}|<F_V^{\rm expt}$ implies

$$\frac{|\lambda^{22}(\lambda^{12})^*|}{M_{\phi_1}^2}<3.6\times10^{-8}\ \mathrm{GeV}^{-2}\ . \tag{52}$$

The constraints from Eqs. (49)–(51) lead to the bounds

$$I_{\rm 1TV}^{1,\rm LQ}<9.5\times10^{-3}\ , \tag{53}$$

for $m_t<200$ GeV. On the other hand the two-spin indicator is not constrained since for the same value of m_t we get

$$I_{\rm 2TV}^{4,\rm LQ}<(\sim1)\times\left[\frac{2|\bar{\mathbf{p}}_\mu|^2|\mathbf{p}_\mu|\sin\theta_{\mu\bar{\mu}}}{(\bar{E}_\mu+m_\mu)(E_\mu\bar{E}_\mu+\mathbf{p}_\mu\cdot\bar{\mathbf{p}}_\mu-m_\mu^2)}\right]\ , \tag{54}$$

which can clearly be larger than the CKM contribution. This should be expected since there are hardly any significant bounds on $\mathrm{Im}[\lambda^{22}(\lambda^{12})^*]$. Finally we note that the results above can be generalized to other leptoquark models.

C. Scalar-pseudoscalar mixing effects

In a class of models, the CP symmetry is violated spontaneously. We will consider the implications of the CP violation in the neutral-Higgs-scalar sector through mixing of scalar and pseudoscalar bosons in multi-Higgs-boson models [29,30] on the T-violation indicators. The simplest model displaying this source was emphasized in Ref. [22].

The most stringent constraint on the parameters of the model comes from the experimental limit on the neutron electric dipole moment (EDM) [31], $d_n^{\rm expt}<1.2\times10^{-25}\ e$ cm. We find that the stronger bound on $X/M_0^2v^2$ is from the chromo-EDM of the quarks at two-loop level [32] which gives

$$\frac{X}{M_0^2v^2}<1.0\times10^{-5}\ \mathrm{GeV}^{-4}\ , \tag{55}$$

where the parameter X is the product of couplings of neutral scalars with the quarks and leptons, and M_0 is the mass of the lightest scalar particles. Here we assume that $M_0\sim1$ GeV $\ll m_t\sim100$ GeV.

The relevant effective interaction is given by

$$L_{\rm eff}^{\rm NH}\sim\frac{G_F}{\sqrt{2}}ig_{\rm SP}\bar{s}(1+\gamma_5)d\bar{\mu}(1+\gamma_5)\mu+\mathrm{H.c.}\ , \tag{56}$$

with

$$g_{\rm SP}\sim\left[\frac{X}{M_0^2v^2}\right]\frac{m_s m_\mu m_c^2}{4\pi^2}f(m_c^2/M_W^2)\sin\theta_C\ , \tag{57}$$

where $f(m_c^2/M_W^2)\sim1$. We have neglected the contribution from the t quark. Using the matrix element

$$\langle\pi^+|\bar{s}(1\pm\gamma_5)d|K^+\rangle\simeq f_+\frac{m_K^2}{m_s}\ , \tag{58}$$

we find that

$$F_S^{\rm NH}=F_P^{\rm NH}\sim i\frac{G_F}{\sqrt{2}}g_{\rm SP}f_+\frac{m_K^2}{m_s}\ ,$$
$$F_V^{\rm NH}=F_A^{\rm NH}\sim 0\ . \tag{59}$$

From these form factors and Eqs. (8) and (13), we can write the neutral-Higgs-boson effects as

$$I_{1\rm TV}^{1,\rm NH}\simeq\left|\frac{{\rm Im}(F_S^{\rm NH})}{m_K F_V^{\rm expt}}\right|\left|\frac{2|\mathbf{p}_\mu||\bar{\mathbf{p}}_\mu|\sin\theta_{\mu\bar\mu}}{E_\mu\bar E_\mu+\mathbf{p}_\mu\cdot\bar{\mathbf{p}}_\mu-m_\mu^2}\right|\ , \tag{60}$$

and,

$$I_{2\rm TV}^{4,\rm NH}\simeq\left|\frac{{\rm Im}(F_P^{\rm NH})}{m_K F_V^{\rm expt}}\right|\left|\frac{2|\bar{\mathbf{p}}_\mu|^2|\mathbf{p}_\mu|\sin\theta_{\mu\bar\mu}}{(\bar E_\mu+m_\mu)(E_\mu\bar E_\mu+\mathbf{p}_\mu\cdot\bar{\mathbf{p}}_\mu-m_\mu^2)}\right|. \tag{61}$$

With the constraint in Eq. (56), we estimate that

$$I_{1\rm TV}^{1,\rm NH}<4.1\times10^{-4}\ , \tag{62}$$

and

$$I_{2\rm TV}^{4,\rm NH}<2.1\times10^{-4}\ . \tag{63}$$

We remark that in contrast with these small values, the *CP*-violating longitudinal muon polarization P_L in the $K_L\to\mu^+\mu^-$ decay is predicted to be at a level of 1% [22].

D. Supersymmetric models

CP violation in supersymmetric models has been studied as a test of effects beyond the standard model [33]. For the minimal standard *CP*-violation SUSY model in which only the CKM phase is considered, one can show that the standard results given in Sec. III basically do not change. In general, there are many more new sources of *CP* violation in addition to the CKM phase. As in Ref. [22], we only discuss the SUSY E_6 model inspired by superstring theory, where *CP* violation comes from loop diagrams with scalar leptoquarks exchanges. The contribution of these loop diagrams turns out to have a similar structure as the neutral Higgs case given in Eq. (57). For this SUSY model, we find

$$I_{1\rm TV}^{1,\rm SUSY}\le5.4\times10^{-3}\ , \tag{64}$$

and

$$I_{2\rm TV}^{4,\rm SUSY}\le2.8\times10^{-3}\ . \tag{65}$$

Here we have used the model which could give the muon polarization in the $K_L\to\mu^+\mu^-$ decay up to 7% as shown in Eq. (45) of Ref. [22].

E. Left-right-symmetric models

CP violation in left-right-symmetric models have been studied extensively in the literature [34]. In contrast with the standard model, the physical phases, contributing to *CP* violation, can be introduced even for two generations of quarks because of the existence of right-handed currents. Without going through the details of the calculation, since the discussion is similar to the neutral-Higgs-boson case, we state our result that the *T*-violation indicators are once again small. More precisely, in the left-right-symmetric models in Ref. [22] we find, from the left-right box diagram shown in Fig. 4 of Ref. [22],

$$I_{1\rm TV}^{1,\rm LR}\le1.6\times10^{-3}\ , \tag{66}$$

and

$$I_{2\rm TV}^{4,\rm LR}\le8.3\times10^{10^{-4}}\ . \tag{67}$$

We note that this model gives $P_L(K_L\to\mu^+\mu^-)\le2\%$.

VI. DISCUSSION AND CONCLUSIONS

Throughout our discussion above we have tacitly assumed that F_V has negligible imaginary part. In the expression given in Eq. (15), it is indeed true. However, if we consider the strong-interaction corrections to it, the situation might alter. Such corrections have been calculated within the context of chiral perturbation theories [35]. There an imaginary part of F_V emerges when the invariant mass of the two-lepton system L is above the two-pion threshold. So we see that if a search for *T*-violation is focused in that part of the phase space where $L<M(\pi\pi)$, then our assumption is perfectly valid. Quantitatively, about 55% of the phase space is below this threshold. However, we see that *T*-violation indicators with two spins take their largest value when $L>M(\pi\pi)$. These two-spin correlations provide the best indicators to explore the CKM mechanism. Therefore one may have to find an optimum area in the phase space. Even above the two-pion threshold, over most of the phase space, the imaginary part has been found in the above calculation [35] to be *at least* about a factor of 2 smaller than the real part for the most of the phase space. Furthermore, the imaginary part and the real part have opposite signs. Therefore, near the "edge" of the phase space, where the imaginary part of the F_V can be important, our predictions for two-spin correlations can be enhanced by about a factor of 2–8. This numerical factor will depend on the value and sign of the undetermined parameter ρ. Therefore measurements near the "edge" of the phase space will be more sensitive to hadronic uncertainties. Away from this region, the effect of including the imaginary part of F_V is not large. As we discussed extensively in Sec. IV, the final-state interactions make negligible contributions to the imaginary part of F_V, and to the indicators for two-spin correlations.

Another important issue is the potential dependence of F_V on L. In extracting F_V from experimental data, we took F_V to be a constant. However, the incorporation of this dependence in our analysis does not represent any special problem. One can extract such dependence by appropriate measurements for the decay $K^+\to\pi^+e^+e^-$, e.g., by measuring $d\Gamma/dL$. Furthermore, within the context of chiral perturbation theory [35] this dependence has been found to be weak. Here it was shown that F_V could change by less than a factor of 2 over allowed range of L. Therefore, our predictions are not expected to change much after this incorporation. Our predictions for two-spin correlations also depend on the form factor F_A. The strong interaction correction to this had been computed, and may change our estimates by about 10–20 %.

The next question that one may ask is how good this process is compared to other proposals to look for $T(CP)$ violation in kaon decays? For this we can refer to the criteria mentioned in the Introduction. Apart from the measurement of ε', the candidates are $K_L\to\pi^0\nu\bar{\nu}$ and $K_L\to\pi^0e^+e^-$, together with the process discussed here, i.e., $K^+\to\pi^+\mu^+\mu^-$. The decay $K^+\to\pi^+e^+e^-$ is ruled out because of the potential experimental difficulties in measuring the polarization of the electron. Measuring the difference in the partial decay rate $K^+\to\pi^+e^+e^-$ and $K^-\to\pi^-e^+e^-$ is difficult due to the small rate and the inability to determine the systematic uncertainties to the required level. Furthermore this asymmetry is too small to be measurable. There are still theoretical as well as experimental controversies about the measurement of ε'. Therefore we look at the last three processes. The first two of these processes, the decays of neutral kaons, are expected to have rather small branching ratios, i.e., of order 10^{-11}. One will also have to separate the effects of direct and indirect CP violation in these two processes. In addition, the exact contribution of the CP-conserving part in $K_L\to\pi^0e^+e^-$ is still not settled. It can be argued that from a theoretical point of view the best processes are $K^+\to\pi^+\mu^+\mu^-$ and $K_L\to\pi^0\nu\bar{\nu}$. This is because the long-distance part of the contribution is under control. Therefore these two processes can be considered to be more attractive candidates from the point of view of our criteria. The relative usefulness of these two processes will depend on the experimental issues involved. In one case, one has to measure the spins of two muons, while in the other there are neutrinos that one cannot observe and one relies solely on detecting π^0 in the final state which is difficult. The appropriate observations in the two cases are in one sense complementary. In the $K_L\to\pi^0\nu\bar{\nu}$ decay one is looking for CP violation, whereas T violation is being looked for in $K^+\to\pi^+\mu^+\mu^-$. In fact, among the processes mentioned here, it is only in $K^+\to\pi^+\mu^+\mu^-$ where one can see T violation directly, rather than taking it as a consequence of CP violation. As far as testing CKM mechanism is concerned, the branching ratio of $K_L\to\pi^0\nu\bar{\nu}$ depends on η^2 while the T-violation indicator in $K^+\to\pi^+\mu^+\mu^-$ depends on η.

In this paper, we focused on the indicators $I^1_{1\mathrm{TV}}$ and $I^4_{2\mathrm{TV}}$. To measure the first, one will have to measure the spin of the antimuon perpendicular to the decay plane. To measure the $I^4_{2\mathrm{TV}}$, apart from the above measurement, the helicity of the muon will have to be measured. The measurements of $I^2_{1\mathrm{TV}}$ and $I^3_{2\mathrm{TV}}$ are similar. However, it appears that unless one can measure the spin of (anti-)muons in each event, it may be harder to measure the indicators $I^1_{2\mathrm{TV}}$ and $I^2_{2\mathrm{TV}}$. These last two indicators are expected to have somewhat larger value than $I^3_{2\mathrm{TV}}$ and $I^4_{2\mathrm{TV}}$, and were considered in Ref. [9]. As we have emphasized in the relevant sections, from the point of view of probing the CKM mechanism, it is the indicators with two-spin correlations that will have experimental significance. The CKM contribution to indicators with one spin are several orders of magnitude smaller. However, these indicators receive significant contribution from the final-state interaction. So the measurement of these indicators is less interesting.

We find that in the extensions of the standard model, the T-violation indicators with one spin are not enhanced significantly. However, the indicator involving two-spin correlations could reach a value of order 1 in the leptoquark models with the present constraints on the parameters. It is also enhanced by the charged-Higgs-boson effect. Other models such as the model with multiple Higgs bosons, supersymmetry, or left-right-symmetric models predict unmeasurably small values for the two-spin indicators. This is to be contrasted with another CP-violation indicator, the longitudinal polarization of muon in $K_L\to\mu\bar{\mu}$, where with a light neutral Higgs boson one can have a signal at the percent level. However, we note that these estimates are model dependent, and can be further reduced when the parameters involved are further constrained with future experimental data.

As mentioned above, the measurement of the interesting indicators require the measurement of the polarizations of two final-state leptons. This means that it will be hard to measure these indicators. However, the polarization of the μ^- and μ^+ have been measured in the past. The measurement of μ^- polarization with sufficient efficiency will be a real challenge. But if this can be accomplished, the rewards are there. If observed, it will not only be the first place where T violation has been observed, but it will also provide an opportunity to test the CKM mechanism of such violation. A kaon factory, e.g., KAON is the most likely place, where such an experiment can be conducted.

In conclusion, we have pointed out that in the process $K^+\to\pi^+\mu^+\mu^-$ the T violation from the CKM phase is large enough so that an experiment for such observation may be feasible. Such measurements will be very valuable because the theoretical predictions are relatively clean. Appropriate measurements can help us to test the CKM mechanism of T violation, and help determine the unknown parameters of the CKM matrix. If a signal larger than the SM prediction is found then it will be a clear signal for the physics beyond the standard model. Furthermore such an observation may very well be the first to see T violation directly.

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[1] See, e.g., T. D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood Academic, New York, 1988). Original references can be found there.

[2] For reviews on the present status of *CP* Violation, see L. Wolfenstein, Annu. Rev. Nucl. Part. Sci. **36**, 137 (1986); *CP* Violation, edited by C. Jarlskog (World Scientific, Singapore, 1989).

[3] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[4] See, e.g., R. D. Peccei, in *The Vancouver Meeting—Particles and Fields '91,* Proceedings of the Joint Meeting of the Division of Particles and Fields of the American Physical Society and the Particle Physics Division of the Canadian Association of Physicists, Vancouver, 1991, edited by D. Axen, D. Bryman, and M. Comyn (World Scientific, Singapore, 1992), p. 3.

[5] J. R. Patterson *et al.*, Phys. Rev. Lett. **64**, 1491 (1990); A. C. Schaffer, talk presented at The Particles and Fields Meeting, Vancouver, Canada, 1991 (unpublished).

[6] G. Barr, in Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics, Geneva, Switzerland, 1991 (unpublished).

[7] For a review see B. Winstein, in *Vancouver Meeting—Particles and Fields '91,* [4], p. 209.

[8] See, e.g., G. Kilcup, *Lattice 90,* Proceedings of the Conference, Tallahassee, Florida, 1990, edited by U. M. Heller, A. D. Kennedy, and S. Sanielevici [Nucl. Phys. B (Proc. (Proc. Suppl.) **20**, 417 (1990)].

[9] P. Agrawal, J. N. Belanger, and C. Q. Geng, Phys. Rev. Lett. **67**, 537 (1991).

[10] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[11] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981); **65**, 1772(E) (1981).

[12] B. Grinstein, M. J. Savage, and M. B. Wise, Nucl. Phys. **B319**, 665 (1989).

[13] C. O. Dib, I. Dunietz, and F. J. Gilman, Phys. Rev. D **39**, 2639 (1989).

[14] J. M. Flynn and L. Randall, Nucl. Phys. **B326**, 31 (1989).

[15] Here we have neglected the L^2 dependence in F_V; including this presents no problem; see the discussion in the last section.

[16] D. S. Beder and G. V. Dass, Phys. Lett. **59B**, 444 (1975).

[17] V. A. Novikov *et al.*, Phys. Rev. D **16**, 223 (1977).

[18] F. Gilman and M. Wise, Phys. Rev. D **20**, 2392 (1979); J. Ellis and J. S. Hagelin, Nucl. Phys. **B217**, 189 (1983).

[19] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983); L. Maiani, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies,* DESY, Hamburg, West Germany, 1977, edited by F. Gutbrod (DESY, Hamburg, 1977).

[20] G. Bélanger and C. Q. Geng, Phys. Rev. D **43**, 140 (1991); see also C. O. Dib *et al., ibid.* **41**, 1522 (1990); and C. S. Kim *et al., ibid.* **42**, 96 (1990).

[21] L. B. Okun and I. B. Khriplovich, Yad. Fiz. **6**, 821 (1966) [Sov. J. Nucl. Phys. **6**, 598 (1968)].

[22] C. Q. Geng and J. N. Ng, Phys. Rev. D **42**, 1509 (1990).

[23] For a review, see G. C. Branco, A. J. Buras, and J.-M. Gérard, Nucl. Phys. **B259**, 306 (1985), and references therein.

[24] S. L. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977).

[25] This has been pointed out by M. J. Savage and M. B. Wise, Phys. Lett. B **250**, 151 (1990). But we disagree with their formula in Eq. (14).

[26] G. Bélanger, C. Q. Geng, and P. Turcotte, Report No. UdeM-LPN-TH-67 (unpublished).

[27] L. J. Hall and L. J. Randall, Nucl. Phys. **B274**, 157 (1986).

[28] G. Bélanger and C. Q. Geng, Phys. Rev. D **44**, 2789 (1991).

[29] S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976); N. Deshpande and E. Ma, Phys. Rev. D **16**, 1583 (1977).

[30] C. Q. Geng and J. N. Ng, Phys. Lett. B **211**, 111 (1988); C. Q. Geng, X. D. Jiang, and J. N. Ng, Phys. Rev. D **38**, 1628 (1988); C. Q. Geng and J. N. Ng, Phys. Rev. Lett. **62**, 2654 (1989).

[31] K. F. Smith *et al.*, Phys. Lett. B **234**, 191 (1990).

[32] J. G. Gunion and D. Wyler, Phys. Lett. B **248**, 170 (1990); D. Chang, K.-Y. Keung, and T. C. Yuan, *ibid.* **251**, 608 (1990).

[33] J.-M. Gérard, G. Grimus, A. Masiero, D. V. Nanopoulos, and A. Raychaudhuri, Nucl. Phys. **B253**, 93 (1985), and reference therein.

[34] For a recent review, see R. N. Mohapatra, in *CP Violation,* [2].

[35] G. Ecker, A. Pich, and E. de Rafael, Nucl. Phys. **B291**, 692 (1987).