Monte Carlo study of quantum number retention in hadron jets

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We present a Monte Carlo study in which we used weighted quantum numbers of hadron jets in an attempt to identify the parent parton of these jets. Two-jet events produced by e^+e^- annihilation were studied using the Lund Monte Carlo program. It was found that the sign of the charge of the leading parton could be determined in a majority of events and that the quark jet could be distinguished from the antiquark jet in a majority of events containing baryons. A careful selection of a subset of the events by making cuts on the value of these weighted quantum numbers increased significantly the accuracy with which both the charge and the baryon number of the leading parton could be determined. Some success was also made in differentiating light-quark from heavy-quark events and in determining the leading quark flavor in the light-quark events. Unfortunately quantum number retention does not differentiate gluon jets from quark jets. The consequences of this for three-jet events and for jet identification in other reactions is discussed.

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I. INTRODUCTION

Probably one of the most challenging aspects of high-energy hadronic interactions is that QCD predicts production cross sections of quarks and gluons whereas experiments detect, instead, jets of hadrons. It is widely believed and there is a large amount of experimental evidence that each jet of hadrons originates from a quark, a multiquark [1] or a gluon jet. Unfortunately it is extremely difficult to identify the particular parton (i.e., quark, multiquark, or gluon) which is the "parent" of any particular jet. Such a determination would certainly be interesting and would often be useful in comparing theoretical predictions with experiment.

In some reactions it is easy to determine the parent of a jet. In deep-inelastic neutrino reactions such as $\nu_\mu p \rightarrow \mu^- + X$ for large values of the Bjorken scaling variable x_B one is nearly certainly seeing an up-quark jet recoiling against a uu multiquark jet. For small x_B, however, and for reactions such as deep-inelastic electroproduction and e^+e^- annihilation the identification of the parent of the jet is much more difficult and until recently not much progress had been made toward this goal.

The main recent progress which has been made has been in the identification of *heavy*-quark (and especially b-quark) jets [2–4] by identifying B and D resonance decays. In the work of Jones [5] jets are classified by their kinematic properties in a way which allows some separation of gluon, light-, and heavy-quark jets. Lönnblad, Peterson, and Rögnvaldsson [6] have applied neural networks to classify jet properties. Their methods apply to light quarks as well as heavy quarks. In this paper we concentrate on quantum number retention as a method for identifying the parent parton of a jet.

One of the earliest ideas in this area was a suggestion by Feynman [7] that any additive quantum numbers of quark, multiquark, and gluon jets should be retained, on average, during the process of their fragmentation into hadrons. According to his idea (as subsequently modified [8–10]) when a parton of charge Q fragments into hadrons the mean value of the charge of the jet produced by this parton [i.e., $\sum_i q_i$ where i runs over all the hadrons in the jet with some (for now arbitrary) low momentum cutoff] should be very closely related to Q. In fact it was predicted to be equal to $Q - \eta_Q$ for quark jets, $Q + \eta_Q$ for antiquark jets, and simply to $Q = 0$ for gluon jets with η_Q representing the relative production of u, d, and s quarks in the sea. (η_Q is typically of order 0.1.)

Feynman made another suggestion which forms the basis of our present study. If we consider a jet with a large number N of hadrons $h_1, ..., h_N$ which we order in rapidity (with h_1 having the highest rapidity) then we can define the weighted charge $Q_{\text{jet}} = \sum_i w_i q_i$ where q_i is the charge of h_i and w_i is an appropriately chosen (monotonically decreasing) weight function (which we shall discuss in more detail later in this paper). Consider the distribution $F(Q_{\text{jet}})$ of events for which the weighted jet charge is Q_{jet}. If the jet originated from a quark of charge Q, then this distribution was predicted to peak at $Q_{\text{jet}} = Q - \eta_Q$ with a width which *decreases* as the number N of hadrons increases. This procedure can be generalized from the determination of the weighted charge of the jet to the determination of the weighted average of any additive quantum number of the jet. The retention of charge and baryon number are the most useful since they are conserved in the decay of any particle in the jet. Thus for very large N these weighted quantum numbers should tell us the quantum numbers (and thus the identity) of the parent parton of the jet.

Unfortunately there are many problems with the practical implementation of Feynman's idea. The core of these problems is that the distribution of values of Q_{jet} is too wide for any practical determination of the quantum numbers of the parent parton. There are several reasons for this. First of all at the lower energies which were available at the time of Feynman's work the hadron multiplicities in the jets were small and thus the distributions were quite broad. This problem improves very slowly with energy since the multiplicities grow only as the logarithm of the energy of jet. Problems with particle identification (including the detection and the identification of neutrals) and with decays of resonances (especially weak and electromagnetic decays) broaden the distribution considerably. Furthermore the gap between the peak at $\frac{2}{3} - \eta_Q$ for the up quark and $\frac{1}{3} + \eta_Q$ for the antidown quark in e^+e^- annihilation is quite small so that it is very difficult even in an ideal situation to disentangle these two kinds of jets.

In the past decade the experimental situation has changed quite dramatically. The energies and thus the multiplicities are much higher and particle identification has improved considerably. In light of these developments and the development of Monte Carlo programs which simulate the experimental data quite well, we take another look at the problem of quantum number retention in the hope that some of the above problems can be overcome.

Our goal in this paper is to use a Monte Carlo simulation of jet production to see if Feynman's method or some variant of his method may work experimentally. The advantage of a Monte Carlo simulation is that we know from the start the parent parton of any jet. To test any particular algorithm for determining the originating parton of a jet we simply apply this algorithm to the hadron jet which is generated by the Monte Carlo simulation and then compare the guess from our algorithm with the correct answer. We hope that any algorithm which makes useful guesses could be used in the future in real experimental situations.

It is of course unrealistic to expect that any algorithm would determine the parent of *all* jets. One simple way to see this is by noting that if there were no strange quarks and if up and down quarks were produced equally in the "sea" then there would be *no way* of distinguishing a u quark from a $\bar{d}$ quark. Thus the presence of strange quarks in the sea provides the only mechanism for distinguishing these two types of jets. This problem is worsened by the fact that kaons and pions cannot always be experimentally distinguished. We thus cannot expect to do perfectly. A more modest but very interesting question is as follows: Can we identify a subset of events for which we can tell with a good probability the parentage of a jet in these events? It is such questions which we study in this paper.

We have investigated several algorithms based on quantum number retention for determining the parentage of a jet. We have studied, among other things, the effects of different weight functions and of studying different quantum numbers. Several techniques other than that of weighted averages were considered as well. Each algorithm was used on data which were simulated by the Lund Monte Carlo [11, 12] event generator and compared with the known parentage of the quark. Although we have simulated only e^+e^- events we expect similar results for other reactions. However as we shall see there are different tricks available to improve the results in these various reactions. In the case of two-jet events in e^+e^- we can make use of the fact that if one jet comes from a quark q then the other comes from its antiquark $\bar{q}$. Combining the information from the two jets improves our determination of the parent quark's identity. This paper contains a summary of our results. A more complete description of our results can be found in Ref. [13].

The remainder of the paper is organized as follows. In the next section we discuss the theoretical background for our Monte Carlo studies. The ideas of quantum number retention are reviewed and various algorithms are introduced. In the following section we summarize the results of our Monte Carlo studies. We conclude with a summary section in which our conclusions and prospects for using these ideas in experimental analysis are discussed.

II. THEORETICAL BACKGROUND

Before delving into a detailed Monte Carlo analysis of quantum number retention we review some of the issues and discuss some strategies for our study. As a basis for our considerations we use the most naive fragmentation model [9] in which (for example) a very energetic leading $q\bar{q}$ pair is produced and subsequently, with a constant rapidity (η) distribution, many $q_i\bar{q}_i$ pairs are produced in the "sea." The final hadrons are, in this simplest model, composed of $q\bar{q}_1$, $q_1\bar{q}_2$, ..., $q_n\bar{q}$ which are then also uniformly distributed in rapidity. This model is, of course, very naive and it serves only as a guide. We shall be constantly concerned with more realistic models. In fact all our ideas are tested on the Lund Monte Carlo program, which is, essentially, a more sophisticated model which agrees well with experiment.

We begin by considering, for definiteness, e^+e^- annihilation in which various leading $q\bar{q}$ pairs are produced with a calculable relative probability. Let $P(q_i)$ be the probability that a $q_i\bar{q}_i$ pair is produced in the fragmentation chain of the leading pair. (In our naive model we neglect any possible dependence of P on rapidity.) We can consider several interesting cases. In case the "sea" is SU(2) symmetric we have $P(u) = P(d)$ and $P(s) = 0$. For the SU(3)-symmetric case we have $P(u) = P(d) = P(s)$. The more realistic possibility is that $P(u) \sim P(d) > P(s) \gg P(c) \gg P(b)$. Consider now some additive quantum number λ of the system (such as charge, baryon number, strangeness, ...). We construct a weighted average of such an additive quantum number by taking a weighted average of the quantum numbers of the hadrons which occur in the final state of the hadron jet [14]. Let w_i be the weight for the ith hadron h_i. Define Λ to be the weighted average for the jet:

$$\Lambda = \sum_{i=1}^{N} [\lambda(h_i)] w_i . \quad (1)$$

In our naive model the hadron h_i is a meson which contains a quark q_{i-1} and an antiquark $\bar{q}_i$ ($i = 0$ corresponds to the parent quark) so that

$$\Lambda = \sum_{i=1}^{N} [\lambda(q_{i-1}) + \lambda(\bar{q}_i)] w_i. \quad (2)$$

Let us consider, for definiteness, the jet produced by a quark (rather than an antiquark). If we define $w_{N+1} = 0$ and $\lambda_i \equiv \lambda(q_i)$ and note that for additive quantum numbers $\lambda(\bar{q}_i) = -\lambda(q_i)$ we find

$$\Lambda = w_1 \lambda_0 + \sum_{i=1}^{N} \lambda_i (w_{i+1} - w_i). \quad (3)$$

When Λ is averaged over many events, the effect of the sea quarks approaches that of the "average" sea quark, i.e.,

$$\langle \lambda_i \rangle \sim \sum P(q_i) \lambda_i \equiv \eta_\lambda. \quad (4)$$

The mean weighted value $\langle \Lambda \rangle$ of Λ becomes

$$\langle \Lambda \rangle = (\lambda_0 - \eta_\lambda) w_1 \quad (5)$$

where λ_0 is the quantum number of the leading quark. (Note that if the leading parton was an antiquark, η_λ would be replaced by $-\eta_\lambda$. This term would vanish for a gluon jet.) If we normalize our weight function so that $w_1 = 1$ then we see that $\langle \Lambda \rangle = (\lambda_0 - \eta_\lambda)$ and, as a result, the distribution of values of Λ will be peaked near $\Lambda = (\lambda_0 - \eta_\lambda)$.

Using Eq. (3), we may compute the mean square $\langle \Lambda^2 \rangle$ and the variance $\sigma^2(\Lambda)$ for Λ. The result is

$$\sigma^2(\Lambda) \equiv \langle \Lambda^2 \rangle - \langle \Lambda \rangle^2 = \sigma_\lambda^2 \left(-w_1^2 + 2 \sum_{i=1}^{N} w_i (w_i - w_{i+1}) \right) \quad (6)$$

where $\sigma_\lambda^2 = \langle \lambda_i^2 \rangle - \langle \lambda_i \rangle^2$. We can find the "best" weight function in this naive model by considering the ratio of the standard deviation to the mean (assuming that the mean does not vanish) and requiring this ratio to be a minimum with respect to variations of all N weights. If we further demand that the w's be normalized so that $w_1 = 1$ the solution to these equations is found to be

$$w_i = \frac{N - i + 1}{N}. \quad (7)$$

This is then the optimum weight function *for our naive model.* For this "best" weight function the mean and ratio of the mean to the variance for the jet weighted average is

$$\langle \Lambda \rangle = \lambda_0 - \eta_\lambda, \quad (8)$$

$$\Gamma = \frac{\sigma(\Lambda)}{\langle \Lambda \rangle} = \frac{\sigma_v}{\lambda_0 - \eta_\lambda} \frac{1}{\sqrt{N}}. \quad (9)$$

This is Feynman's original result. With the weight function (7) and for a large number of hadrons N, the distribution of weighted charges is sharply peaked at $q_0 - \eta_Q$ where q_0 is the charge of the parent quark since the width of the distribution falls as $1/\sqrt{N}$. Thus for large N it should, in principle, be possible to determine the parent parton of a hadron jet. In fact a *minimal* requirement for this idea to work is both high energy and high multiplicity since for relatively low energy but high multiplicity events the particle containing the leading quark is not as well separated from the others in momentum space.

In an SU(3)-symmetric model in which $d\bar{d}$, $u\bar{u}$, and $s\bar{s}$ pairs are produced in the "sea" with equal probabilities the correction factor η_Q for charge retention vanishes. The distribution of weighted electric charges over many events in which the leading quark had the same charge would be peaked at the charge of those leading quarks. In an SU(2)-symmetric model in which only $d\bar{d}$ and $u\bar{u}$ pairs are produced and in equal quantities Eq. (4) gives $\eta_Q = \frac{1}{6}$. In this case the peaks of the weighted charge distribution would be at $\frac{2}{3} - \frac{1}{6} = \frac{1}{2}$ for up quarks and $-\frac{1}{3} - \frac{1}{6} = -\frac{1}{2}$ for down quarks. Hence the u-quark peak and the $\bar{d}$ peaks would be at the same point, and u-quark jets would be indistinguishable from $\bar{d}$-antiquark jets by the charge retention test. In fact in the absence of strange quarks these two jets are entirely indistinguishable. To see this note [15] that in the absence of strange and heavier quarks, all mesons are left invariant under the combined operations of isospin rotation and charge conjugation. Under this transformation, the light quarks transform as $(d, \bar{d}, u, \bar{u}) \longrightarrow (\bar{u}, u, \bar{d}, d)$, and all pion states are unchanged. Therefore d and $\bar{d}$ jets are indistinguishable from $\bar{u}$ and u jets, respectively. We are saved, of course, by the fact that strange quarks *are* produced which can help (at least in principle) distinguish these two types of jets.

The above model is clearly a huge oversimplification of what really occurs in the production of a quark jet. The weight function which is the optimum for this simple model will almost certainly *not* be the optimum weight function for the real system. It is thus necessary to use the Monte Carlo program itself to determine the best weight function. One option is to seek a weight function which is dependent on the kinematical variables of the particles in the jet rather than simply their ordering in the jet. We expect that the larger the momentum that a particle carries the greater the probability that it contains (or decayed from the particle which contained) the leading quark. Thus, as before, we construct a weighted average of the additive quantum number λ for a jet but we now allow the weight assigned to each particle to depend on the fraction x of the total longitudinal momentum (or possibly the fraction η of the total rapidity) of the jet that it carries. The most successful weight functions which we found were

$$w(x) = x^k \text{ and } w(\eta) = \eta^\kappa, \quad (10)$$

where k and κ are constants. Values of k between 0.29 and 0.33 and κ between 0.90 and 0.94 proved to be the most successful in extracting the parent parton from hadron jet events generated by the Monte Carlo program.

We discussed previously that the most interesting experimental question is the following: Given a jet observed in an experiment, what is the probability with which we can guess certain of its quantum numbers correctly?

This probability depends, of course, on the details of the experiment since the relative rates of production of the various quark flavors depend on the circumstances. We therefore choose *not* to present our results in this way. In order to make our results (which are computed for e^+e^- interactions) applicable to a wide variety of interactions we present our results on a flavor-by-flavor basis. We answer the question: Given that a jet originates from a quark of a given flavor, what is the probability that we will guess its properties correctly? These results can then be combined with the rates of production of various flavors to answer the original question which is of more direct experimental interest.

III. THE MONTE CARLO ANALYSIS

A. Generation of events

Two-jet events in electron-positron annihilation were simulated using the Lund Monte Carlo program JETSET 7.1. Most of the simulations were done at 80-GeV center-of-mass energy just below energies of the CERN e^+e^- collider LEP and not near the Z^0 resonance. The energy dependence of quantum number retention in the range 20 to 100 GeV was also studied. The Lund Monte Carlo program [11,12] has been very successful at reproducing a wide variety of experimental results and is commonly used for experimental analyses involving jets. It also provides several optional models which can be used for generating those events. We chose not to use the default parton fragmentation model. Instead, event generation was done using the method of Petersen [16] which is based on second-order QCD matrix elements. This option was chosen in order to facilitate identification of the quark and gluon parentage of the jets examined. Jet reconstruction of the generated events was done with an iterative cluster algorithm in the subroutine LUCLUS, with options to make it conform to the analysis routine introduced by Bethke [17] at the JADE Collaboration. The parameter $y_{\rm cut}$ which controls the point at which the cluster algorithm terminates was set at $y_{\rm cut} = 0.015$. This value provides a good correspondence between the number of initial partons and the number of jets which are reconstructed.

In events in which two jets were reconstructed but more than two partons were present in the initial state, the jet which subtended the smallest angle with the leading quark was deemed the quark jet, while the other was associated with the antiquark.

We have found it convenient to compute not only the weighted charge Q of the generated events but also weighted down, up, strange, and lepton [18] characteristics of the jets. We define, for example, the "down characteristic" of a quark to be 1 if it is a d quark, -1 if it is a $\bar{d}$ antiquark, and 0 otherwise. The down characteristic of composite particles is defined to be the sum of the down characteristic of its components. The up, strange, and lepton characteristics are defined in an analogous manner, and we consider only charged leptons. The weighted quark and lepton characteristics of the jet are then computed by taking a weighted average of the corresponding characteristics for the particles in the jet. Thus the down weight of an N particle jet in which particle h_i has rapidity η_i would be

$$D_{\rm jet} = \sum_{i=1}^{N} w(\eta_i) d(h_i) \tag{11}$$

with $U_{\rm jet}$, $S_{\rm jet}$, and $L_{\rm jet}$ defined similarly. The weighted electric charge can easily be extracted from the weighted quark and lepton characteristics via

$$Q_{\rm jet} = -\tfrac{1}{3}(D_{\rm jet} + S_{\rm jet}) + \tfrac{2}{3}U_{\rm jet} - L_{\rm jet} . \tag{12}$$

Other weighted quantum numbers such as strangeness and baryon number can be similarly extracted.

One final point is that contamination from heavy lepton (such as τ) decays is insignificant since a cut can be made on the invariant mass of the jets. Following suggestions by Brandelik [19], we reject two-jet events in which the invariant mass of both jets is less than the mass of the τ, and find the number rejected is negligible (the cut done used the τ mass 1784.1 + 3(2.7) MeV [20]). The mass of a single τ is too small to produce two "jets" at 80 GeV, so no cuts were made on three-jet events.

B. Charge retention

In this section we present our Monte Carlo results for two-jet events arising from electron-positron annihilation at 80 GeV. In electron-positron annihilation events in which two jets are formed by a quark and an antiquark, the leading parton in one jet will be positively charged while the other will be negatively charged. We thus expect that the mean charge of the two jets should have the same magnitude but opposite sign, and that the standard deviation of the weighted charges should be the same. Thus, if we subtract the weighted charge of one jet from the other, then we will double the mean while the standard deviation will increase by a factor $\sqrt{2}$ [21]. This should therefore increase the accuracy of determining the charge of a jet. Figure 1 shows the distribution of the difference between the weighted charges of positively and negatively charged leading partons of the five possible quark flavors using the weight function $w(\eta) = \eta^\kappa$ with $\kappa = 0.92$. As mentioned earlier, this value of the parameter κ was found to be the most successful in determining the properties of the leading quark in the jet. It compares with the value $\kappa = 1.0$ used by Stuart [22]. The distributions for different flavors have been shifted vertically for readability.

From the figure, it is evident that it should be possible to determine which of the two jets resulted from the positively charged leading parton in the majority of cases. In addition, by considering only those events for which the magnitude of the weighted charge is greater than some cutoff, we should be able to differentiate the positive jet from the negative jet for a subset of events quite accurately. From the locations of the peaks, we see that events in which the leading quark–antiquark pair have charge $\pm\frac{2}{3}$ should be easier to tag than those which

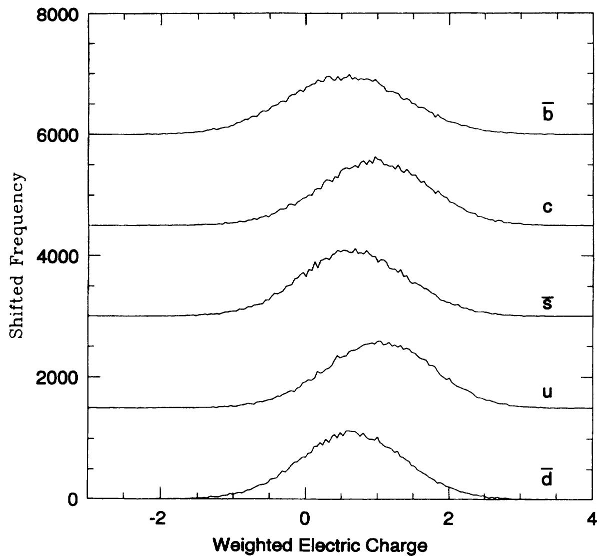

FIG. 1. Weighted charge distributions for positively charged leading partons in two-jet events for each of the five flavors. Distributions are plotted using the weight function $w(\eta) = \eta^{\kappa}$ with $\kappa = 0.92$.

have charge $\pm\frac{1}{3}$, because the former are peaked farther from the vertical axis. Finally we note that the proximity and width of the peaks are such that it is not possible to differentiate charge $\pm\frac{2}{3}$ ($u\bar{u}$ and $c\bar{c}$) events from charge $\pm\frac{1}{3}$ ($d\bar{d}$, $s\bar{s}$, and $b\bar{b}$) events using the weighted charge.

A more quantitative view of these results can be obtained by looking at Table I. In producing this table we have generated 50 000 events for each of the five flavors of quarks. For each flavor we compare several algorithms for guessing the charge of the leading quark and we see how often we guess correctly. Table I shows the results of three such algorithms. In the first algorithm we compute the weighted charge of both jets separately. If these charges are opposite (i.e., the jets "agree") then we guess that the jet with the positive weighted charge contained the quark or antiquark with positive charge. If the jets do not agree we make no guess. In the second and third algorithms we subtract the weighted charges of one jet from the other jet. In the second case we make our guess based on the sign of this charge difference. In the third algorithm we put a cut on the weighted charge difference and only guess if $|Q_{\text{jet}}| \geq 0.912$ (which is approximately the value of the mean weighted charge for all events). The results for all five flavors are presented in Table I.

We see from the table that, as expected, it is easier to tag the sign of the charge of the leading parton when it has magnitude $\frac{2}{3}$ (i.e., if it is a u or c quark) than when it has magnitude $\frac{1}{3}$ (i.e., if it is a d, s, or b quark). Also, by applying cuts on the magnitude of the weighted charge, we can guess correctly on a sizable fraction of the events. We found that by combining the weighted charge of the two jets and applying cuts on the magnitude, we can obtain a higher percentage correct for the same number guessed than we do by considering the two jets separately and guessing only when the two jets have opposite signs. Notice, however, that experimentally there is an advantage to considering the two jets separately since comparing the number of events on which the jets agree to those in which they do not gives some *experimental* handle on how often one is guessing correctly.

C. Baryon-number retention

Several experimental collaborations have recently been attempting to measure the charge asymmetry in hadron

TABLE I. Determination of the sign of the electric charge of leading partons in two-jet events at 80 GeV with $w(\eta) = \eta^{0.92}$. Q_{jet} is the *difference* of the weighted charge of the two jets.

Method of identification	Quark flavor	% of total events guessed	% guessed correctly
Jets agree	Down	65.6	88.5
Sign of Q_{jet}	Down	100	81.8
$\lvert Q_{\text{jet}}\rvert \geq 0.912$	Down	38	95.7
Jets agree	Up	76.1	95.8
Sign of Q_{jet}	Up	100	91.3
$\lvert Q_{\text{jet}}\rvert \geq 0.912$	Up	56.6	99.0
Jets agree	Strange	66.0	88.4
Sign of Q_{jet}	Strange	100	81.8
$\lvert Q_{\text{jet}}\rvert \geq 0.912$	Strange	38.6	95.8
Jets agree	Charm	73.9	94.4
Sign of Q_{jet}	Charm	100	89.1
$\lvert Q_{\text{jet}}\rvert \geq 0.912$	Charm	52.7	98.3
Jets agree	Bottom	61.8	81.8
Sign of Q_{jet}	Bottom	100	75.0
$\lvert Q_{\text{jet}}\rvert \geq 0.912$	Bottom	37.6	90.1

jets from e^+e^- annihilation [22–26], particularly in heavy-quark events. The most common method is to try to tag the charge of the leading quark and compare the results with theoretical predictions. However, as pointed out by Stuart [22], the charge asymmetry is more sensitive to the *baryon number* of the parton than to its charge. In other words the u, c, d, s, and b quarks have an asymmetry of the same sign but opposite to the sign of their antiquarks. Thus for the purposes of such experiments it is more useful to tag the baryon number of the partons rather than their charge. For this reason, we consider the weighted baryon number $B_{\rm jet}$ of e^+e^- annihilation events, which can be computed from the weighted flavor characteristics by

$$B_{\rm jet} = \tfrac{1}{3}(D_{\rm jet} + U_{\rm jet} + S_{\rm jet}). \quad (13)$$

This quantity is conserved by weak interactions, and the only nonzero contribution to it will be from baryons and antibaryons in the final state. At 80 GeV, it was found that about 40% of all jets and 60% of all two-jet events which were generated in the Monte Carlo program had a non-negligible weighted baryon number. In events for which $B_{\rm jet} \neq 0$ the *sign* of $B_{\rm jet}$ can be used to differentiate a quark from an antiquark jet. In the case of e^+e^- annihilation the ability to differentiate these jets can be improved either by demanding that the two jets agree on the sign of $B_{\rm jet}$ or by combining the two jets (i.e., subtracting their value of $B_{\rm jet}$). We can further improve the differentiation by considering only events for which $|B_{\rm jet}|$ is larger than some minimum value. Clearly each such cut will reduce the number of events for which we attempt to guess the baryon number of the jet but it improves significantly the percentage of events which we guess correctly. The results of this analysis are summarized in Table II.

The large percentage (of order 85%) of correctly guessing which jet is the quark jet indicates that, according to the Lund Monte Carlo program, baryons tend to be produced closer in rapidity to the quark than to the antiquark. This "prediction" could possibly be tested in deep-inelastic neutrino reactions at large x_B or in e^+e^- events in which two baryon-antibaryon pairs well separated in rapidity are produced, and comparing the number of events with rapidity ordering $B_1\bar{B}_1\ B_2\bar{B}_2$ to those with $B_1\bar{B}_1\ \bar{B}_2B_2$.

We have seen that charge retention can be used to distinguish $u, c, \bar{d}, \bar{s}$, and $\bar{b}$ from the other quarks and antiquarks and that baryon number can be used to distinguish u, c, c, s, and b quarks from their antiquarks. Thus if we combine both charge and baryon-number retention we should be able to differentiate $d\bar{d}$, $s\bar{s}$, and $b\bar{b}$ events from $u\bar{u}$ and $c\bar{c}$ events. This would allow separate measurement of the two asymmetry curves in the paper by Stuart [22].

Table III shows the frequency with which *both* the sign of the charge *and* sign of the baryon number are correctly guessed. The weighted quantum numbers for all events are computed using the difference between the weighted quantum numbers for the two jets. In addition, cuts are made on the magnitudes of $Q_{\rm jet}$ and $B_{\rm jet}$. Results for the $u\bar{u}$ and $c\bar{c}$ events are better than those of $d\bar{d}$, $s\bar{s}$, and $b\bar{b}$ because the sign of their charge is easier to tag.

D. Flavor retention

The next, more difficult question, is whether or not we can determine the flavor of the leading quark and antiquark in electron-positron annihilation events using a weighted average of the quark flavor of the final-state hadrons. The main problem in doing this is that the flavor is not conserved in the weak interactions. This is especially a problem for charm and bottom quarks which will decay long before any hadrons containing them arrive at the detector. Thus we cannot expect to tag charm and bottom quark jets by considering flavor retention alone. The situation can be improved by also considering the weighted lepton number of a jet. This will help identify high-momentum leptons which occur in semileptonic decays of heavy quarks. In fact, instead of considering the weighted lepton number it is more useful to consider separately the weighted lepton number $L^+_{\rm jet}$ and $L^-_{\rm jet}$ for positively and negatively charged leptons.

TABLE II. Differentiation of the quark and antiquark jet in 50 000 two-jet events per flavor at 80 GeV with $w(\eta) = \eta^{0.92}$. The sign of $B_{\rm jet}$ is used and its value for the two jets is combined.

Method of identification	Quark flavor	% of total events guessed	% of events guessed correctly
$B_{\rm jet} > 0$	Down	58.6	84.9
$\|B_{\rm jet}\| \geq 0.234$	Down	30.5	97.4
$B_{\rm jet} > 0$	Up	58.6	85.0
$\|B_{\rm jet}\| \geq 0.234$	Up	30.2	97.3
$B_{\rm jet} > 0$	Strange	57.2	85.5
$\|B_{\rm jet}\| \geq 0.234$	Strange	30.4	97.5
$B_{\rm jet} > 0$	Charm	48.2	86.3
$\|B_{\rm jet}\| \geq 0.234$	Charm	27.1	97.7
$B_{\rm jet} > 0$	Bottom	30.7	87.5
$\|B_{\rm jet}\| \geq 0.234$	Bottom	20.5	97.5

TABLE III. Simultaneous determination of the sign of the charge and baryon number in 50 000 two-jet events per flavor at 80 GeV. Cut 1 refers to $|Q_{\text{jet}}|, |B_{\text{jet}}| > 0$ and cut 2 refers to $|Q_{\text{jet}}| > 0.912$ and $|B_{\text{jet}}| > 0.234$.

Method of identification	Quark flavor	% of total events guessed	% of events guessed correctly
Cut 1	Down	58.6	68.3
Cut 2	Down	10.2	90.1
Cut 1	Up	58.6	79.2
Cut 2	Up	19.2	97.5
Cut 1	Strange	57.2	68.7
Cut 2	Strange	10.3	89.9
Cut 1	Charm	48.3	79.2
Cut 2	Charm	15.8	97.5
Cut 1	Bottom	30.7	61.5
Cut 2	Bottom	6.5	79.1

(For example L^+_{jet} is the weighted average of the lepton number of all the positively charged leptons.) Now instead of subtracting L^-_{jet} from L^+_{jet} we *add* them and consider $L^+_{\text{jet}} + L^-_{\text{jet}}$. We do this in order to prevent cancellation of the weighted lepton characteristic in events containing decay chains where opposite-sign charged leptons are produced (such as $b \to c\,l_1^-\,\bar{\nu}_{l_1} \to s\,l_2^+\,\nu_{l_2}\,l_1^-\,\bar{\nu}_{l_1}$ or $c \to s\,l_1^+\,\nu_{l_1} \to u\,l_2^-\,\bar{\nu}_{l_2}\,l_1^+\,\nu_{l_1}$).

As mentioned previously we do not attempt to distinguish b from c jets. Instead, for a given jet we determine which of $|D_{\text{jet}}|$, $|U_{\text{jet}}|$, $1.3|S_{\text{jet}}|$, or $4(L^+_{\text{jet}} + L^-_{\text{jet}})$ is largest and guess that the event is either $d\bar{d}$, $u\bar{u}$, $s\bar{s}$, or a heavy-quark (either $c\bar{c}$ or $b\bar{b}$) event accordingly. If the jet turns out to originate from a light quark we guess at its flavor and then we use the *sign* of the corresponding weighted flavor to distinguish the quark jet from the antiquark jet. (The factor of 1.3 by which we multiply S_{jet} was introduced to compensate the effect of hadronic decays which reduce the strange-quark characteristic of the outgoing particles. It was chosen so that the mean weighted strange-quark characteristic of $s\bar{s}$ events is approximately equal to the mean weighted down- and up-quark characteristic of $d\bar{d}$ and $u\bar{u}$ events, respectively. The factor of 4 by which we multiply the weighted lepton characteristics was chosen to balance the fraction of heavy-quark events correctly identified with the fraction of light-quark events misidentified as heavy-quark events. Raising this factor will not increase the number of heavy quarks tagged substantially because there is only a limited probability that heavy quarks will decay semileptonically; many decay modes are strictly hadronic.) In practice we do not simply pick the largest of the weighted flavors. We may put some cut on their values, we may combine the values of the two jets, or we may demand that the two jets agree in their determination of the flavor. In such cases we only attempt a guess at the flavor for a fraction of the events but, for those events guessed, the chances of being correct are much higher.

In Table IV we present the results for a Monte Carlo simulation in which 50 000 events were generated for each flavor. For this table we have made several cuts. Firstly we consider only events in which no leptons are identified and in which the total strangeness in the event vanished. The table shows results for two representative cases. In

TABLE IV. Determination of leading quark flavor in 50 000 two-jet events at 80 GeV with $w(\eta) = \eta^{0.92}$. The various cuts used are discussed in the text. For each generated flavor we show the percentage of events which we guess either correctly or as any other given flavor.

Fl	% guessed	% d	% $\bar{d}$	% u	% $\bar{u}$	% s	% $\bar{s}$
d cut 2	42.4	70.4	2.3	8.7	12.6	2.0	3.9
d cut 1	6.5	87.1	0.4	5.0	2.8	2.3	2.5
u cut 2	42.3	8.3	13.9	70.2	1.8	0.9	4.9
u cut 1	6.5	5.2	2.8	86.8	0.3	0.5	4.3
s cut 2	24.8	35.4	3.7	5.5	11.1	43.1	1.2
s cut 1	9.2	27.3	0.2	1.8	0.9	69.5	0.5
c cut 2	20.4	4.7	36.4	38.0	1.0	18.2	1.7
c cut 1	2.2	3.8	2.7	59.4	0.2	32.3	1.7
b cut 2	10.4	29.0	6.8	7.3	15.2	39.2	2.4
b cut 1	1.9	25.6	0.0	4.2	0.3	68.6	1.3

the first case we demand that both jets agree on the determination of the flavor and that the largest weighted flavor $|f_{\rm jet}|$ be larger than 0.3 times the sum of the absolute values of the nonselected weighted quark flavors. For example, if $|D_{\rm jet}|$ were the largest, we would only accept the event if $|D_{\rm jet}| > 0.3(|U_{\rm jet}| + |S_{\rm jet}|)$. We call this "cut 1." In the second case we combine the weighted flavors of both jets and demand that the largest $|f_{\rm jet}|$ be larger than 0.6 times the sum of the absolute values of the other flavors. We call this "cut 2." We have tried many other values for this cut. These results are described in Ref. [13]. Notice from the table that if one is willing to make severe cuts on the data which catch only a few percent of the events, then one can guess the flavor with reasonable accuracy. The accuracy reduces significantly if we loosen the cuts to catch about 40% of the events. Note that ideally we would like the percentage guessed in c- and b-quark events to be as low as possible since these are background to the light-quark events.

E. Gluon jets

In this section we discuss the identification of gluon jets using quantum number retention. We have chosen to do this by generating three-jet events in e^+e^- annihilation since these events consist of a quark, an antiquark, and a gluon. Using the Monte Carlo program, the distribution of the weighted quantum numbers of each of the three types of jets was computed. Figure 2 shows the distribution of weighted charges for the quark, antiquark, and gluon jets originating from each of the five flavors at a center-of-mass energy of 80 GeV. The distributions for the quark and antiquark jets are, of course, similar to those in the two-jet case, except that they are slightly wider. The gluon jet's charge distribution is peaked at zero as expected, but it is too wide to effectively distinguish it from the quark and the antiquark jets. Evidently quantum number retention alone is not sufficient to identify a gluon jet and thus some other method is needed to determine which of the three jets originated from a gluon. Once this is done, quantum number retention can be used to identify the parent partons of the remaining jets.

There are several techniques available for identifying the gluon jet. Firstly, in e^+e^- annihilation with three-jet final states, the jet opposite the two jets which subtend the largest angle between them is most often the gluon jet. Marshall [27] points out that the transverse momentum and multiplicity of gluon jets tends to be higher than

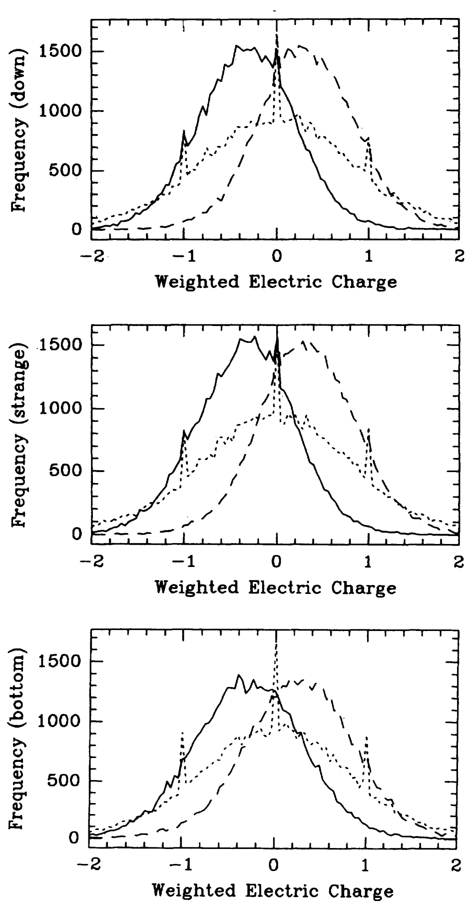

FIG. 2. Weighted charge distributions in three-jet events originating from a quark and an antiquark with each of the five flavors and a gluon. The quark, antiquark, and gluon jet distributions are represented by solid, dashed, and dotted lines, respectively. The total center-of-mass energy is 80 GeV.

light-quark jets of the same energy. Mättig [28] notes that charm quark jets are similar to light-quark jets in this respect, and while bottom quark jets have higher multiplicity and transverse momentum than those of the lighter-quark jets, theirs are still lower than gluon jets. Mjaed and Proriol [29] and Fodor [30] have studied differentiation of quark and gluon jets by defining moments [31]:

$$M_{nm} = \sum_i \left(\frac{p_{i_T}}{p_{\rm jet}}\right)^n \eta_i^m , \tag{14}$$

where the summation runs over all particles in the jet and p_{i_T} is the transverse momentum of particle i. The value of some of these moments tends to be larger in gluon jets of a given energy than quark jets of the same energy, while some tend to be smaller. Combining these two ideas we found that we could distinguish the gluon jet with 95% accuracy in more than 52% of d, u, s, and c events, and with 91% accuracy in 37% of b events. (These results do not include the effects of experimental resolution or acceptance.)

Once the identity of the gluon jet is determined, it is possible to apply the same quantum number retention techniques which were discussed for two-jet events to the two remaining jets. The accuracy with which the quantum numbers of the two remaining jets can be determined is similar but slightly lower than the two-jet results at the same c.m. system (c.m.s.) energy. This is partly due to the fact that the energy of the quark and antiquark jets is lower in three-jet events than in two-jet events at the same center-of-mass energy. We show in the next section that identification of the parent quark is more difficult at lower energies. Furthermore the algorithm for identifying the gluon jet is far from perfect and this will also affect the accuracy of the identification of the quark jet.

It is evident from our results that the quantum number properties of quark jets are "universal"; i.e., they are independent of the details of how the jet was produced. This is hardly surprising since our results are based on the Lund Monte Carlo program in which this property occurs naturally. The main lesson from our analysis of three-jet events is that it is essential to separate the gluon jet using alternate methods (i.e., methods unrelated to quantum number retention). This may be possible in experiments such as electroproduction but it will be much more difficult for jets produced in $\bar{p}$-p and p-p reactions. In such cases quantum number retention will be of much more limited use.

F. Effect of center-of-mass energy

The naive model of quantum numbers retention predicts that the ratio of the standard deviation to the mean of the weighted charge should decrease as the number of particles increases. In particular $\sigma(Q_{\rm jet})/\langle Q_{\rm jet}\rangle \sim 1/\sqrt{N}$. As the c.m.s. energy of the events increases, the multiplicity increases as $\ln(E_{\rm c.m.s.})$, and this ratio should decrease allowing better identification of properties of the leading partons. For any *given* c.m. energy we might also expect that the higher-multiplicity events give better results. There are, however, several factors to consider. First note that hadronic decays play a major role in breaking the ordered structure of the naive model. Now at any given energy both the total available rapidity and the mean multiplicity grow like $\ln(N)$. As a result the rapidity distribution is constant on the average and at most very weakly dependent on energy. Thus even at very high energy the high-multiplicity events find the first generation of hadrons (i.e., those produced before any decays occur) spaced very closely in multiplicity. The effect of hadronic decays on the ordering of these events in rapidity will thus be large. For events whose multiplicity is near the mean we expect no such effect and thus as the energy increases the ability of quantum number retention to distinguish the parent partons increases (albeit quite slowly).

In Table V we show examples of the energy dependence

TABLE V. Determination of the sign of the electric charge and the sign of the baryon number of the leading partons in two-jet events as a function of c.m.s. energy using $w(\eta) = \eta^{0.92}$.

$E_{\rm c.m.s.}$ (GeV)	Flavor	Charge % guessed	Charge % correct	Baryon number % guessed	Baryon number % correct
20	Down	69.2	82.0	37.7	88.9
20	Up	78.7	91.3	37.3	88.7
20	Strange	69.8	82.9	36.3	88.7
20	Charm	75.7	88.2	27.0	92.2
60	Down	70.8	88.2	43.6	91.1
60	Up	82.2	95.8	44.2	91.1
60	Strange	70.9	88.9	43.3	91.3
60	Charm	79.8	93.9	36.6	92.7
60	Bottom	70.5	81.1	22.2	93.1
100	Down	68.0	91.5	45.0	93.2
100	Up	81.8	97.3	45.6	93.0
100	Strange	69.2	91.5	44.5	93.2
100	Charm	79.5	96.3	39.2	93.8
100	Bottom	67.5	84.4	27.1	93.6

of the quantum number retention analysis. More details can be found in Ref. [13]. In these tables we present results for both charge retention and baryon-number retention which are used to guess the charge and the baryon number of the quarks, respectively. In these tables we do not demand that the two jets agree on the quantum number but instead we combine the results of both by subtracting their weighted quantum number. In both cases we have put a cut on the data by demanding that the weighted charge or baryon number be greater than half its mean value. The table shows the percentage of the total events guessed and the percentage of those guessed which are guessed correctly for center-of-mass energies of 20, 60, and 100 GeV.

We see from the table that as the energy increases, the rate at which we correctly identify the sign of the quarks leading the jet increases. We have also studied the energy dependence of flavor retention. Both the tagging of heavy quarks using weighted lepton number and flavor and the identification of light quarks increase with energy, as expected. Details can be found in Ref. [13].

IV. DISCUSSION

We have presented a detailed Monte Carlo study of quark quantum number retention in two- and three-jet events produced by e^+e^- annihilation. In particular we have investigated the possibility of identifying the parent parton of a jet by computing various weighted quantum numbers of the jet. Our analysis provides only a guide since experimental factors such as acceptance, resolution, and particle identification issues were not included in the Monte Carlo program. They will certainly make this identification much more difficult and a Monte Carlo program which includes these experimental effects is necessary for any specific experiments. We have, however, included the effects of particle decays.

It is clear that one is not able to determine the parent parton of a jet in *all* events. Our goal was to identify a subset of events (based on the weighted quantum numbers of the jets in the event) for which this identification is possible with a reasonably good accuracy. It is also clear that if we make more severe cuts on the values of these quantum numbers the accuracy of identification improves. We found that the weighted charge and the weighted baryon number could be used to distinguish positively charged from negatively charged parent partons and to distinguish quark from antiquark parents with very good accuracy in a large majority of cases. Details are shown in Tables II and III.

The determination of the exact identity of the quark is more difficult. We studied how the combination of weighted flavor number and weighted lepton number could be used to distinguish heavy from light quarks and, to identify the quark in case it is light. Because heavy quarks decay weakly before reaching the detector, they provide the major source of background in determination of the flavors of the leading quarks in light-quark events. We have attempted to circumvent this problem by considering not only the weighted flavor number of any jet but also the weighted value of positively and negatively charged leptons. Using techniques described in this paper which were based on these weighted quantum numbers we are able to identify the parent quark of a jet with good accuracy but only if we make severe cuts on the data. The details are described in Ref. [13] and representative results are shown in Table IV.

The effect of heavy quarks can, of course, be reduced by other means. Since their discovery extensive work has been done to characterize the properties of heavy-quark jets. The decays of the D mesons have been studied [32] and the decays can be reconstructed in some events. Both the semileptonic [33] and the nonleptonic [34] decays of the B meson have been studied, and detection of B mesons by reconstruction of the D mesons to which they decay is also possible [2]. Thus if a larger fraction of the heavy-quark events could be tagged by reconstruction of some of the decays, then the background to the light-quark events would be reduced. Thus although we have found that using weighted quantum numbers alone provides some limited success in the identification of heavy-quark jets, more sophisticated techniques would certainly improve the analysis.

We have studied the possibility of identifying gluon jets by studying three-jet events in e^+e^- annihilation. We find it impossible to distinguish a gluon jet from a quark jet (on an event-by-event basis) using quantum number retention alone. In experiments such as e^+e^- annihilation there are many techniques available to single out this jet so that it does not interfere with the identification of the parent quark of the other jets. However in other experiments such as pp collisions quantum number retention will be of very limited utility if gluon jets cannot be separated using other techniques.

We have studied the use of baryon-number retention to determine the sign of the baryon number of the quark which initiated the jet. This study has direct applications to analyses of the forward-backward asymmetries in $e^+e^- \rightarrow$ hadrons, as well as the measurement of the axial-vector and vector couplings of the quarks. Using baryon-number retention instead of charge retention permits measurements of the sum of the two asymmetries instead of their difference [22], thus avoiding cancellation. In addition, by combining charge and baryon-number retention together or using quark flavor retention, it should be possible to measure the asymmetry of the d-type and u-type quark flavors separately.

Although we have studied quantum number retention in jets produced by electron-positron annihilation our analysis can also be applied to other high-energy interactions which produce hadronic jets. In e^+e^- annihilation we increased the quality of our results by considering simultaneously the quantum numbers of both jets. This takes advantage of the fact that the leading parton in one jet is the antiparticle of the leading parton in another jet. On the other hand heavy-quark decays made identification of $c\bar{c}$ and $b\bar{b}$ jets difficult, and provided large backgrounds in determination of light-quark events. In other interactions one can make use of other advantages. In leptoproduction for example, if the Bjorken scaling variable x_B is large, we can be fairly certain that the lepton has interacted with a valence quark instead of a sea quark. This reduces the possible identities of the leading parton. In the interaction $\nu_\mu p \to \mu^- + X$ for large x_B we can be fairly certain that the leading parton in the jet is a u quark with a uu fragment in the target-fragmentation region, and in the interaction $\bar{\nu}_\mu p \to \mu^+ + X$ it is likely that a d quark creates the jet leaving a du fragment. In leptoproduction with a *charged* lepton (such as $e^- p \to e^- X$), if we hit a valence quark it will be either a u or d quark. We saw in e^+e^- annihilations that it was easier to differentiate d jets from u jets than from $\bar{u}$ jets. Thus if we know that our jet is either a u or a d its identification is simplified. In fact these events should often be distinguishable using charge retention alone.

Other experiments may have other tricks for improving the success of quantum number retention. The most difficult case is that of pp and $\bar{p}p$ collisions. In this case we have little *a priori* knowledge of the possible types of partons. If we could be sure that we do not have a gluon jet we could use the quantum number analysis on a single jet with cuts. Thus for these experiments, some algorithm for excluding gluon jets becomes a crucial component in applying an analysis based on quantum number retention.

ACKNOWLEDGMENTS

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