Two-photon width of singlet positronium and quarkonium with arbitrary total angular momentum

E. S. Ackleh
Department of Physics, University of Tennessee, Knoxville, Tennessee 37996-1501

T. Barnes
Physics Division and Center for Computationally Intensive Physics, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6373 and Department of Physics, University of Tennessee, Knoxville, Tennessee 37996-1501

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In this paper we derive $\Gamma_{\gamma\gamma}$ for $S=0$ positronium and quarkonium states with all allowed quantum numbers ($J^{PC}=0^{-+},2^{-+},4^{-+},\ldots$), in both nonrelativistic and relativistic regimes. $\gamma\gamma$ partial widths have previously been published only for fermion-antifermion states with $J^{PC}=0^{\pm+},1^{++}$ (off-shell photons), and $2^{\pm+}$. The topic of higher-spin $\gamma\gamma$ partial widths is of current interest in part because a recent nonrelativistic $q\bar{q}$ calculation for 2^{-+} finds a much smaller $\gamma\gamma$ partial width than the ≈ 1 keV observed for the $I=1$, $J^{PC}=2^{-+}$ state $\pi_2(1670)$. We find very large relativistic corrections to the nonrelativistic (contact) approximation for this width as well as large systematic uncertainties in the procedure used to incorporate the physical resonance mass. Our relativistic $q\bar{q}$ estimate for $\Gamma_{\gamma\gamma}(\pi_2(1670))$ is 0.1–0.3 keV, which is somewhat smaller than the experimental value. This discrepancy may only reflect inaccuracies in the theoretical technique. Finally, we quote relativistic numerical results for the $\gamma\gamma$ widths of various $0^{-+},2^{-+}$, and 4^{-+} $(u\bar{u}-d\bar{d})/\sqrt{2}$, $c\bar{c}$ and $b\bar{b}$ states in a Coulomb-plus-linear potential model.

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I. INTRODUCTION

It is now well established that two-photon couplings provide a useful probe of the internal structure of mesons. In particular, the "classic" light 0^{-+} and 2^{++} states π^0, η, η', f_2 (1270), $a_2(1320)$, and $f'_2(1525)$ and the $c\bar{c}$ states $\eta_c(2980)$ and $\chi_2(3555)$ have been observed at e^+e^- machines with $\gamma\gamma$ partial widths consistent with quark-model predictions [1–10]. Although spin-1 states cannot be produced from two on-shell photons, the $1^{++}q\bar{q}$ state $f_1(1285)$ has been observed with a $\gamma\gamma$ coupling consistent with quark model estimates for $Q^2\neq 0$ photons [6,10-13]. Conversely, mesons thought to be non-$q\bar{q}$ states generally have $\gamma\gamma$ widths far from expectations for a $q\bar{q}$ state. These anomalous states include the $f_0(975)$ and $a_0(980)$ $K\bar{K}$-molecule candidates [14,15], the $\eta(1440)$ and the $f_2(1720)$. Mesons that have $\gamma\gamma$ widths far from $q\bar{q}$ quark model predictions should evidently be considered candidate non-$q\bar{q}$ states. For this reason it is clearly important to have accurate quark model predictions of $\gamma\gamma$ widths for all experimentally accessible $q\bar{q}$ mesons. This paper attempts to derive such $\gamma\gamma$ widths for the spin-singlet case.

We should caution the reader that the absolute scale of $\gamma\gamma$ widths of a light $q\bar{q}$ multiplet is a rather sensitive quantity. One can easily be misled into believing that $\gamma\gamma$-width calculations are relatively "stable" by models that reproduce well-established 0^{-+} and 2^{++} widths with accuracies of typically $\sim 30\%$. This however represents the quality of fit possible given good experimental data, and the overall scale is actually quite sensitive to relativistic effects, the quark mass assumed, and the prescription used to incorporate the physical resonance mass. Without experimental data to constrain the overall width scale, order-of-magnitude uncertainties are typically encountered in $\gamma\gamma$ width predictions for $l>0$ light $q\bar{q}$ states. (See, for example, the relativistic $q\bar{q}$ calculation of Bergström, Hulth, and Snellman [16], which predates accurate data on 2^{++} decays.) For recently observed states such as the light 2^{-+} $q\bar{q}$ system, the overall theoretical $\gamma\gamma$ width should only be considered an order-of-magnitude estimate, whereas the relative rates such as $I=0/I=1$ are of course much more reliably predicted. This uncertainty of overall scale generally increases with increasing l and decreases with increasing quark mass; the accuracy of $\gamma\gamma$ partial width predictions will presumably improve as data on more well-established $q\bar{q}$ states becomes available.

The immediate motivation for this study is the recent measurement of the two-photon width of the $I=1$, $J^{PC}=2^{-+}$ state $\pi_2(1670)$ by the CELLO [17] and Crystal Ball [18] Collaborations; their results are

$$\Gamma(\pi_2(1670)\to\gamma\gamma)=0.8\pm0.3\pm0.12\ \text{keV} \quad (1)$$

and

$$\Gamma(\pi_2(1670)\to\gamma\gamma)=1.45\pm0.23\pm0.28\ \text{keV}\ , \quad (2)$$

respectively, which combined in quadrature give

$$\Gamma(\pi_2(1670)\to\gamma\gamma)=1.13\pm0.24\ \text{keV}\ . \quad (3)$$

A partial width of ≈ 1 keV for an $I=1$ pseudotensor state in this mass region appears plausible *a priori* for a $q\bar{q}$ state, so both experimental groups cited their measurements as evidence that the $\pi_2(1670)$ is a conventional $^1D_2$ $(u\bar{u}-d\bar{d})/\sqrt{2}$ quarkonium state. [Compare the Particle Data Group [10] average value for the well-established

$I=1$ tensor $a_2(1320)$, $\Gamma_{\gamma\gamma}=0.902\pm0.151$ keV.] Subsequent to these experimental measurements, Anderson, Austern, and Cahn [19] published a result for the $\gamma\gamma$ width of a nonrelativistic 2^{-+} quarkonium resonance of mass M_R, which is

$$\Gamma_{\rm NR}(q\bar{q}(2^{-+})\to\gamma\gamma)=192\alpha^2\left|\left\langle\frac{e_q^2}{e^2}\right\rangle\right|^2\frac{|\psi''(0)|^2}{M_R^6} . \tag{4}$$

For their parameters this led to the surprisingly small width estimate of

$$\Gamma_{\rm NR}(\pi_2(q\bar{q})\to\gamma\gamma)=1.2-9.4\,{\rm eV} , \tag{5}$$

which is two to three orders of magnitude smaller than the experimental results for the $\pi_2(1670)$. Either the experimental results are inconsistent with a $q\bar{q}$ assignment or the estimate (5) is too small and the theoretical width requires more careful investigation.

We consider it highly unlikely that the $\pi_2(1670)$ is not a $q\bar{q}$ state, so the assumptions which led to (4) and (5) are presumably inaccurate. Although there are expectations of $q\bar{q}g$ hybrid mesons with these quantum numbers [8,20,21] and others [22] in this general mass region, the proximity of the 3^{--} $\rho_3(1690)$ and 1^{--} $\rho(1700)$ strongly supports the identification of the $\pi_2(1670)$ with the $^1D_2$ member of an $l=2$ $q\bar{q}$ multiplet. Its detection in $\gamma\gamma$ is a very useful development, as we must have an accurate understanding of the couplings of higher-mass conventional $q\bar{q}$ states if we are to distinguish them from hybrid mesons or other exotics. One should note in this regard that Close and Li [13] have calculated $2^{-+}/1^{-+}/0^{-+}$ hybrid couplings to $\gamma\gamma$, and expect the 2^{-+} hybrid to couple quite weakly to $\gamma\gamma$.

In this paper we present results from a study of the $\gamma\gamma$ widths of positronium bound states and $q\bar{q}$ mesons. We derive a general relativistic result for the $\gamma\gamma$ width of a singlet ($S=0$) state of arbitrary total angular momentum, and show that it agrees with the relativistic formula of Bergström, Snellman, and Tengstrand [23] and Hayne and Isgur [24], who considered the special case $J^{PC}=0^{-+}$. We then derive the nonrelativistic limit of our general result, and recover the 2^{-+} $\Gamma_{\gamma\gamma}$ contact formula of Anderson, Austern, and Cahn (4) as a special case. The $\gamma\gamma$ widths of all spin-singlet positronium bound states are obtained in closed form in this limit. We also evaluate the $\gamma\gamma$ width of a relativistic 2^{-+} state with a Coulomb-plus-linear wave function numerically, and conclude that (a) the relativistic formula gives a $\gamma\gamma$ width of about $\frac{1}{10}$ to $\frac{1}{3}$ times that observed for the $\pi_2(1670)$, and (b) the nonrelativistic (contact) approximation (4) is inaccurate for these states. This conclusion regarding the contact approximation for light-quark $q\bar{q}$ states has been reported previously [4,16]. Other reasons for the disagreement between our numerical results and those of Anderson, Austern, and Cahn are also discussed. Finally we quote numerical results for $\Gamma_{\gamma\gamma}$ of $I=1$, $c\bar{c}$, and $b\bar{b}$ states with $J^{PC}=0^{-+}$, 2^{-+}, and 4^{-+} in a Coulomb-plus-linear potential model, and discuss the prospects for detecting these states experimentally.

II. DERIVATION OF $\Gamma_{\gamma\gamma}$ FOR ARBITRARY J

The $\gamma\gamma$ partial width of a positronium bound state is given by [25]

$$\frac{d\Gamma}{d\Omega_{\rm k}}=\frac{1}{32\pi^2}\sum_{\lambda_1,\lambda_2}|\mathcal{M}(\lambda_1,\lambda_2)|^2 , \tag{6}$$

where the bound-state amplitude $\mathcal{M}(\lambda_1,\lambda_2)$ is related to the invariant amplitude $\mathcal{M}_{fi}$ and the positronium wave function $\Phi(\mathbf{p})=\phi(p)Y_{lm}(\Omega_{\mathbf{p}})$ [which has an implicit spin-singlet wave function $(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)/\sqrt{2}$] by

$$\mathcal{M}(\lambda_1,\lambda_2)=\frac{1}{\sqrt{(2\pi)^3}}\int d\mathbf{p}\left[\frac{m^2}{E_1E_2}\right]^{1/2}\mathcal{M}_{fi}\Phi(\mathbf{p}) , \tag{7}$$

and the arguments (λ_1,λ_2) refer to the polarizations of the final photons. Our normalization condition for the positronium wave function is $\int_0^\infty dp\,p^2|\phi(p)|^2=1$. We approximate the invariant amplitude $\mathcal{M}_{fi}$ by the Feynman diagrams of Fig. 1:

$$\mathcal{M}_{fi}=\mathcal{M}_1+\mathcal{M}_2 , \tag{8}$$

where

$$\mathcal{M}_1=-ie^2\bar{v}_{\mathbf{p}_2s_2}\not{\epsilon}_2^*\frac{1}{\not{q}_1-m}\not{\epsilon}_1^*u_{\mathbf{p}_1s_1} \tag{9}$$

and

$$\mathcal{M}_2=-ie^2\bar{v}_{\mathbf{p}_2s_2}\not{\epsilon}_1^*\frac{1}{\not{q}_2-m}\not{\epsilon}_2^*u_{\mathbf{p}_1s_1} . \tag{10}$$

The momenta satisfy $q_1=p_1-k_1=k_2-p_2$ and $q_2=p_1-k_2=k_1-p_2$. Specializing to the c.m. frame, we define $\mathbf{p}_1=-\mathbf{p}_2\equiv\mathbf{p}$, $\mathbf{k}_1=-\mathbf{k}_2\equiv\mathbf{k}$, $E_1=E_2\equiv E=\sqrt{\mathbf{p}^2+m^2}$ and $k_1^0=k_2^0=|\mathbf{k}_1|=|\mathbf{k}_2|\equiv\omega$. (Note that using the free-quark $\mathcal{M}_{fi}$ actually introduces an ambiguity in the relativistic decay rate, since the momentum constraints imply $E_p=\omega$, but we fix ω at $M/2$ and E_p varies when we integrate over $\mathbf{p}$. This ambiguity is an artifact of the neglect of binding in the free-quark $\mathcal{M}_{fi}$. Fortunately this ambiguity only affects the relativistic results and appears at a high order in β.)

To evaluate $\mathcal{M}_{fi}$ we introduce explicit Dirac spinors

$$u_{\mathbf{p}s_1}=\eta\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{p}}{E+m}\end{bmatrix}u_{s_1} , \tag{11}$$

$$v_{-\mathbf{p}s_2}=\eta\begin{bmatrix}-\dfrac{\sigma\cdot\mathbf{p}}{E+m}\\ 1\end{bmatrix}v_{s_2} , \tag{12}$$

where the overall constant η is

$$\eta=\left[\frac{E+m}{2m}\right]^{1/2} \tag{13}$$

and the rest-frame Pauli spinors are

$$u_\uparrow=\begin{bmatrix}+1\\0\end{bmatrix},\quad u_\downarrow=\begin{bmatrix}0\\+1\end{bmatrix} , \tag{14}$$

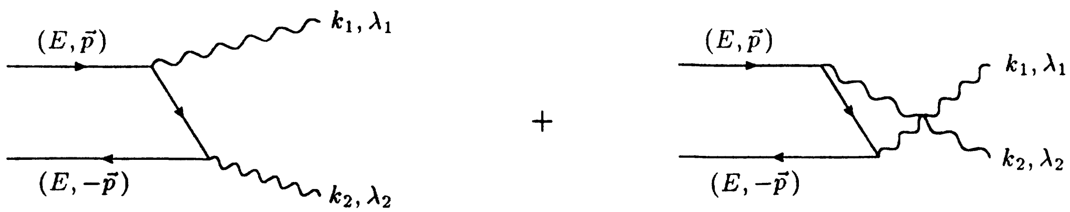

FIG. 1. Diagrams incorporated in our $\Gamma_{\gamma\gamma}$ calculation.

$$v_\uparrow=\begin{bmatrix}0\\+1\end{bmatrix},\quad v_\downarrow=\begin{bmatrix}-1\\0\end{bmatrix}. \tag{15}$$

We may write the invariant amplitude $\mathcal{M}_{fi}$ in terms of Pauli spinors and 2×2 matrices as

$$\mathcal{M}_{fi}=-\eta^2 v_{s_2}^\dagger\left[\frac{\sigma\cdot\mathbf{p}}{E+m}\mathcal{O}_{11}+\frac{\sigma\cdot\mathbf{p}}{E+m}\mathcal{O}_{12}\frac{\sigma\cdot\mathbf{p}}{E+m}+\mathcal{O}_{21}+\mathcal{O}_{22}\frac{\sigma\cdot\mathbf{p}}{E+m}\right]u_{s_1}, \tag{16}$$

where $\mathcal{O}_{11}\cdots\mathcal{O}_{22}$ are 2×2 submatrices of the Dirac matrix

$$\mathcal{O}=-ie^2\left\{\not\epsilon_2^*\left[\frac{1}{\not q_1-m}\right]\not\epsilon_1^*+\not\epsilon_1^*\left[\frac{1}{\not q_2-m}\right]\not\epsilon_2^*\right\}, \tag{17}$$

and are defined by

$$\mathcal{O}=\begin{bmatrix}\mathcal{O}_{11} & \mathcal{O}_{12}\\ \mathcal{O}_{21} & \mathcal{O}_{22}\end{bmatrix}.$$

From (17) (and using the notation $\tilde{a}\equiv\sigma\cdot\mathbf{a}$), one can show that

$$\mathcal{O}_{11}=\frac{ie^2}{2Ew}(E-w-m)\left[\frac{\tilde\epsilon_2^*\tilde\epsilon_1^*}{1-\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}+\frac{\tilde\epsilon_1^*\tilde\epsilon_2^*}{1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}\right], \tag{18}$$

$$\mathcal{O}_{22}=-\frac{ie^2}{2Ew}(E-w+m)\left[\frac{\tilde\epsilon_2^*\tilde\epsilon_1^*}{1-\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}+\frac{\tilde\epsilon_1^*\tilde\epsilon_2^*}{1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}\right], \tag{19}$$

and

$$\mathcal{O}_{12}=-\mathcal{O}_{21}=\frac{ie^2}{2Ew}\left[\frac{\tilde\epsilon_2^*\tilde q_1\tilde\epsilon_1^*}{1-\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}+\frac{\tilde\epsilon_1^*\tilde q_2\tilde\epsilon_2^*}{1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}\right], \tag{20}$$

where $\beta=|\mathbf{p}|/E$. Using (16) and (18)–(20), we find that the invariant amplitude $\mathcal{M}_{fi}$ for electron and positron polarization states s_1 and s_2 is

$$\mathcal{M}_{fi}(s_1,s_2)=\mathcal{A}v_{s_2}^\dagger I u_{s_1}+\mathcal{B}\cdot v_{s_2}^\dagger\sigma u_{s_1}, \tag{21}$$

where the expressions $\mathcal{A}$ and $\mathcal{B}$ are

$$\mathcal{A}=\frac{e^2\hat{\mathbf{k}}\cdot(\epsilon_1^*\times\epsilon_2^*)}{2E}\left[\frac{1}{1-\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}+\frac{1}{1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}\right] \tag{22}$$

and

$$\begin{aligned}\mathcal{B}=&\frac{ie^2}{2m}\frac{1}{1-\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}\left\{\hat{\mathbf{k}}\,\epsilon_1^*\cdot\epsilon_2^*+\beta\left[\epsilon_1^*\,\epsilon_2^*\cdot\hat{\mathbf{p}}+\left[\frac{2E}{w}-1\right]\epsilon_2^*\,\epsilon_1^*\cdot\hat{\mathbf{p}}\right]\right.\\&\left.-\beta^2\frac{E}{E+m}\hat{\mathbf{p}}\,\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}\,\epsilon_1^*\cdot\epsilon_2^*-\beta^3\frac{2E^2}{w(E+m)}\hat{\mathbf{p}}\,\epsilon_1^*\cdot\hat{\mathbf{p}}\,\epsilon_2^*\cdot\hat{\mathbf{p}}\right\}+(\epsilon_1^*\leftrightarrow\epsilon_2^*,\hat{\mathbf{k}}\to-\hat{\mathbf{k}})\\=&\frac{ie^2}{m}\frac{1}{1-\beta^2(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2}\left\{\beta\left[\hat{\mathbf{k}}\epsilon_1^*\cdot\epsilon_2^*\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}+\frac{E}{w}(\epsilon_1^*\,\epsilon_2^*\cdot\hat{\mathbf{p}}+\epsilon_2^*\,\epsilon_1^*\cdot\hat{\mathbf{p}})\right]\right.\\&+\beta^2\left[1-\frac{E}{w}\right]\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}(\epsilon_1^*\,\epsilon_2^*\cdot\hat{\mathbf{p}}-\epsilon_2^*\,\epsilon_1^*\cdot\hat{\mathbf{p}})\\&\left.-\beta^3\frac{E}{E+m}\hat{\mathbf{p}}\left[\frac{2E}{w}\epsilon_1^*\cdot\hat{\mathbf{p}}\epsilon_2^*\cdot\hat{\mathbf{p}}+\epsilon_1^*\cdot\epsilon_2^*(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2\right]\right\}.\end{aligned} \tag{23}$$

(Note that E and ω are formally related in this amplitude, as discussed previously.) For spin-singlet positronium the matrix element of σ vanishes and $v_{s_2}^\dagger I u_{s_1}$ contributes an overall factor of $-\sqrt{2}$, so that (21) becomes

$$\mathcal{M}_{fi}(S=0)=-\frac{e^2\hat{\mathbf{k}}\cdot(\epsilon_1^*\times\epsilon_2^*)}{\sqrt{2}E}\left[\frac{1}{1-\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}+\frac{1}{1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}}\right]. \tag{24}$$

This is a relativistically correct result given (9) and (10); the use of Pauli matrices and spinors is simply a computational convenience.

To derive the $\gamma\gamma$ decay rate it is useful to note that the angular dependence of the final helicity-zero two-photon state with $J^{PC}=\text{even}^{-+}$ and $J_z=M$ is unique, and can be written in terms of helicity eigenstates $|\lambda_1\lambda_2\rangle$ as [26]

$$|\gamma\gamma;JM\rangle = \frac{1}{\sqrt{2}}\int d\Omega\, Y_{JM}(\Omega)(|p,\Omega;++\rangle-|p,\Omega;--\rangle)\ . \tag{25}$$

From (25) one can see that the photon angular probability distribution in this state is proportional to $|Y_{JM}(\Omega_{\mathbf{k}})|^2$, so the distribution of emitted photons in singlet-positronium decay must also be proportional to $|Y_{JM}(\Omega_{\mathbf{k}})|^2$:

$$\frac{d\Gamma^{JM}}{d\Omega_{\mathbf{k}}}=2\Gamma^{J}_{\gamma\gamma}|Y_{JM}(\Omega_{\mathbf{k}})|^2\ . \tag{26}$$

We may therefore obtain $\Gamma^J_{\gamma\gamma}$ directly from the angular distribution (26) with a particular choice for M and $\Omega_{\mathbf{k}}$. Here we choose $M=0$ and $\hat{\mathbf{k}}=\hat{\mathbf{z}}$, which gives

$$\Gamma^J(e^+e^-\to\gamma\gamma)=\frac{1}{2|Y_{J0}(\hat{\mathbf{z}})|^2}\frac{d\Gamma^{J0}}{d\Omega_{\mathbf{k}}}(e^+e^-\to\gamma\gamma)\bigg|_{\hat{\mathbf{k}}=\hat{\mathbf{z}}}\ . \tag{27}$$

On combining (6), (7), and (24), this gives a $\gamma\gamma$ decay rate of

$$\Gamma^J(e^+e^-\to\gamma\gamma)=\frac{1}{64\pi^2|Y_{J0}(\hat{\mathbf{z}})|^2}\times\sum_{\lambda_1\lambda_2}\left|\int\frac{d\mathbf{p}}{\sqrt{(2\pi)^3}}\left[\frac{-me^2\hat{\mathbf{k}}\cdot(\boldsymbol{\epsilon}_1^*\times\boldsymbol{\epsilon}_2^*)}{\sqrt{2}E^2}\left[\frac{1}{1-\beta\cos(\theta)}+\frac{1}{1+\beta\cos(\theta)}\right]\phi(p)Y_{J0}(\theta,\phi)\right]\right|^2\ . \tag{28}$$

The angles θ and ϕ in the integrand refer to the direction of $\hat{\mathbf{p}}$ within the positronium wave function, not to the axis $\hat{\mathbf{k}}\equiv\hat{\mathbf{z}}$ of the outgoing photons, so that $\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}=\hat{\mathbf{z}}\cdot\hat{\mathbf{p}}=\cos\theta$. The internal angular integral can be carried out in closed form using the identity $Q_J(t)=\frac{1}{2}\int_{-1}^{+1}P_J(\mu)(t-\mu)^{-1}d\mu$, and after summing over photon polarizations we find

$$\Gamma^J(e^+e^-\to\gamma\gamma)=\frac{2\alpha^2}{\pi m^2}\left|\int_0^\infty dp\, p^2\phi(p)\left[\frac{1-\beta^2}{\beta}\right]Q_J(\beta^{-1})\right|^2\ , \tag{29}$$

where $Q_J(x)$ is the Legendre function of the second kind. This is an exact relativistic result for singlet positronium, given the diagrams of Fig. 1, the decay formulas (6) and (7) and the amplitude (24).

III. NONRELATIVISTIC LIMIT

From our relativistic formula (29) and the asymptotic form of $Q_J(\beta^{-1})$ for $\beta\to 0$, we have

$$\Gamma^J_{\mathrm{NR}}(e^+e^-\to\gamma\gamma)\frac{2\alpha^2}{\pi m^2}\left|\int_0^\infty dp\, p^2\phi(p)\left[\frac{1-\beta^2}{\beta}\right]\frac{2^J(J!)^2}{(2J+1)!}\beta^{J+1}\right|^2\ . \tag{30}$$

On substituting the nonrelativistic form $\beta\approx p/m$ and assuming $\beta\ll 1$, this becomes

$$\Gamma^J_{\mathrm{NR}}(e^+e^-\to\gamma\gamma)=\frac{2^{2J+1}(J!)^4}{\pi(2J+1)!^2}\frac{\alpha^2}{m^{2J+2}}\left|\int_0^\infty dp\, p^{J+2}\phi(p)\right|^2\ . \tag{31}$$

This may also be written in terms of derivatives of the position-space wave function $\psi(r)$ [normalized to $\int_0^\infty r^2|\psi(r)|^2dr=1$] at the origin, using the result

$$\int_0^\infty dp\, p^{J+2}\phi(p)=\left[\frac{\pi}{2}\right]^{1/2}(-i)^J\frac{(2J+1)!!}{J!}\psi^{(J)}(0)\ . \tag{32}$$

This gives the remarkably simple form

$$\Gamma^J_{\mathrm{NR}}(e^+e^-\to\gamma\gamma)=\frac{\alpha^2}{m^{2J+2}}|\psi^{(J)}(0)|^2\ . \tag{33}$$

For $J=l=2$ this is

$$\Gamma_{\mathrm{NR}}({}^1D_2\to\gamma\gamma)=\frac{\alpha^2}{m^6}|\psi''(0)|^2\ , \tag{34}$$

which agrees with the result (4) of Anderson, Austern, and Cahn [19] after their quarkonium formula is divided by a color factor of 3 and a flavor factor of $|\langle e_q^2/e^2\rangle|^2$, and their M_R is replaced by $2m$.

For positronium with a pure $-\alpha/r$ potential, the derivatives at contact can be evaluated analytically, and substitution into (33) gives our general nonrelativistic result for the $\gamma\gamma$ decay width of singlet positronium with radial quantum number n and angular momentum $J=l$;

$$\Gamma^{J}_{\mathrm{NR}}(e^+e^-\to\gamma\gamma) = \frac{1}{2(l+1)} \frac{\begin{pmatrix} n+l \\ 2l+1 \end{pmatrix}}{\begin{pmatrix} 2l+1 \\ l \end{pmatrix}} n^{-(2l+4)} m\alpha^{2l+5} . \quad (35)$$

In the $l=0$ case this reduces to the familiar result first derived by Pirenne [27]:

$$\Gamma_{\mathrm{NR}}(n\,{}^1S_0\to\gamma\gamma)=\frac{1}{2n^3}m\alpha^5 . \quad (36)$$

IV. APPLICATION TO QUARKONIUM

A. Transcription to $q\bar{q}$: mass dependence

We may use our relativistic result (29) to model meson decays to $\gamma\gamma$ under the usual quark model assumption that the initial meson is a pure $q\bar{q}$ state with a wave function $\Phi(\mathbf{p})$, which we obtain from a nonrelativistic Coulomb-plus-linear potential model. Of course this is only an approximate method, and additional important corrections may arise from effects such as gluon exchange, non-$q\bar{q}$ components in the meson wave function and corrections to the free quark propagator assumed in $\mathcal{M}_{fi}$.

Application of our results to quarkonium requires multiplication of (29) by a color factor of 3, a flavor factor of $|\langle e_q^2/e^2\rangle|^2$, the replacement of m by m_q in the amplitude, and an experimentally motivated treatment of the dependence of $\Gamma_{\gamma\gamma}$ on the resonance mass M_R (which we shall discuss subsequently). For $\pi^0\to\gamma\gamma$ this transcription gives

$$\begin{aligned}\Gamma(\pi^0\to\gamma\gamma) &= \frac{\alpha^2}{3\pi m_q^2}\left|\int_0^\infty dp\, p^2\phi(p)\left[\frac{1-\beta^2}{\beta}\right]Q_0(\beta^{-1})\right|^2 \\ &= \frac{\alpha^2}{3\pi m_q^2}\left|\int_0^\infty dp\, p^2\phi(p)\left[\frac{1-\beta^2}{\beta}\right]\frac{1}{2}\ln\left[\frac{1+\beta}{1-\beta}\right]\right|^2 \\ &= \frac{\alpha^2}{3\pi m_q^2}\left|\int_0^\infty dp\, p^2\phi(p)\left[\frac{m_q^2}{pE}\right]\ln\left[\frac{E+p}{m_q}\right]\right|^2 . \end{aligned} \quad (37)$$

This is identical to the relativistic result of Bergström, Snellman, and Tengstrand [23] and Hayne and Isgur [24], except that Hayne and Isgur then multiply the rate by an *ad hoc* overall factor of $(M_R/M_{\mathrm{ref}})^3$ to impose an expected approximate M_R^3 mass dependence:

$$\Gamma(\pi^0\to\gamma\gamma)=\Gamma(\pi^0\to\gamma\gamma)|_{\text{quark model}}\left[\frac{M_R}{M_{\mathrm{ref}}}\right]^3 . \quad (38)$$

Multiplication of the rate (37) by $(M_R/M_{\mathrm{ref}})^3$ is primarily motivated by comparison with experiment. The $\gamma\gamma$ widths of the light pseudoscalars π^0, η, and η' seen in e^+e^- experiments [6] are known to increase rapidly with M_R:

$$\Gamma_{\gamma\gamma}(\pi^0)=7.29\pm0.19 \text{ eV} , \quad (39)$$

$$\Gamma_{\gamma\gamma}(\eta)=0.524\pm0.031 \text{ keV} , \quad (40)$$

$$\Gamma_{\gamma\gamma}(\eta')=4.25\pm0.19 \text{ keV} . \quad (41)$$

In contrast, direct evaluation of the theoretical decay rate (37) (with the appropriate charge factor of $|\langle e_q^2/e^2\rangle|^2$) would incorrectly give comparable $\gamma\gamma$ widths for these resonances, because this formula makes no direct reference to the physical resonance mass M_R.

Note that the observed $\gamma\gamma$ widths are approximately proportional to M_R^3:

$$\Gamma_{\gamma\gamma}(\pi^0)/M_{\pi^0}^3=2.96\pm0.08 \text{ keV GeV}^{-3} , \quad (42)$$

$$\Gamma_{\gamma\gamma}(\eta)/M_{\eta}^3=3.17\pm0.19 \text{ keV GeV}^{-3} , \quad (43)$$

$$\Gamma_{\gamma\gamma}(\eta')/M_{\eta'}^3=4.84\pm0.22 \text{ keV GeV}^{-3} . \quad (44)$$

This approximate M_R^3 dependence can be motivated by consideration of a simple pseudoscalar-$\gamma\gamma$ effective Lagrangian,

$$\mathcal{L}_I=\tfrac{1}{2}g\phi F_{\mu\nu}\tilde{F}_{\mu\nu} , \quad (45)$$

which leads to a 0^{-+} $\gamma\gamma$ width proportional to M_R^3,

$$\Gamma_{\gamma\gamma}=\frac{1}{64\pi}g^2M_R^3 . \quad (46)$$

The problem with the quark model calculation is that it does not incorporate the physical mass of the initial state, which must be restored as a phenomenological input. In effect we treat the quark model calculation as a determination of g in (45) (for 0^{-+}), and then impose the overall M_R^3 dependence found in (46). There is clearly a need for an improved calculation of these rates at the quark level which incorporates the physical meson mass *ab initio*.

Our prescription is thus to "correct" the M_R dependence of the quark model $\Gamma_{\gamma\gamma}$ calculation by introducing an overall multiplicitive factor, as in (38). This requires specification of a "reference" mass $M_{\rm ref}$, which is a characteristic mass scale in the $q\bar{q}$ model wave function. Hayne and Isgur [24] originally suggested setting $M_{\rm ref}$ equal to the mass actually calculated in the quark model. This has the advantage of eliminating this $(M_R/M_{\rm ref})^p$ correction as models become more accurate, but the disadvantage that adding a constant to $V(r)$ changes $M_{\rm ref}$ and hence the predicted decay rate. A choice for $M_{\rm ref}$ that does not suffer this unphysical feature is the expected rest plus kinetic energy; this was used by Godfrey and Isgur [28], in the relativistic form $2\sqrt{{\rm p}^2+m^2}$. There is clearly considerable arbitrariness in the choice of $M_{\rm ref}$, and hence a corresponding uncertainty in the overall scale of decay rates.

For the $\gamma\gamma$ decays of $l=1$ $q\bar{q}$ states and the $l=2$, $J^{PC}=2^{-+}$ state we also expect an M_R^3 dependence in $\Gamma_{\gamma\gamma}$, since the effective Lagrangians

$$\mathcal{L}_I^{(2^{++})}=\tfrac{1}{2}g f_{\mu\nu}F_{\mu\alpha}F_{\nu\alpha} \,, \tag{47}$$

$$\mathcal{L}_I^{(0^{++})}=\tfrac{1}{2}g\phi F_{\mu\nu}F_{\mu\nu} \,, \tag{48}$$

$$\mathcal{L}_I^{(2^{-+})}=\tfrac{1}{2}g f_{\mu\nu}F_{\mu\alpha}\tilde{F}_{\nu\alpha} \tag{49}$$

have the same number of derivatives. For higher-spin initial mesons, more derivatives will be required to construct an effective Lagrangian, which will lead to a higher power of M_R. For example, in the 4^{-+} case two additional derivatives are required, so we incorporate an $(M_R/M_{\rm ref})^7$ dependence in $\Gamma(4^{-+}\to\gamma\gamma)$. The higher-spin $\gamma\gamma$ partial widths are evidently much more sensitive to the choice of the reference mass $M_{\rm ref}$, and their absolute scale is correspondingly less well determined.

B. The $\pi_2(1670)$ $\gamma\gamma$ width

For our numerical estimate of the $\pi_2(1670)$ decay rate we take $M_{\rm ref}$ to be the rms relativistic kinetic energy of the quarkonium state,

$$M_{\rm ref}=2\langle\psi|({\rm p}^2+m_q^2)|\psi\rangle^{1/2} \,, \tag{50}$$

in terms of which our $\gamma\gamma$ partial width is

$$\Gamma(\pi_2\to\gamma\gamma)=\frac{6\alpha^2}{\pi m_q^2}\left|\left\langle\frac{e_q^2}{e^2}\right\rangle\right|^2\left|\int_0^\infty dp\, p^2\phi(p)\left[\frac{1-\beta^2}{\beta}\right]Q_2(\beta^{-1})\right|^2\left[\frac{M_R}{M_{\rm ref}}\right]^3 , \tag{51}$$

where the Legendre function is

$$Q_2(x)=\frac{1}{4}(3x^2-1)\ln\left[\frac{x+1}{x-1}\right]-\frac{3}{2}x \,.$$

We evaluate this decay rate numerically for Coulomb-plus-linear wave functions, with a "standard" light-quark parameter set $\alpha_s=0.6$, $a=0.18$ GeV^2 and $m_q=0.33$ GeV; the wave-function integral is found to be

$$\int_0^\infty dp\, p^2\phi(p)\left[\frac{1-\beta^2}{\beta}\right]Q_2(\beta^{-1})=2.617\times10^{-2}\,{\rm GeV}^{3/2} \,, \tag{52}$$

and the rms relativistic kinetic energy (50) is

$$M_{\rm ref}=1.164\,{\rm GeV} \,. \tag{53}$$

Combining these and taking $M_R=1.67$ GeV, we find a $\gamma\gamma$ partial width of

$$\Gamma(\pi_2(1670)\to\gamma\gamma)=0.105\,{\rm keV} \,, \tag{54}$$

which is approximately an order of magnitude smaller than the measured rates (1) and (2).

In view of the discrepancy between our result (54) and experiment, it is important to investigate the stability of this $\Gamma_{\gamma\gamma}$ with respect to changes in the parameters α_s, a, m_q, and $M_{\rm ref}$. We find that the $\gamma\gamma$ width is rather insensitive to α_s and a but depends strongly on m_q and $M_{\rm ref}$; for small changes from the "standard" parameters quoted above we find

$$\frac{\delta\Gamma_{\gamma\gamma}}{\Gamma_{\gamma\gamma}}=0.06\frac{\delta\alpha_s}{\alpha_s}+0.32\frac{\delta a}{a}-2.61\frac{\delta m_q}{m_q} \,, \tag{55}$$

so that the strongest dependence by far is on m_q. ($M_{\rm ref}$ is implicitly a function of α_s, a, and m_q here, as we have set it equal to the rms relativistic kinetic energy.) If we vary the definition of $M_{\rm ref}$ with fixed α_s, a, and m_q, we find

$$\frac{\delta\Gamma_{\gamma\gamma}}{\Gamma_{\gamma\gamma}}=-3\frac{\delta M_{\rm ref}}{M_{\rm ref}} \,, \tag{56}$$

so the scale of partial widths also depends strongly on the $M_{\rm ref}$ assumed.

The sensitivity of decay widths to m_q was previously noted by Hayne and Isgur [24] in a study of weak and electromagnetic quarkonium decays in a relativistic quark model. They concluded that a better numerical fit to the data followed from an "extended" parameter set with a light-quark mass of $m_q=220$ MeV. As we have no particular reason to prefer the conventional value of $m_q=330$ MeV, we also quote results for this lighter m_q. This gives

$$\Gamma(\pi_2(1670)\to\gamma\gamma)=0.275\,{\rm keV} \,, \tag{57}$$

which motivates our final estimate of

$$\Gamma(\pi_2(1670)\to\gamma\gamma)=0.1-0.3\,{\rm keV} \,. \tag{58}$$

This is somewhat smaller than the experimental result of ≈ 1 keV, but in view of the remaining freedom in the choice of $M_{\rm ref}$ and the rather *ad hoc* incorporation of M_R in $\Gamma_{\gamma\gamma}$, we do not believe that (58) argues against a $q\bar{q}$ assignment for the $\pi_2(1670)$. The appropriate test of this assignment is a search for the $I=0$ partner $\eta_2(\approx 1700)$,

TABLE I. Estimated and observed $I=1$ quarkonium $\gamma\gamma$ widths for the range $m_q=330$ MeV $\rightarrow$220 MeV.

0^{-+}		2^{-+}		4^{-+}	
$\pi^0(135)$	3.4→6.4 eV				
Expt. [6]	7.29±0.19 eV				
$\pi(1300)$	0.43→49 keV	$\pi_2(1670)$	0.11→0.27 keV		
		Expt. [17,18]	1.13±0.24 keV		
$\pi(3S)(1880?)$	0.74→1.00 keV	$\pi_2(2D)(2130?)$	0.10→0.16 keV	$\pi_4(1G)(2330?)$	0.21→1.6 keV

which should have a $\gamma\gamma$ width of about 2–3 keV $[\frac{25}{9}\Gamma_{\gamma\gamma}(\pi_2)]$. If the discrepancy between (58) and experiment is confirmed, this will emphasize the importance of developing more realistic techniques for the calculation of $\gamma\gamma$ widths. A method in which the approximate M_R^3 dependence arises naturally in $\Gamma_{\gamma\gamma}$ would be a particularly useful improvement in technique.

As our estimate (58) for the $\gamma\gamma$ width of the $\pi_2(1670)$ differs considerably from the previous numerical estimate of Anderson, Austern, and Cahn (5), it is instructive to compare our decay formulas to understand the origin of this discrepancy. One important difference is their use of the nonrelativistic "contact" approximation in evaluating the decay amplitude. This assumes that it is a good approximation to keep only the leading term in a series expansion of the amplitude in (51) in powers of p/m_q, which for $J=2$ requires

$$\frac{\mathcal{J}_0}{\mathcal{J}}=\frac{\int_0^\infty dp\, p^2\phi(p)\left[\frac{2p^2}{15m_q^2}\right]}{\int_0^\infty dp\, p^2\phi(p)\left[\left[\frac{1-\beta^2}{\beta}\right]Q_2(\beta^{-1})\right]}\approx 1\ . \quad (59)$$

For light-quark parameters $\alpha_s=0.6$, $a=0.18$ GeV^2 and $m_q=0.33$ GeV, we find that this ratio is actually

$$\frac{\mathcal{J}_0}{\mathcal{J}}=\frac{\int_0^\infty dp\, p^2\phi(p)\left[\frac{2p^2}{15m_q^2}\right]}{\int_0^\infty dp\, p^2\phi(p)\left[\left[\frac{1-\beta^2}{\beta}\right]Q_2(\beta^{-1})\right]}=7.996\ , \quad (60)$$

so the contact approximation is a poor one for light 2^{-+} decays. Similar conclusions have been reported for other light-quark $q\bar{q}$ states [4,16]. Note however that (60) implies that the contact approximation should *overestimate* the $\gamma\gamma$ decay rate by about a factor of 64, whereas Anderson, Austern, and Cahn underestimate it by two to three orders of magnitude [assuming the $\pi_2(1670)$ is indeed the $^1D_2$ $q\bar{q}$ state]. The compensating factor is their identification of the quark mass m_q with half the resonance mass, $m_q=M_R/2\approx 835$ MeV, instead of the usual constituent mass $m_q\approx 330$ MeV. Since the nonrelativistic decay formula (4) scales as m_q^6, replacing 330 MeV by 835 MeV reduces their $\Gamma_{\gamma\gamma}$ by a factor of $(835/330)^6=262$. The combination of these two large factors scales their decay rate down from our relativistic result by an overall factor of ≈ 4. Our phenomenological $(M_R/M_{\rm ref})^3$ multiplier in (51) increases the predicted $\Gamma_{\gamma\gamma}$ of the $\pi_2(1670)$ by about a factor of 3. This together with the ≈ 4 above accounts for the order-of-magnitude difference between our result of 0.11 keV for $m_q=0.33$ GeV in a Coulomb-plus-linear potential and their result of 6 eV in the very similar Cornell model. [They also give results for several less well motivated potentials, which leads to their quoted range of 1.2 eV (power law) -9.4 eV (logarithmic).]

C. $\Gamma_{\gamma\gamma}$ for other $S=0$ $q\bar{q}$ states

As we have derived a relativistic $S=0$ $\Gamma_{\gamma\gamma}$ formula for arbitrary total J, it may be of interest to present numerical results for other higher-spin singlet states as well as for radial excitations. We give results for $I=1$ 0^{-+}, 2^{-+}, and 4^{-+} states in Table I (we take $\alpha_s=0.6$, $a=0.18$ GeV^2, and quote a range which follows from $m_q=330$ MeV $\rightarrow$220 MeV), for the corresponding $c\bar{c}$ states in Table II and for $b\bar{b}$ states in Table III. The $c\bar{c}$ and $b\bar{b}$ parameter sets used are $(\alpha_s,a,m_Q)=(0.4,\ 0.18$ GeV^2, 1.4 GeV) and (0.15, 0.18 GeV^2, 4.5 GeV) respectively; these are typical of values found in fits to the spectrum of states. For levels without well-established experimental candidates we use the mass estimates of Godfrey and Isgur [28], which are indicated by question marks. For the $^1G_4$ $c\bar{c}$ and $b\bar{b}$ states, which are not discussed by Godfrey and Isgur, we use an approximate mass extrapo-

TABLE II. Estimated and observed singlet charmonium $\gamma\gamma$ widths.

0^{-+}		2^{-+}		4^{-+}	
$\eta_c(2980)$	4.8 keV				
Expt. [30]	≈8±2 keV				
$\eta_c(3590)$	3.7 keV	$\eta_c(1D)(3840?)$	20 eV		
$\eta_c(2S)(4060?)$	3.3 keV	$\eta_c(2D)(4210?)$	35 eV	$\eta_c(1G)(4350?)$	0.92 eV

TABLE III. Estimated singlet $b\bar{b}$ $\gamma\gamma$ widths.

0^{-+}		2^{-+}		4^{-+}	
$\eta_b(1S)(9400?)$	0.17 keV				
$\eta_b(2S)(9980?)$	0.13 keV	$\eta_b(1D)(10150?)$	33 meV		
$\eta_b(3S)(10340?)$	0.11 keV	$\eta_b(2D)(10450?)$	69 meV	$\eta_b(1G)(10500?)$	59 μeV

lated from their $l<4$ results. We caution the reader that the 4^{-+} rates incorporate an $(M_R/M_{\rm ref})^7$ factor, so the $M_{\rm ref}$ ambiguity discussed previously is quite large, and these rates are at best order-of-magnitude estimates. The predictions are all more stable for $c\bar{c}$ and $b\bar{b}$, although the freedom to adjust m_c and introduce a compensating V_0 can plausibly lead to factor-of-two changes in the $c\bar{c}$ widths.

A qualitative conclusion regarding heavy quarks is that $\Gamma_{\gamma\gamma}$ is quickly suppressed by orbital excitation. This orbital suppression grows more pronounced as the quark mass increases, so that each step of $\Delta l=2$ results in a decrease in $\Gamma_{\gamma\gamma}$ by about two orders of magnitude for $c\bar{c}$ and about three orders of magnitude for $b\bar{b}$. As the current statistical accuracy in resolving $c\bar{c}$ states in $\gamma\gamma$ collisions is at the keV level, Tables II and III show that it is unrealistic to expect to observe $l\geq 2$ singlet $c\bar{c}$ and $b\bar{b}$ states at e^+e^- machines.

In contrast, in Table I we do not find significant suppression of light-quark $q\bar{q}$ $\gamma\gamma$ widths with increasing l, which is consistent with the observation of the $a_2(1320)$ and the $\pi_2(1670)$ with comparable $\gamma\gamma$ widths. One may plausibly expect to observe higher-l states such as the $l=3$ $4^{++}/3^{++}/2^{++}$ multiplet at $\approx$2050 MeV and the $l=4$ $\pi_4(2330?)$ in $\gamma\gamma$ collisions. Our preliminary results suggest that the $^3F_4$ should couple dominantly to $\lambda=2$ $\gamma\gamma$ states, whereas the $^3F_2$ should couple significantly to both $\lambda=0$ and $\lambda=2$ $\gamma\gamma$ states [29].

We do not find significant $\Gamma_{\gamma\gamma}$ suppression with radial excitation in any of the $q\bar{q}$ systems considered here. This suggests that the $\pi(1300)$, radially excited light η states, the $\eta_c(3590)$, radially excited $2\,^3P_2$ $q\bar{q}$ states at $\approx$1800 MeV, and a radial $\pi_2(2130?)$ may all be observable in $\gamma\gamma$. These states may provide particularly sensitive tests of the quark model calculations presented here.

V. CONCLUSIONS

We have derived relativistic and nonrelativistic results for the $\gamma\gamma$ partial widths of singlet $(S=0)$ positronium and quarkonium states with arbitrary total angular momenta. Our $\Gamma_{\gamma\gamma}$ formula agrees with previous relativistic calculations of the 0^{-+} width. There are, however, additional relativistic effects and binding corrections which are not incorporated in this calculation; some of these are noted in the text. Our nonrelativistic formula leads to a closed form result for the $\Gamma_{\gamma\gamma}$ of any singlet positronium bound state. For the 2^{-+} case, our nonrelativistic formula agrees with the result of Anderson, Austern, and Cahn, although we find large relativistic corrections, and our relativistic 2^{-+} $\Gamma_{\gamma\gamma}$ is numerically much closer to the observed $\pi_2(1670)\to\gamma\gamma$ partial width. We find that light-quark 2^{-+} and 4^{-+} $q\bar{q}$ states should be observable in $\gamma\gamma$ collisions, but that $l\geq 2$ $c\bar{c}$ and $b\bar{b}$ singlet states have very small $\gamma\gamma$ couplings. For all these systems we find that moderate radial excitation does not significantly decrease the $\gamma\gamma$ coupling. These results suggest several interesting possibilities for experimental searches.

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